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SOME GENERALIZATIONS OF CONTINUITY AND QUASI-CONTINUITY OF MULTIVALUED MAPS

1. Introduction

A subset A of a topological space X is said to be:

— semi-open [9], if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,

— pre-open [10], if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,

— pre-semi-open [1], if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$,

— an α-set [15], if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$.

The union of all semi-open (resp. pre-open, pre-semi-open) sets contained in A is called the semi-interior [3] (resp. pre-interior [11], pre-semi-interior [1]) of A and it is denoted by $s\,\mathrm{Int}(A)$ (resp. $p\,\mathrm{Int}(A)$, $ps\,\mathrm{Int}(A)$). Semi-closed (resp. pre-closed, pre-semi-closed) sets and semi-closure (resp. pre-closure, pre-semi-closure) are defined in a manner analogous to the corresponding concepts of closed sets and closure [3], [11], [1]. The semi-closure (resp. pre-closure, pre-semi-closure) of A is denoted by $s\,\mathrm{Cl}(A)$ (resp. $p\,\mathrm{Cl}(A)$, $ps\,\mathrm{Cl}(A)$). It was observed in [15] that the collection T^α of all subsets of a space (X,T) which are α-sets is a topology on X. The interior and the closure of A with respect to the topology T^α will be denoted by $\alpha\,\mathrm{Int}(A)$ and $\alpha\,\mathrm{Cl}(A)$, respectively. In [1], sets having the property $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$ are called semi-pre-open. We use the term "pre-semi-open" since $A \subset s\,\mathrm{Int}(s\,\mathrm{Cl}(A))$ if and only if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$, but a subset A satisfying $A \subset p\,\mathrm{Cl}(p\,\mathrm{Int}(A))$ according to the term "semi-pre-open" need not be pre-semi-open. In [1] is proved that

LEMMA 1.1. *For a subset $A \subset X$ we have*:

(i) $s\,\mathrm{Int}(A) = A \cap \mathrm{Cl}(\mathrm{Int}(A))$,

(ii) $p\,\mathrm{Int}(A) = A \cap \mathrm{Int}(\mathrm{Cl}(A))$,

(iii) $\alpha\,\mathrm{Int}(A) = A \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,

(iv) $ps\,\mathrm{Int}(A) = A \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$.

If F is a multivalued map from a topological space X into a topological space Y, written $F : X \to Y$, then for any sets $A \subset X$ and $B \subset Y$ we use the following notations:

$$F^+(B) = \{x \in X : F(x) \subset B\}, \quad F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\},$$

and $F(A) = \{F(x) : x \in A\}$.

A multivalued map $F : X \to Y$ is called quasi-continuous [16] at $x \in X$ if for any open sets $V, U \subset Y$ such that $F(x) \subset V$ and $F(x) \cap U \neq \emptyset$, and for any open set W containing x there exists a nonempty open set $G \subset W$ such that $F(z) \subset V$ and $F(z) \cap U \neq \emptyset$ for each $z \in G$.

By $sC(F)$ we denote the set of all points at which F is quasi-continuous. A map F is called quasi-continuous if $sC(F) = X$.

Clearly, if a map F is of the form $F(x) = \{f(x)\}$ for some singlevalued map $f : X \to Y$, then the above definition reduce to the Kempisty definition of quasi-continuity [8]:

A map $f : X \to Y$ is said to be quasi-continuous at $x \in X$ if for each open set V containing $f(x)$ and each open set U containing x, there exists an open nonempty set $G \subset U$ such that $f(G) \subset V$.

In ([14], Theorem 1.1), it is shown that a map f is quasi-continuous at each point $x \in X$, if and only if it is semi-continuous [9], i.e., for every open set $V \subset Y$, $f^{-1}(V)$ is a semi-open subset of X.

Denote by $\mathcal{A}(Y)$ the collection of all nonempty subsets of Y. As it is well known [13], the collection of all classes of the form $\langle O_1, O_2, \ldots, O_n \rangle = \{A \in \mathcal{A}(Y) : A \subset \bigcup\{O_k : k = 1, 2, \ldots, n\}; A \cap O_k \neq \emptyset, k = 1, 2, \ldots, n\}$ with $O_1, O_2, \ldots, O_n$ all open in Y, is a base for the Vietoris topology on $\mathcal{A}(Y)$. A subbase for this topology is the collection consisting of all classes having one of the following forms:

$$O^+ = \{A \in \mathcal{A}(Y) : A \subset O\},$$
$$O^- = \{A \in \mathcal{A}(Y) : A \cap O \neq \emptyset\}, \text{ with } O \text{ open in } Y.$$

In [6] it is shown that a multivalued map $F : X \to Y$ is quasi-continuous at $x \in X$ if and only if for any open sets $V, U \subset Y$ such that $F(x) \subset V$ and $F(x) \cap U \neq \emptyset$ we have $x \in s\,\mathrm{Int}(F^+(V) \cap F^-(U))$.

Also in [6] it is shown that if $F : X \to \mathcal{A}(Y)$ is quasi-continuous as a single-valued map (i.e., with respect to the Vietoris topology on $\mathcal{A}(Y)$), then it is quasi-continuous as a multivalued map $F : X \to X$, but the converse is not true.

We say that a multivalued map $F : X \to Y$ is continuous at $x \in X$, if it is continuous at x as a single-valued map $F : X \to \mathcal{A}(Y)$.

The set of all points at which a multivalued map $F : X \to Y$ is continuous will be denoted by $C(F)$. A map $F : X \to Y$ is called continuous if $C(F) = X$.

It is easy to prove that a map $F : X \to \mathcal{A}(Y)$ is continuous at $x \in X$ with respect to the Vietoris topology if and only if for any open sets V, $U \subset Y$ such that $F(x) \in V^+ \cap U^-$ we have $x \in \mathrm{Int}(F^+(V) \cap F^-(U))$.

In this paper we introduce some forms of cliquishness of multivalued maps $F : X \to Y$. These forms are weaker than continuity and quasi-continuity with respect to the Vietoris topology in $\mathcal{A}(Y)$.

2. Definitions and preliminaries

DEFINITION 2.1. A multivalued map $F : X \to Y$ is said to be:

— pre-continuous at $x \in X$ if for any open sets V, $U \subset Y$ such that $F(x) \in V^+ \cap U^-$ we have $x \in p\,\mathrm{Int}(F^+(V) \cap F^-(U))$;

— pre-semi-continuous at $x \in X$ if for any open sets V, $U \subset Y$ such that $F(x) \in V^+ \cap U^-$ we have $x \in ps\,\mathrm{Int}(F^+(V) \cap F^-(U))$;

— α-continuous at $x \in X$ if for any open sets V, $U \subset Y$ such that $F(x) \in V^+ \cap U^-$ we have $x \in \alpha\,\mathrm{Int}(F^+(V) \cap F^-(U))$.

The sets of all points at which a map F is pre-continuous (resp. pre-semi-continuous, α-continuous) will be denoted by $pC(F)$ (resp. $psC(F)$, $\alpha C(F)$).

A map $F : X \to Y$ is called pre-continuous (resp. pre-semi-continuous, α-continuous) if $pC(F) = X$ (resp. $psC(F) = X$, $\alpha C(F) = X$).

If a map $F : X \to Y$ is given by $F(x) = \{f(x)\}$ for some single-valued map $f : X \to Y$, then the above definition reduces to the following:

A map $f : X \to Y$ is said to be pre-continuous [10] (resp. pre-semi-continuous, α-continuous [18]) at $x \in X$ if for each open set V containing $f(x)$ there exists a pre-open (resp. pre-semi-open, an α-) set A containing x such that $f(A) \subset V$.

