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ORBITS OF  $K^+$ -ACTION ON NORMAL VARIETIES

Let  $K$  be an algebraically closed field of characteristic zero. In this paper  $K^+$  denotes the additive group of  $K$  and  $P^n$  the projective space of dimension  $n$ . Let  $K^+$  acts on a normal variety  $X$ . We will prove the following theorem.

**Theorem.** Let  $X$  be a normal variety with  $K^+$ -action on it. If  $K^+(a)$  is an orbit of a point  $a$ , then  $K^+(a)$  is a smooth curve.

**Proof.** By [1], there exists a  $K^+$ -invariant quasi-projective neighbourhood of  $\overline{K^+(a)}$ , also, by [1], we can embed it equivariantly in some  $P^{n-1}$  with a linear action on it. Hence, it is sufficient to prove the theorem in the case of  $X = P^{n-1}$ .

Let  $K^+$  act algebraically on  $P^{n-1}$  by an algebraic morphism  $\varphi$  i.e.  $\varphi: K^+ \times P^{n-1} \rightarrow P^{n-1}$ . Then there exists a nilpotent matrix  $A$  such that  $\varphi(t, x) = e^{At}x$  for any  $t \in K^+$  and  $x \in P^{n-1}$ , [2]. We may assume that the matrix  $A$  is in its Jordan form and dimension of blocks of  $A$  decreases and the highest dimension of blocks is  $m$  and number of the biggest blocks is  $k$ , [2]. Let  $a = [a_1, \dots, a_n] \in P^{n-1}$ . We will show that  $K^+(a)$  is a smooth curve. We consider two cases:

1. There exists  $i$ ,  $1 \leq i \leq k$ , such that  $a_{m(k-i+1)} \neq 0$  i.e. there exists a non zero coordinate which lies on the last place in one of the biggest blocks.

2. For any  $i$ ,  $1 \leq i \leq k$ , there is  $a_{m(k-i+1)} = 0$ .

In the first case we may assume that  $a_m \neq 0$  (in the opposite case we change the base on  $P^{n-1}$ ). The second case can be

reduced to the first one, since the set  $\{a \in P^{n-1} : a_{m(k-i+1)} = 0 \text{ for any } i, 1 \leq i \leq k\}$  is  $K^+$ -invariant and isomorphic to the projective space  $P^{n-k-1}$ . If the point  $a$  in the space  $P^{n-k-1}$  is not of the form as in the point 1, we continue the same method. At the end we find the projective space  $P^j$  in which the point  $a$  is of the form as in the case 1 and  $P^j$  is embedded in  $P^{n-1}$  as a  $K^+$ -invariant subspace.

So to complete the proof it remains to prove that if  $a_m \neq 0$ , then  $K^+(a)$  is smooth. We shall assume  $a_m = 1$ .

Let  $x \in K^+(a)$ , then there exists such  $t \in K^+$  that  $x_1 = f(t)$ ,  $x_2 = f'(t), \dots, x_m = f^{(m-1)}(t)$ , where  $f(t) = t^{m-1} + b_{m-2}t^{m-2} + \dots + b_1t + b_0$  and  $b_i = \frac{a_{i+1}(m-1)!}{i!}$  for  $i = 0, 1, \dots, m-1$ .

By the smoothness of the orbit  $K^+(a)$ , it is sufficient to prove that the point  $\lim_{t \rightarrow \infty} ta$  (which lies in the closure of the orbit  $K^+(a)$ ) is nonsingular in  $\overline{K^+(a)}$ .

Since  $a_m \neq 0$ , the point  $\lim_{t \rightarrow \infty} ta$  has the first coordinate not equal to zero.

We will consider the equations of  $\overline{K^+(a)}$  in some neighbourhood of the point  $\lim_{t \rightarrow \infty} ta$ . Let  $U = \{x \in P^{n-1} : x_1 \neq 0\}$ . Then  $\lim_{t \rightarrow \infty} ta \in U \cap \overline{K^+(a)}$ . The set  $U \cap K^+(a)$  contains almost all points of  $\overline{K^+(a)}$ ; it does not contain only these points for which  $f(t) = 0$ . So  $U \cap \overline{K^+(a)}$  is a neighbourhood of the point  $\lim_{t \rightarrow \infty} ta$  in  $\overline{K^+(a)}$ .

Let  $x = [x_1, \dots, x_n] \in U \cap \overline{K^+(a)}$ , then  $x_1 = 1$ ,  $x_2 = \frac{f'}{f}$ ,  $\dots, x_m = \frac{f^{(m-1)}}{f}$ . Putting  $u = \frac{1}{t}$ , we get  $\lim_{t \rightarrow \infty} ta = \lim_{u \rightarrow 0} \frac{a}{u}$ .

We will show that there exist constants  $c_1, \dots, c_m$  such that:  $f(t) = c_1t f'(t) + c_2t f''(t) + \dots + c_{m-1}t f^{(m-1)}(t) + c_m f^{(m-1)}(t)$ . Coefficients at the same power of  $t$  of the left and right-hand side of this equation are equal, hence:

$$1 = c_1(m-1),$$

$$b_{m-2} = b_{m-2}(m-2)c_1 + c_2(m-2)(m-1),$$

$$\vdots$$

$$b_1 = b_1c_1 + 2b_2c_2 + 2 \cdot 3b_3c_3 + \dots + (m-1)!c_{m-1},$$

$$b_0 = c_m(m-1)!.$$

We get  $m$  equations in  $m$  variables. The determinant of this system of equations is not equal to zero, so there exists a unique solution of it. Hence, we get  $1 = c_1t x_2 + \dots + c_{m-1}t x_m + c_m x_m$  implying

$$t = \frac{1 - c_m x_m}{c_1 x_2 + \dots + c_{m-1} x_m} \text{ and } u = \frac{c_1 x_2 + \dots + c_{m-1} x_m}{1 - c_m x_m}.$$

Let  $U_1 = \{[x_1, \dots, x_m] \in U : c_m x_m \neq 1\}$ ,  $U_1$  is an open neighbourhood of  $\lim_{t \rightarrow \infty} ta$ . This gives the injection  $u : U_1 \cap \overline{K^+(a)} \rightarrow K$ , where  $K$  denotes affine space of dimension 1. It is easy to check that the inverse is also regular. Hence, it is an isomorphism of  $U_1 \cap \overline{K^+(a)}$  with an open subset in  $K$ , so  $\overline{K^+(a)}$  is a smooth curve.

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