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ON DECOMPOSITION OF FORMULAS OF A FIRST ORDER LANGUAGE

1. Introduction

We shall be concerned here with that part of the syntax of a first order language which deals with the decomposition of formulas of that language. More precisely, we shall be interested in the relative partial decomposition of formulas, a device to be described in the sequel in full detail. For the beginning it suffices to say that there will be given some instructions for decomposition of formulas according to which there proceeds the elimination of certain (thence relative decomposition) connectives and quantifiers from formulas which stops at some chosen formulas of the language (thence partial decomposition) and not always at elementary ones.

First attempts in this direction go back most likely to mathematical folklore. However, in [2] (pp.221-223), [4] (pp.229-306) and in [3] trees were connected with formulas in another context, namely, as a useful tool in proof theory but in [3] and [4] with sequences of subformulas of a given formula as elements and not with subformulas themselves. It seems that in [1] (pp.128-131) the trees of occurences of subformulas were firstly studied in their own right.

Now to the language itself. Let the alphabet Θ of our language $\mathcal{L}$ be composed of mutually disjoint sets of signs as follows: a countable set X of individual variables x_i (i.e. for any natural i), a countable set C of individual constants C_i (for any natural i), a countable set $\mathcal{F}$ of function letters F_i^k (for any natural i and k), a countable set $\mathcal{P}$ of predicate letters P_i^k (for any natural i and k) except for P_1^2 in place of which there is the sign = adopted, sets of sentential connectives $I = \{\neg, \vee, \&, \Rightarrow, \equiv\}$, quantifiers $Q = \{\forall, \exists\}$ and brackets $\{ (\, , \,) \}$.

Throughout this paper we accept the notational system based on inner brackets (Cf. [1], p.128).

Let in the sequel $Z \in W(\theta)$ mean that Z is a word in the alphabet θ, i.e. a finite sequence of sings from θ. Let further

$$(1.1) \qquad t : x_i$$

be read as "the word t, $t \in W(\theta)$, is of the form x_i". Now the terms of the language $\mathcal{L}$ can be characterized as precisely those words $t \in W(\theta)$ for which $t : x_i$, $t : C_i$ or $t : F_i^k(u_1 \ldots u_k)$, where $u_1, \ldots, u_k$ are terms of $\mathcal{L}$. Having terms of $\mathcal{L}$ already defined, the definition of formulas of $\mathcal{L}$ proceeds in a usual way.

Definition 1.1. (Definition of formulas of $\mathcal{L}$)

1° For any predicate letter P_i^k and any k terms of $\mathcal{L}$ the word $P_i^k(t_1 \ldots t_k)$ of $W(\theta)$ is a formula of $\mathcal{L}$.

2° For any two terms t_1 and t_2 of $\mathcal{L}$ the word $= (t_1 \, t_2)$ of $W(\theta)$ is a formula of $\mathcal{L}$. The words of $W(\theta)$, which are formulas of $\mathcal{L}$ according to 1° or 2°, are called elementary formulas of $\mathcal{L}$.

3° If the word A of $W(\theta)$ is a formula of $\mathcal{L}$, then so are the following three words of $W(\theta)$: $\neg(A)$, $\forall x_i(A)$ and $\exists x_i(A)$, where, in latter two words, x_i is any individual variable of $\mathcal{L}$.

4° If the words A and B of $W(\theta)$ are formulas of $\mathcal{L}$, then so are the following four words of $W(\theta)$: $(A) \& (B)$, $(A) \vee (B)$, $(A) \Longrightarrow (B)$ and $(A) \equiv (B)$.

5° The set of formulas of $\mathcal{L}$ is the least set of words of $W(\theta)$ that contains elementary formulas of $\mathcal{L}$ and is closed under the formation rules 3° and 4° above.

6° A word $T \in W(\theta)$ is said to be a subword of a word $Z \in W(\theta)$ if there are words $U_1, U_2 \in W(\theta)$, possibly empty, such that $Z:U_1\ T\ U_2$. Subformulas are defined as subwords being formulas. If A is a subformula of B, then B is also said to be a superformula of A.

7° The sign $\neg$, $\forall$ or $\exists$ is said to be the leading connective of a word $A \in W(\theta)$ whenever $A : \neg(B)$, $A : \forall x_i(B)$ or $A : \exists x_i(B)$ for some $B \in W(\theta)$. So are the signs $\&$, v, $\Longrightarrow$ and $\equiv$ whenever $A:(B)\ \&\ (C)$, $A:(B) \vee (C)$, $A:(B) \Rightarrow (C)$ or $A:(B) \equiv (C)$ for some $B, C \in W(\theta)$.