DEFINITION 2.2. A multivalued map $F : X \to Y$ is said to be:

— basically continuous (resp. basically α-continuous, basically quasi-continuous, basically pre-continuous, basically pre-semi-continuous) at $x \in X$ if there exists $z \in X$ such that for each pair of open sets V, $U \subset Y$ satisfying $F(z) \in V^+ \cap U^-$ we have

$$x \in \mathrm{Int}(F^+(V) \cap F^-(U)) \text{ (resp. } x \in \alpha\,\mathrm{Int}(F^+(V) \cap F^-(U)),$$
$$x \in s\,\mathrm{Int}(F^+(V) \cap F^-(U)), \quad x \in p\,\mathrm{Int}(F^+(V) \cap F^-(U)),$$
$$x \in ps\,\mathrm{Int}(F^+(V) \cap F^-(U)));$$

— basically α-cliquish (resp. basically s-cliquish, basically pre-α-cliquish, basically pre-s-cliquish) at $x \in X$ if there exists $z \in X$ such that for each pair of open sets V, $U \subset Y$ satisfying $F(z) \in V^+ \cap U^-$ we have $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V) \cap F^-(U))))$ (resp. $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V) \cap F^-(U)))$, $x \in \mathrm{Int}(\mathrm{Cl}(F^+(V) \cap F^-(U)))$, $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V) \cap F^-(U)))))$;

— basically cliquish (resp. basically pre-cliquish) at $x \in X$ if for any open set W containing x there exists $z \in X$ such that for each pair of open sets V, $U \subset Y$ satisfying $F(z) \in V^+ \cap U^-$ we have

$$W \cap \mathrm{Int}(F^+(V) \cap F^-(U)) \neq \emptyset \quad (\text{resp. } W \cap \mathrm{Int}(\mathrm{Cl}(F^+(V) \cap F^-(U))) \neq \emptyset).$$

The set of all points at which a map F is

— basically continuous (resp. basically α-continuous, basically quasi-continuous, basically pre-continuous, basically pre-semi-continuous) will be denoted by $BC(F)$ (resp. $\alpha BC(F)$, $sBC(F)$, $pBC(F)$, $psBC(F)$);

— basically α-cliquish (resp. basically s-cliquish, basically pre-α-cliquish, basically pre-s-cliquish) will be denoted by $\alpha BA(F)$ (resp. $sBA(F)$, $p\alpha BA(F)$, $psBA(F)$);

— basically cliquish (resp. basically pre-cliquish) will be denoted by $BA(F)$ (resp. $pBA(F)$).

A map $F : X \to Y$ is called

— basically continuous (resp. basically α-continuous, basically quasi-continuous, basically pre-continuous, basically pre-semi-continuous) if $BC(F) = X$ (resp. $\alpha BC(F) = X$, $sBC(F) = X$, $pBC(F) = X$, $psBC(F) = X$);

— basically α-cliquish (resp. basically s-cliquish, basically pre-α-cliquish, basically pre-s-cliquish) if $\alpha BA(F) = X$ (resp. $sBA(F) = X$, $p\alpha BA(F) = X$, $psBA(F) = X$);

— basically cliquish (resp. basically pre-cliquish) if $BA(F) = X$ (resp. $pBA(F) = X$).

PROPOSITION 2.1. *For a multivalued map $F : X \to Y$ the following hold*:

(i) *F is basically continuous* (*resp. basically α-continuous, basically quasi-continuous, basically pre-continuous, basically pre-semi-continuous*) *at $x \in X$ if and only if for any collections $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$, we have $x \in \bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$* (*resp. $x \in \bigcup\{\alpha\,\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$, $x \in \bigcup\{s\,\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$, $x \in \bigcup\{p\,\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$, $x \in \bigcup\{ps\,\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$*).

(ii) *F is basically α-cliquish* (*resp. basically s-cliquish, basically cliquish*) *at $x \in X$ if and only if for any collections $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$, we have $x \in \bigcup\{\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s)))) : s \in S\}$* (*resp. $x \in \bigcup\{\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s))) : s \in S\}$, $x \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\})$*).

(iii) *F is basically pre-α-cliquish* (*resp. basically pre-s-cliquish, basically pre-cliquish*) *at $x \in X$ if and only if for any collections $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- :$*

$s \in S\}$, *we have* $x \in \bigcup\{\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))) : s \in S\}$ (*resp.* $x \in \bigcup\{\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s)))) : s \in S\}$, $x \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))) : s \in S\})$).

Proof. (i). The necessity is evident. To prove the converse implicattion assume that F is not basically continuous at $x \in X$. Then, for every $z \in X$ we take an open sets $V_z, U_z \subset Y$ such that $F(z) \in V_z^+ \cap U_z^-$ and $x \notin \mathrm{Int}(F^+(V_z) \cap F^-(U_z))$. Let us take $\{V_z : z \in X\}$ and $\{U_z : z \in X\}$. Clearly, $F(X) \subset \bigcup\{V_z^+ \cap U_z^- : z \in X\}$ and $x \notin \bigcup\{\mathrm{Int}(F^+(V_z) \cap F^-(U_z)) : z \in X\}$ and the proof for basically continuity is complete.

The rest of the proof of (i) is analogous.

The quite similar proofs of (ii) and (iii) we omit.

A multivalued map F of a topological space X into a uniform space Y with a uniformity $\mathcal{U}$ is said to be cliquish [4] at a point $x \in X$ if for every $V \in \mathcal{U}$ and for every open set W containing x there exists an open nonempty set $G \subset W$ such that $[F(x') \times F(x'')] \cap V \neq \emptyset$ for any x', $x'' \in G$.

If a uniformity $\mathcal{U}$ is induced by a metric on Y and $f : X \to Y$ is a single-valued map, then the cliquishness of the map $F : X \to Y$ given by $F(x) = \{f(x)\}$ means the well known cliquishness of f [2, 19].

A multivalued map $F : X \to Y$ is said to be lower feeble T_1-cliquish [17] at a point $x \in X$ if for every open cover $\mathcal{A}$ of Y and for every open set W containing x there exists an open nonempt set $G \subset W$ such that for any x', $x'' \in G$ there exists $U \in \mathcal{A}$ satisfying $F(x') \cap U \neq \emptyset$ and $F(x'') \cap U \neq \emptyset$.

In [17] it is shown that the class of cliquish multivalued maps is greater than the class of lower feeble T_1-cliquish maps. We have the following

PROPOSITION 2.2. *Any basically cliquish map* $F : X \to Y$ *is lower feeble* T_1*-cliquish.*

Proof. Let $x \in X$, let W be an open set containing x and let $\mathcal{A}$ be an open cover of Y. Since F is basically cliquish at x, there exists $z \in X$ such that for each pair of open sets V, $U \subset Y$ satisfying $F(z) \in V^+ \cap U^-$ we have $W \cap \mathrm{Int}(F^+(V) \cap F^-(U)) \neq \emptyset$. Clearly, $F(z) \in U^-$ for some $U \in \mathcal{A}$. Thus, $F(z) \in V^+ \cap U^-$, where $V = Y$. Consequently, we have nonempty open set G, namely $G = W \cap \mathrm{Int}(F^+(V) \cap F^-(U))$, such that $G \subset W$ and for every $p \in G$ we have $F(p) \cap U \neq \emptyset$. Thus, F is lower feeble T_1-cliquish at x.

The converse of the above proposition is not true as the following example shows.

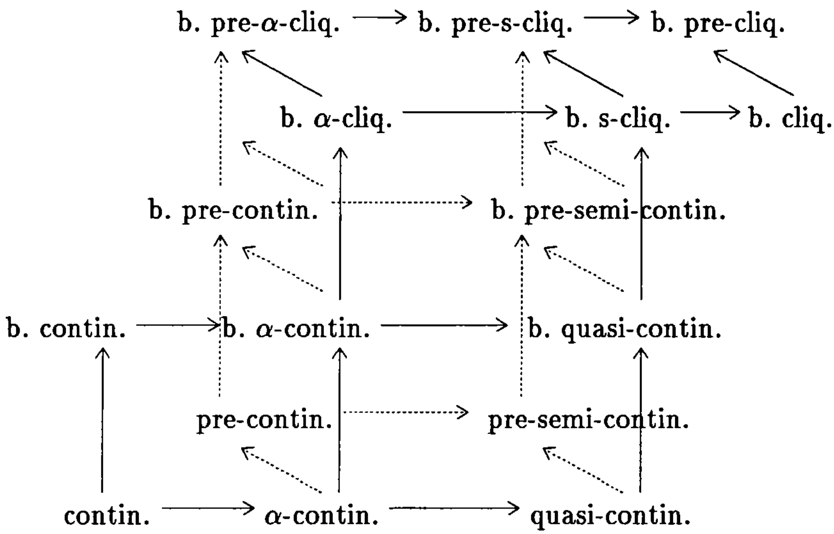
EXAMPLE 2.1. Let X be the set of all real numbers with the natural topology and let Y be the same set with the topology generated by $\{(-\infty, 1), \{1\}, (1, \infty)\}$. Define $F : X \to Y$ by the formula: $F(x) = \{1, 2\}$ for $x \in Q$, and $F(x) = \{0, 1\}$ for $x \notin Q$, where Q is the set of all rational numbers.

