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ON SOME APPLICATIONS OF LIMIT ELEMENTS OF MAPPINGS

In the article, the notion of limit elements of mappings is applied to studying closed mappings, those having closed graphs and w-Darboux mappings (cf. [2]-[6]). Moreover, the purpose of the paper is to investigate the notion of points of closedness of mappings which was introduced by R. Pawlak in [6]. To begin with, let us establish some notation and recall the most important definitions.

All mappings to be considered here are defined on Hausdorff spaces and take values in Hausdorff spaces. All spaces are assumed to be Hausdorff. The space of real numbers with the natural topology is denoted by R. As usual, N denotes the set of positive integers. The derived set of A is denoted by A^d. The fact that f is a mapping of a space X to a space Y is written in symbols as $f: X \to Y$ (we do not require the continuity of f). The graph of $f: X \to Y$ is denoted by $G(f)$. All other symbols are standard. Generally, the notation here conforms to that of [1].

D e f i n i t i o n s (cf. [6]). Let $f: X \to Y$.

(1) A point $y \in Y$ is called a limit element of f at a point $x \in X$ if there exists a net $\{x_s;\ s \in S\} \subset X \setminus \{x\}$ such that $x = \lim_{s\in S} x_s$ and $y = \lim_{s\in S} f(x_s)$. Denote by $L(f,x)$ the set of all limit elements of f at x.

(2) A point $y \in L(f,x)$ is called a $(*)$-limit element of f at x if, for any net $\{x_s;\ s \in S\} \subset X$ such that $x = \lim_{s\in S} x_s$ and

$y = \lim_{s\in S} f(x_s)$, there exists $s_0 \in S$ for which $f(x_s) = y$ whenever $s \geqslant s_0$. Denote by $L^*(f,x)$ the set of all (*)-limit elements of f at x.

(3) If $L(f,x) \setminus \{f(x)\} \subset L^*(f,x)$, then we say that x is a point of closedness of f or, equivalently, that f is closed at x.

(4) If $L(f,x) \subset \{f(x)\}$, then we say that the graph of f is closed at x.

Theorem 1. For every mapping $f: X \to Y$, any point $x \in X$ and any base $\mathcal{U}(x)$ of neighbourhoods of x, we have:

(i) $L(f,x) = \bigcap \{\overline{f(U \setminus \{x\})}: U \in \mathcal{U}(x)\}$, so $L(f,x)$ is a closed subset of Y (cf. [3; pp.51-52]).

(ii) $L^*(f,x) = L(f,x) \cap \bigcup \{f(U) \setminus f(U)^d: U \in \mathcal{U}(x)\}$.

(iii) $L^*(f,x) \cap L(f,x)^d = \emptyset$, so $L^*(f,x)$ is a discrete subspace of Y.

(iv) If x is a point of closedness of f, then $L(f,x) \setminus \{f(x)\}$ is a discrete subspace of Y.

Proof. (ii) For a point $y \in Y$, let $\mathcal{V}(y)$ be a base of neighbourhoods of y. Suppose first that $y \in f(U)^d$ whenever $U \in \mathcal{U}(x)$. To each $U \in \mathcal{U}(x)$ and $V \in \mathcal{V}(y)$ we can assign some $x_{(U,V)} \in U$ such that $f(x_{(U,V)}) \in V \setminus \{y\}$. Let us define, for $U_n \in \mathcal{U}(x)$ and $V_n \in \mathcal{V}(y)$ $(n=1,2)$, that $(U_1,V_1) \leqslant (U_2,V_2)$ if $U_1 \supset U_2$ and $V_1 \supset V_2$. The set $S = \mathcal{U}(x) \times \mathcal{V}(y)$ is directed by the relation $\leqslant$. The net $\{x_{(U,V)}; (U,V) \in S\}$ converges to x and the net $\{f(x_{(U,V)}); (U,V) \in S\}$ converges to y; therefore $y \notin L^*(f,x)$.

Conversely, if $y \in L(f,x) \setminus L^*(f,x)$, then there exists a net $\{x_t; t \in T\} \subset X$ such that $f(x_t) \neq y$ for any $t \in T$, $\lim_{t\in T} x_t = x$ and $\lim_{t\in T} f(x_t) = y$, so $y \in f(U)^d$ whenever $U \in \mathcal{U}(x)$. This completes the proof of (ii).

