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ALMOST WEAKLY CONTINUOUS FUNCTIONS

1. Introduction

Levine [6] introduced the notion of weakly continuous function between topological spaces. Husain [4] introduced and studied the notion of almost continuous functions. In [9], almost continuity is called precontinuity by Mashhour et al. Recently, Janković [5] has introduced the notion of almost weakly continuous functions. Almost weak continuity is implied by both almost continuity and weak continuity which are independent of each other.

The purpose of the present paper is to obtain several characterizations of almost weakly continuous functions and to improve some of results established by Mashhour et al. [9] and the first author [15] of this paper. In §3, we obtain several characterizations of almost weakly continuous functions. In §4, we obtain some sufficient conditions for an almost weakly continuous function to be almost continuous. In §5, we show that the assumption on "almost continuous" in several results established in [9] and [15] can be replaced by "almost weakly continuous".

2. Preliminaries

Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A is said to be <u>regular closed</u> (resp. <u>regular open</u>) if A=Cl(Int(A)) (resp. A=Int(Cl(A))). A subset A is said to be <u>preopen</u> [9] (resp. <u>semi-open</u> [7], <u>α-open</u> [14]) if A⊂Int(Cl(A)) (resp. A⊂Cl(Int(A)), A⊂Int(Cl(Int(A)))). It is shown in [10, Lemma 3.1] that A is

α-open in X if and only if A is preopen and semi-open in X. The family of all preopen sets in X is denoted by PO(X). For a point $x \in X$, we set $PO(X,x)=\{U | x \in U \in PO(X)\}$. The complement of a preopen set (resp. α-open set) is said to be <u>preclosed</u> (resp. α-closed). The <u>preclosure</u> [2] of A, denoted by Pcl(A), is defined by the intersection of all preclosed sets of X containing A. We notice that $x \in Pcl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in PO(X,x)$. The <u>preinterior</u> of A, denoted by Pint(A), is defined by the union of all preopen sets of X contained in A. A point $x \in X$ is said to be <u>θ-adherent</u> to A if $A \cap Cl(U) \neq \emptyset$ for every open set U of X containing x. The set of all θ-adherents points of A is called the <u>θ-closure</u> [18] of A and is denoted by $Cl_\theta(A)$. It is shown in [18] that $Cl_\theta(U)=Cl(U)$ for every open set U of X and $Cl_\theta(U)$ is closed for every subset A of X. If $Cl_\theta(A)=A$, then A is said to be <u>θ-closed</u>. The complement of a θ-closed set is said to be <u>θ-open</u>.

Definition 2.1. A function $f:X \to Y$ is said to be <u>weakly continuous</u> [6] if for each $x \in X$ and each open set V containing f(x), there exists an open set U containing x such that $f(U) \subset Cl(V)$.

Definition 2.2. A function $f:X \to Y$ is said to be <u>almost continuous</u> [4] if for each $x \in X$ and each open set V containing f(x), $Cl(f^{-1}(V))$ is a neighborhood of x.

In [17, Theorem 4], Rose showed that a function $f:X \to Y$ is almost continuous if and only if $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$ for every open set V of Y. Mashhour et al. defined a function $f:X \to Y$ to be <u>precontinuous</u> [9] if $f^{-1}(V) \in PO(X)$ for every open set V of Y. It is obvious that precontinuity is equivalent to almost continuity. We shall utilize the term "almost continuous" in the sequel.

Definition 2.3. A function $f:X \to Y$ is said to be <u>almost weakly continuous</u> [5] (briefly <u>a</u>.<u>w</u>.<u>c</u>.) if $f^{-1}(V) \subset Int(Cl(f^{-1}(Cl(V))))$ for every open set V of Y.

It is shown in [6, Theorem 1] that a function $f:X\to Y$ is weakly continuous if and only if $f^{-1}(V)\subset Int(f^{-1}(Cl(V)))$ for every open set V of Y. Therefore, it follows from Examples 5.8 and 5.10 of [11] that almost weak continuity is strictly weaker than both weak continuity.

