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NONLINEAR COMPOUND PROBLEM FOR ELLIPTIC SYSTEM IN THE PLANE

1. Introduction

Let Γ_0 be a boundary of the Liapunov region D given on the plane $\mathbb{C}$ of the complex variable and let Γ be a contour inside D. Let us denote by D^+ the two connected domain bounded by Γ_0 and Γ, by D^- the inner region bounded by Γ and by $\bar{D}$ the region $D^- \cup D^+ \cup \Gamma \cup \Gamma_0$. For positive direction on Γ and Γ_0 we admit that which leaves D^+ on the left. The origin of the coordinate system belongs to D^-.

Consider the following problem: Determine a function w satisfying in $D^+ \cup D^-$ the generalized Cauchy-Riemann system in the complex form

(1) $$\partial_{\bar{z}} w = F(z,w)^{*)},$$

whose limiting values satisfy the following boundary conditions

(2) $$w^+ = G\, w^- + g(z, w^+, w^-) \quad \text{on } \Gamma,$$

(3) $$\mathrm{Re}(e^{i\tau} w) = \psi(z,w) \quad \text{on } \Gamma_0,$$

*) $\partial_{\bar{z}} = \partial_x + i\partial_y$, $w(z)$ and $F(z,w)$ stand for $w(z,\bar{z})$ and $F(z,\bar{z},w,\bar{w})$ respectively.

and the side conditions

$$(4) \qquad \int_{\Gamma_0} \operatorname{Im}(e^{i\tau} w)\, \sigma\, d\xi = k(w)$$

$$(5) \qquad w(z_j) = a_j(w), \quad j=1,2,\dots,|\varkappa_1+\varkappa_2|, \; z_j \in D^+ \cup D^- .$$

The functions F, G, g, ψ, σ, τ and the functionals k, a_j are given, and

$$(6) \qquad \varkappa_1 = \operatorname{ind} G,$$

$$(7) \qquad \varkappa_2 = \operatorname{ind} e^{i\tau},$$

$\varkappa_1 + \varkappa_2$ being the index of the compound problem (1)-(5). The problem (1), (2), (3) for analytic functions was solved in the linear case by I.S.Rogozhina [9], and Lu Chien Ke [3] and in the case of the generalized Cauchy-Riemann equations in the papers [13], [15]. More general elliptic complex equations were treated by A. Mamourian [6], [7] and the nonlinear equations of the type $\partial_{\bar{z}} w = F(z,w,\partial_z w)$ by Fang Ainoing [4]. A special case of compound problems for holomorphic functions in the unit disc was considered in [12]. The nonlinear equations (1) with nonlinear boundary conditions of type (2) or (3) were solved by W.Wendland [11], A.Begehr and G.Hsiao [2], W.Begehr and G.Hile [1] and others by a combination of an imbedding with Newton's method, whose origin is due to Wacker [10]. In the paper [14] this method is extended to the problem (1)-(5) in some simpler case of $G \equiv 1$.

2. A priori estimate and assumptions

Throughout the paper we admit that the contours Γ and Γ_0 and the index of the problem satisfy the following conditions:

I. $\qquad \Gamma, \Gamma_0 \in C^{1+\alpha}(\mathbb{C}), \quad 0 < \alpha \leq 1,$

II. The index of the problem i.e. $\varkappa = \varkappa_1 + \varkappa_2$ is negative.

In this paper we shall use the same approach as in [14], and base on the following lemma.