Property (iii) follows from (i) and (ii). Property (iv) is a consequence of (iii).

Let us recall that a mapping $f: X \to Y$ is called closed provided f transforms each closed subset of X onto a closed subset of Y.

R. Pawlak proved in [6; Theorem 1(b)] that if each point of a compact space X is a point of closedness of $f: X \to Y$, then f is closed. We shall show that the converse of the above-mentioned theorem does not hold.

Example 1. Let us consider the one-point compactification ωR of R and the space Y obtained from R by identifying the set N with a point y_0. Denote by x_0 the point of $\omega R \setminus R$ and define

$$f(x) = \begin{cases} x & \text{for } x \in R \setminus N, \\ y_0 & \text{for } x \in N, \\ -1 & \text{for } x = x_0. \end{cases}$$

Of course, the mapping $f: \omega R \to Y$ is closed. If U is an open neighbourhood of x_0 in ωR and V is an open neighbourhood of y_0 in Y, then $(V \setminus \{y_1\}) \cap f(U \setminus \{x_0\}) \neq \emptyset$ because $R \setminus U$ is compact in R; hence, by Theorem 1 (i)-(ii), $y_0 \in L(f,x_0) \setminus [\{f(x_0)\} \cup L^*(f,x_0)]$, so f is not closed at x_0. As $L(f,x_0) = \{y_0\}$, this example also points out that the converse of Theorem 1(iv) does not hold.

It is easily seen that the assumption of the compactness of X is needless in the second part of Proposition 3 of [6]; accordingly, we have

Theorem 2. Let f be a mapping of a space X to a space Y. Then $G(f)$ is a closed subset of $X \times Y$ if and only if $G(f)$ is closed at each point $x \in X$.

Let us present the following local version of a well-known result concerning mappings which take values in compact spaces:

Theorem 3. Suppose that f is a mapping of a space X to a compact space Y. Then $x \in X$ is a point of continuity of f if and only if $G(f)$ is closed at x (cf. [1; Exercise 3.1.D]).

T h e o r e m 4. Let f be a closed mapping of a regular space X to a space Y. For any $x \in X$, the following conditions are equivalent:

(i) $G(f)$ is closed at x;

(ii) $f^{-1}(y)$ is closed whenever $y \in L(f,x) \setminus \{f(x)\}$.

P r o o f . Assume (ii) and suppose that $y \in L(f,x)\setminus\{f(x)\}$. There exist disjoint open subsets U and W of X such that $x \in U$ and $f^{-1}(y) \subset W$. Since f is closed, the set $V = Y\setminus f(X\setminus W)$ is an open neighbourhood of y and, moreover, $V \cap f(U) = \emptyset$. Hence $y \notin \overline{f(U)}$, which is impossible by Theorem 1(i).

The assumption of the regularity of X is essential in Theorem 4.

E x a m p l e 2. Let Y be the space described in Example 1. In the set $X = \omega R$ consider the topology generated by all the sets that either are open in ωR or are of the form $U \setminus N$ where U is open in ωR. Then X is a non-regular Hausdorff space. Let f defined in Example 1 be considered as a mapping of X to Y. Then f is closed and, moreover, all fibers of f are closed in X. However, the graph of f is not closed at x_0.

From our Theorems 3 and 4 we immediately obtain the following local version of Theorem 4.9 of [2]:

T h e o r e m 5. A closed mapping f of a regular space X to a compact space Y is continuous at a point $x \in X$ if and only if $f^{-1}(y)$ is closed whenever $y \in L(f,x) \setminus \{f(x)\}$.

T h e o r e m 6. Let f be a closed mapping of a first-countable space X to a first-countable space Y. For any $x \in X$, the following conditions are equivalent:

(i) $G(f)$ is closed at x;

(ii) $f^{-1}(y)$ is closed whenever $y \in L(f,x) \setminus \{f(x)\}$.

P r o o f . In view of [6; Theorem 1(a)], x is a point of closedness of f, so $x \in \overline{f^{-1}(y)}$ whenever $y \in L(f,x)$.

Theorems 3 and 6 imply

T h e o r e m 7. A closed mapping f of a first-countable space X to a first-countable compact space Y is continuous at a point $x \in X$ if and only if $f^{-1}(y)$ is closed whenever $y \in L(f,x) \setminus \{f(x)\}$.

