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PROFESSOR ROMAN SIKORSKI (1920-1983)

Professor Roman Sikorski was born on July 11, 1920 at Mszczonów. He graduated from a high school in Żyrardów in 1937. One of the teachers of this school wrote on 16.06.1937 in the grade book the following opinion about the pupil Roman Sikorski: "Brilliant mathematical and natural science abilities. A good organizer. Honest character. Outstanding inteligence".

Already in the high school Roman Sikorski revealed - admired many years later at meetings of the Polish Mathematical Society - his literary abilities and uncommon wit. Namely he was the editor of a school journal entitled "Young Pens", in which he wrote many pointed articles on the topics of importance for the youth at that time.

However, the further career of Professor Roman Sikorski was permanently connected with mathematics: this stemmed from it and harvested for it. Professionally he was associated with the University of Warsaw and the Polish Academy of Sciences. His social aspirations were realized through his activity in the Polish Mathematical Society.

Roman Sikorski graduated in 1947 with M.Sc. in mathematics from the University of Warsaw. As soon as in 1949 he was awarded Ph.D. in mathematics and one year later he passed his privat docent colloquium. He became extraordinary Professor of Mathematics in 1954, and in 1957 he was promoted to full professorship. In 1962 he was elected to corresponding membership in the Polish Academy of Sciences, in 1969 he was raised

to full membership. This unusually fast scientific career he owed not only to his outstanding abilities, but also to his inexhaustible laboriousness. He used to say that only the conjunction of these two traits leads to a success in science. Owing to this attitude even the years of the war did not cause a significant breach in his scientific career - during this time he gained, by self-study, "a broad basic mathematical education", as he himself wrote later in his curriculum vitae.

In 1950 he assumed the position of a substituting professor of mathematics at the Faculty of Chemistry of Technical University of Warsaw. He held this position for three years not breaking his formal ties with University of Warsaw. Since 1953 he had lectures as professor at the Faculty of Mathematics and Mechanics of University of Warsaw. From the beginning of the State Mathematical Institute and also later when this Institute become a part of the Polish Academy of Sciences - Professor Roman Sikorski conducted there his famous seminar on functional analysis with very wide thematic, where many young mathematicians were educated. Besides this seminar Professor Sikorski conducted jointly with Professor Helena Rasiowa a seminar on logic and lattice theory at the University of Warsaw. At this seminar many papers on Boolean algebras and Post algebras were reported. The influence of this seminar was spread to Technical University of Warsaw, from where many mathematicians participated in it. In recognition of his achievement, he was awarded in 1955 with a state prize. Besides scientific and didactic activity, Professor Roman Sikorski found time for public activity in the framework of Polish Mathematical Society. In the years 1965-1977 he was the president of this society. In 1980 he was conferred a honorary membership in the Polish Mathematical Society. Despite the heavy illness, Professor Roman Sikorski till the last moments of his life participated in mathematical activities and was in contact with his students and collaborators. He died in Warsaw on September 12, 1983.

The main field of scientific research of Professor Roman Sikorski was the theory of Boolean algebras and its applications. In 1948 he published in Fundamenta Mathematicae a remarkable theorem on the representation of Boolean σ-algebras. This theorem was independently proved by Loomis. The theorem says that every Boolean σ-algebra A is isomorphic to the quotient algebra, F/Δ, where F is the σ-field generated by open-closed subsets in the Stone space of A, and Δ is the σ-ideal of the set of the first category of Bair. Equally remarkable result is the theorem saying that the injective objects in the category of Boolean algebras are complete algebras. This theorem implies two important corollaries:

1. A complete Boolean algebra B is the completion of a Boolean algebra A if and only if it contains a dense subalgebra B_0 isomorphic with A.