The map F is lower feeble T_1-cliquish since for every open cover $\mathcal{A}$ of Y, $1 \in U$ for some $U \in \mathcal{A}$ and, consequently, for every open nonempty set $G \subset X$ we have $F(x') \cap U \neq \emptyset$ and $F(x'') \cap U \neq \emptyset$ for any $x', x'' \in G$ since $1 \in F(x') \cap F(x'')$. However, F is not basically cliquish since for $z \in Q$ there exists a pair of open sets V, $U \subset Y$, namely $V = Y$ and $U = (1, \infty)$ such that $F(z) \in V^+ \cap U^-$ and $\emptyset = \mathrm{Int}(Q) = \mathrm{Int}(F^+(V) \cap F^-(U))$; analogously, for $z \notin Q$ putting $V = Y$ and $U = (-\infty, 1)$ we have $F(z) \in V^+ \cap U^-$ and $\mathrm{Int}(F^+(V) \cap F^-(U)) = \emptyset$.

3. The comparison of types of multivalued maps

From definitions we immediately obtain the following

Diagram 3.1.



The following series of examples show that all the implications cannot be reversed.

EXAMPLE 3.1. A pre-continuous map need not be basically cliquish. Let F be such as in Example 2.1. We shall show that F is pre-continuous. If $x \in Q$ (resp. $x \notin Q$) and $F(x) \in V^+ \cap U^-$ for some open sets V, $U \subset Y$, then $x \in Q \subset F^+(V) \cap F^-(U)$ (resp. $x \in X \setminus Q \subset F^+(V) \cap F^-(U)$) and, consequently, $x \in p\,\mathrm{Int}(F^+(V) \cap F^-(U))$ since the sets Q and $X \setminus Q$ are pre-open.

EXAMPLE 3.2. A basically continuous map need not be pre-semi-continuous. Let X be such as in Example 2.1 and let Y be the same set with the topology $T = \{Y, \emptyset, (1,2)\}$. Define $F : X \to Y$ by: $F(1) = (1,2)$ and

$F(x) = (3,4)$ if $x \neq 1$. Let $z \neq 1$. If $V, U \subset Y$ is a pair of open sets such that $F(z) \in V^+ \cap U^-$, then $V = U = Y$ and, consequently $\mathrm{Int}(F^+(V) \cap F^-(U)) = X$. Thus, F is basically continuous. But F is not pre-semi-continuous at $x = 1$ since for $V = U = (1,2)$ we have $F(x) \in V^+ \cap U^-$ and $ps\,\mathrm{Int}(F^+(V) \cap F^-(U)) = \emptyset$.

EXAMPLE 3.3. A basically α-cliquish map need not be basically pre-semi-continuous. Let X be such as in Example 2.1 and let $F : X \to X$ be such as in Example 3.2. Then F is basically α-cliquish since for $z \neq 1$ and for any open sets V, $U \subset X$ such that $F(z) \in V^+ \cap U^-$ we have $W \subset F^+(V) \cap F^-(U)$, where $W = X \setminus \{1\}$, so $X = \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(W))) \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V) \cap F^-(U))))$. However, for $z = 1$ (resp. $z \neq 1$) and for $V = U = (1,2)$ (resp. $V = U = (3,4)$) we have $F(z) \in V^+ \cap U^-$ and $1 \notin ps\,\mathrm{Int}(F^+(V) \cap F^-(U)) = \emptyset$ (resp. $1 \notin ps\,\mathrm{Int}(F^+(V) \cap F^-(U)) = X \setminus \{1\}$). Thus, F is not basically pre-semi-continuous at $x = 1$.

EXAMPLE 3.4. A quasi-continuous map need not be basically pre-α-cliquish. Let X be such as in Example 2.1. Define $F : X \to X$ by $F(x) = (k, k+1)$ for $x \in [k, k+1]$, k is an integer. If $x \in X$ and $F(x) \in V^+ \cap U^-$ for some open sets V, $U \subset X$, then $x \in [k, k+1)$ for some k and $x \in [k, k+1) \subset F^+(V) \cap F^-(U)$. Consequently, $x \in s\,\mathrm{Int}(F^+(V) \cap F^-(U))$ since the set $[k, k+1)$ is semi-open. Thus F is quasi-continuous. However, if $z \in X$, then $z \in [k, k+1)$ for some k and for $V = U = (k, k+1)$ we have $F(z) \in V^+ \cap U^-$ and $0 \in \mathrm{Int}(\mathrm{Cl}(F^+(V) \cap F^-(U))) = (k, k+1)$. Thus, F is not basically pre-α-cliquish at $x = 0$.

EXAMPLE 3.5. A basically cliquish map need not be basically pre-s-cliquish. Let X be such as in Example 2.1. Define $F : X \to Y$ by the formula

$$F(x) = \begin{cases} \left(1, 1 + \frac{1}{2}\right) & \text{if } x \in [1, \infty) \\ \left(n+1, n + \frac{3}{2}\right) & \text{if } x \in \left[\frac{1}{n+1}, \frac{1}{n}\right) \\ \left(0, \frac{1}{2}\right) & \text{if } x = 0 \\ \left(-(n+1), -\left(n + \frac{1}{2}\right)\right) & \text{if } x \in \left(-\frac{1}{n}, -\frac{1}{n+1}\right] \\ \left(-1, -\frac{1}{2}\right) & \text{if } x \in (-\infty, -1], \quad n = 1, 2, \ldots \end{cases}$$

If $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ are a collections of open subsets of X such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$, then $X \setminus (\{\frac{1}{n} : n = 1, 2, \ldots\} \cup$

$\{-\frac{1}{n} : n = 1,2,\ldots\} \cup \{0\}) \subset \bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}$. So $X = \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\})$ and, by Proposition 2.1 (ii), F is basically cliquish. But F is not basically pre-s-cliquish at $x = 0$. Indeed, for $\{V_s : s \in S\} = \{U_s : s \in S\} = \{(n+1, n+\frac{3}{2} : n = 1,2,\ldots\} \cup \{(-(n+1), -(n+\frac{1}{2})) : n = 1,2,\ldots\} \cup \{(1, 1+\frac{1}{2}), (0, \frac{1}{2}), (-1, -\frac{1}{2})\}$ we have $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$ and $0 \notin \bigcup\{\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s)))) : s \in S\}$ and, by Proposition 2.1 (iii), F is not basically pre-s-cliquish at $x = 0$.

EXAMPLE 3.6. An α-continuous map need not be basically continuous. Let X be such as in the Example 2.1 and let Y be the same set with the topology $T = \{(a, \infty) : a \in Y\} \cup \{Y, \emptyset\}$. Define $F : X \to Y$ as follows

$$F(x) = \begin{cases} \left(1, \frac{3}{2}\right) & \text{if } x \notin \left\{\frac{1}{n} : n = 1,2,\ldots\right\} \\ \left(-n, -n+\frac{1}{2}\right) & \text{if } x = \frac{1}{n}, \quad n = 1,2,\ldots \end{cases}$$

Clearly, F is α-continuous since the set $Y \setminus \{\frac{1}{n} : n = 1,2,\ldots\}$ is an α-set but it is not open, and for every n', the set $Y \setminus \{\frac{1}{n'+n} : n = 1,2,\ldots\}$ is an α-set, but not open. However, for every $a \in Y$, $0 \notin \mathrm{Int}(F^+((a,\infty)) \cap F^-((a,\infty)))$. So, F is not basically continuous.

A multivalued map $F : X \to Y$ is said to be barely continuous if for every nonempty closed set $M \subset X$ the restriction F/M has at least one point of the continuity.

When $F(x) = \{f(x)\}$ for a single-valued map $f : X \to Y$, then above definition coincides with the well known definition of the barely continuity [12].