For a mapping $f: X \to Y$ and a point $y \in Y$, denote

$$T(f,y) = \{x \in X: y \in L(f,x)\}$$

(cf. [3; p.63] and [4; Definition 3.3]).

Let us observe that if $x \in T(f,y)^d$, then, for any neighbourhoods U and V of x and y, resp., we have $f(U \setminus \{x\}) \cap V \neq \emptyset$, so, by Theorem 1(i), $x \in T(f,y)$; therefore the set $T(f,y)$ is closed (cf. [4; Theorem 3.3]).

T h e o r e m 8. Let f be a closed mapping of a locally sequentially compact space X to a Fréchet space Y and let $x \in X$. If $x \notin T(f,y)^d$ for any $y \in L(f,x) \setminus \{f(x)\}$, then x is a point of closedness of f.

P r o o f . Suppose that $y \in L(f,x)\setminus[\{f(x)\} \cup L^*(f,x)]$. There exists a neighbourhood W of x such that $W \cap T(f,y) = \{x\}$. We can find a neighbourhood U of x such that $\bar{U}$ is sequentially compact and $\bar{U} \subset W$. According to Theorem 1(ii), $y \in f(U)^d$, so there exists a sequence $\{x_n;\ n \in N\}$ of elements of U such that $\lim_{n\in N} f(x_n) = y$ and $f(x_n) \neq y$ for any $n \in N$. As $\bar{U}$ is sequentially compact, the sequence $\{x_n;\ n \in N\}$ contains a subsequence $\{x_{n_k};\ k \in N\}$ which converges to some point $z \in \bar{U}$. Then $z = x$ because $y \in L(f,z)$ and $\bar{U} \cap T(f,y) = \{x\}$. The set $A = \{x\} \cup \bigcup_{k\in N} \{x_{n_k}\}$ is closed; however, $y \in \overline{f(A)} \setminus f(A)$. The contradiction obtained concludes the proof.

T h e o r e m 9. Let f be a mapping of a sequentially compact regular space X to a Fréchet space Y. Assume that, for any $x \in X$, we have $x \notin T(f,y)^d$ whenever $y \in L(f,x)\setminus\{f(x)\}$. Then f is closed if and only if each point of X is a point of closedness of f.

P r o o f . If each point of X is a point of closedness of f, then arguing similarly as in the proof of Theorem 1(b) in [6], we deduce that f is closed. Since every sequentially compact regular space is locally sequentially compact, Theorem 8 completes the proof.

Without any difficulties one can check that the space Y defined in Example 1 is a Fréchet space (cf. [1; Example 1.6.18]). Therefore, in view of Example 1, the requirement that $x \in T(f,y)^d$ whenever $y \in L(f,x) \setminus \{f(x)\}$ cannot be omitted in Theorems 8 and 9.

T h e o r e m 10. Let f be a mapping of a sequentially compact space X to a Fréchet space Y. Assume that, for any $x \in X$, the set $T(f,y) \setminus \{x\}$ is compact whenever $y \in L(f,x) \setminus \{f(x)\}$. Then f is closed if and only if each point of X is a point of closedness of f.

P r o o f . Suppose that $y \in L(f,x) \setminus [\{f(x)\} \cup L^*(f,x)]$ for some $x \in X$. By [1; Theorem 3.1.6], there exists a neighbourhood U of x such that $\bar{U} \cap T(f,y) = \{x\}$. Since $\bar{U}$ is sequentially compact, according to the proof of Theorem 8, we obtain that f is not closed.

C o r o l l a r y 1. Let f be a mapping of a sequentially compact space X to a Fréchet space Y. Assume that, for any $x \in X$, the set $T(f,y)$ is countable and $x \notin T(f,y)^d$ whenever $y \in L(f,x) \setminus \{f(x)\}$. Then f is closed if and only if each point of X is a point of closedness of f.

P r o o f . We have observed that $T(f,y)$ is closed for any $y \in Y$. If $T(f,y)$ is countable, then Theorem 3.10.30 of [1], together with the theorem given in [1; Exercise 3.10.A], implies that $T(f,y)$ is compact. To complete the proof, it suffices to use Theorem 10.

