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CYCLICAL LIFE TESTS WITH GAMMA DISTRIBUTION AS THE TEST MODEL

1. Introduction

Exponential, gamma and Weibull distributions are generally considered to be suitable models in life tests. Extensive works are available under these models such as Evans and Nigm [4], Klein and Basu [5], Lawless [6], Lingappaiah [7, 8, 9, 10, 11, 12]. Using these models, many forms of life tests have been discussed such as accelerated life tests, DeGroot and Goel [2], Moshe Shaked [17], Singpurwala and Al-Khayyal [21], Nelson and Meeker [18]. Another type is, life tests in batches of items, some batches missing in between, Lingappaiah [13], single and double censored and truncated life tests, Lingappaiah [9], Nelson and Meeker [18]. Also, sequential life tests are of another kind, discussed in Chen and Wardrop [1], Lingappaiah [14]. Generalised life tests based on Weibull model is considered by Lingappaiah [15]. Nonparametric approach in life tests is taken up in Padgett and Wei [19]. Lingappaiah [16] considers the analysis of shift in shape parameter of a model in life tests.

In this paper, another type of life test is treated. That is, a cyclical life test with k cycles each with p-phases. Average life is taken as the same for all cycles in each phase. While different phases have different average lives. Life test is here based on gamma model. First of all, a very general form of life test is dealt with, where the test time period for each cycle and each phase is different. Next as a special case, time periods in each phase for all cycles is taken as same. That is, now, there are only p time periods compared to kp time periods before. As a further special case, exponential model is considered when the analysis becomes simpler. Using the Bayesian approach, estimates of the average lives are obtained. Works of Srivastava [22] and Patel and Gajjar [20] are shown to be particular cases of our analysis here. The computation of these estimates by the Bayesian method seems to be straightforward and more exact as compared to ML approach used in [22] and [20] which include iterative process. Estimation procedure here involves only successive sums along with certain coefficients whose values are extensively available with the author. Finally, certain recurrence relations in the means of different cycles and phases are given. The main idea of this paper is to set up a most general form of cyclical life test under gamma model with many phases and cycles and then use Bayesian approach for the estimation purpose and also to show that many special cases are possible, where the computation becomes much easier.

2. Distribution

Consider a cyclical life test with k cycles and p-phases in each cycle. Let $\theta_1,\ldots,\theta_p$ represent the failure rates in phases $1,2,\ldots,p$, respectively, for all cycles and T_{ij} represent the test time period in i-th phase and j-th cycle. Then the test periods can be put in the form as follows:

(0)

	Phases						
Cycles	1,	2,	...,	h,	...,	p
cycle 1	T_{11}	T_{21}	...	T_{h1}	...	T_{p1}
cycle 2	T_{12}	T_{22}	...	T_{h2}	...	T_{p2}
⋮						
cycle j	T_{1j}	T_{2j}	...	T_{hj}	...	T_{pj}
⋮						
cycle k	T_{1k}	T_{2k}	...	T_{hk}	...	T_{pk}

Now the probability density function of the variable t, the life of an item in h-th phase and j-th cycle can be written as

$$(1)\quad f_h^{(j)}(t) = \left[\theta_h\, e^{-\theta_h(t-T_{(h-1)j})} \left[\theta_h(t-T_{(h-1)j})\right]^{\alpha-1} (\Gamma(\alpha))^{-1}\right] \times$$

$$\times \left[\phi(\underset{\sim}{\theta})\psi(\theta_1,\dots,\theta_{h-1})\right], \quad T_{(h-1)j} < t < T_{hj},$$

where $\underset{\sim}{\theta} = (\theta_1,\dots,\theta_p)$ and

$$(1a)\quad \begin{cases} \phi(\underset{\sim}{\theta}) = \prod_{i=1}^{p}\prod_{l=1}^{j-1} Z_{il}, \quad \psi(\theta_1,\dots,\theta_{h-1}) = \prod_{i=1}^{h-1} Z_{ij}, \\ Z_{il} = \sum_{q=0}^{\alpha-1} e^{-\theta_i v_{il}}(\theta_i v_{il})^q\,(q!)^{-1}, \\ v_{il} = T_{il}-T_{(i-1)l}, \; v_{1l} = T_{1l}-T_{p(l-1)}, \; T_{p0}=0, \; T_{0j}=T_{p(j-1)}, \\ \qquad i=2,3,\dots,p;\; l=1,2,\dots,k. \end{cases}$$