2. Decomposition of formulas

To decompose a given formula F of $\mathcal{L}$ will mean to determine certain (possibly all) occurrences of some (possibly all) subformulas of F. This is achieved by eliminating sistematically the leading connectives from F together with an appropriate pair (or pairs) of outermost standing brackets and variables which immediately follow some quantifier and storing the information about the commited transformation. However, the definition of decomposition will be given in a slightly more general fashion, for words of $W(\theta)$, as to serve equally well as a criterion of wellformedness for formulas of $\mathcal{L}$.

D e f i n i t i o n 2.1. (Definition of decomposition) The decomposition $\mathcal{D}(Z)$ of a word $Z \in W(\theta)$ is any sequence of subwords E_i of Z, called members or occurrences of $\mathcal{D}(Z)$, together with an appropriate instruction to be assigned to each E_i, which is defined as follows.

1° Each E_i is supplied either by a STOP-instruction, written as (STOP), or by one among the following seven, called the instructions for decomposition or elimination: $(\neg e\ x)$, $(\forall x_i\ e\ x)$, $(\exists x_i\ e\ x)$, $(\&\ e\ x)$, $(\vee\ e\ x)$, $(\Rightarrow e\ x)$ and $(\equiv e\ x)$. All the instructions will be attached to the occurrences they are applied to.

2° If $E_i : P_m^k(t_1 \ldots, t_k)$ or $E_i := (t_1\ t_2)$, then each such E_i is supplied by a STOP-instruction.

3° Suppose E_i is the least member put into $\mathcal{D}(Z)$ at some stage of generating $\mathcal{D}(Z)$, i.e. the member with minimal i which is not yet supplied by any instruction, and E_j is the last member put into $\mathcal{D}(Z)$ by then, i.e. the member with maximal j. There are three cases to be distinguished.

$3^{\circ}1$ If $E_i : \neg(F)$, $\forall x_i(F)$ or $\exists x_i(F)$, then each such E_i is supplied either by a STOP-instruction or, respectively by an instruction: $(\neg e\ x)$, $(\forall x_i\ e\ x)$ or $(\exists x_i\ e\ x)$. In the latter case put E_{j+1}:F.

$3^{\circ}2$ If E_i:(F) & (G), (F) v (G), $(F) \Rightarrow (G)$ or $(F) \equiv (G)$, then each such E_i is supplied either by a STOP-instruction or, respectively, by an instruction: (& e x), (v e x), $(\Rightarrow e\ x)$ or $(\equiv e\ x)$. In the latter case put E_{j+1}:F and E_{j+2}:G.

$3^{\circ}3$ If E_i is not of the form mentioned in $3^{\circ}1$ or $3^{\circ}2$, then each such E_i is supplied by a STOP-instruction.

4° E_1:F.

By 4°, $\mathcal{D}(Z)$ contains at least one element. $\mathcal{D}(Z)$ is always a finite set because the process of decomposing a given word $Z \in W(\Theta)$ necessarily stops by 2° or by $3^{\circ}3$. Furthermore, by 3° one sees that $\mathcal{D}(Z)$ is uniquely determined.

D e f i n i t i o n 2.2. (Definition of various kinds of decompositions):

1° Every occurrence E_i of some $\mathcal{D}(Z)$, which is supplied by a STOP-instruction, is called a terminal occurrence of that $\mathcal{D}(Z)$.

2° If in a decomposition $\mathcal{D}(Z)$ there is at least one E_i of the form quoted in $3^{\circ}1$ or $3^{\circ}2$, which is supplied by a STOP-instruction, then every such $\mathcal{D}(Z)$ is called a partial decomposition of that $Z \in W(\Theta)$. If no occurrences of some $\mathcal{D}(Z)$ of the form $3^{\circ}1$ or $3^{\circ}2$ are supplied by a STOP-instruction, then every such $\mathcal{D}(Z)$ is called a total decomposition of that $Z\ \mathcal{D}W(\Theta)$.