D e f i n i t i o n 5. Let $f: X \to Y$. We shall say that $x \in X$ is a d-point of f if there exists a base $\mathcal{U}(x)$ of neighbourhoods of x such that, for any $U \in \mathcal{U}(x)$, either $f(U)$ is dense in itself or $f(U) = \{f(x)\}$. If each point of X is a d-point of f, then f will be called a d-mapping.

T h e o r e m 11. Let $f: X \to Y$ and suppose that $x \in X$ is a d-point of f. If $L^*(f,x) \neq \emptyset$, then there exists a neighbourhood U of x such that $f(U) = \{f(x)\}$, so f is continuous at x.

P r o o f . Take a base $\mathcal{U}(x)$ of neighbourhoods of x such that, for any $W \in \mathcal{U}(x)$, either f(W) is dense in itself or $f(W) = \{f(x)\}$. If $y \in L^*(f,x)$, then, by Theorem 1(ii), we can find $U \in \mathcal{U}(x)$ such that $y \in f(U) \setminus f(U)^d$; hence $f(U) = \{f(x)\}$ because f(U) is not dense in itself.

Let us note a few consequences of Theorem 11.

T h e o r e m 12. Let f be a d-mapping of a connected space X to a space Y. If $L^*(f,x) \neq \emptyset$ for any $x \in X$ then f is constant.

P r o o f . Consider any $x_0 \in X$. By Theorem 11, the set $A = f^{-1}[f(x_0)]$ is clopen in X, so A = X.

T h e o r e m 13. Let $f: X \to Y$ and suppose that $x \in X$ is a d-point of f. Then x is a point of closedness of f if and only if the graph of f is closed at x.

Theorems 2, 8 and 13 imply

T h e o r e m 14. Let f be a closed d-mapping of a locally sequentially compact space X to a Fréchet space Y. Then G(f) is a closed subset of $X \times Y$ if and only if, for any $x \in X$, we have $x \notin T(f,y)^d$ whenever $y \in L(f,x) \setminus \{f(x)\}$.

T h e o r e m 15. Let f be a closed mapping of a locally sequentially compact space X to a compact Fréchet space y. A d-point $x \in X$ of f is a point of continuity of f if and only if $x \notin T(f,y)^d$ whenever $y \in L(f,x) \setminus \{f(x)\}$.

P r o o f . It suffices to apply Theorems 3, 8 and 13.

Theorems 2, 10 and 13 yield

T h e o r e m 16. Let f be a d-mapping of a sequentially compact space X to a Fréchet space Y. Then G(f) is a closed subset of $X \times Y$ if and only if f is closed and, for any $x \in X$, the set $T(f,y) \setminus \{x\}$ is compact whenever $y \in L(f,x) \setminus \{f(x)\}$.

T h e o r e m 17. A d-mapping f of a sequentially compact space X to a compact Fréchet space Y is continuous if and only if f is closed and, for any $x \in X$, the set $T(f,y) \setminus \{x\}$ is compact whenever $y \in L(f,x) \setminus \{f(x)\}$.

P r o o f . The proposition follows from Theorems 3 and 16 (cf. [1; Exercise 3.1.D]).

D e f i n i t i o n 6. Let $f: X \to Y$. We shall say that $x \in X$ is a w-Darboux point of f if there exists a base $\mathcal{U}(x)$ of neighbourhoods of x such that $\overline{f(U)}$ is connected for any $U \in \mathcal{U}(x)$. If each point of X is a w-Darboux point of f, then f will be called a w-Darboux mapping (cf. [5; Definition 1]).

Clearly, every w-Darboux mapping is a d-mapping.

Let us recall that a rimcompact space is a Hausdorff space having a base of open sets with compact boundaries.

Now, we are in a position to extend and generalize both Theorems 2 and 4 of [6] (cf. also [5; Theorem 1]).

T h e o r e m 18. Suppose that f is a mapping of a space X to a rimcompact space Y. If $x \in X$ is a w-Darboux point of f, then the following conditions are equivalent:

(i) x is a point of continuity of f;

(ii) x is a point of closedness of f;

(iii) the graph of f is closed at x;

(iv) $|L(f,x)| \leqslant 1$;

(v) $L(f,x)$ is finite;

(vi) $L(f,x)$ is a discrete subspace of Y;

(vii) $L(f,x)$ is either empty or not dense in itself;

(viii) $f(x)$ is not an accumulation point of $L(f,x)$.