L e m m a . If:

1^0 a,b satisfy the inequalities $\|a\|_0$, $\|b\|_0 \leq K$ in D,

2^0 $G,\ g \in C^{\alpha}(\Gamma)$, $\psi \in C^{\alpha}(\Gamma_0)$, $0 < \alpha \leq 1$,

3^0 $\sigma \in C(\Gamma_0)$ and $\sigma > 0$,

4^0 $e^{i\tau} \in C^{1+\alpha}(\Gamma_0)$ and τ is realvalued,

5^0 $k \in \mathbf{R}$, $a_j \in \mathbf{C}$, $j=1,2,\dots,|\varkappa_1+\varkappa_2|$,

conditions I, II being fulfilled, then there exist constants γ, $\tilde{\gamma}_1$, $\tilde{\gamma}_2$, $\tilde{\gamma}_3$, $\tilde{\gamma}_4$, depending on G, K, Γ, Γ_0, τ, σ such that the solution of the problem

(8) $$\partial_{\bar{z}} w = 0, \quad \text{in } D^+ \cup D^-,$$

(9) $$w^+ = G\, w^- + g, \quad \text{on } \Gamma,$$

(10) $$\mathrm{Re}(e^{i\tau} w) = \psi, \quad \text{on } \Gamma_0,$$

(11) $$\int_{\Gamma_0} \mathrm{Im}(e^{i\tau} w)\, \sigma\, d\xi = k,$$

(12) $$w(z_j) = a_j, \quad z_j \in D^+ \cup D^-, \quad j=1,2,\dots,|\varkappa_1+\varkappa_2|$$

satisfies in $D^+ \cup \Gamma$ and $D^- \cup \Gamma$ respectively the inequality

(13) $$\|w\|_\alpha \leq \tilde{\gamma}_1 \|\psi\|_\alpha + \tilde{\gamma}_2 |k| + \tilde{\gamma}_3 \sum_{j=1}^{\varkappa_1+\varkappa_2} |a_j| + \tilde{\gamma}_3 \sum_{j=1}^{\varkappa_1+\varkappa_2} |V(z_j)| + \tilde{\gamma}_4 \|g\|_\alpha + \gamma \|\partial_{\bar{z}} w - aw - b\bar{w}\|_0,$$

where $\|w\|_\alpha = \max\{\|w^+\|_\alpha, \|w^-\|_\alpha\}$ in $C^\alpha(D^+ \cup \Gamma) \cap C^\alpha(D^- \cup \Gamma)$ and V is denoted by (21)*).

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*) We have denoted $\|f\|_0 = \sup_D |f|$, $\|f\|_\alpha = \|f\|_0 + \sup_{\substack{z \neq \tilde{z} \\ z, \tilde{z} \in D}} \dfrac{|f(\tilde{z}) - f(z)|}{|\tilde{z} - z|^\alpha}$.

P r o o f . Let $w = V + XU$, where the function V continuous to Γ_0 is the solution of the problem

$$(14) \qquad \partial_{\bar z} V = 0 \quad \text{in} \quad D^+ \cup D^-,$$

$$(15) \qquad V^+ = G\, V^- + g \quad \text{on } \Gamma,$$

and the function $U \in C^1(\bar D)$ fulfils the conditions

$$(16) \qquad \mathrm{Re}(e^{i\tau} X\, U) = \psi - \mathrm{Re}(e^{i\tau} V) \quad \text{on } \Gamma_0,$$

$$(17) \qquad \int_{\Gamma_0} \mathrm{Im}(e^{i\tau} X\, U)\, \delta\, d\xi = k - \int_{\Gamma_0} \mathrm{Im}(e^{i\tau} V)\, \delta\, d\xi,$$

$$(18) \qquad X\, U(z_j) = a_j - V(z_j), \quad z_j \in D^+ \cup D^-, \quad j = 1, 2, \dots, |\varkappa_1 + \varkappa_2|.$$

The function X means the canonical solution of the homogeneous problem (14), (15), is of the form (cf. [5], [8]):

$$(19) \qquad X(z) = \begin{cases} e^{\Gamma(z)} & \text{in } D^+ \\ z^{-\varkappa_1} e^{\Gamma(z)} & \text{in } D^-, \end{cases}$$

and

$$(20) \qquad \Gamma(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\ln\left[\tau^{\varkappa_1} G(\tau)\right] d\tau}{\tau - z} .$$

Thus the solution of the problem (14), (15) is

$$(21) \qquad V(z) = \frac{X(z)}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^+(\tau)} \frac{d\tau}{\tau - z} + \Phi,$$

Φ being an arbitrary analytic function in $\bar D$. Without loss of generality we admit $\Phi \equiv 0$ [cf. [3], [13]].
The function V satisfies the inequality

$$(22) \qquad \|V\|_\alpha \leqslant M_g \|g\|_\alpha,$$

where the positive constant M_g depends on Γ and X.