3^{o} Let J be the set of all instructions for decomposition as given in 1^{o} of the Definition 2.1. If there is some subset K of J omitted by some $\mathcal{D}(Z)$, i.e. if there is always applied a STOP-instruction to every occurrence E_i of that $\mathcal{D}(Z)$ whose leading connective is dealt with in K, then every such $\mathcal{D}(Z)$ is called a relative decomposition, more precisely, a decomposition relatively to the set of instructions L = J - K. If no instructions from J are omitted, then every such $\mathcal{D}(Z)$ is said to be an absolute decomposition of that Z.

An interesting case thereof is obtained when the set L consists of all the instructions for elimination of all sentential connectives so that $K = \{(\forall x_i \text{ e } x), (\exists x_i \text{ e } x)\}$.

There are many possibilities, even mutually interdependent, for a decomposition $\mathcal{D}(Z)$ to be a relative partial decomposition of that $Z \in W(\theta)$. The two marginal possibilities are obtained either when no instructions for decomposition are used at all, i.e. when $L = \emptyset$, so that $\mathcal{D}(Z)$ reduces to the sequence: 1. Z(STOP), or when L = J so that $\mathcal{D}(Z)$ turns out to be an absolute total decomposition of Z.

$\mathcal{D}(Z)$ is essentially a sequence so let d(Z) denote the set whose members are all the occurrences of $\mathcal{D}(Z)$ and let $d^{*}(Z)$ be the set of all the subformulas of Z which occur in d(Z). Obviously, to each occurrence E_i of $\mathcal{D}(Z)$ there corresponds a unique word $E_i^{*} \in W(\theta)$, the word of this E_i. Note that E_i^{*} may be identical with some E_j^{*} though $E_i \neq E_j$ in $\mathcal{D}(Z)$.

In the sequel, if not otherwise stated, we shall usually take into consideration some given relative partial decomposition, written for short RPD. Apparently, the procedure of decomposing a given word $Z \in W(\theta)$ is designed so that it might single out the formulas of $\mathcal{L}$ within the set $W(\theta)$.

T h e o r e m 2.1. A word $F \in W(\theta)$ is a formula of $\mathcal{L}$ if and only if every terminal occurrence in any $\mathcal{D}(F)$, an RPD of F, is a formula.

Note that 3^o1 and 3^o2, when applied to formulas, yield again formulas and that, again by 3^o1, 3^o2 and 3^o3, every occurrence E_i in whatsoever RPD of F will be supplied by a STOP-instruction if it would not be a formula. Hence, each such E_i is a terminal occurrence of any $\mathcal{D}(F)$. An easy proof of the fact that all the occurrences E_i of any $\mathcal{D}(F)$ are formulas of $\mathcal{L}$ if and only if F is a formula of $\mathcal{L}$ is now by induction and therefore can be omitted. Henceforth we have the following

Corollary 2.1. A word $F \in W(\Theta)$ is a formula of $\mathcal{L}$ if and only if all terminal occurrences in its absolute total decomposition $\mathcal{D}^0(F)$ are elementary formulas of $\mathcal{L}$.

The next theorem is stated for formulas, resp. for subformulas, of $\mathcal{L}$ but it applies equally well to words, resp. subwords, of $W(\Theta)$.

Theorem 2.2. Let $\mathcal{D}(F)$ be a given relative partial decomposition of a formula F of $\mathcal{L}$ and d(F) the set of all those occurrences which appear in $\mathcal{D}(F)$. Define in d(F):

(2.1) $E_i \leq E_j$ if and only if the occurrence E_i of $\mathcal{D}(F)$ is a subformula of the occurrence E_j of $\mathcal{D}(F)$.

Then it holds:

(1) by (2.1) there is defined a relation of partial order in d(F),

(2) $E_i \leqslant E_j$ holds in d(F) if and only if there is an RPD $\mathcal{D}(E_j)$, being relative and partial under the same conditions as $\mathcal{D}(F)$, such that the occurrence E_i of $\mathcal{D}(F)$ is a term in $\mathcal{D}(E_j)$ and

(d) $(d(F), \leqslant)$ is a tree.

The proof of (1). $A \leqslant A$ holds by 6^o in Definition 1.1. If $B:U_1\ A\ U_2$ and $C:V_1\ B\ V_2$, then $C:V_1\ U_1\ A\ U_2\ V_2$, i.e. $A \leqslant B$ and $B \leqslant C$ imply $A \leqslant C$. Suppose $A \leqslant B$. Were $A \neq B$, B would be a proper superformula of A and hence B would contain some sign missing in A for, going through the instructions for elimination 3^o1 and 3^o2, one sees that by passing from a formula to its proper subformula some signs of the first one are always missing. But all sings of B must be signs of A if $B \leqslant A$ should hold. Therefore, $A \leqslant B$ and $B \leqslant A$ imply $A = B$.