If $\alpha=1$, then (1) reduces to

$$(1b)\quad f_h^{(j)}(t) = \left[\theta_h e^{-\theta_h(t-T_{(h-1)j})}\, \bar{\phi}(\underset{\sim}{\theta})\bar{\psi}(\theta_1,\dots,\theta_{h-1})\right],$$

where $\bar{\phi}(\underset{\sim}{\theta}) = \prod_{i=1}^{p}\prod_{l=1}^{j-1} e^{-\theta_i v_{il}}$, $\bar{\psi}(\theta_1,\dots,\theta_{h-1}) = \prod_{i=1}^{h-1} e^{-\theta_i v_{ij}}$;
$\phi(\underset{\sim}{\theta})$, $\bar{\phi}(\underset{\sim}{\theta})$ correspond to first $(j-1)$ cycles and $\psi(\theta_1,\dots,\theta_{h-1})$, $\bar{\psi}(\theta_1,\dots,\theta_{h-1})$ correspond to j-th cycle only.

Let

$$(2)\quad Q_h^{(j)} = P\left[T_{(h-1)j} < t < T_{hj}\right] = \int_{T_{(h-1)j}}^{T_{hj}} f_h^{(j)}(t)\,dt =$$

$$= \left[1 - \sum_{q=0}^{\alpha-1} e^{-\theta_h v_{hj}}(\theta_h v_{hj})^q (q!)^{-1}\right] \phi(\underset{\sim}{\theta})\psi(\theta_1,\dots,\theta_{h-1}).$$

So (2) is of the form $Q_h(j) = (1-\Sigma)\Phi\psi$ and can be rewritten as

$$(3) \qquad Q_h^{(j)} = \Phi(\underset{\sim}{\theta}) \left[\psi(\theta_1,\dots,\theta_{h-1}) - \psi(\theta_1,\dots,\theta_h)\right] =$$

$$= \Phi(\underset{\sim}{\theta})\, Z_{1j} \dots Z_{(h-1)j}(1-Z_{hj}).$$

Now the distribution corresponding to (1) is

$$(4) \quad F_h\binom{j}{t} = \sum_{i=1}^{p}\sum_{l=1}^{j-1} Q_i^{(l)} + \sum_{i=1}^{h-1} Q_i^{(j)} + \int_{T_{(h-1)j}}^{t} f_h\binom{j}{t}\,dt =$$

$$= (1-Z_{11}) + (Z_{11}-Z_{11}Z_{21}) + (Z_{11}Z_{21}-Z_{11}Z_{21}Z_{31}) + \dots$$

$$\dots + \left(\prod_{i=1}^{p}\prod_{l=1}^{j-1} Z_{il}\right)\left(\prod_{i=1}^{h-2} Z_{ij} - \prod_{i=1}^{h-1} Z_{ij}\right) + \int_{T_{(h-1)j}}^{t} f_j\binom{j}{t}\,dt =$$

$$= 1 - \Phi(\underset{\sim}{\theta})\psi(\theta_1,\theta_2,\dots,\theta_{h-1}) + \int_{T_{(h-1)j}}^{t} f_h\binom{j}{t}\,dt$$

for $T_{(h-1)j} < t < T_{hj}$ and with

$$(4a) \qquad \psi(Q_0) = 1.$$

Now, using (1) in (4), we get

$$(4b) \quad F_h\binom{j}{t} = 1 - \left[\sum_{q=0}^{\alpha-1} e^{-\theta_h(t-T_{(h-1)j})}\left\{\theta_h(t-T_{(h-1)j})\right\}^q (q!)^{-1}\right] \times$$

$$\times\, \Phi(\underset{\sim}{\theta})\psi(\theta_1,\dots,\theta_{h-1}).$$

So (4b) is of the form

$$F_h\binom{j}{t} = 1 - \Sigma\Phi\psi.$$

Special case (a). Now, in (0) if

$$T_{ij} = (j-1)T + \sum_{l=1}^{i} T_l, \tag{5}$$

where $T = T_1 + \dots + T_p$, then

$$v_{il} = T_i \quad \text{for all} \quad l = 1,2,\dots,k,\ i = 1,2,\dots,p. \tag{5a}$$