Observe that the function X on the contour Γ_0 is of the form

$$X(z) = |X(z)| e^{it(z)},$$

where

$$t(z) = \arg e^{\Gamma(z)} + \arg z^{-\varkappa_1}.$$

The function t is real-valued and

$$\operatorname{ind}\left[e^{i\tau}X\right] = \operatorname{ind}\left[|X| e^{i(\tau+t)}\right] = \varkappa_1 + \varkappa_2.$$

Conditions (16), (17) may be transformed as follows

$$(16') \qquad \operatorname{Re}\left[e^{i(\tau+t)} U\right] = \frac{\psi}{|X|} - \frac{1}{|X|} \operatorname{Re}\left[e^{i\tau}V\right],$$

$$(17') \qquad \frac{1}{|X|} \int_{\Gamma_0} \operatorname{Im}\left[e^{i(\tau+t)}\right] U \ \sigma |X| d\xi = \frac{k}{|X|} - \frac{1}{|X|} \int_{\Gamma_0} \operatorname{Im}(e^{i\tau}V)\ \sigma\, d\xi,$$

the condition (18 remaining unchanged.

The problem (16′), (17) with condition (18) is equivalent to the problem discussed by H. Begehr and G.Hsiao in [1] and in view of their results we have for the function U

$$(23) \quad \|U\|_\alpha \leq \frac{\gamma_1}{m_X} \|\psi - \operatorname{Re}(e^{i\tau}V)\|_\alpha + \frac{\gamma_2}{m_X}\left|k - \int_{\Gamma_0} \operatorname{Im}(e^{i\tau}V)\ \sigma\, d\xi\right| +$$
$$+ \frac{\gamma_3}{m_X} \sum_{j=1}^{|\varkappa_1+\varkappa_2|} |a_j - V(z_j)| + \gamma \|\partial_{\bar z}U - aU - b\bar U\|_0,$$

where $m_X < |X| < M_X$.

Observe that

$$(24) \qquad \|\operatorname{Re}(e^{i\tau}V)\|_\alpha = \|\operatorname{Re}(e^{i\tau}V)\|_0 +$$
$$+ \sup_{\substack{z\neq\tilde z\\ z,\tilde z\in D^+\cup D^-}} \frac{|\operatorname{Re}\ e^{i\tau(z)}V(z) - e^{i\tau(\tilde z)}V(\tilde z)|}{|z - \tilde z|} \leq$$
$$\leq (1+M_\tau)\|V\|_0 + \sup_{\substack{z\neq\tilde z\\ z,\tilde z\in D^+\cup D^-}} \frac{|V(z) - V(\tilde z)|}{|z - \tilde z|} \leq (1+M_\tau)\|V\|_\alpha$$

the constant M_τ being of the form

$$(25)\qquad M_\tau = \sup_{\substack{z\neq\tilde z\\ z,\tilde z\in D^+\cup D^-}} \frac{\left|e^{i\tau(z)} - e^{i\tau(\tilde z)}\right|}{|z-\tilde z|^\alpha}$$

and depends on the domain $D^+\cup D^-$ and the function τ.

In the same way we get

$$(26)\qquad \left|\int_{\Gamma_0} \mathrm{Im}(e^{i}\,V)\,\sigma\,d\xi\right| \leq \hat M_\tau \|V\|_\alpha ,$$

where the constant $\hat M_\tau$ depends on Γ_0, τ and σ.