P r o o f . The implications (i) $\Rightarrow$ (ii) and (iii) $\Rightarrow$ (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi) $\Rightarrow$ (vii) are obvious. That (ii) $\Rightarrow$ (iii) follows from Theorem 13. Hence it suffices to show that (vii) $\Rightarrow$ (viii) $\Rightarrow$ (i).

First of all, let us fix any base $\mathcal{U}(x)$ of neighbourhoods of x such that $\overline{f(U)}$ is connected for each $U \in \mathcal{U}(x)$. Suppose that (viii) does not hold. Take an arbitrary $y \in L(f,x) \setminus \{f(x)\}$ and consider any neighbourhood V of y. Since Y is rimcompact, there exists an open neighbourhood W of y such that $f(x) \notin \overline{W} \subset V$ and $K = \mathrm{Fr}\, W$ is compact. For each $U \in \mathcal{U}(x)$, we have $f(U) \setminus W \neq \emptyset$ and $f(U) \cap W \neq \emptyset$; hence, the connectedness of $\overline{f(U)}$ implies that $\overline{f(U)} \cap K \neq \emptyset$. As $f(x) \notin K$, then $\overline{f(U\setminus\{x\})} \cap K \neq \emptyset$ for any $U \in \mathcal{U}(x)$ and, consequently, $\bigcap \{\overline{f(U \setminus \{x\})} : U \in \mathcal{U}(x)\} \cap K \neq \emptyset$

because K is compact. It follows from Theorem 1(i) that $K \cap L(f,x) \neq \emptyset$; therefore y is an accumulation point of $L(f,x)$, so (vii) $\Rightarrow$ (viii).

Assume (viii) and suppose that (i) does not hold. There exists a neighbourhood V of f(x) such that $(V \setminus \{f(x)\}) \cap L(f,x) = \emptyset$ and, moreover, $f(U) \setminus V \neq \emptyset$ for any $U \in \mathcal{U}(x)$. Let us take an open neighbourhood W of f(x) such that $\overline{W} \subset V$ and Fr W is compact. Then $\overline{f(U)} \cap \mathrm{Fr}\, W \neq \emptyset$ for any $U \in \mathcal{U}(x)$. Using the same arguments as in the proof of the implication (vii) $\Rightarrow$ (viii), we obtain that $L(f,x) \cap \mathrm{Fr}\, W \neq \emptyset$. This contradicts the fact that $(V \setminus \{f(x)\}) \cap L(f,x) = \emptyset$. Hence (viii) $\Rightarrow$ (i).

The example of [5; p. 772] points out that the assumption of rimcompactness is needed in the above theorem.

Theorem 18, along with Theorem 2, implies

C o r o l l a r y 2. A w-Darboux mapping of a space X to a rimcompact space Y is continuous if and only if G(f) is a closed subset of $X \times Y$.

The following two corollaries can be regarded as generalizations of Theorem 4 of [3; p.63]:

C o r o l l a r y 3. Let f be a mapping of a space X to a rimcompact space Y. If $x_0 \in X$ is a w-Darboux point of f, the set $A = \{x \in X: x_0 \in T(f,f(x))\}$ is finite and, moreover, $f(x_0)$ is not an accumulation point of $L(f,x_0) \setminus f(X)$, then f is continuous at x_0.

P r o o f . Let us observe that $A = f^{-1}[L(f,x_0)]$, so $L(f,x_0) = f(A) \cup [L(f,x_0) \setminus f(X)]$. Theorem 18 completes the proof.

C o r o l l a r y 4. Suppose that f is a mapping of a space X to a rimcompact space Y. If f(X) is a closed subset of Y, $x_0 \in X$ is a w-Darboux point of f and, moreover, the set $\{x \in X: x_0 \in T(f,f(x))\}$ is finite, then f is continuous at x_0.

P r o o f . In the case where f(X) is closed in Y, we have $L(f,x_0) \subset f(X)$; hence the proposition follows from Corollary 3.

Our next theorem is a generalization of Theorem 2.3(c) of [2].

T h e o r e m 19. Let Y be a Tychonoff space which has a compactification with the remainder of cardinality $< 2^{\aleph_0}$. Let f be a mapping of a space X to Y and suppose that $x \in X$ is a w-Darboux point of f. Then f is continuous at x if and only if $|L(f,x)| < 2^{\aleph_0}$.