Now the distribution function (4b) is

$$F_h^{(j)}(t) = 1 - \left[\sum_{q=0}^{\alpha-1} e^{-\theta_h[t-\varepsilon(h,j)]} \left[\theta_h\{t-\varepsilon(h,j)\}\right]^q (q!)^{-1} \right] \times \tag{6}$$

$$\times \left[\phi^o(\underset{\sim}{\theta})\right]^{j-1} \psi^o(\theta_1,\dots,\theta_{h-1}),\ \varepsilon(h,j) < t < \varepsilon(h+1,j).$$

So (6) is of the form

$$F_h^{(j)}(t) = 1 - \Sigma^o (\phi^o)^{j-1} \psi^o, \tag{6a}$$

where Σ^o is Σ in (4b) under special case (a).

In (6) we have

$$\begin{cases} \phi^o(\underset{\sim}{\theta}) = \prod_{i=1}^{p} Z_i^o,\ \varepsilon(h,j) = (j-1)T + \sum_{i=1}^{h-1} T_i, \\ \psi^o(\theta_1,\dots,\theta_{h-1}) = \prod_{i=1}^{h-1} Z_i^o, \\ Z_i^o = \sum_{q=0}^{\alpha-1} e^{-\theta_i T_i}(\theta_i T_i)^q (q!)^{-1}. \end{cases} \tag{6b}$$

Special case (b). Suppose $\alpha = 1$ in special case (a), then (6) reduces to

$$(7)\quad F_h^{(j)}(t) = 1-\exp\left\{-\theta_h\left[t-\varepsilon(h,j)\right]\right\}\exp\left[-(j-1)\theta T+\sum_{i=1}^{h-1}\theta_i T_i\right] =$$

$$= 1-\exp\left[-\theta_h t+(j-1)\sum_{i=1}^{p}(\theta_h-\theta_i)T_i+\sum_{i=1}^{h}(\theta_h-\theta_i)T_i\right],$$

$$\varepsilon(h,j) < t < \varepsilon(h+1,j),$$

where $\theta T = \theta_1 T_1 + \dots + \theta_p T_p$.

Now, (7) is exactly the same as in Patel and Gajjar [20]. If $p = 2$, in special case (b), then (7) reduces to

$$(8)\quad \begin{cases} F_1^{(j)}(t) = 1 - e^{-\theta_1 t+(j-1)(\theta_1-\theta_2)T_2}, & \varepsilon(1,j) < t < \varepsilon(2,j), \\ F_2^{(j)}(t) = 1 - e^{-\theta_2 t-j(\theta_1-\theta_2)T_1}, & \varepsilon(2,j) < t < \varepsilon(3,j) = j(T_1+T_2). \end{cases}$$

Again (8) corresponds to those in Srivastava [22].

3. Bayesian estimates

Now, consider special case (a); from (1), the probability of a failure in the j-th cycle and h-th phase can be written as

$$(9)\quad P_{hj} = \left[\int_{\varepsilon(h,j)}^{\varepsilon(h+1,j)} \theta_h e^{-\theta_h\left[t-\varepsilon(h,j)\right]}\left[\theta_h\left\{t-\varepsilon(h,j)\right\}\right]^{\alpha-1}\left[\Gamma(\alpha)\right]^{-1}dt\right]\times$$

$$\times\left[\phi^o(\underset{\sim}{\theta})\right]^{j-1}\left[\psi^o(\theta_1,\dots,\theta_{h-1})\right] =$$

$$= \left[1-\sum_{q=0}^{\alpha-1} e^{-\theta_h T_h}(\theta_h T_h)^q (q!)^{-1}\right]\left[\phi^o(\underset{\sim}{\theta})\right]^{j-1}\left[\psi^o(\theta_1,\dots,\theta_{h-1})\right].$$