Denoting

$$(27)\qquad \tilde\gamma = \frac{\gamma_1}{m_X}(1+M_\tau) + \frac{\gamma_2}{m_X}\hat M_\tau$$

we obtain the inequality

$$(28)\qquad \|U\|_\alpha \leq \frac{\gamma_1}{m_X}\|\psi\|_\alpha + \frac{\gamma_2}{m_X}|k| + \tilde\gamma M_g\|g\|_\alpha + \frac{\gamma_3}{m_X}\sum_{j=1}^{\varkappa_1+\varkappa_2}|a_j| + $$
$$+ \frac{\gamma_3}{m_X}\sum_{j=1}^{\varkappa_1+\varkappa_2}|V(z_j)| + \gamma\|\partial_{\bar z}U - aU - b\bar U\|_0 .$$

Let us evaluate the solution w of the problem (8)-(12).

$$(29)\qquad \|w\|_\alpha = \|V + XU\|_\alpha \leq \|V\|_\alpha + \|XU\|_\alpha \leq$$
$$\leq {}_g\|g\|_\alpha + \frac{\gamma_1}{m_X}M_X\|\psi\|_\alpha + \frac{\gamma_2}{m_X}M_X|k| + \tilde\gamma M_g M_X\|g\|_\alpha +$$
$$+ \frac{\gamma_3}{m_X}M_X\sum_{j=1}^{|\varkappa_1+\varkappa_2|}|a_j| + \frac{\gamma_3}{m_X}M_X\sum_{j=1}^{|\varkappa_1+\varkappa_2|}|V(z_j)| +$$
$$+ \gamma\|\partial_{\bar z}XU - aXU - b\overline{XU}\|_0$$

$$(1 + \tilde{\gamma} M_g M_X)\|g\|_\alpha + \tilde{\gamma}_1 \|\psi\|_\alpha + \tilde{\gamma}_2 |k| + \tilde{\gamma}_3 \sum_{j=1}^{|\varkappa_1+\varkappa_2|} |a_j| +$$

$$+ \tilde{\gamma}_3 \sum_{j=1}^{|\varkappa_1+\varkappa_2|} |V(z_j)| + \gamma \|\partial_{\bar z} w - aw - b\bar w + aV + b\bar V\|_0 .$$

Thus we have

$$(30) \quad \|w\|_\alpha \leqslant \tilde{\gamma}_1 \|\psi\|_\alpha + \tilde{\gamma}_2 |k| + \tilde{\gamma}_3 \sum^{|\varkappa_1+\varkappa_2|} |a_j| + \gamma_3 \sum_{j=1}^{|\varkappa_1+\varkappa_2|} |V(z_j)| +$$

$$+ \tilde{\gamma}_4 \|g\|_\alpha + \gamma \|\partial_{\bar z} w - aw - b\bar w\|_0 ,$$

where we have denoted

$$(31) \quad \tilde{\gamma}_1 = \frac{\gamma_1}{m_X} M_X,\ \tilde{\gamma}_2 = \frac{\gamma_2}{m_X} M_X,\ \tilde{\gamma}_3 = \frac{\gamma_3}{m_X} M_X,\ \tilde{\gamma}_4 = (1+\tilde{\gamma} M_X + 2K) M_g .$$

To solve the problem (1)-(5) we admit the assumptions I, II, and the following ones.

(i) $F \in C^\alpha(D)$ for each fixed $w \in \mathbf{C}$. F is continuously differentiable up to the second order with respect to $\bar w$ and w and these derivatives are bounded in $\bar D \in \mathbf{C}$ by a constant K,

(ii) ψ is real-valued and for each $w \in C^{1+\alpha}$ the function $\psi(z,w)$ is in $C^{1+\alpha}$, besides $\|\psi(z,\tilde w) - \psi(z,w)\|_\alpha \leqslant M_1 \|w - \tilde w\|_\alpha$, for all $w, \tilde w \in \mathbf{C}$,

(iii) k is real-valued functional on $C^{1+\alpha}$ and $|k(w) - k(\tilde w)| \leqslant$ $\leqslant M_2 \|w - \tilde w\|_\alpha$ for all $w, \tilde w \in \mathbf{C}$,