P r o o f . Let us take an arbitrary compactification αY of Y such that $|\alpha Y \setminus Y| < 2^{\aleph_0}$. Suppose that $|L(f,x)| < 2^{\aleph_0}$ but f is not continuous at x. Let $f_\alpha = f$ be considered as a mapping of X to αY. Of course, x is a w-Darboux point of f_α. Denote by $\mathcal{U}(x)$ any base of neighbourhood of x such that the closure $\overline{f_\alpha(U)}$ of $f_\alpha(U)$ in αY is connected for each $U \in \mathcal{U}(x)$ (until the end of the proof, the bar denotes the closure in αY). Since f_α is not continuous at x, by virtue of Theorem 18, $f_\alpha(x)$ is an accumulation point of $L(f_\alpha,x)$. It follows from Theorem 1 that $\overline{f_\alpha(U \setminus \{x\})} = \overline{f_\alpha(U)}$ for any $U \in \mathcal{U}(x)$. Therefore, by [1; Theorem 6.1.18 and Example 2.5.4], the set $L(f_\alpha,x) = \bigcap\{\overline{f_\alpha(U)}: U \in \mathcal{U}(x)\}$ is connected. Hence, by [1; Corollary 6.1.3], $|L(f_\alpha,x)| \geq 2^{\aleph_0}$. On the other hand, $L(f_\alpha,x) \subset L(f,x) \cup (\alpha Y \setminus Y)$, so $|L(f_\alpha,x)| < 2^{\aleph_0}$. The contradiction obtained completes the proof.

Finally, let us note without proofs a few corollaries to Theorem 19.

C o r o l l a r y 5. Let f be a mapping of a space X to a locally compact space Y. Suppose that $x \in X$ is a w-Darboux point of f. Then f is continuous at x if and only if $|L(f,x)| < 2^{\aleph_0}$.

C o r o l l a r y 6. Let Y be a Tychonoff space which has a compactification with the remainder of cardinality $< 2^{\aleph_0}$. Let f be a mapping of a space X to Y and suppose that $x_0 \in X$ is a w-Darboux point of f. If $|L(f,x_0) \setminus f(X)| < 2^{\aleph_0}$ and $|\{x \in X: x_0 \in T(f,f(x))\}| < 2^{\aleph_0}$, then f is continuous at x_0.

Corollary 7. Let f be a mapping of a space X to a locally compact space Y and suppose that $x_0 \in X$ is a w-Darboux point of f. If $|L(f,x_0) \setminus f(X)| < 2^{\aleph_0}$ and $|\{x \in X: x_0 \in T(f,f(x))\}| < 2^{\aleph_0}$, then f is continuous at x_0.

Corollary 8. Let Y be a Tychonoff space which has a compactification with the remainder of cardinality $< 2^{\aleph_0}$. Suppose that f is a mapping of a space X to Y such that f(X) is a closed subset of Y. If $x_0 \in X$ is a w-Darboux point of f and $|\{x \in X: x_0 \in T(f,f(x))\}| < 2^{\aleph_0}$, then f is continuous at x_0.

Corollary 9. Let f be a mapping of a space X to a locally compact space Y. If f(X) is a closed subset of Y, $x_0 \in X$ is a w-Darboux point of f and $|\{x \in X: x_0 \in T(f,f(x))\}| < 2^{\aleph_0}$, then f is continuous at x_0.

Corollary 10. Let Y be a Tychonoff space which has a compactification with the remainder of cardinality $< 2^{\aleph_0}$. Then a w-Darboux mapping f of a space X to Y is continuous if and only if G(f) is a closed subset of $X \times Y$.

REFERENCES

[1] R. Engelking: General Topology. PWN, Warszawa 1977.

[2] T.R. Hamlett: Cluster sets in general topology, J. London Math. Soc. (2), 12 (1976) 192-198.

[3] J. Jędrzejewski: Własności funkcji związane z pojęciem spójności, Acta Univ. Lodz. (1984) 1-84.

[4] P.E. Long: Connected mappings, Duke Math. J. 35 (4) (1968) 677-682.

[5] H. Pawlak, R. Pawlak: On continuity and limit points of w-Darboux functions, Demonstratio Math. 16 (1983) 771-775.

[6] R. P a w l a k : On local characterization of closed functions and functions with closed graphs, Demonstratio Math. 19 (1986) 181-188.

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