Censored above and below. Let r items fail before the (m+1)-th cycle and s items survive after (m+k) cycles, then, by (4), the likelihood function can be written as

$$(10)\quad L(\underset{\sim}{\theta},\underset{\sim}{T}) = C\left[F_p^{(m)}(mT)\right]^r \left[\prod_{i=1}^{p}\prod_{j=m+1}^{m+k} P_{ij}^{n_{ij}}\right]\left[1-F_p^{(m+k)}\{(m+k)T\}\right]^s,$$

where n_{ij} items fail in the j-th cycle and i-th phase and $\underset{\sim}{T} = (T_1,\dots,T_p)$, C is a constant. Two terms $\left[F_p^{(m)}(mT)\right]^r$, $\left[1-F_p^{(m+k)}\{(m+k)T\}\right]^s$ in (10) can be written, using (6b), as

$$(11)\quad \sum_{u=0}^{r}\binom{r}{u}(-1)^u\left[\prod_{i=1}^{p} Z_i^o\right]^{y+s+u}$$

where

$$(11a)\quad y = (m+k-1)s+(m-1)u.$$

Let $A_i(\alpha,n)$ denote the coefficient of t^i in the expansion of $\left(\sum_{i=0}^{\alpha-1}\frac{t^i}{i!}\right)^n$. Then the $A_i(\alpha,n)$ satisfy the following recurrence relation as given in Lingappaiah [8]

$$(12)\quad A_i(\alpha,n) = \sum_{j=0}^{\alpha-1} A_{i-j}(\alpha,n-1)(j!)^{-1}$$

and, from this recurrence relation, tables of values of the $A_i(\alpha,n)$ are generated for various values of α (integer) and n. These extensive tables are available with the author. Entire computation of the results of this paper depends mainly on these coefficient and, as such, these tables are essential for this result. Now (11) can be expressed as

$$(13)\quad \sum_{u=0}^{r}\binom{r}{u}(-1)^u\left[\prod_{i=1}^{p}\sum_{v_i=0}^{(\alpha-1)(s+u+y)} A_{v_i}(\alpha,s+u+y)(\theta_i T_i)^{v_i}\right]\times$$

$$\times\left[e^{-(\theta T)(s+u+y)}\right],$$

where $\theta T = \theta_1 T_1 + \dots + \theta_p T_p$; the part involving $P_{ij}^{n_{ij}}$ in (10) can be written as

$$(14)\quad \prod_{i=1}^{p} \prod_{j=m+1}^{m+k} P_{ij}^{n_{ij}} = \left[\prod_{i=1}^{p} (1-Z_i^o) \right]^{n_i} \cdot$$

$$\cdot \left[\prod_{i=1}^{p} \prod_{j=1}^{k} \{\phi^o(\underset{\sim}{\theta})\}^{(m+j-1)n_{i(j+m)}} \right] \cdot \left[\prod_{i=1}^{p} \{\psi^o(\theta_1,\dots,\theta_{i-1})\}^{n_i} \right],$$

where

$$(14a)\quad n_i = \sum_{j=1}^{k} n_{ij},\ N = n_1+\dots+n_p,\ n_j' = n_{1j'}+\dots+n_{pj'},$$
$$j' = m+j,\ N_i = n_1+\dots+n_i,\ N_p = N,\ N = n_1'+\dots+n_k',$$

(N is the total number of observations from (m+1)-th to (m+k)-th cycles). Eq. (14a) can be put as follows:

Cycle		
m+1	$n_{1(m+1)} \dots n_{i(m+1)} \dots n_{p(m+1)}$	$n'_{(m+1)} = n'_1$
⋮		
m+j	$n_{1(m+j)} \dots n_{i(m+j)} \dots n_{p(m+j)}$	$n'_{(m+j)} = n'_j$
⋮		
m+k	$n_{1(m+k)} \dots n_{i(m+k)} \dots n_{p(m+k)}$	$n'_{(m+k)} = n'_k$
	$n_1 \dots n_i \dots n_p$	$=N$

Now in (14) the middle square bracket term is

$$(15)\quad \prod_i \prod_j \left[\phi^o(\underset{\sim}{\theta})\right]^{(m+j-1)n_{ij'}} = \prod_{i=1}^{p} (Z_i^o)^{\sum_j (m+j-1)n_j'} =$$

$$= \prod_{i=1}^{p} (Z_i^o)^W = \prod_{i=1}^{p} \left[\sum_{r_i=0}^{(\alpha-1)W} A_{r_i}(\alpha,W)(\theta_i T_i)^{r_i} \right] \cdot e^{-(\theta T W)},$$

where $W = N(m-1) + \sum_{j=1}^{k} j(n_j')$.