The proof of (2). Observe that the formula F is the greatest element of the set $d(F)$ when it is ordered by (2.1). Hence, $A \leqslant B$ implies $A \in d(B)$ so A occurs in $\mathcal{D}(B)$ and conversely whenever the conditions on decomposition in $\mathcal{D}(F)$ stay unchanged in $\mathcal{D}(B)$.

The proof of (3). In view of (2), it suffices to show that the set $O(H) = \{X \in d(F): X \geqslant H\}$ is a chain in $(d(F), \leqslant)$ for every $H \in d(F)$. Firstly, let $O(H)$ be a chain. Since $F \in O(H)$, F is a maximal element of $O(H)$, hence the chain $O(H)$ is well-ordered. Finally, $O(H)$ is a chain indeed because for any two occurrences A and B in $O(H)$, $A \neq B$, it holds by (2): if not $A \leqslant B$, then the occurrence A does not appear in $\mathcal{D}(B)$. Hence, the occurrence H cannot appear in $\mathcal{D}(B)$ too because H is a subformula of the occurrence A. Therefore, either $A \leqslant B$ or $B \leqslant A$ holds for any two occurrences A and B of $O(H)$.

E x a m p l e 2.1. Let us determine the absolute total decomposition $\mathcal{D}^0(F)$ of the formula $F: (\forall x_1((A) \,\&\, ((A) \Longrightarrow (B)))) \Longrightarrow ((A) \,\&\, ((A) \Longrightarrow (B)))$, where $A: P_2^2(x_1 C_3)$ and $B: P_2^2(x_1 C_2)$.

Here is the corresponding $\mathcal{D}(F)$:

1.	$(\forall x_1((A) \,\&\, ((A) \Longrightarrow (B)))) \Longrightarrow ((A) \,\&\, ((A) \Longrightarrow (B)))$	$(\Rightarrow e\ x)$
2.	$\forall x_1((A) \,\&\, ((A) \Longrightarrow (B)))$	$(\forall x_1\ e\ x)$
3.	$(A) \,\&\, ((A) \Longrightarrow (B))$	$(\&\ e\ x)$
4.	$(A) \,\&\, ((A) \Longrightarrow (B))$	$(\&\ e\ x)$
5.	A	(STOP)
6.	$(A) \Longrightarrow (B)$	$(\Longrightarrow e\ x)$
7.	A	(STOP)
8.	$(A) \Longrightarrow (B)$	$(\Longrightarrow e\ x)$

9. A (STOP)

10. B (STOP)

11. A (STOP)

12. B (STOP)

E x a m p l e 2.2. The next Diagram 2.1 represents the tree $(d^{o}(F), \leqslant)$ of the formula F of Example 2.1. Instead of quoting all the occurrences of $\mathcal{D}^{o}(F)$ at the appropriate knots of the tree, these knots are rather labeled by natural number i under which a particular E_i appears in $\mathcal{D}^{o}(F)$. The terminal occurrences of $\mathcal{D}^{o}(F)$ are shadowed in the diagram.

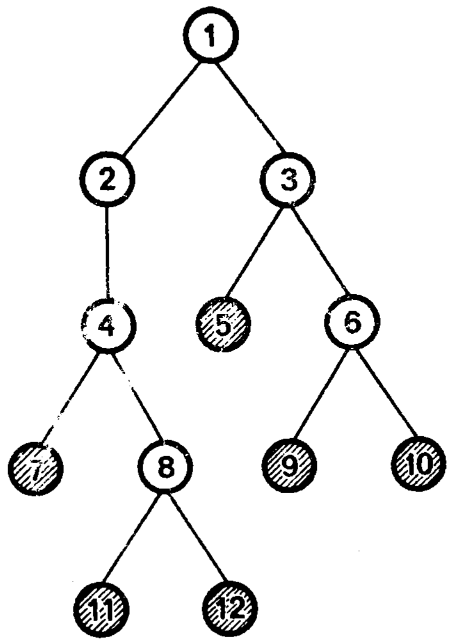

Diagram 2.1

3. Properties of decompositions

Here there will be stated some equivalent conditions for an RPD of a formula F of $\mathcal{L}$ to be determined by all its terminal occurrences. The following two definitions enable us to formulate our next theorem.