3. Characterizations

In this section, we obtain several characterizations of a.w.c. functions.

Theorem 3.1. The following are equivalent for a function $f:X\to Y$:

(a) f is a.w.c.

(b) $Cl(Int(f^{-1}(V)))\subset f^{-1}(Cl(V))$ for every open set V of Y.

(c) $Pcl(f^{-1}(V))\subset f^{-1}(Cl(V))$ for every open set V of Y.

(d) $f^{-1}(V)\subset Pint(f^{-1}(Cl(V)))$ for every open set V of Y.

(e) For each point $x\in X$ and each open set V containing f(x), there exists $U\in PO(X,x)$ such that $f(U)\subset Cl(V)$.

Proof. (a)$\Rightarrow$(b): This is shown in [11, Theorem 3.1].

(b)$\Rightarrow$(c): It is shown in [1, Theorem 1.5] that $Pcl(A)=A\cup Cl(Int(A)$ for every subset A of X. Therefore, we obtain $Pcl(f^{-1}(V))\subset f^{-1}(V))$ for every open set V of Y.

(c)$\Rightarrow$(d):Let V be any open set of Y. Then Y-Cl(V) is open in Y and we have

$$X-Pint(f^{-1}(Cl(V)))=$$
$$=Pcl(f^{-1}(Y-Cl(V)))\subset f^{-1}(Cl(Y-Cl(V)))\subset X-f^{-1}(V).$$

Therefore, we obtain $f^{-1}(V)\subset Pint(f^{-1}(Cl(V)))$.

(d)$\Rightarrow$(e): Let $x\in X$ and V be any open set of Y containing f(x). We have $x\in f^{-1}(V)\subset Pint(f^{-1}(Cl(V)))$. Set $U=Pint(f^{-1}(Cl(V)))$, then we obtain $U\in PO(X,x)$ and $f(U)\subset Cl(V)$.

(e)$\Rightarrow$(a): Let V be any open set of Y and x any point of $f^{-1}(V)$. There exists $U\in PO(X,x)$ such that $f(U)\subset Cl(V)$. Therefore, we obtain $U\subset f^{-1}(Cl(V))$ and hence $x\in Int(Cl(f^{-1}(Cl(V))))$. This shows that $f^{-1}(V)\subset Int(Cl(f^{-1}(Cl(V))))$.

Remark 3.2. For every $V\in PO(Y)$, we have $Cl(V)= = Cl(Int(Cl(V)))$ and hence the condition "open set V" in each statement of Theorem 3.1 can be replaced by "preopen set V".

In [5, Theorem 4.7], Janković showed that a function $f:X\to Y$ is a.w.c. if and only if $f(Cl(U))\subset Cl_\theta(f(U))$ for each open set U of X. We shall obtain analogous characterizations of a.w.c. functions.

Theorem 3.3. The following are equivalent for a function $f:X\to Y$:

(a) f is a.w.c.

(b) $f(Pcl(A))\subset Cl_\theta(f(A))$ for every subset A of X.

(c) $Pcl(f^{-1}(B))\subset f^{-1}(Cl_\theta(B))$ for every subset B of Y.

(d) $Pcl(f^{-1}(Int(Cl_\theta(B))))\subset f^{-1}(Cl_\theta(B))$ for every subset B of Y.

(e) $Pcl(f^{-1}(Int(Cl(V))))\subset f^{-1}(Cl(V))$ for every open set V of Y.

(f) $Pcl(f^{-1}(Int(Cl(V))))\subset f^{-1}(Cl(V))$ for every $V\in PO(Y)$.

(g) $Pcl(f^{-1}(Int(F)))\subset f^{-1}(F)$ for every regular closed set F of Y.