PROPOSITION 3.1. *Any barely continuous map $F : X \to Y$ is basically cliquish.*

Proof. Assume that F is basically continuous. Let $x \in X$ and let W be an open set containing x. Then, by the assumption, there exists a point $z \in \mathrm{Cl}(W)$ at which map $F/\mathrm{Cl}(W)$ is continuous. Thus, for any open sets $V, U \subset Y$ such that $F(z) \in V^+ \cap U^-$, there exists an open set B containing z such that $F(p) \in V^+ \cap U^-$ for every $p \in B \cap \mathrm{Cl}(W)$. The set $G = B \cap W$ is nonempty and $G \subset W \cap \mathrm{Int}(F^+(V) \cap F^-(U))$, what imples the basically cliquishness of F at x.

In [5] it is shown that quasi-continuity and barely continuity of single-valued maps are independent properties. We shall show that barely continuity of multivalued maps and the four forms of cliquishness (namely basically α-cliquishness, basically s-cliquishness, basically pre-α-cliquishness and basically pre-s-cliquishness) are independent.

EXAMPLE 3.7.

(i) A basically α-cliquish map need not be barely continuous. Let us consider the set $X = [0, \infty)$ with the topology $T = \{\emptyset, X\} \cup \{(r, \infty) : r > 0\}$ and let Y be the set of all real numbers with the natural topology. Define $F : X \to Y$ as follows:

$$F(x) = \begin{cases} \left\{1, \dfrac{1}{2}\right\} & \text{if } x \in (0,1) \cap Q \\ \left\{-1, -\dfrac{1}{2}\right\} & \text{if } x \in (0,1) \setminus Q \\ \{0\} & \text{if } x = 0 \\ \left\{\dfrac{1}{n+1}, \dfrac{1}{n+2}\right\} & \text{if } x \in [n, n+1) \cap Q \\ \left\{-\dfrac{1}{n+1}, -\dfrac{1}{n+2}\right\} & \text{if } x \in [n, n+1) \setminus Q,\ n = 1, 2, \ldots; \end{cases}$$

where Q is the set of all rational numbers.

We take $z = 0$. If $F(z) \in V^+ \cap U^-$ for some open subsets $V, U \subset Y$, then $0 \in U \cap V$ and for some k we have $F(x) \in V^+ \cap U^-$ for any $x \in (k, \infty)$. Clearly, $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}((k, \infty)))) = \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V) \cap F^-(U)))) = X$, what implies basically α-cliquishness of F. However, for $M = [0, 1]$ and for $x = 0$ (resp. $x = 1$, $x \in (0, 1)$) we have $V = U = (-\frac{1}{4}, \frac{1}{4})$ (resp. $V = (\frac{1}{4}, 1)$ and $U = (\frac{1}{3}, 1)$; $V = (\frac{1}{2}, 2)$ and $U = (1, 2)$) such that $F(x) \in V^+ \cap U^-$ and the interior of $M \cap (F^+(V) \cap F^-(U))$ in the subspace M is empty. So F is not barely continuous.

(ii) Let $F : X \to Y$ be such as in Example 3.5. The map F is not basically pre-s-cliquish. It is easy to prove that F is barely continuous.

Recall that a topological space X is said to be extremally disconnected if for each open subset $U \subset X$, $\mathrm{Cl}(U)$ is open.

PROPOSITION 3.2. *If X is an extremally disconnected space, then every quasi-continuous (resp. pre-semi-continuous, basically quasi-continuous, basically pre-semi-continuous, basically s-cliquish, basically pre-s-qliquish) map $F : X \to Y$ is α-continuous (resp. pre-continuous, basically α-continuous, basically pre-continuous, basically α-cliquish, basically pre-α-cliquish).*

Proof. It is sufficient to see that extremally disconnectedness of X implies $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(B))) = \mathrm{Cl}(\mathrm{Int}(B))$, $\mathrm{Int}(\mathrm{Cl}(B) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))$ for any $B \subset X$ and, consequently, $s\,\mathrm{Int}(B) = \alpha\,\mathrm{Int}(B)$ and $ps\,\mathrm{Int}(B) = p\,\mathrm{Int}(B)$.

PROPOSITION 3.3. *If $F : X \to Y$ is a basically cliquish (resp. basically pre-cliquish) map such that the set $F(X) \subset \mathcal{A}(Y)$ is compact with respect to* *the Vietoris topology on $\mathcal{A}(Y)$, then F is basically s-cliquish (resp. basically pre-s-cliquish).*

Proof. Assume that F is basically cliquish. Let $x \in X$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. Clearly, the collection $\{V_s^+ \cap U_s^- : s \in S\}$ is an open cover of the set $F(X) \subset \mathcal{A}(Y)$. Since $F(X)$ is compact, there exists a finite subset $S' \subset S$ such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S'\}$ and, by Proposition 2.1 (ii) we have $x \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S'\}) = \bigcup\{\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s))) : s \in S'\}$. Thus, by Proposition 2.1 (ii), F is basically s-cliquish at x.

PROPOSITION 3.4. *If Y is a T_1-space and $F : X \to Y$ is basically continuous (resp. basically α-continuous, basically quasi-continuous, basically pre-continuous, basically pre-semi-continuous) multivalued map with closed values, then F is continuous (resp. α-continuous, quasi-continuous, pre-continuous, pre-semi-continuous).*

Proof. Assume that F is basically continuous. Let $x \in X$ and let V, $U \subset Y$ be a pair of open subsets such that $F(x) \in V^+ \cap U^-$. For every $z \in X$ such that $F(z) \neq F(x)$ we take $V_z = U_z = Y \setminus \{y_z\}$ for some $y_z \in F(x) \setminus F(z)$ (resp. $V_z = Y$ and $U_z = Y \setminus F(x)$) if $F(x) \not\subset F(z)$ (resp. if $F(x) \subset F(z)$). It is easy to see that for every $z \in X$ such that $F(z) \neq F(x)$ we have $F(z) \in V_z^+ \cap U_z^-$ and $F(x) \notin V_z^+ \cap U_z^-$. So, $F(X) \subset V^+ \cap U^- \cup \bigcup\{V_z^+ \cap U_z^- : z \in X, F(z) \neq F(x)\}$. Thus, by Proposition 2.1 (i) we have $x \in \mathrm{Int}(F^+(V) \cap F^-(U)) \cup \bigcup\{\mathrm{Int}(F^+(V_z) \cap F^-(U_z)) : z \in X, F(z) \neq F(x)\}$ and, consequently, $x \in \mathrm{Int}(F^+(V) \cap F^-(U))$ since $x \notin \mathrm{Int}(F^+(V_z) \cap F^-(U_z))$ for every $z \in X$ such that $F(z) \neq F(x)$. Hence F is continuous at x.

The rest of the proof is similar.

4. Characterizations of the set of continuity and cliquishness of multivalued maps

The familly of all subsets of a space (X,T) which are semi-open (resp. pre-semi-open) is denoted by $SO(X,T)$ (resp. $PSO(X,T)$).

THEOREM 4.1. *For every multivalued map $F : (X,T) \to (Y,T')$ there exists collections $\{\mathcal{A}_s \subset SO(X,T) : s \in S\}$ and $\{\mathcal{K}_s \subset PSO(X,T) : s \in S\}$ such that*

(i) $BC(F) = \bigcap_{s\in S} \bigcup\{\mathrm{Int}(A) : A \in \mathcal{A}_s\} = \bigcap_{s\in S} \bigcup\{\mathrm{Int}(K) : K \in \mathcal{K}_s\}$,

(ii) $sBC(F) = \bigcap_{s\in S} \bigcup\{A : A \in \mathcal{A}_s\}$,

(iii) $\alpha BA(F) = \bigcap_{s\in S} \bigcup\{\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A))) : A \in \mathcal{A}_s\}$,

(iv) $sBA(F) = \bigcap_{s\in S} \bigcup\{\mathrm{Cl}(\mathrm{Int}(A)) : A \in \mathcal{A}_s\}$,

(v) $BA(F) = \bigcap_{s\in S} \mathrm{Cl}(\bigcup\{\mathrm{Int}(A) : A \in \mathcal{A}_s\})$,

(vi) $psBC(F) = \bigcap_{s\in S} \bigcup\{K : K \in \mathcal{K}_s\}$,

(vii) $p\alpha BA(F) = \bigcap_{s\in S} \bigcup\{\mathrm{Int}(\mathrm{Cl}(K)) : K \in \mathcal{K}_s\}$,

(viii) $psBA(F) = \bigcap_{s\in S} \bigcup\{\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(K))) : K \in \mathcal{K}_s\}$,

(ix) $pBA(F) = \bigcap_{s\in S} \mathrm{Cl}(\bigcup\{\mathrm{Int}(\mathrm{Cl}(K)) : K \in \mathcal{K}_s\})$.