D e f i n i t i o n 3.1. Let $\mathcal{D}(F)$ be a given RPD of a formula F of $\mathcal{L}$. Let $H_1,\ldots,H_n$ be all terminal occurrences of $\mathcal{D}(F)$ and let $G_1,\ldots,G_k$ be the corresponding subformulas[1] $H_1^*,\ldots,H_n^*$ of F. Then we say that the formula F is relatively[2] composed of the occurrences $H_1,\ldots,H_n$ of its subformulas $G_1,\ldots,G_k$. Let us denote any such decomposition by $\mathcal{D}_F(H_1,\ldots,H_n)$ and the corresponding set of all occurrences by $d_F(H_1,\ldots,H_n)$.

D e f i n i t i o n 3.2. Let $\gamma^o(F)$ be that tree which, in view of (3) of Theorem 2.2, corresponds to the total absolute decomposition $\mathcal{D}^o(F)$ of the formula F of $\mathcal{L}$. An initial segment of $\gamma^o(F)$, determined by the occurrences $H_1,\ldots,H_n$ of subformulas $G_1,\ldots,G_k$ of F, is the set $\gamma_F(H_1,\ldots,H_n)$ defined as follows:

$$\gamma_F(H_1,\ldots,H_n) = \{X \in \gamma^o(F) : X \geqslant H_i \text{ for some } H_i \text{ in } \gamma^o(F)\},$$

where the order relation is that of (2.1).

It can be easily shown that the set $\gamma_F(H_1,\ldots,H_n)$, when ordered by the relation $\leqslant$ in (2.1), is a tree because the set $O(G) = \{X \in \gamma_F(H_1,\ldots,H_n) : X \geqslant G\}$ is a chain for any $G \in \gamma_F(H_1,\ldots,H_n)$. Indeed, since $G \geqslant H_i$ for some H_i and $A,B \in O(G)$ by supposition, hence $A \geqslant H_i$ and $B \geqslant H_i$ by transitivity. The proof now proceeds as in the proof of (3) in Theorem 2.2.

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1) Cf. Example 2.1 and Diagram 2.1 where there are six different terminal occurrences of only two formulas A and B.

2) I.e., relatively to the same set L of instructions for decomposition to which $\mathcal{D}(F)$ is known to be a relative partial decomposition. Of course, any chosen partial decomposition remains unchanged throughout this definition.

T h e o r e m 3.1. The following three assertions are mutually equivalent for any formula F of $\mathcal{L}$ whenever $n \geqslant 2$:

(1) The formula F is relatively composed of the occurrences $H_1,\ldots,H_n$ of its subformulas $G_1,\ldots,G_k$;

(2) $d_F(H_1,\ldots,H_n) = \gamma_F(H_1,\ldots,H_n)$;

(3) The occurrences $H_1,\ldots,H_n$ of subformulas $G_1,\ldots,G_k$ of F from a maximal antichain in $\gamma^o(F)$.

The case $n = 1$ should be exempt from the formulation because (3) obviously does not hold for any singleton $\{H_1\}$. However, the equivalence between (1) and (2) still holds in this case. This follows immediately from the fact that the corresponding tree reduces to a single chain if $n = 1$.

The proof consists of proofs of the threee consecutive implications.

(1) implies (2). There are given an RPD $\mathcal{D}_F(H_1,\ldots,H_n)$, where in $H_1,\ldots,H_n$ are all its terminal occurrences, and two sets: $d_F(H_1,\ldots,H_n)$ and $\gamma_F(H_1,\ldots,H_n)$. Let $X \in \gamma_F(H_1,\ldots,H_n)$, i.e. let $X \geqslant H_i$ for some $H_i \in$ $\in d_F(H_1,\ldots,H_n)$. Since the occurrence X, being a superformula of H_i, necessarily belongs to the decomposition $\mathcal{D}_F(X_1,\ldots,H_n)$, hence $X \in d_F(H_1,\ldots,H_n)$. Conversely, let $X \in d_F(H_1,\ldots,H_n)$, i.e. let X appear in $\mathcal{D}_F(H_1,\ldots,H_n)$. Since X is a superformula of some terminal occurrence H_i of $\mathcal{D}_F(H_1,\ldots,H_n)$, hence the RPD $\mathcal{D}(X)$ contains H_i and henceforth it holds by (2) of Theorem 2.2, that $X \geqslant H_i$. Thus $X \in \gamma_F(H_1,\ldots,H_n)$.