2. The completion of a Boolean algebra A is a complete field of sets if and only if A is atomic.

In 1950 in Fundamenta Mathematicae there appeared an article by R. Sikorski and H. Rasiowa in which the first algebraic proof of the theorem of Gödel on the completeness of predicate calculus of classical logic was given. The crutial step of this proof was provided by the famous "lemma of Rasiowa and Sikorski". This lemma says that in every Boolean algebra there exists a prime ideal containing an arbitrary fixed non-zero element and preserving any fixed countable family of meets and joins in this algebra. Rasiowa-Sikorski's lemma has been later used several times by authors in the proofs of completeness for non-classical logics. Using Rasiowa-Sikorski's lemma Dana Scott has found another proof of the famous theorem of Cohen on the independence of the continuum hypothesis.

The investigations of Roman Sikorski carried individually and jointly with H. Rasiowa on applications of topology and Boolean algebras in logic led both authors to writing the monograph entitled "Mathematics of Metamathematics" published by Polish Scientific Publishers in 1963. This book had already three English editions and one Russian. It presents a new me-

thod of logic lecture according to which all logic theorems are derived from corresponding algebraic and topological theorems. The connection between logic and algebra is provided by Lindenbaum-Tarski algebras.

In the years 1950-52 Roman Sikorski worked on the theory of m-products ($m \geqslant \aleph_0$) of Boolean algebras. The results were published in Fundamenta Mathematicae and Studia Mathematica. The definition of free Boolean product introduced by him turned out to be equivalent with the definition of coproduct in the category of Boolean algebras. There exists, up to an isomorphism, a unique such product. However, the situation is different in the case of σ-product of σ-algebras. There are many such products, but they can be naturally ordered and of special importance there are minimal and maximal σ-products. For example, the minimal σ-product of σ-fields of set is isomorphic to the measure product, known from measure theory. On the other hand the maximal σ-product of σ-Boolean algebras has a useful and elegant homomorphism extension property. For this reason the maximal σ-product is also called the free σ-product. Conducting these investigations R. Sikorski showed that the Banach theorem on the extension of measures defined on σ-independent σ-fields can be derived from the theorem on existence of coproduct measure. This theorem does not hold for arbitrary σ-Boolean algebras with σ-measures.

The culminating place in the achievements of Professor Sikorski is occupied by his monograph "Boolean algebras" published by Springer-Verlag. This book is undoubtedly best among all books written by R. Sikorski. In the years 1958-69 there were published three English and one Russian editions, which indicates great interest of mathematical world in this book.

Professor Roman Sikorski was also interested in the theory of functions of real variable. This interest led him to writing the text-book "Real Functions" published by Polish Scientific Publishers. The book was written according to his lectures he held on this subject at the University of Warsaw. The

book contains many detailed results on real functions and is still used by students and scientific workers. The interest of Professor Sikorski in the theory of real functions stems from theory of Boolean algebras, where he widely used real functions methods (as a matter of fact, the monograph "Boolean Algebra" was published in the series "Reelle Funktionen" of the "Ergebnisse der Mathematic und iher Granzgebiete of Springer-Verlag).

Professor Roman Sikorski has also made significant contribution to the theory of distribution. He wanted to base the theory of distribution on the notions more elementary than those used in the works of S.L. Sobolev and L. Schwartz. He proposed to define a distribution as the equivalence class of an ordered pair (p,f), where p is a non-negative integer an f a function continuous on an open set Q. Two such pairs (p,f) and (q,g) are equivalent if $p>q$, the derivative $f^{(p-q)}$ is continuous and the difference $f^{(p-q)}-g$ is a polynomial of degree less than q, or if $q>p$ and the difference $g^{(q-p)}-f$ is a polynomial of degree less than p. Such an approach to the theory of distributions is presented in the textbook "Real Functions". Another, also elementary, approach to the theory of distributions was developed in the papers of Professor Sikorski written jointly with Jan Mikusiński. In these papers the distributions are defined as the equivalence classes of fundamental sequence of functions. Namely, a sequence of smooth functions (f_n) is fundamental if there exist functions (F_n) and order k such $F_n^{(k)} = f_n$ and the sequence (F_n) is almost uniformly convergent on the real line. Two sequences (f_n) and (g_n) are equivalent if the sequence $f_1,g_1,f_2,g_2,\ldots$ is fundamental. Such an approach to the theory of distribution turned out to be fruitful and simple. The papers of R. Sikorski and J. Mikusiński presenting the sequential approach to the theory of distribution have been translated into Russian, French, Polish and Chinese and published in the form of a book. Professor Roman Sikorski developed later further the theory of distribution defining convolution and Fourier transform of di-