Proof. (a)$\Rightarrow$(b): Let A be any subset of X. Let $x\in Pcl(A)$ and W be any open set containing f(x). Since f is a.w.c., by Theorem 3.1 there exists $U\in PO(X,x)$ such that $f(U)\subset Cl(W)$. Since $x\in Pcl(A)$, $U\cap A\neq\emptyset$ and hence $\emptyset\neq f(U\cap A)\subset Cl(W)\cap f(A)$. Therefore, we obtain $f(x)\in Cl_\theta(f(A))$. This shows that $f(Pcl(A))\subset \subset Cl_\theta(f(A))$.

(b)$\Rightarrow$(c): Let B be any subset of Y. Then we have

$$f(Pcl(f^{-1}(B)))\subset Cl_\theta(f(f^{-1}(B)))\subset Cl_\theta(B).$$

Therefore, we obtain $Pcl(f^{-1}(B))\subset f^{-1}(Cl_\theta(B))$.

(c)$\Rightarrow$(d): Let B be any subset of Y. Since $Cl_\theta(B)$ is closed in Y and $Cl_\theta(V)=Cl(V)$ for every open set V of Y [18, Lemma 2], we have

$$Pcl(f^{-1}(Int(Cl_\theta(B))))\subset f^{-1}(Cl_\theta(Int(Cl_\theta(B)))) =$$
$$= f^{-1}(Cl(Int(Cl_\theta(B))))\subset f^{-1}(Cl(Cl_\theta(B))) = f^{-1}(Cl_\theta(B)).$$

(d)$\Longrightarrow$(e): This is obvious since $Cl_\theta(V)=Cl(V)$ for every open set V.

(e)$\Longrightarrow$(f): This follows from $Cl(Int(Cl(V)))=Cl(V)$ for every $V\in PO(Y)$.

(f)$\Longrightarrow$(g): Let F be any regular closed set of Y. Then we have $Int(F)\in PO(Y)$ and hence

$$Pcl(f^{-1}(Int(F))) = Pcl(f^{-1}(Int(Cl(Int(F)))))\subset f^{-1}(Cl(Int(F))) = = f^{-1}(F).$$

(g)$\Longrightarrow$(a): Let V be any open set of Y. Then Cl(V) is regular closed in Y. Therefore, we obtain

$$Pcl(f^{-1}(V))\subset Pcl(f^{-1}(Int(Cl(V))))\subset f^{-1}(Cl(V)).$$

It follows from Theorem 3.1 that f is a.w.c.

Corollary 3.4. Let Y be a regular space. The following are equivalent for a function $f:X\longrightarrow Y$:

(a) f is almost continuous.

(b) $f^{-1}(Cl_\theta(B))$ is preclosed in X for every subset B of Y.

(c) $f^{-1}(F)$ is preclosed in X for every θ-closed set F of Y.

(d) $f^{-1}(V)$ is preopen in X for every θ-open set V of Y.

(e) f is a.w.c.

Proof. Since Y is regular, $Cl_\theta(B)=Cl(B)$ for every subset B of Y. Therefore, a subset F is θ-closed (resp. θ-open) in Y if and only if it is closed (resp. open) in Y. Janković [5] remarks that a.w.c. functions into regular spaces are almost continuous. Therefore, Theorem 1 of [15] completes the proof.

Lemma 3.5 (Mashhour et al. [9]). Let X be a topological space and A and X_0 subsets of X.

(1) If $A\in PO(X)$ and X_0 is semi-open in X, then $A\cap X_0\in PO(X_0)$.

(2) If $X_0\in PO(X)$ and $A\in PO(X_0)$, then $A\in PO(X)$.

It is shown in [11, Theorem 6.2.9] that if $f:X\longrightarrow Y$ is a.w.c. and X_0 is open in X then the restriction $f|X_0:X_0\longrightarrow Y$ is a.w.c. The following theorem is a slight improvement of this result.

Theorem 3.6. If a function $f:X\longrightarrow Y$ is a.w.c. and X_0 is semi-open in X, then the restriction $f|X_0:X_0\longrightarrow Y$ is a.w.c.