Proof. This is an immediate consequence of Proposition 2.1 and of the fact for every subset $B \subset X$ we have $\mathrm{Int}(s\,\mathrm{Int}(B)) = \mathrm{Int}(B) = \mathrm{Int}(ps\,\mathrm{Int}(B))$ and $\mathrm{Int}(\mathrm{Cl}(ps\,\mathrm{Int}(B))) = \mathrm{Int}(\mathrm{Cl}(B))$. It is sufficient to take:
for any collections $\mathcal{V}_s = \{V_{s,z} : z \in Z\}$ and $\mathcal{U}_s = \{U_{s,z} : z \in Z\}$ of open subsets of Y such that $F(X) \subset \bigcup\{V_{s,z}^+ \cap U_{s,z}^- : z \in Z\}$, $\mathcal{A}_s = \{s\,\mathrm{Int}(F^+(V_{s,z}) \cap F^-(U_{s,z})) : z \in Z\}$ and $\mathcal{K}_s = \{ps\,\mathrm{Int}(F^+(V_{s,z}) \cap F^-(U_{s,z})) : z \in Z\}$.

COROLLARY 4.1. *For a multivalued map $F : X \to Y$ the following hold*:

(i) *The sets $BA(F)$ and $pBA(F)$ are closed.*

(ii) *If one of the sets $C(F)$, $\alpha C(F)$, $sC(F)$, $\alpha BA(F)$, $sBA(F)$ (resp. $pC(F)$, $psC(F)$, $p\alpha BA(F)$, $psBA(F)$) is dense, then F is basically cliquish (resp. basically precliquish).*

(iii) *If X is a Baire space and the $X \setminus C(F)$ (resp. $X \setminus pC(F)$) is of the first category, then F is cliquish (resp. pre-cliquish).*

We say that a collection $\mathcal{A}$ of subsets of Y has property P_c if there exists sequences $\{\mathcal{V}_n : n = 1,2,\ldots\}$, $\{\mathcal{U}_n : n = 1,2,\ldots\}$ of families of open subsets of Y such that:

If $\mathcal{A} \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$ for a collections $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ of open subsets of Y, then there exists a number n such that $\mathcal{A} \subset \bigcup\{V^+ \cap U^- : V \in \mathcal{V}_n, U \in \mathcal{U}_n\} \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$.

The following example shows some collection with the property P_c.

EXAMPLE 4.1. Let X be the space of real numbers with the natural topology. Let $\mathcal{A} = \{[1,2],[3,4]\}$. If $\mathcal{A} \subset \bigcup\{U_s^- : s \in S\}$ for some collection $\{U_s : s \in S\}$ of open subsets of X, there exist rational numbers k, p such that for some positive integer n holds $(k-\frac{1}{n}, k+\frac{1}{n}) \subset (1,2)\cap U_s$, $(p-\frac{1}{n}, p+\frac{1}{n}) \subset (3,4) \cap U_s$, for some $s, s' \in S$. It is obvious that for $\mathcal{U}_{k,p,n} = \{(k-\frac{1}{n}, k+\frac{1}{n}), (p-\frac{1}{n}, p+\frac{1}{n})\}$ we have $\mathcal{A} \subset \bigcup\{U^- : U \in \mathcal{U}_{k,p,n}\} \subset \bigcup\{U_s^- : s \in S\}$. Let $\mathcal{V}_n = \{(1-\frac{1}{n}, 2+\frac{1}{n}), (3-\frac{1}{n}, 4+\frac{1}{n})\}$, $n = 1,2,\ldots$ If $\mathcal{A} \bigcup\{V_s^+ : s \in S\}$ for some collection $\{V_s : s \in S\}$ of open subsets of X, there exists a number

n such that $\mathcal{A} \subset \bigcup\{V^+ : V \in \mathcal{V}_n\} \subset \bigcup\{V_s^+ : s \in S\}$. Thus, $\mathcal{A}$ has the property P_c.

Remark 4.1. It is easy to observe that if the collection $F(X)$ has property P_c, then in the proof of Theorem 4.1 it may be taken $\mathcal{A}_n = \{s\,\mathrm{Int}(F^+(V) \cap F^-(U)) : V \in \mathcal{V}_n, U \in \mathcal{U}_n\}$, $n = 1,2,\ldots$ and $\mathcal{K}_n = \{ps\ \mathrm{Int}(F^+(V) \cap F^-(U)) : V \in \mathcal{V}_n, U \in \mathcal{U}_n\}$, $n = 1,2,\ldots$, where $\{\mathcal{V}_n : n = 1,2,\ldots\}$ and $\{\mathcal{U}_n : n = 1,2,\ldots\}$ are a sequences from the definition of the property P_c.

Thus, from Theorem 4.1. we obtain the following

COROLLARY 4.2. *For a multivalued map $F : X \to Y$ such that $F(X)$ has the property P_c, the following hold*:

(i) *The sets $BC(F)$, $\alpha BA(F)$ and $p\alpha BA(F)$ are G_δ.*

(ii) The sets $BA(F) \setminus BC(F)$ and $pBA(F) \setminus p\alpha BA(F)$ are of the first category.

Proof. The proof of (i) is obvious. For the proof of (ii), according to remark 4.1 and Theorem 4.1, take $W_n = \bigcup\{B : B \in \mathcal{B}_n\}$ where $\mathcal{B}_n = \{\mathrm{Int}(F^+(V) \cap F^-(U)) : V \in \mathcal{V}_n, U \in \mathcal{U}_n\}$ for $n = 1,2,\ldots$, and $\{\mathcal{V}_n : n = 1,2,\ldots\}$, $\{\mathcal{U}_n : n = 1,2,\ldots\}$ are the sequence from the definition of the propert P_c. Then $BA(F) \setminus BC(F) = \bigcap\{\mathrm{Cl}(W_n) : n = 1,2,\ldots\} \setminus \bigcap\{W_n : n = 1,2,\ldots\} \subset \bigcup\{\mathrm{Cl}(W_n) \setminus W_n : n = 1,2,\ldots\}$ and, since for every number n, the set $\mathrm{Cl}(W_n) \setminus W_n$ is nowhere dense, the set $BA(F) \setminus BC(F)$ is of the first category.

The proof for the set $pBA(F) \setminus p\alpha BA(F)$ is similar.

Applying Proposition 3.4 and the above corollary, we obtain

COROLLARY 4.3. *If Y is a T_1-space and $F : X \to Y$ is a multivalued map with closed values such that $F(X)$ has the property P_c, then the followin hold*:

(i) *The set $C(F)$ is G_δ.*

(ii) *The set $BA(F) \setminus C(F)$ is of the first category.*

Form Proposition 3.4, Corollary 4.1 and from Corollary 4.3 we have

THEOREM 4.2. *If Y is a T_1-space, X is a Baire space and $F : X \to Y$ is a multivalued map with closed values such that $F(X)$ has property P_c, then F is basically cliquish if and only if the set $X \setminus C(F)$ is of the first category.*

A topological space X is said to be Baire space in the narrow sense if every closed subspace of X is a Baire space [7].

THEOREM 4.3. *If X is a Baire space in the narrow sense, Y is a T_1-space and $F : X \to Y$ is a multivalued map with closed values such that $F(X)$*

has the property P_c, then F is barely continuous if and only if for every nonempty closed set $M \subset X$, the map F/M is basically cliquish.

