DEMONSTRATIO MATHEMATICA
Vol. XV No 2 1982

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VECTOR FIELDS ON R^2 WITHOUT OSCILLATIONS ARE GENERIC

This paper deals with a problem which has arisen from one of the central research themes in dynamical systems in the last twenty years, i.e. from Ω-stability and structural stability. This problem can be formulated as follows: does every nonwandering point of an Ω-stable or structural stable dynamical system belong to the closure of the union of all its periodic orbits. So far an answer has been given by C.Pugh (see [5]) for dynamical systems of compact manifolds. We give a partial answer to the noncompact case i.e. we prove that there exists a residual set of vector fields on R^2 which satisfy $\Omega = \mathrm{Per}$. Thus this property is a necessary condition for Ω-stable or for structural stable vector fields on R^2. For the other consequences of this theorem see [2].

We introduce the following notation:

$H^r(R^2)$ - the space of C^r $(r \geq 1)$ vector fields on R^2 which generate flows endowed with the strong (Whitney) C^r-topology,

Y - an element of $H^r(R^2)$,

$T^Y : R^2 \times R \to R^2$ - the flow of the complete vector field Y,

$T^Y(x) = \{T^Y(x,t) : t \in R\}$ - the Y-orbit of x,

$T^Y_+(x) = \{T^Y(x,t) : t \geq 0\}$,

$T^Y_-(x) = \{T^Y(x,t) : t \leq 0\}$,

$T^Y(A) = \bigcup\{T^Y(x,t) : x \in A, t \in R\}$ for $A \subset R^2$,

$\omega(T^Y(x)) = \{y \in R^2 : \exists\, t_n \to +\infty \;.\ni.\; T^Y(x,t_n) \to y\}$,

$\alpha(T^Y(x)) = \{y \in R^2 : \exists\, t_n \to -\infty \;.\ni.\; T^Y(x,t_n) \to y\}$,

$W_Y^{u(s)}(x)$ - the global unstable (stable) manifold of a fixed point x,

$\Omega(Y)$ - the set of all nonwandering points of Y,

Per Y - the union of all closed orbit and all critical points of Y,

$[a,b]$ - segment of the transverse section with ends a,b,

(a,b) - segment of the transverse section without ends a,b,

For definition of $W_Y^{u(s)}(x)$, $\Omega(Y)$ see [3].

Definition 1. a) $T_{\pm}^{Y}(x)$ is bounded if it is contained in some compact set $C \subset R^2$.

b) $T_{\pm}^{Y}(x)$ escapes to infinity if for each compact set $C \subset R^2$ there exists a point $y \in T_{\pm}^{Y}(x)$ for which $T_{\pm}^{Y}(y) \cap C = \emptyset$.

c) $T_{\pm}^{Y}(x)$ oscillates if it is neither bounded nor escapes to infinity.

Definition 2. The oscillation region of Y will be a region $E \subset R^2$ with the following properties:

(i) E is simply connected and unbounded

(ii) FrE (the boundary of E) has at most countable number of components, each of them is unbounded and separates R^2.

(iii) a component of FrE is either an orbit which escapes to infinity for $t \to -\infty$ and for $t \to +\infty$ or a set consisting of a saddle p, a branch of $W_Y^u(p)$ which escapes to infinity for $t \to +\infty$ and a branch of $W_Y^s(p)$ which escapes to infinity for $t \to -\infty$.

(iv) if $x \in$ FrE and $Y(x) \neq 0$ then for any small enough transverse section P_1xP_2 to Y at x there exists a sequence $x_n \in P_1xP_2$, $n = 1,2,\dots$, such that $x_n \to x$. For any n, x_{n+1} lies between x_n and x, $T^Y(x_n,t_n)$ is the first common point of $T_+^Y(x_n)$ and $P_1xP_2 \cap E$, $T^Y(x_n,t_n) \to y$ and $t_n \to +\infty$. Moreover the regions A_n are bounded by a sum of arcs $a_n = \{T^Y(x_n,t) : 0 \leq t \leq t_n\}$ and $[x_n, T^Y(x_n,t_n)]$, form an increasing sequence and $\bigcup_{n=1}^{\infty} A_n = E$.

The properties of oscillation regions are described in [1].
$G_1^r(R^2) = \{Y \in H^r(R^2)$: all critical points of Y are hyperbolic$\}$.

We denote the Kupka-Smale vector fields in $H^r(R^2)$ by $G_2^r(R^2)$ i.e. for $Y \in G_2^r(R^2)$:

(i) all critical points and closed orbits are hyperbolic.