(iv) the a_j's are complex valued functionals on $C^{1+\alpha}(D^+ \cup D^-)$ such that

$$\sum_{j=1}^{|\varkappa_1+\varkappa_2|} |a_j(w) - a_j(\tilde w)| \leqslant M_3 \|w - \tilde w\|_\alpha \quad \text{for all } w, \tilde w \in \mathbf{C},$$

(v) for each w_1, w_2 in $C^\alpha(\Gamma)$ the function $g(z,w_1,w_2)$ is in $C^\alpha(\Gamma)$ as a function of z, moreover there exists a non-negative constant M_4 such that for all $w_1, w_2, \tilde w_1, \tilde w_2$ in $\mathbf{C}$ we have

$$\|g(z,w_1,w_2) - g(z,\tilde w_1,\tilde w_2)\|_\alpha \leqslant M_4 \left[\|w_1 - \tilde w_1\|_\alpha + \|w_2 - \tilde w_2\|_\alpha\right]$$

(vi) $\sigma \in C(\Gamma_0)$, $\sigma > 0$; $e^{i\tau} \in C^{1+\alpha}(\Gamma_0)$, τ is realvalued.

(vii) The constants M_1, M_2, M_3, M_4 are independent of w and $\tilde{w}$ and have to be so small, that

$$\sum_{j=1}^{4} \tilde{\gamma}_j M_j < 1,$$

where $\tilde{\gamma}_j$ are the constants appearing in the inequality (13).

3. The imbedding method

Consider for each $t \in \langle 0,1 \rangle$ the following problems:

(32) $$\partial_{\bar{z}} w = tF(z,w) \quad \text{in } D^+ \cup D^-$$

(33) $$Gw^- + tg(z,w^+,w^-) \quad \text{on } \Gamma,$$

(34) $$\mathrm{Re}(e^{i\tau} w) = t\,\psi(z,w) \quad \text{on } \Gamma_0,$$

(35) $$\int_{\Gamma_0} \mathrm{Im}(e^{i\tau} w)\,\sigma\, d\xi = tk(w)$$

(36) $$w(z_j) = ta_j(w), \quad j=1,2,\ldots,|\varkappa_1+\varkappa_2|, \quad z_j \in D^+ \cup D^-$$

in which (1)-(5) is imbedded for t=1.

For t=0 the problem (32)-(36) is a linear one and corresponds to the homogeneous problem (1), (2), (3), whose solution is obtained in [3], [9] and is unique in view of conditions (4), (5).

Let us assume $w(z,t_0)$ to be a solution of the problem (31)-(36) for a given t_0, $0 \leqslant t_0 < 1$ and let $w_{n+1}(z,t)$ be a solution of the following problem ($w_0(z,t) := w(z,t_0)$):

(37) $$\partial_{\bar{z}} w_{n+1} = tF_w(z,w_n)(w_{n+1} - w_n) + tF_{\bar{w}}(z,w_n)(\bar{w}_{n+1} - \bar{w}_n) + tF(z,w_n) \quad \text{in } D^+ \cup D^-,$$

(38) $$w^+_{n+1} = Gw^-_{n+1} + tg(z,w^+_n,w^-_n) \quad \text{on } \Gamma,$$

(39) $$\mathrm{Re}(e^{i\tau} w_{n+1}) = t\psi(z,w_n) \quad \text{on } \Gamma_0,$$

$$(40)\qquad \int_{\Gamma_0} \operatorname{Im}(e^{i\tau} w_{n+1})\, \sigma\, d\xi = t k(w_n),$$

$$(41)\qquad w_{n+1}(z_j, t) = t a_j(w_n), \quad j=1,2,\dots,|\varkappa_1+\varkappa_2|, \quad z_j \in D^+ \cup D^-.$$