stributions. He also introduced the notion of "continuous part of a distribution" and proved an interesting formula for the representation of multidimensional Dirac's delta. The sequential theory of distribution is very straightforward and characterizes the relation th physics.

The work of Professor Roman Sikorski in the theory of distributions brings a significant contribution to functional analysis. Independently of this, R. Sikorski dealt with an algebraic approach to functional analysis. Namely, he investigated and developed the theory of determinants in Banach spaces. To this topic he devoted 11 papers, among them his last paper (joint with A. Buraczewski) published in 1980. The purpose of the determinant theory was to give analytic formulas which are generalizations of the classic formulas of Cramer, and to find criteria for solving the equation $Tx = \acute{y}$ for a linear operator T acting in a Banach space. Of course, such a theory can be developed only for some class of operators which are close to the identity operator, namely, for the class of those operators which determine linear functionals on finite dimensional operators were independently introduced by Grothendieck and Leżański. Professor Sikorski called them quasi-nuclear operators and developed for them a determinant theory. He constructed for them the so-called determinant systems. He explains why the determinant system constitute a natural generalization of the classical theory of linear equations. R. Sikorski discovered many new analytical formulas for determinants and developed an algebraic theory of determinant systems indicating to its natural conection with the theory of Fredholm operators in Banach spaces. The investigations of Roman Sikorski in the determinant theory have had a strong influence on other mathematicians, in particular they have been further carried on by his pupil Adam Buraczewski.

Independently of the determinant theory, Professor Roman Sikorski wrote several other papers in functional analysis. In these papers he investigated the existence of generalized limits invariant with respect to acting of an abelian semi-

-group for bounded scalar functions defined on some abstract set on which this semi-group acts as a semi-group of transformations, he gave a simple proof of the Mazur-Orlicz theorem on linear inequalities, and he found a general schema of summability of scalar and vector functions, the special cases of which are the methods of Toeplitz and continuous methods.

A more detailed presentation of the works of Roman Sikorsk was given in the articles of H. Rasiowa, T. Traczyk, E. Grzegorek and Cz. Ryll-Nardzewski, P. Antosik and A. Pełczyński published in Vol. 24 (1982) of "Wiadomości Matematyczne". Also there a list of publications of Roman Sikorski can be found.

By publishing this issue of Demonstratio Mathematica devoted to the memory of Professor Roman Sikorski, the Editorial Committee would like to commemorate the scientific didactic, and social activity of the Professor and to pay tribute to him In particular, we would like to emphasize the merits of Professor Roman Sikorski for the developements of mathematical research in Technical University of Warsaw. Professor Sikorski had initiated in this school the investigations in the fields of abstract algebra and lattice theory, which later developed into a seminar on algebra acting up to the present times. In the framework of this seminar there are carried on works in the domains of the theory of Boolean and Post algebras, lattic theory and the theory of partially ordered sets and their applications in engineering, computer science and quantum mechanics. The seminar has educated 2 professors, 4 associate professors and many Ph.D.'s. Almost in all proceedings for scientific degrees in algebra at the Institute there can be traced the influence of works of Professor Roman Sikorski, who as an adviser, referee, or the author of cited papers has shaped the final form of the work. His influence can also be traced in other seminars of the Institute of Mathematics, for example in the seminar on differential geometry, where the thory of differential spaces introduced by him is further developed.

The work of Professor Roman Sikorski is being carried on at the Technical University of Warsaw by his pupils and pupils of his pupils. The memory of him and the gratitude to him will be always alive at the Technical University of Warsaw, and his name will be mentioned among the names of the most distinguished professors of this school.

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