Proof. Let $x \in X_0$ and V be any open set of Y containing f(x). Since f is a.w.c., by Theorem 3.1 there exists $U \in PO(X,x)$ such that $f(U) \subset Cl(V)$. By Lemma 3.5, we have $U \cap X_0 \in PO(X_0,x)$ and $(f|X_0)(U \cap X_0) \subset f(U) \subset Cl(V)$. This shows that $f|X_0$ is a.w.c.

Theorem 3.7. A function $f:X \to Y$ is a.w.c. if, for each $x \in X$, there exists $X_0 \in PO(X,x)$ such that the restriction $f|X_0:X_0 \to Y$ is a.w.c.

Proof. Let $x \in X$ and V be any open set of Y containing f(x). There exists $X_0 \in PO(X,x)$ such that $f|X_0:X_0 \to Y$ is a.w.c. and hence $(f|X_0)(U) \subset Cl(V)$ for some $U \in PO(X_0,x)$. By Lemma 3.5, $U \in PO(X,x)$ and $f(U) \subset Cl(V)$. This shows that f is a.w.c.

Corollary 3.8. Let $\{U_\alpha | \alpha \in \nabla\}$ be a cover of a space X by α-open sets of X. Then a function $f:X \to Y$ is a.w.c. if and only if the restriction $f|U_\alpha:U_\alpha \to Y$ is a.w.c. for each $\alpha \in \nabla$.

Proof. This is an immediate consequence of Theorem 3.6 and 3.7.

4. Sufficient conditions for a.w.c. functions to be almost continuous

Lemma 4.1. Let X be a topological space. If A is α-open in X and $B \in PO(X)$, then $A \cap B \in PO(X)$.

Proof. If O is open in X, then $O \cap Cl(A) \subset Cl(O \cap A)$ for any subset A of X. By utilizing this result, we obtain

$$A \cap B \subset Int(Cl(Int(A))) \cap Int(Cl(B)) \subset Int[Cl(Int(A)) \cap Int(Cl(B))] \subset$$
$$\subset Int[Cl[Int(A) \cap Cl(B)]] \subset Int[Cl[A \cap B]].$$

This shows that $A \cap B$ is preopen in X.

A function $f:X \to Y$ is said to be <u>weak* continuous</u> [6] if $f^{-1}(Fr(V))$ is closed in X for each open set V of Y, where Fr(V) denotes the frontier of V. A function $f:X \to Y$ is said to be <u>locally weak* continuous</u> [16] if there exists an open basis $\mathcal{B}$ for the topology on Y such that $f^{-1}(Fr(V))$ is closed in X for each V in $\mathcal{B}$.

Theorem 4.2. An a.w.c. function $f:X \to Y$ is almost continuous if there exists an open basis $\mathcal{B}$ for the topology on Y such that $f^{-1}(Fr(V))$ is α-closed for each V in $\mathcal{B}$.

Proof. Let $f:X\longrightarrow Y$ be an a.w.c. function which satisfies the condition of Theorem 4.2. Let $x\in X$ and W be any open set of Y containing f(x). There exists $V\in\mathcal{B}$ such that $f(x)\in V\subset W$. Since f is a.w.c., by Theorem 3.1 there exists $U\in PO(X,x)$ such that $f(U)\subset Cl(V)$. By the hypothesis, $X-f^{-1}(Fr(V))$ is α-open in X. Set $U_0=U\cap[X-f^{-1}(Fr(V))]$. Then since $f(x)\bar{\in}Fr(V)$, we obtain $x\in U_0\in PO(X)$ by Lemma 4.1. Moreover, we have

$$f(U_0)=f(U)\cap(Y-Fr(V))\subset Cl(V)\cap(Y-Fr(V))=V\subset W.$$

It follows from [15, Theorem 1] that f is almost continuous.

The following two corollaries are immediate consequences of Theorem 4.2.