(2) implies (3). Observe that any set $\{H_1,\ldots,H_n\}$ of some, not necessarily all, terminal occurrences in a given RPD $\mathcal{D}(F)$ always forms an antichain when ordered by the relation (2.1) for $H_i < H_j$ implies H_j is not a minimal element of the set $d_F(H_1,\ldots,H_n)$ when ordered by (2.1). It remains to prove that $H_1,\ldots,H_n$ is a maximal antichain of $\gamma^o(F)$ if $\gamma_F(H_1,\ldots,H_n) = d_F(H_1,\ldots,H_n)$. Consider the set $\{H_1,\ldots,H_n,K\}$ for some $K \in \gamma^o(F)$ and such that $K \neq$ $\neq H_1,\ldots,K \neq H_n$. If $K \in \gamma_F(H_1,\ldots,H_n)$, then $K \geqslant H_i$ for

some H_i so that $\{H_1,\dots,H_n,K\}$ is not an antichain. If $K \notin \mathcal{T}_F(H_1,\dots,H_n)$, then $K \notin d_F(H_1,\dots,H_n)$ for $\mathcal{T}_F(H_1,\dots,H_n) = d_F(H_1,\dots,H_n)$. Hence, $K \in \mathcal{T}^o(F)$ must be a proper subformula of some subformula H_i of F, i.e. K belongs to some $\mathcal{D}(H_i)$ and henceforth it holds $K < H_i$ for some H_i by (2) of Theorem 2.2. Therefore, no proper superset of the set $\{H_1,\dots,H_n\}$ is an antichain of $\mathcal{T}^o(F)$.

(3) implies (1). Let us determine the absolute total decomposition $\mathcal{D}^o(F)$ of the formula F of $\mathcal{L}$. Replace therein each instruction of every occurrence $H_1,\dots,H_n$ by a STOP-instruction unless it is already attached there. Next erase in $\mathcal{D}^o(F)$ all successors of any among the occurrences $H_1,\dots,H_n$, i.e. all the occurrences X with $X < H_i$ for some H_i. What is left over is $\mathcal{D}_F(H_1,\dots,H_n)$. Were this not the case, the set S obtained from $\mathcal{D}^o(F)$ by the above deletion would contain at least one element K such that $K \notin d_F(H_1,\dots,H_n)$, i.e. such that for no H_i it would hold $K \geqslant H_i$. Consequently, K must be incomparable with every occurrence $H_1,\dots,H_n$ because the case $K < H_i$, for some H_i, is ruled out by construction and the case $K \geqslant H_i$, for some H_i, by supposition. Hence, $H_1,\dots,H_n$ can be extended to an antichain $\{H_1,\dots,H_n,K\}$.

E x a m p l e 3.1. Apparently, each RPD of some given formula F of $\mathcal{L}$ determines in $\mathcal{T}^o(F)$ the maximal antichain of all of its terminal occurrences, but, in view of Theorem 3.1, the converse as also true: each maximal antichain in $\mathcal{T}^o(F)$, say $\{H_1,\dots,H_n\}$, determines a RPD of F, namely the decomposition $\mathcal{D}_F(H_1,\dots,H_n)$, which is obtained from $\mathcal{T}_F(H_1,\dots,H_n)$ according to (2) in Theorem 3.1.

Therefore, in order to determine all possible RPD's of a formula F of $\mathcal{L}$, it is sufficient and necessary to list all the maximal antichains in $\mathcal{T}^o(F)$ increased by the singleton $\{F\}$. Here is the list of all maximal antichains of $\mathcal{T}^o(F)$ of the formula F of Example 2.1 (see also Example 2.2 and Diagram 2.1): $\{2,3\}$, $\{4,3\}$, $\{4,5,6\}$, $\{2,5,6\}$, $\{2,5,9,10\}$, $\{4,5,9,10\}$, $\{7,8,3\}$, $\{7,8,5,6\}$, $\{7,8,5,9,10\}$, $\{7,11,12,3\}$, $\{7,11,12,5,6\}$, $\{7,11,12,5,9,10\}$.

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