For each $w_n \in C^{1+\alpha}(D^+ \cup D^-)$ this linear problem is uniquely solvable. Let us investigate the difference $w_{n+1} - w_n$:

$$(42)\qquad \partial_{\bar z}(w_{n+1} - w_n) = t\Big\{F_w(w_{n+1} - w_n) + F_{\bar w}(\bar w_{n+1} - \bar w_n) +$$

$$+ \frac{1}{2} F_{(w,w)}(w_n - w_{n-1})^2 + F_{(w,\bar w)}|w_n - w_{n-1}|^2 +$$

$$+ \frac{1}{2} F_{(\bar w,\bar w)}(\bar w_n - \bar w_{n-1})^2\Big\},$$

$$(43)\qquad w^+_{n+1} - w^+_n = G(w^-_{n+1} - w^-_n) + tg(z, w^+_n, w^-_n) - tg(z, w^+_{n-1}, w^-_{n-1}),$$

$$(44)\qquad \operatorname{Re}\left[e^{i\tau}(w_{n+1} - w_n)\right] = t\left[\psi(z, w_n) - \psi(z, w_{n-1})\right],$$

$$(45)\qquad \int_{\Gamma_0} \operatorname{Im}\left(e^{i\tau}(w_{n+1} - w_n)\right) \sigma\, d\xi = tk(w_n) - tk(w_{n-1}),$$

$$(46)\qquad w_{n+1}(z_j, t) - w_n(z_j, t) = t\left[a_j(w_n) - a_j(w_{n-1})\right].$$

The expression $F_{(w,w)}$ in (42) is defined by the mean value formula (cf. [11]),

$$(47)\qquad F_{(w,w)} = \int_0^1 F_{(w,w)}(z, \tau w_n + (1-\tau) w_{n-1}, \tau \bar w_n + (1-\tau)\bar w_{n-1})\, \tau\, d\tau$$

and the expressions $F_{(w,\bar w)}$, $F_{(\bar w,\bar w)}$ are defined similarly.

Basing on the assumptions I, II and (i)-(vii) and the a priori estimate (13) we get

$$(48)\qquad \|w_{n+1} - w_n\|_\alpha \leq t\left[\tilde\gamma_1 M_1 + \tilde\gamma_2 M_2 + \tilde\gamma_3 M_3 + \tilde\gamma_4 M_4 + 2K\gamma\|w_n - w_{n-1}\|_\alpha\right]$$

$$\cdot \|w_n - w_{n-1}\|_\alpha.$$

The convergence of $\{w_{n+1} - w_n\}$ may be proved if the initial approximation satisfies

$$(49) \qquad t\left[\sum_{j=1}^{4} \tilde{\gamma}_j M_j + 2K\gamma\|w_1 - w_o\|_\alpha\right] < 1.$$

Thus the constants appearing in the suppositions i - vii must be sufficiently small.

For the difference $(w_1 - w_o)$ we obtain

$$(50) \qquad \|w_1 - w_o\|_\alpha \leq (t - t_o)\left[\beta\|w_o\|_\alpha + \beta_o\right],$$

where we have denoted

$$(51) \qquad \beta = \sum_{j=1}^{4} \tilde{\gamma}_j M_j$$

$$\beta_o = \gamma K + \tilde{\gamma}_1\|\psi(z,w^x)\|_\alpha + \tilde{\gamma}_4\|g(z,w^x)\|_\alpha + \tilde{\gamma}_3 \sum_{j=1}^{|\varkappa_1+\varkappa_2|} |a_j(w^x)| + \tilde{\gamma}_2|k(w^x)|$$

and w^x is the solution of (32)-(36) for t=0.

In a similar way we get

$$(52) \qquad \|w_o\|_\alpha \leq \frac{t_o\beta_o}{1 - t_o\beta}.$$

Substituting (52) in the inequality (50) we obtain

$$(53) \qquad \|w_1 - w_o\|_\alpha \leq \frac{t - t_o}{1 - \beta}\beta_o.$$

The convergence of $\{w_n\}$ to w with respect to the α-norm can now be established from the inequality (48) in the same way as in [1], [2], [14]. Similarly one can prove that the function w satisfies the equation (1) and the conditions (2), (3). Proof of the uniqueness of the solution of the problem (1)-(5) is based on the transformation of the compound problem to the Hilbert one (cf. [3], [9], [13]) and on the fact that the function Φ in the representation (21) has no influence on the final result.

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