Corollary 4.3. If a function $f:X\longrightarrow Y$ is a.w.c. locally weak* continuous, then f is almost continuous.

Corollary 4.4. If a function $f:X\longrightarrow Y$ is a.w.c. weak* continuous, then f is almost continuous.

A topological space Y is said to be <u>strongly locally compact</u> [16] if each point of Y has a closed compact neighborhood. A topological space Y is said to be <u>rim-compact</u> if there exists an open basis for the topology on Y such that Fr(V) is compact for each V in $\mathcal{B}$.

Corollary 4.5. If Y is a rim-compact space and $f:X\longrightarrow Y$ is an a.w.c. function with a closed graph G(F), then f is almost continuous.

Proof. Since Y is rim-compact, there exists an open basis $\mathcal{B}$ for the topology on Y such that Fr(V) is compact for each V in $\mathcal{B}$. Since G(f) is closed, it follows from [3, Theorem 3.6] that $f^{-1}(Fr(V))$ is closed in X for each V in $\mathcal{B}$. Therefore, f is locally weak* continuous and hence by Corollary 4.3 f is almost continuous.

Corollary 4.6. If $f:X\longrightarrow Y$ is an a.w.c. function into a strongly locally compact space Y and has a closed graph, then f is continuous.

Proof. Since every strongly locally compact space is rim--compact, this follows immediately from Corollary 4.5 and [16, Theorem 7].

5. Hausdorff spaces and a.w.c. functions

In this section, we shall show that the assumption "almost continuous" in some of results established in [9] and [15] can be replaces by "almost weakly continuous"

Theorem 5.1. If $f:X\longrightarrow Y$ is an a.w.c. function and Y is a Hausdorff space, then the graph $G(f)$ is preclosed in $X\times Y$.

Proof. Let (x,y) be any point of $X\times Y-G(f)$. Then $y\neq f(x)$ and there exist open sets V and W in Y such that $f(x)\in V$, $y\in W$ and $V\cap W=\emptyset$; hence $Cl(V)\cap W\neq\emptyset$. Since f is a.w.c., by Theorem 3.1 there exists $U\in PO(X,x)$ such that $f(U)\subset Cl(V)$. Therefore, we obtain $f(U)\cap W=\emptyset$ and hence $(U\times W)\cap G(f)=\emptyset$. Since $U\times W$ is a preopen set of $X\times Y$ containing (x,y), $(x,y)\overline{\in}Pcl(G(f))$ and hence $Pcl(G(f))=G(f)$. It follows from [2, Lemma 2.3] that $G(f)$ is preclosed in $X\times Y$.

Corollary 5.2. (Mashhour et al. [9]; Popa [15]). If $f:X\longrightarrow Y$ is almost continuous and Y is Hausdorff, then $G(f)$ is preclosed in $X\times Y$.

A function $f:X\longrightarrow Y$ is said to be <u>almost α-continuous</u> [13] (resp. α-continuous [10]) if $f^{-1}(V)$ is α-open in X for every regular open (resp. open) set V of Y. In [13, Remark 2.5], it is shown that almost α-continuity is strictly weaker than α-continuity.

Theorem 5.3. Let $f_1,f_2:X\longrightarrow Y$ be functions into a Hausdorff space Y. If f_1 is almost α-continuous and f_2 is a.w.c., then the set $\{x\in X|f_1(x)=f_2(x)\}$ is preclosed in X.

Proof. Let $A=\{x\in X|f_1(x)=f_2(x)\}$ and suppose that $x\in X-A$. Then $f_1(x)\neq f_2(x)$ and there exist open sets V_1 and V_2 such that $f_1(x)\in V_1$, $f_2(x)\in V_2$ and $V_1\cap V_2=\emptyset$; hence $Int(Cl(V_1))\cap\cap Cl(V_2)=\emptyset$. Since f_1 is almost α-continuous, there exists an α-open set U_1 of X containing x such that $f(U_1)\subset Int(Cl(V_1))$

[13, Theorem 3.2]. Since f_2 is a.w.c., by Theorem 3.1 there exists $U_2 \in PO(X,x)$ such that $f_2(U_2) \subset Cl(V_2)$. We have $F_1(U_1) \cap \cap f(U_2) = \emptyset$ and $U_1 \cap U_2 \in PO(X,x)$ by Lemma 4.1. Therefore, we obtain $(U_1 \cap U_2) \cap A = \emptyset$ and hence $x \in X - Pcl(A)$. This shows that A is preclosed in X.

