

SEMINAR ON DIFFERENTIAL SPACES

The concept of differentiable manifold remains crucial in modelling many physical phenomena; in particular space-times of all major physical theories are supposed to share properties of a sufficiently smooth manifold. However, in some areas of research there is the necessity to go beyond this assumption. For instance, there are poor reasons for supporting the view that, in the quantum gravity regime of the very early Universe, the differentiable manifold structure will continue to play its role as an arena for physical processes. And even in classical singularities (i.e. without taking into account any quantum gravity effects) the manifold structure is expected to brake down. The high degree of homogeneity, inherent in the manifold concept, seem to be both very limiting and, in some cases, quite arbitrary assumption.

When giving up the locally Euclidean character of physical spaces, their numerical description must be preserved, i.e. the one in terms of real numbers or real valued functions. This is because all measurement results are always given as real numbers.

In the beginning of the sixties the idea was born to systematically investigate possible generalizations of the manifold concept. This suggestion was spelled out by A. Grothendieck, and was first discussed among mathematicians working in the field of algebraic geometry. Some mathematicians took over the idea, and several similar concepts were introduced generalizing the notion of manifold. They became known under the common name of differential spaces.

In 1965 Roman Sikorski at the University of Warsaw had an introductory course on differential geometry entirely based on the concept of differential space. In 1972 a book appeared,

being an extended version of this course notes, bearing the title "Introduction to Differential Geometry" (in Polish). From the mid sixties one can observe the increase of number of works dealing with differential spaces (in the late seventies differential spaces appeared in the AMS classification (58A40)). At that time R. Sikorski and W. Waliszewski conducted a seminar at the Institute of Mathematics of the Polish Academy of Sciences devoted to differential spaces. In the period 1972-1981 this seminar was run in cooperation with the Institute of Mathematics of the Warsaw Technical University. From 1982 on, it is a seminar of the Warsaw Technical University.

In the early eighties the Cracow Group of Cosmology begun its activity. Some participants of this group, having accidentally read Sikorski's book, got an idea to use the differential space concept in order to construct a generalized model of space-time which would possibly contain also classical space-time singularities. First attempts proved to be promising, and the project crystalized to investigate this possibility in a more systematic way. In this situation establishing contacts with the Warsaw seminar seemed desirable and natural. The cooperation started in 1987; common seminars took place in Warsaw and in Cracow interchangeably. Several summer and winter schools were organized in Pasierbiec near Limanowa.

The present issue of the "Demonstratio Mathematica" contains works presented at the Summer School "Differential Spaces and Their Applications" which took place in Pasierbiec, 20-26 September 1990. Besides Polish participants the following invited speakers took part in the Summer School: Prof. Dr. K. Buchner (München). Prof. Dr. P. Dombrowski (Köln), Prof. Dr. A. Frölicher (Genève). and Prof. Dr. W. Gähler (Berlin).

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