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A GRAPH WITH MEAN DISTANCE BEING A GIVEN RATIONAL

A distance $d(x,y)$ of two vertices x and y of a connected graph G is the length of a shortest path joining x and y. A mean distance $\mu(G)$ of a connected graph G is defined as $\sum_{x,y\in V(G)} d(x,y)/n(n-1)$, where n is the number of vertices of G.

The following problem was stated by Plesnik in [1] (Remark 12, p.20): Given a rational $t \geq 1$, does there exist a graph G with $\mu(G) = t$? Our note gives a positive answer to this question.

Let $G_{k,p}$ $(k \geq p)$ be a graph obtained from a cycle C_{2k+1} by adding edges between all pairs of vertices whose distance in C_{2k+1} is less than or equal to p. In particular, $G_{k,1} = C_{2k+1}$ and $G_{k,k} = K_{2k+1}$.

One can easily check that

(1) $\mu(G_{k,p}) = (p+2p+\ldots+\lfloor k/p\rfloor p + (k-\lfloor k/p\rfloor p)\lceil k/p\rceil)/k.$

Suppose that $t = a/b$ for relatively prime positive integers a and b, $a \geq b$, and let c be an integer such that

(2) $$2/(c+2) < b/a \leq 2/(c+1).$$

Since $a \geq b$, we have $c \geq 1$. Define

(3) $$p = 2((c+1)b-a)$$

and

(4) $$d = c(2a-(c+1)b).$$

It follows from (2) that $0 \leqslant d < p$. Finally put

(5) $$k = cp + d.$$

(1), (3), (4) and (5) imply that

$$\mu(G_{k,p}) = a/b = t.$$

REFERENCES

[1] J. Plesník: On the sum of all distances in a graph or digraph, J. Graph Theory 8 (1984), 1-21.

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