Corollary 5.4 (Popa [15]). Let X be a topological space and Y a Hausdorff space. Let f_1 and f_2 be functions of X into Y. If f_1 is continuous and f_2 is almost continuous, then the set $\{x \in X | f_1(x) = f_2(x)\}$ is preclosed in X.

Proof. This is an immediate consequence of Theorem 5.3.

A function $f:X \to Y$ is said to be <u>weakly α-continuous</u> [12] if for each $x \in X$ and each open set V containing $f(x)$, there exists an α-open set U containing x such that $f(U) \subset Cl(V)$.

Theorem 5.5. Let $f_1, f_2 : X \to Y$ be functions into a Urysohn space Y. If f_1 is weakly α-continuous and f_2 is a.w.c., then the set $\{x \in X | f_1(x) = f_2(x)\}$ is preclosed in X.

Proof. The proof is quite similar to that of Theorem 5.3.

Theorem 5.6. Let X be a Hausdorff space and A a subset of X. If $f:X \to A$ is an a.w.c. function such that the restriction $f|_A$ is the identity function, then A is preclosed in X.

Proof. Suppose that A is not preclosed. There exists a point $x \in Pcl(A) - A$. Since $x \neq f(x)$, there exist open sets U and V in X such that $x \in U$, $f(x) \in V$ and $U \cap V = \emptyset$; hence $U \cap Cl(V) = \emptyset$. Since f is a.w.c., there exists $G \in PO(X,x)$ such that $f(G) \subset \subset Cl_A(V \cap A) \subset Cl(V)$ where $Cl_A(V \cap A)$ denotes the closure of an open set $V \cap A$ in the subspace A. Since $G \cap U \in PO(X,x)$ and $x \in Pcl(A)$, we have $(G \cap U) \cap A \neq \emptyset$. Let $a \in (G \cap U) \cap A$. We have $f(a) = a \in U$ and hence $f(a) \in X - Cl(V)$. This shows that $f(G) \not\subset Cl(V)$. This is a contradiction.

Corollary 5.7 (Mashhour et al. [9]). Let A be a subset of a Hausdorff space X. If a function $f:X \to A$ is precontinuous and $f|_A$ is the identity function, then A is a preclosed set of X.

Proof. Since every precontinuous function is a.w.c., this follows immediately from Theorem 5.6.

For a function $f:X\longrightarrow Y$, the graph $G(f)=\{(x,f(x)|x\in X\}$ is said to be strongly closed [8] if for each $(x,y)\in X\times Y-G(f)$, there exist open sets U and V containing x and y, respectively, such that $[U\times Cl(V)]\cap G(F)=\emptyset$.

Theorem 5.8. If $f:X\longrightarrow Y$ is a.w.c. injection with a strongly closed graph, then X is Hausdorff.

Proof. Let x_1 and x_2 be any distinct points of X. Since f is injective, $f(x_1)\neq f(x_2)$ and by [8, Lemma 1] there exist open sets U and V such that $x_1\in U$, $f(x_2)\in V$ and $f(U)\cap Cl(V)=\emptyset$. Therefore, we have $U\cap f^{-1}(Cl(V))=\emptyset$ and hence $U\cap Int(Cl(f^{-1}(Cl(V))))=\emptyset$. Since f is a.w.c., $x_2\in f^{-1}(V)\subset Int(Cl(f^{-1}(V)))$ and hence X is Hausdorff.

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