The first square bracket term in (14) is

$$(16)\qquad \prod_{i=1}^{p} (1-Z_i^o)^{n_i} =$$

$$= \prod_{i=1}^{p} \left[\sum_{l_i=0}^{n_i} \binom{n_i}{l_i} (-1)^{l_i} \sum_{z_i=0}^{(\alpha-1)l_i} A_{z_i}(\alpha, l_i)\cdot(\theta_i T_i)^{z_i} e^{-\theta_i T_i l_i} \right].$$

The last square bracket term in (14) is

$$(17)\qquad \prod_{i=1}^{p} \left[\prod_{v=1}^{i-1} (Z_v^o) \right]^{n_i} = \prod_{i=1}^{p} (Z_i^o)^{N-N_i} = \exp\left[- \sum_{i=1}^{p} \theta_i T_i (N-N_i) \right] \times$$

$$\times \prod_{i=1}^{p} \left[\sum_{t_i=0}^{(\alpha-1)(N-N_i)} A_{t_i}(\alpha, N-N_i)(\theta_i T_i)^{t_i} \right].$$

Note in (17), for $i = p$, it reduces to 1.

Now, by (13), (15), (16), (17), eq. (10) can be written as

$$(18)\qquad L(\underset{\sim}{\theta}, \underset{\sim}{T}) = C \sum_{u=0}^{r} \binom{r}{u} (-1)^u \prod_{i=1}^{p} \left[\sum_{l_i} \binom{n_i}{l_i} (-1)^{l_i} \cdot \right.$$

$$\left. \cdot \sum_{v_i} A_{v_i} \sum_{r_i} A_{r_i} \sum_{t_i} A_{t_i} \sum_{z_i} A_{z_i} (\theta_i T_i)^{M_i} \right] \left[e^{-\sum_{i=1}^{p} \theta_i T_i H_i} \right],$$

where $H_i = [(N-N_i)+(s+u+v)+l_i+W]$, $M_i = r_i+z_i+t_i+v_i$, $t_p = 0$, $N_p = N$, and sums are read as $\sum_{v_i} A_{v_i}(\theta_i T_i)^{v_i}$, $\sum_{r_i} A_{r_i}(\theta_i T_i)^{r_i}$ and so on. Limits of the sums in (18) are given in (13), (15), (16) and (17). If $\alpha = 1$ in (18), (special case (b)), then r_i, v_i, t_i, z_i all vanish and we have

$$\text{(18a)}\quad L(\underset{\sim}{\theta},\underset{\sim}{T}) = C\left[\left(1-e^{-m(T\theta)}\right)^{r} e^{-s(m+k)(T\theta)}\right] \times$$

$$\times \left[\prod_{i=1}^{p}\left(1-e^{-\theta_i T_i}\right)^{n_i} e^{-(T_i\theta_i)(N-N_i)}\right]\left[e^{-T\theta \sum\limits_i \sum\limits_j (m+j-1)n_{ij'}}\right];$$

$\frac{\partial}{\partial\theta_i}\log L = 0$ in (18a) gives $\hat{\theta}_i$ of Patel and Gajjar [20].

Taking the priors for $(\theta_1,\ldots,\theta_p)$ as

$$\text{(19)}\qquad g(\underset{\sim}{\theta}) = \prod_{i=1}^{p}\left[e^{-\theta_i}\,\theta_i^{g_i-1}\left[\Gamma(g_i)\right]^{-1}\right]$$

we get the Bayesian estimate $\hat{\theta}_h$ as

$$\text{(20)}\qquad \hat{\theta}_h = \frac{\int \theta_h\, L(\underset{\sim}{\theta},\underset{\sim}{T})g(\underset{\sim}{\theta})d\underset{\sim}{\theta}}{\int L(\underset{\sim}{\theta},\underset{\sim}{T})g(\underset{\sim}{\theta})d\underset{\sim}{\theta}},$$

where $d\underset{\sim}{\theta} = d\theta_1 \ldots d\theta_p$, and $\int$ is p-fold.

