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CENTRAL PROJECTION OF A SEGMENT PERPENDICULAR TO THE GIVEN PLANE

Introduction

The presented construction does not use the measure point for lines perpendicular to the given plane and can be applied without any changes to the perspective on slope image plane (see: B. Grochowski [1]).

1. Consider a plane π (Fig.1), two parallel lines l, m none of them lying on π, an ideal point (i.e. a point at infinity or a direction) N^{∞} belonging neither to the plane π nor to lines l, m and two equal segments $AB \subset l$, $CD \subset m$.

The parallel projections of the segments AB, CD on the plane π from the direction N^{∞} are parallel and equal segments $A'B'$, $C'D'$. Hence it follows that $A'B'$ can be obtained by parallel translation of $C'D'$ by the vector $\overrightarrow{C'A'}$.

Taking into account that the reciprocal projection of $A'B'$ on the line l from the direction N^{∞} gives AB we have the following:

S t a t e m e n t.

1° If the lines l, m are parallel and a point $A \in l$ and a segment $CD \subset m$ are parallel projected from a direction N^{∞} on a plane π into the point A' and the segment $C'D'$ respectively (compare Fig.1),

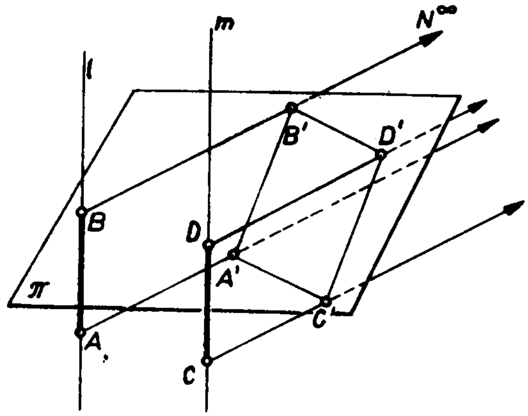

Fig.1

2^0 if the segment $C'D'$ is then parallel translated by the vector $\overrightarrow{C'A'}$ into the segment $A'B'$,

3^0 and if the segment $A'B'$ is parallel projected from the direction N^∞ on the line l into the segment AB,

then AB = CD.

2. Let us now be given a circle c with center O^τ and radius δ(Fig.2) as the distance circle of central projection.

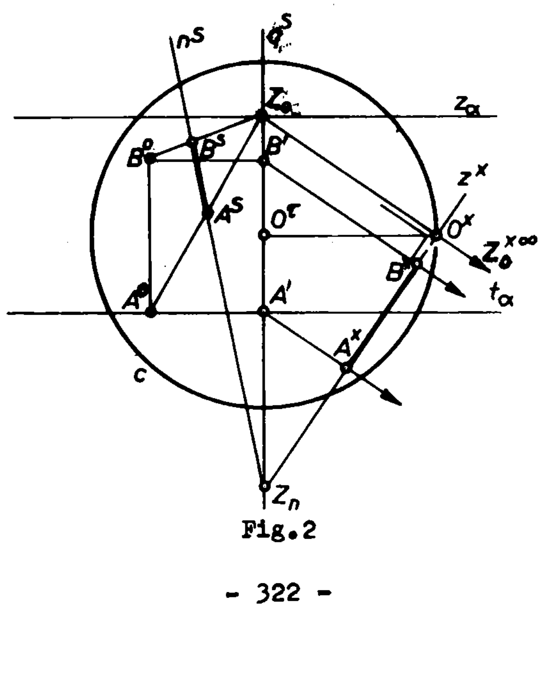

Fig.2

Let a plane α be determined by its parallel traces $t_\alpha \parallel z_\alpha$. Picking arbitrary $A^s \notin z_\alpha$ as the projection of a proper point $A \in \alpha$ we shall construct the projection A^sB^s of the segment $AB \perp \alpha$ having the given distance $|AB|$.

First we draw through O^τ the line q^s perpendicular to z_α intersecting it in the point $Z_o = q^s \cap z_\alpha$. The rabatment O^x of the center of projection O lies on the circle c and satisfies the condition $O^\tau O^x \parallel z_\alpha$. The line z^x through O^x perpendicular to Z_oO^x is the rabatment of the projecting line z perpendicular to the plane α. Hence it follows that the point $Z_n = q^s \cap z^x$ is the convergence point for the lines perpendicular to the plane α.

Such a line n through A has projection n^s passing through the points Z_n and A^s. To construct the projection A^sB^s of the segment $AB \subset n$ of distance $|AB|$ we take into consideration the following three points:

$$A^o = t_\alpha \cap Z_oA^s, \quad A' = t_\alpha \cap q^s \quad \text{and} \quad A^x = z^x \cap A'Z_o^{x\infty}$$

where $Z_o^{x\infty}$ is the direction of the line Z_oO^x.

Then we proceed as follows:

1° We measure off on the line z^x the segment A^xB^x of the given distance $|AB|$.

2° We project A^xB^x from the direction $Z_o^{x\infty}$ of the line Z_oO^x on the line q^s into the segment $A'B'$.

3° We translate parallel $A'B'$ in the direction of the vector $\overrightarrow{A'A^o}$ into the segment $A^oB^o \perp t_\alpha$.

4° We project A^oB^o from the point Z_o on the line n^s obtaining the segment A^sB^s.

The segment A^sB^s is the central projection of the segment AB having the given distance $|AB|$ perpendicular to the plane α.

It is so by virtue of Statement of section 1. Then we have here two parallel lines perpendicular to the plane α: the projecting line z passing through the center of projection O and the line n passing through the given point $A \in A$. The segment A^xB^x of distance $|AB|$ is measured off (in rabatment) on the line z and projected on the image plane τ from the rabatment $Z_0^{x\infty}$ of the direction Z_0^{∞} of the line OZ_0; then after properly translation it is projected (in perspective) on the line n from the perspective Z_0 of the same direction Z_0^{∞}.

REFERENCE

[1] B. Grochowski: Nets for the perspective on the slope projecting plane, Bull. Inst. Math. Kraków Technical University, Janowice 1985.

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