Proof. If F is barely continuous and $M \subset X$ is a closed nonempty set and the map F/M is clearly barely continuous and, by Proposition 3.1, F/M is basically cliquish. Conversely, if $M \subset X$ is a closed nonempty set and the map F/M is basically cliquish, then by Theorem 4.2, the set $M \setminus C(F/M)$ is of the first category in the subspace M, so the set $C(F/M)$ is dense in M since M is a Baire space. Consequently, $C(F/M) \neq \emptyset$ what implies the barely continuity of F.

5. Restrictions

We will denote the closure and the interior of a subset K of a subspace $Z \subset X$ by $Z - \mathrm{Cl}(K)$ and $Z - \mathrm{Int}(K)$, respectively.

A collection $\mathcal{A}$ of nonempty subsets of Y is said to be dense in a collection $\mathcal{B}$ of nonempty subsets of Y [6] if for any $B \in \mathcal{B}$ and any two open sets V, $U \subset Y$ such that $B \in V^+ \cap U^-$ we have $V^+ \cap U^- \cap \mathcal{A} \neq \emptyset$.

We say that a collection $\mathcal{A}$ is dense in a collection $\mathcal{B}$ at a set $B \in \mathcal{B}$ if for any two open sets V, $U \subset Y$ such that $B \in V^+ \cap U^-$ we have $V^+ \cap U^- \cap \mathcal{A} \neq \emptyset$.

It is obvious that $\mathcal{A}$ is dense in $\mathcal{B}$ if and only if for every $B \in \mathcal{B}$, $\mathcal{A}$ is dense i $\mathcal{B}$ at B.

THEOREM 5.1. *Let $F : X \to Y$ be a multivalued map and let $Z \subset X$ be a subspace such that for every $F(p) \in F(X \setminus Z)$, the collection $F(Z)$ is not dense in $F(X \setminus Z)$ at $F(p)$. Then the following hold:*

(i) *If F is basically cliquish and Z is pre-semi-open, then the restriction F/Z is basically cliquish.*

(ii) *If F is basically pre-cliquish and Z is semi-open, then the restriction F/Z is basically pre-cliquish.*

(iii) *If F is basically s-cliquish and Z is pre-open, then the restriction F/Z is basically s-cliquish.*

(iv) *If F is basically pre-s-cliquish and Z is an α-set, then the restriction F/Z is basically pre-s-cliquish.*

(v) *If F is basically α-cliquish and Z is pre-semi-open, then the restriction F/Z is basically α-cliquish.*

(vi) *If F is basically pre-α-cliquish and Z is semi-open, then the restriction F/Z is basically pre-α-cliquish.*

Proof. (i) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collection of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By Proposition 2.1 (ii) it is suffices to show that $x \in Z - \mathrm{Cl}(\bigcup\{Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) : s \in S\})$.

Let W be an open set containing x. Then $x \in W \cap Z \subset W \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(Z)))$, what implies $W \cap \mathrm{Int}(\mathrm{Cl}(Z)) \neq \emptyset$. Take a point $x' \in W \cap Z \cap \mathrm{Int}(\mathrm{Cl}(Z))$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V^+ \cap U^- \cap F(Z) = \emptyset$. It is obvious that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$. Thus, by Proposition 2.1 (ii) we have $x' \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\}) \cup \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_p) \cap F^-(U_p)) : p \in X \setminus Z\})$ and, consequently, either $W \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap \mathrm{Int}(F^+(V_s) \cap F^-(U_s)) \neq \emptyset$ for some $s \in S$, or $W \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap \mathrm{Int}(F^+(V_p) \cap F^-(U_p)) \neq \emptyset$ for some $p \in X \setminus Z$. Thus, either $G \subset F^+(V_s) \cap F^-(U_s)$ for some $s \in S$ and some open nonempty set $G \subset W \cap \mathrm{Int}(\mathrm{Cl}(Z))$, or $G \subset F^+(V_p) \cap F^-(U_p)$ for some $p \in X \setminus Z$ and some open nonempty set $G \subset W \cap \mathrm{Int}(\mathrm{Cl}(Z))$. So we have a set G', namely $G' = G \cap Z$, such that $\emptyset \neq G' \subset W \cap Z \cap F^+(V_s) \cap F^-(U_s)$ for some $s \in S$ since for every $p \in X \setminus Z$ we have $F(G') \cap V_p^+ \cap U_p^- \subset F(Z) \cap V_p^+ \cap U_p^- = \emptyset$. Thus, $G' \subset W \cap Z \cap \bigcup\{Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) : s \in S\}$ since G' is open in the subspace Z. So $x \in Z - \mathrm{Cl}(\bigcup\{Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) : s \in S\})$ and the proof of (i) is complete.

(ii) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. If W is an open set containing x, then $x \in W \cap Z \subset W \cap \mathrm{Cl}(\mathrm{Int}(Z))$, what implies $W \cap \mathrm{Int}(Z) \neq \emptyset$. Take a point $x' \in W \cap \mathrm{Int}(Z)$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V_p^+ \cap U_p^- \cap F(Z) = \emptyset$. Clearly, $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$. By Proposition 2.1 (iii) we have $x' \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))) : s \in S\}) \cup \mathrm{Cl}(\bigcup\{\mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))) : p \in X \setminus Z\})$. Thus, either $W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))) \neq \emptyset$ or some $s \in S$, or $W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))) \neq \emptyset$ for some $s \in S$, or $W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))) \neq \emptyset$ for some $p \in X \setminus Z$. But $W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))) = \emptyset$ since $Z \cap F^+(V_p) \cap F^-(U_p) = \emptyset$. Consequently we have a subset G, namely $G = W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s)))$, nonempty, open in the subspace Z, $G \subset W \cap Z$ and such that $G \subset Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z))$. Indeed , for each $z \in G$ and every open set O containing z we have $O \cap Z \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$ since $\emptyset \neq O \cap G \subset O \cap Z$ and $O \cap G \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Therefore $x \in Z - \mathrm{Cl}(\bigcup\{Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)) : s \in S\})$ and, by Proposition 2.1 (iii) F/Z is basically pre-cliquish at x.

(iii) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V_p^+ \cap U_p^- \cap F(Z) = \emptyset$. It is obvious that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$. Thus, by

Proposition 2.1 (ii) we have either $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s)))$ for some $s \in S$, or $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p)))$ for some $p \in X \setminus Z$. It is easy to see that $x \notin \mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p)))$ for every $p \in X \setminus Z$. Indeed, if we assume on the contrary that $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p)))$ for some $p \in X \setminus Z$, then $x \in \mathrm{Int}(\mathrm{Cl}(Z)) \cap \mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p)))$ since Z is pre-open; consequently, $F(Z) \cap V_p^+ \cap U_p^- \neq \emptyset$, which is impossible. Therefore, $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s)))$ for some $s \in S$ and, for every open set W containing x there exists a subset G, namely $G = W \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap \mathrm{Int}(F^+(V_s) \cap F^-(U_s))$, such that $G \cap Z \subset W \cap Z$, $G \cap Z \neq \emptyset$, and $G \cap Z \subset F^+(V_s) \cap F^-(U_s) \cap Z$. So $W \cap Z \cap Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) \neq \emptyset$ and, consequently, $x \in Z - \mathrm{Cl}(Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))$. Thus, F/Z is basically s-cliquish by Proposition 2.1 (ii).

(iv) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V_p^+ \cap U_p^- \cap F(Z) = \emptyset$. Clearly, $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$ and, by Proposition 2.1 (iii) we have either $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))))$ for some $s \in S$, or $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))))$ for some $p \in X \setminus Z$. At first we prove that $x \notin \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))))$ for every $p \in X \setminus Z$. Assume on the contrary that $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p))))$. Then $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z))) \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_p) \cap F^-(U_p)))) \neq \emptyset$, so $F(Z) \cap V_p^+ \cap U_p^- \neq \emptyset$ which is impossible. Thus, $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))))$ for some $s \in S$. Since $Z \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z)))$, for every open set W containing x there exists a set G, namely $G = W \cap \mathrm{Int}(Z) \cap \mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s)))$, such that $\emptyset \neq G \cap Z \subset W \cap Z$ and, for each $z \in G \cap Z$ and every open set O containing z we have $O \cap Z \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Therefore, $x \in Z - \mathrm{Cl}(Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)))$ and, by Proposition 2.1 (iii), F/Z is basically pre-s-cliquish.