From (18), (19) and (20), we get

$$\text{(21)}\quad \hat{\theta}_h = \frac{\sum\limits_u \binom{r}{u}\cdot(-1)^u \prod\limits_{i=1}^{p}\left[\left(\prod\limits_{l=1}^{5}\Sigma_{li}\right)\binom{n_i}{l_i}(-1)^{l_i}\dfrac{\Gamma(M_i+g_i+\delta_i)\,T_i^{M_i}\left\{1+\delta_i(g_i-1)\right\}}{(1+T_i H_i)^{M_i+\delta_i+g_i}}\right]}{\sum\limits_u \binom{r}{u}(-1)^u \prod\limits_{i=1}^{p}\left[\left(\prod\limits_{l=1}^{5}\Sigma_{li}\right)\binom{n_i}{l_i}(-1)^{l_i}\dfrac{T_i^{M_i}\,\Gamma(M_i+g_i)}{(1+T_i H_i)^{M_i+g_i}}\right]},$$

where $\delta_i = \begin{cases}1 & \text{if } i = h\\ 0 & \text{if } i \neq h\end{cases}$ and Σ_{1i}, Σ_{2i}, Σ_{3i}, Σ_{4i}, Σ_{5i} are $\sum\limits_{l_i}$, $\sum\limits_{v_i}$, $\sum\limits_{r_i}$, $\sum\limits_{t_i}$, $\sum\limits_{z_i}$, respectively, in (18). If $\alpha = 1$, (special case (b)), then all v_i, r_i, t_i, z_i vanish and we have (21) as

$$(22)\quad \hat{\theta}_h = \frac{\sum_u \binom{r}{u}(-1)^u \left[\prod_{i=1}^{p} \sum_{l_i} \binom{n_i}{l_i}(-1)^{l_i} \frac{\{1+\delta_i(g_i-1)\}}{(1+T_iH_i)^{g_i+\delta_i}} \right]}{\sum_u \binom{r}{u}(-1)^u \prod_{i=1}^{p} \left[\sum_{l_i} \binom{n_i}{l_i}(-1)^{l_i} \frac{1}{(1+T_iH_i)^{g_i}} \right]} .$$

4. Recurrence relations. From (1), when $\alpha=1$, we get the r-th moment in j-th cycle and h-th phase as

$$(23)\quad \mu_{hr}^{(j)} = \int_{\varepsilon(h,j)}^{\varepsilon(h+1,j)} \theta_h t^r e^{-\theta_h[t-\varepsilon(h,j)]} Q(h,j)\, dt =$$

$$= Q(h,j) \sum_{i=0}^{r} \binom{r}{i} [\varepsilon(h,j)]^{r-i} \left(\frac{i!}{\theta_h^i}\right) \cdot$$

$$\cdot \left[1 - \sum_{q=0}^{i} e^{-\theta_h T_h} (\theta_h T_h)^q (q!)^{-1}\right],$$

where

$$(23a)\qquad Q(h,j) = \exp\left[-\left\{(\theta T)(j-1) + \sum_{i=1}^{h-1} T_i\theta_i\right\}\right].$$

From (23) we get with $r = 1$

$$(24)\quad \mu_h^{(j)} = Q(h,j) \left[\left\{\varepsilon(h,j) + T_h + \frac{1}{\theta_h}\right\}\left(1 - e^{-\theta_h T_h}\right) - T_h\right]$$

which implies the following recurrence relations (for $j > 1$)

$$(25)\quad \mu_h^{(j)} = \mu_{h-1}^{(j)} \exp\left(-\theta_{h-1}T_{h-1}\right) \frac{B(h,j)}{B(h-1,j)} ,$$

where

$$(25a) \qquad B(h,j) = \left(1 - e^{-\theta_h T_h}\right)\left[S_h + (j-1)T + \frac{1}{\theta_h}\right] - T_h .$$

For $j = 1$, use (24) and $S_h = T_1 + \dots + T_h$, $S_p = T$, $h=1,2,\dots,p$.
Similarly from (24), we get

$$(26) \qquad \mu_h^{(j)} = \mu_h^{(j-1)} e^{-(\theta T)}[B(h,j)/B(h,j-1)], \; j > 1 .$$

For $j = 1$, use again (24).

5. Comments: a) There is quite a large number of parameters such as $\theta_1,\dots,\theta_p$; $T_1,\dots,T_p$; (T_{ij}, $i=1,\dots,p$, $j=1,\dots,k$ in the general case), $g_1,\dots,g_p,m,r,s,\alpha,p,k,n_{ij}'$, $i=1,\dots,p$, $j=1,2,\dots,k$. This shows that a large number of tables can be generated by varying any two parameters and holding the remaining fixed.

b) Instead of independent priors for the θ_i, one can take dependent prior as

$$(27) \qquad g(\underset{\sim}{\theta}) = e^{-\theta_p}, \; 0 < \theta_1 < \theta_2 < \dots < \theta_p < \infty .$$

But now evaluation of integrals in (20) to integrate out the θ_i will be slightly complex.

c) Similar to (21), if the θ_i are known, one can get the estimates of the T_i with proper priors for the T_i, in terms of known θ_i.

d) Computer programme for (22) is available for $p = 2$. From this $\hat{\theta}_1$ and $\hat{\theta}_2$ are tabulated for different T_1, T_2 and other parameters. Because of the length of the paper, tables are not included here. But they are available with the author.

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REFERENCES

[1] E.E. Chen, R.L. Wardrop: Bayes sequential estimation in a life test and asymptotic properties, Comm. Statist. 9(A) (1980), 659-672.

[2] M.H. DeGroot, K.P. Goel: Bayesian estimation and optimal designs in partially accelerated life testing. Naval Res. Logist. Quart. 26 (1979), 223-225.

[3] I.R. Dunsmore: The Bayesian predictive distribution in life testing models, Technometrics, 16 (1974), 455-460.

[4] I.G. Evans, A.M. Nigm: Bayesian prediction for two parameter Weibull life time models, Comm. Statist. 9(A) (1980), 649-658.

[5] J.P. Klein, A.P. Basu: Weibull accelerated life tests when there are competing causes of failure, Comm. Statist. 10(A) (1981), 2073-2100.

[6] J.F. Lawless; A prediction problem concerning samples from the exponential distribution, with applications to life testing, Technometrics, 13 (1971), 725-730.

[7] G.S. Lingappaiah: Prediction in exponential life testing, Canad. J. Statist. 1 (1973), 113-117.

[8] G.S. Lingappaiah: Prediction in the samples from the gamma distribution as applied to life testing, Austral. J. Statist. 16 (1974), 30-32.

[9] G.S. Lingappaiah: Bayesian approach to prediction in complete and censored samples from Weibull population, Metron, 35 (1977), 167-179.

[10] G.S. Lingappaiah: Bayesian approach to the prediction problem in the exponential population, IEEE Trans. Reliab. R-28 (1978), 222-225.

[11] G.S. Lingappaiah: Bayesian approach to the prediction problem in gamma population, Demonstratio Math. 11 (1978), 907-920.

[12] G.S. L i n g a p p a i a h : Bayesian approach to prediction and the spacings in the exponential distribution, Ann. Inst. Statist. Math. 31 (1979), 391-401.

[13] G.S. L i n g a p p a i a h : Intermittent life testing and Bayesian approach to prediction with spacings in the exponential model, Statistica, 40 (1980), 477-490.

[14] G.S. L i n g a p p a i a h : Sequential life testing with spacings. Exponential model. IEEE Trans. Reliab. R-30 (1981), 370-374.

[15] G.S. L i n g a p p a i a h : Prediction and testing in a generalised life test. Demonstratio Math. 14 (1981), 471-482.

[16] G.S. L i n g a p p a i a h : Shifting shape parameter in life tests, a Bayes approach. IEEE Trans. Reliab. R-32 (1983).

[17] M o s h e S h a k e d : Accelerated life testing for a class of linear hazard rate type distribution. Technometrics, 20 (1978) 457-466.

[18] W. N e l s o n , Q.W. M e e k e r : Theory for optimum accelerated censored life tests for Weibull and extreme value distributions. Technometrics, 20 (1978), 171-178.

[19] W.J. P a d g e t , L.J. W e i : A Bayesian non parametric estimator of survival probability assuming increasing failure rate. Comm. Statist. 10(A) (1981), 49-66.

[20] I.D. P a t e l , A.V. G a j j a r : Trivariate and multivariate extension of life test with periodic change in failure rate-group censored observations. Vidya, 20(B) (1977), 55-71.

[21] D.N. S i n g p u r w a l a , A.F. A l - K h a y y a l: Accelerated life tests using the Power law model for the Weibull distribution. Theory and Appl. Reliab., Academic Press, N.Y. 1977.

[22] T.N. S r i v a s t a v a : Life tests with periodic change in failure rate-grouped observations. J. Amer. Statist. Assoc. 70, (1975), 394-397.

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