(v) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V_p^+ \cap U_p^- \cap F(Z) = \emptyset$. Clearly, $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$ and, by Proposition 2.1 (ii) we have either $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s))))$ for some $s \in S$, or $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p))))$ for some $p \in X \setminus Z$. We shall show that $x \notin \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p))))$ for every $p \in X \setminus Z$. Assume in the contrary that $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_p) \cap F^-(U_p))))$ for some $p \in X \setminus Z$. Since $x \in Z \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(Z)))$, $Z \cap \mathrm{Int}(F^+(V_p) \cap F^-(U_p)) \neq \emptyset$. So $F(Z) \subset V_p^+ \cap U_p^- \neq \emptyset$, which is impossible. Therefore, $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s)))$ for some $s \in S$. The set $Z \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s))))$ is open in the subspace Z and, for each $z \in Z \cap \text{Int}(\text{Cl}(\text{Int}(F^+(V_s) \cap F^-(U_s))))$ and for every open set 0 containing z we have an open subset G, namely $G = O \cap \text{Int}(\text{Cl}(Z)) \cap \text{Int}(F^+(V_s) \cap F^-(U_s))$, such that $\emptyset \neq G \cap Z \subset O \cap Z$ and $G \cap Z \subset F^+(V_s) \cap F^-(U_s) \cap Z$. It implies $Z \cap \text{Int}(\text{Cl}(\text{Int}(F^+(V_s) \cap F^-(U_s)))) \subset Z - \text{Cl}(Z - \text{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))$, so $x \in \bigcup\{Z - \text{Int}(Z - \text{Cl}(Z - \text{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))) : s \in S\}$ and, by Proposition 2.1 (ii), F/Z is basically α-cliquish.

(vi) Let $x \in Z$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(Z) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By the assumption, for every $p \in X \setminus Z$ there exists a pair of open sets $V_p, U_p \subset Y$ such that $F(p) \in V_p^+ \cap U_p^-$ and $V_p^+ \cap U_p^- \cap F(Z) = \emptyset$. Clearly, $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\} \cup \bigcup\{V_p^+ \cap U_p^- : p \in X \setminus Z\}$ and, by Proposition 2.1 (iii) we have $x \in \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s)))$ for some $s \in S$, or $x \in \text{Int}(\text{Cl}(F^+(V_p) \cap F^-(U_p)))$ for some $p \in X \setminus Z$. We see that $x \notin \text{Int}(\text{Cl}(F^+(V_p) \cap F^-(U_p)))$ for every $p \in X \setminus Z$. Indeed, if for some $p \in X \setminus Z$ we have $x \in \text{Int}(\text{Cl}(F^+(V_p) \cap F^-(U_p)))$, then $\text{Int}(Z) \cap F^+(V_p) \cap F^-(U_p) \neq \emptyset$ since $x \in Z \subset \text{Cl}(\text{Int}(Z))$. So $F(Z) \cap V_p^+ \cap U_p^- \neq \emptyset$, which is impossible. Therefore $x \in \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s)))$ for some $s \in S$. It easy to see that the set $Z \cap \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s)))$ is open in the subspace Z and, for each $z \in Z \cap \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s)))$ and for every open set O containing z we have $O \cap Z \cap \text{Int}(Z) \cap \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s))) \neq \emptyset$, so $O \cap Z \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. It implies $Z \cap \text{Int}(\text{Cl}(F^+(V_s) \cap F^-(U_s))) \subset Z - \text{Int}(Z - \text{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z))$ and, consequently, $x \in Z - \text{Int}(Z - \text{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z))$. Thus, by Proposition 2.1 (iii), F/Z is basically pre-α-cliquish. The proof of Theorem 5.1 is complete.

THEOREM 5.2. *For a multivalued map $F : X \to Y$ and a subspace $Z \subset X$, the following hold*:

(i) *If the restriction F/Z is basically α-cliquish (resp. basically s-cliquish), then F is basically α-cliquish (resp. basically s-cliquish) at each point of the set $\alpha\,\text{Int}(Z)$.*

(ii) *If the restriction F/Z is basically pre-α-cliquish (resp. basically pre-s-cliquish), then F is basically pre-α-cliquish (resp. basically pre-s-cliquish) at each point of the set $p\,\text{Int}(Z)$.*

(iii) *If the restriction F/Z is basically cliquish, then F is basically cliquish at each point of the set $s\,\text{Int}(Z)$.*

(iv) *If the restriction F/Z is basically pre-cliquish, then F is basically pre-cliquish at each point of the set $ps\,\text{Int}(Z)$.*

Proof. (i) Assume that F/Z is basically α-cliquish. Let $x \in \alpha\,\text{Int}(Z)$ and let $\{V_s : s \in S\}$ and $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. Then, by Lemma 1.1 (iii) and Proposition 2.1 (ii) we have $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z))) \cap Z - \mathrm{Int}(Z - \mathrm{Cl}(Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z)))$ for some $s \in S$. Hence there exists an open set O such that $x \in O \cap Z \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z))) \cap Z - \mathrm{Cl}(Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))$. It is easy to see that for each $z \in O \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z)))$ and for every open set W containing z we have $O \cap \mathrm{Int}(Z) \cap W \subset Z - \mathrm{Cl}(Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))$, so there exists an open set G such that $\emptyset \neq G \cap Z \subset O \cap \mathrm{Int}(Z) \cap W$ and $G \cap Z \subset F^+(V_s) \cap F^-(U_s) \cap Z$. Thus, the set $G' = G \cap O \cap \mathrm{Int}(Z) \cap W$ is open, nonempty, $G' \subset W$ and $G' \subset F^+(V_s) \cap F^-(U_s)$. Therefore, $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s))))$ and, by Proposition 2.1 (ii), F is basically α-cliquish at x.

Now we assume that F/Z is basically s-cliquish. Let $x \in \alpha\,\mathrm{Int}(Z)$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. Then, by Lemma 1.1 (iii) and by Proposition 2.1 (ii), there exists $s \in S$ such that $x \in \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z))) \cap Z - \mathrm{Cl}(Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z))$ and, for every open set O containing x we have $O \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z))) \cap Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) \neq \emptyset$, what is equivalent to the existence of open set G such that $\emptyset \neq G \cap Z \subset F^+(V_s) \cap F^-(U_s) \cap Z$ and $G \cap Z \subset O \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(Z)))$. Thus, the set $G' = G \cap O \cap \mathrm{Int}(Z)$ is open, nonempty, $G' \subset O$ and $G' \subset F^+(V_s) \cap F^-(U_s)$. So $x \in \mathrm{Cl}(\mathrm{Int}(F^+(V_s) \cap F^-(U_s)))$ and, by Proposition 2.1 (ii), F is basically s-cliquish at x.

(ii) Assume that F/Z is basically pre-α-cliquish. Let $x \in p\,\mathrm{Int}(Z)$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. Then, by Lemma 1.1 (ii) and, by Proposition 2.1 (iii), there exists $s \in S$ such that $x \in \mathrm{Int}(\mathrm{Cl}(Z)) \cap Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z))$ and consequently, there exists an open set O such that $x \in O \cap Z \subset \mathrm{Int}(\mathrm{Cl}(Z)) \cap Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$. Let us put $O' = O \cap \mathrm{Int}(\mathrm{Cl}(Z))$. Clearly, O' is open, $x \in O'$ and; if $z \in O'$, then for every open set W containing z we have $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Indeed, because $z \in W \cap O \cap \mathrm{Int}(\mathrm{Cl}(Z))$, the set $K = W \cap O \cap Z$ is open in the subspace Z, nonempty and $K \subset Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$, what implies $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. So, we have $x \in \mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s)))$ and, by Proposition 2.1 (iii), F is basically pre-α-cliquish at x.

Now we assume that F/Z is basically pre-s-cliquish. Let $x \in p\,\mathrm{Int}(Z)$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. By Lemma 1.1 (ii) and Proposition 2.1 (iii), there exists $s \in S$ such that $x \in \mathrm{Int}(\mathrm{Cl}(Z))) \cap Z - \mathrm{Cl}(Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)))$. This implies that for every open set O containing x there exists an open, nonempty set G such that $\emptyset \neq G \cap Z \subset O \cap \mathrm{Int}(\mathrm{Cl}(Z))$ and $G \cap Z \subset Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$. Observe that the set $G' = G \cap O \cap \mathrm{Int}(\mathrm{Cl}(Z))$ is open, nonempty, $G' \subset O$ and, for each $z \in G'$ and for every open set W containing z we have $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Indeed; $W \cap G' \cap Z \subset W$, $W \cap G' \cap Z \neq \emptyset$ and $W \cap G' \cap Z \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$ since $W \cap G' \cap Z \subset Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$. So $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Therefore $x \in \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(F^+(V_s) \cap F^-(U_s))))$ and, by Proposition 2.1 (iii), F is basically pre-s-cliquish at x.

(iii) Assume that F/Z is basically cliquish. Let $x \in s\,\mathrm{Int}(Z)$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. If O is an open set containing x, then $x \in O \cap \mathrm{Cl}(\mathrm{Int}(Z))$ and $\emptyset \neq O \cap \mathrm{Int}(Z) \subset Z - \mathrm{Cl}(\bigcup\{Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) : s \in S\})$ by Lemma 1.1 (ii) and, by Proposition 2.1 (ii). Then there exists $s \in S$ such that $O \cap \mathrm{Int}(Z) \cap Z - \mathrm{Int}(F^+(V_s) \cap F^-(U_s) \cap Z) \neq \emptyset$, what implies the existence of open nonempty set G such that $\emptyset \neq G \cap Z \subset O \cap \mathrm{Int}(Z)$ and $G \cap Z \subset F^+(V_s) \cap F^-(U_s) \cap Z$. Observe that the set $G' = G \cap O \cap \mathrm{Int}(Z)$ is open, nonempty, $G' \subset F^+(V_s) \cap F^-(U_s)$ and $G' \subset O$. Thus $x \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(F^+(V_s) \cap F^-(U_s)) : s \in S\})$ and, by Proposition 2.1 (ii), F is basically cliquish at x.

(iv) Assume that F/Z is basically pre-cliquish. Let $x \in ps\,\mathrm{Int}(Z)$ and let $\{V_s : s \in S\}$, $\{U_s : s \in S\}$ be a collections of open subsets of Y such that $F(X) \subset \bigcup\{V_s^+ \cap U_s^- : s \in S\}$. If O is an open set containing x, then $x \in O \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(Z)))$ and $\emptyset \neq O \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap Z \subset Z - \mathrm{Cl}(\bigcup\{Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V - s) \cap F^-(U_s) \cap Z)) : s \in S\})$ by Lemma 1.1 (iv) and Proposition 2.1 (iii). Then there exists $s \in S$ such that $O \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap Z \cap Z - \mathrm{Int}(Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)) \neq \emptyset$. Consequently, there exists an open nonempty set G such that $G \cap Z \neq \emptyset$ and $G \cap Z \subset O \cap \mathrm{Int}(\mathrm{Cl}(Z)) \cap Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$. Observe that the set $' = G \cap O \cap \mathrm{Int}(\mathrm{Cl}(Z))$ is open, nonempty, $G' \subset O$ and, for each $z \in G'$ and for every open set W containing z we have $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Indeed, we have $G' \cap W \cap Z \neq \emptyset$, $G' \cap W \cap Z \subset W$, $G' \cap W \cap Z \subset G \cap Z \subset Z - \mathrm{Cl}(F^+(V_s) \cap F^-(U_s) \cap Z)$ and $G' \cap W \cap Z$ is open in the subspace Z, so $G' \cap W \cap Z \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. Thus $W \cap F^+(V_s) \cap F^-(U_s) \neq \emptyset$. From it follows $x \in \mathrm{Cl}(\bigcup\{\mathrm{Int}(\mathrm{Cl}(F^+(Vs) \cap F^-(U_s))) : s \in S\})$ and, by Proposition 2.1 (iii), F is basically pre-cliquish at x. This completes the proof of Theorem 5.2.

COROLLARY 5.1. *For a multivalued map $F : X \to Y$, the following hold*:

(i) *If F/D is basically pre-cliquish (resp. basically pre-s-cliquish, basically pre-α-cliquish) for some dense subset $D \subset X$, then F is basically pre-cliquish (resp. the set $psBA(F)$ is dense, the set $p\alpha BA(F)$ is dense).*

(ii) *If F/Z is basically cliquish (resp. basically s-cliquish, basically α-*

cliquish) for some semi-open dense subset $Z \subset X$, then F is basically cliquish (resp. the set $sBA(F)$, the set $\alpha BA(F)$ is dense).

Proof. It follows from Corallary 4.1, Theorem 5.2 and from the fact for every dense subset $D \subset X$ (resp. semi-open dense subset $Z \subset X$ we have $p\,\mathrm{Int}(D) = ps\,\mathrm{Int}(D) = D$ (resp. $\alpha\,\mathrm{Int}(Z) = s\,\mathrm{Int}(Z) = Z$).

From Theorem 5.2 we also have the following

COROLLARY 5.2. *For a multivalued map $F : X \to Y$, the following hold:*

(i) *If there exists a cover $\mathcal{A}$ of X by α-sets such that for every $A \in \mathcal{A}$, the restriction F/A is basically α-cliquish (resp. basically s-cliquish), then F is basically α-cliquish (resp. basically s-cliquish).*

(ii) *If there exists a cover $\mathcal{A}$ of X by pre-open sets such that for every $A \in \mathcal{A}$, the restriction F/A is basically pre-α-cliquish (resp. basically pre-s-cliquish), then F is basically pre-α-cliquish (resp. basically pre-s-cliquish).*

(iii) *If there exists a cover $\mathcal{A}$ of X by semi-open sets (resp. pre-semi-open sets) such that for every $A \in \mathcal{A}$, the map F/A is basically cliquish (resp. basically pre-cliquish), then F is basically cliquish (resp. basically pre-cliquish).*

References

[1] D. Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38 (1986), 24–32.

[2] W. Bledsoe, *Neighborly functions*, Proc. Amer. Math. Soc. 3 (1952), 114–115.

[3] S. G. Crossley and S. K. Hildebrand, *Semi-closure*, Texas J. Sci., 22 (1971), 99–112.

[4] J. Ewert, *On quasi-continuous and cliquish maps with values in uniform spaces*, Bull. Acad. Polon. Sci. 32 (1984), 81–88.

[5] J. Ewert, *On barely continuous and cliquish maps*, Demonstratio Math. 17 (1984), 331–338.

[6] J. Ewert and T. Noubrunn, *On quasi-continuous multivalued maps*, Demonstratio Math. 21 (1988), 697–711.

[7] Z. Frolik, *Baire space and some generalizations of complete metric spaces*, Czech. Math. J. 86 (1961), 237–248.

[8] S. Kempisty, *Sur les fonctions quasicontinues*, Fund. Math. 19 (1932), 184–197.

[9] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36–41.

[10] A. S. Mashhour, M. E. El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings*, Proc. Math. Soc. Egypt, 51 (1981).

[11] A. S. Mashhour, M. Abb El-Mansef and J. A. Husain, *On pretopological spaces*, Bull. Math. Soc. Math. R.S.R., 28 (1984), 39–45.

[12] E. Michael and I. Namioka, *Barely continuous functions*, Bull. Acad. Polon. Sci., 24 (1976), 889–892.

[13] E. Michael, *Topologies on spaces of subsets*, Trans. Amer. Math. Soc., 71 (1951), 152–182.
[14] A. Neubrunnova, *On certain generalizations of notion of continuity*, Mat. Casopis, 23 (1973), 374–380.
[15] O. Njåstad, *On classes of nearly open sets*, Pacific J. Math., 15 (1965), 961–970.
[16] V. Popa, *Certaines characterisations des fonctions multivoques quasicontinues et faiblement continues*, Stud. Cerc. Mat., 37 (1985), 77–82.
[17] M. Przemski, *On feeble T_1-continuity and feeble T_1-cliquishness of multivalued maps*, Comment. Math., 28 (1989), 331–344.
[18] I. Z. Reilly and M. K. Vamanamurthy, *On α-continuity in topological spaces*, University of Auckland Report Series, 193 (1982).
[19] S. P. Thielman, *Types of functions*, Amer. Math. Monthly, 60 (1953), 156–161.

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