(ii) the unstable and stable manifolds of saddles are in general position i.e. have no common points.

It is clear that $G_2^r(R^2) \subset G_1^r(R^2)$. $G_1^r(R^2)$ and $G_2^r(R^2)$ are residual subset of $H^r(R^2)$. Moreover $G_1^r(R^2)$ is open in $H^r(R^2)$ (see [4]).

T h e o r e m 1. If $Y \in G_2^r(R^2)$ then $\Omega(Y) = \mathrm{Per}\, Y$ iff Y has no oscillation regions.

P r o o f . Suppose that $Y \in G_2^r(R^2)$ has an oscillation region E. By Th.2 [1] the boundary of E is contained in $\Omega(Y)$. Thus there exists an orbit which escapes to infinity for $t \to +\infty$ or for $t \to -\infty$. This implies that there exists an orbit of nonwandering points which does not belong to Per Y. We get a contradiction which finishes the proof of necessity. Assume that $Y \in G_2^r(R^2)$ has no oscillation regions. Suppose that $\Omega(Y) \not\subset \mathrm{Per}\, Y$ (converse inclusion is always true). Let $x \in \Omega(Y) - \mathrm{Per}\, Y$. Therefore there exist a transverse section P_1xP_2 to Y at x, a sequence of points $x_n \in P_1xP_2$, a sequence $t_n \to +\infty$ such that $x_n \to x$, $T^Y(x_n,t_n)$ is the first common point of $T_+^Y(x_n)$ and P_1xP_2 and $T^Y(x_n,t_n) \to y$. Moreover for any n, x_{n+1} lies between x_n and x. Let A_n be a region bounded by a sum of arcs $a_n = \{T^Y(x_n,t) : 0 \leqslant t \leqslant t_n\}$ and $[x_n, T^Y(x_n,t_n)]$. Then the regions A_n form an increasing sequence. Let us denote $\bigcup_{n=1}^{\infty} A_n$ by E. Now we describe the orbit $T^Y(x)$. By Th.3 and Remark 1 [1] $T^Y(x)$ cannot oscillate. Thus $T_+^Y(x)$ either is bounded or escapes to infinity. If $T_+^Y(x)$ is bounded then using the assumption that Y is Kupka-Smale vector field and Lemma 1 [1], we get that $\omega(T^Y(x))$ is either a closed hyperbolic stable orbit or a critical hyperbolic point: sink or saddle. Because $x \in \Omega(Y)$, so $\omega(T^Y(x))$ can be neither a stable closed orbit nor a sink. Therefore if $T^Y(x)$ is bounded, then there exists a saddle p such that $\omega(T^Y(x)) = \{p\}$ and $x \in W^s_Y(p)$. It is not difficult to see that $T^Y(z)$ - a branch of $W^u_T(p)$ is also a set of accumulation points of a_n, $n = 1,2,\ldots$. Hence $T^Y(z) \subset \mathrm{Fr}E$. Using again the argument that $Y \in G^r_2(R^2)$ we get that $T^Y(z)$ escapes to infinity for $t \to +\infty$. Analogously one can prove that $T^Y_-(x)$ either escapes to infinity for $t \to -\infty$ or $\alpha(T^Y(x))$ is a saddle q and $T^Y(\bar{z})$ - a branch of $W^s_Y(q)$ is contained in the same component of $\mathrm{Fr}E$ as $T^Y(x)$. Therefore E is simply connected and unbounded region. Moreover any component of $\mathrm{Fr}E$ is like in Def.2. Thus E is an oscillation region of Y and we get a contradiction which finishes the proof.

Now we formulate and prove some technical lemmas which we need to proof of next theorem.

D e f i n i t i o n 3. A flowbox for $Y \in H^r(M)$ we call a closed quadrilateral $F \subset R^2$ containing no restpoints of Y, with two (opposite) edges $S_\pm$ transverse to Y and the other two edges Y-orbit segments, each joining an endpoint of S_+ to an endpoint of S_-.

We call S_+ the entrance set and S_- the exit set of Y. In what follows, $T^Y[x,y]$ denotes the closed Y-orbit segment from x to y.

L e m m a 1. Suppose $S_\pm$ are transverse sections to $Y \in H^r(R^2)$ such that:

(i) S_+ is compact

(ii) the forward semi-orbits of each point x of S_+ intersects S_-, and the first intersection $P_Y(x)$ is interior to S_-.

Then the Poincaré map $P_Y : S_+ \to \mathrm{int}\, S_-$ is as smooth as Y, and there exists a compact-open C^r-neighbourhood U of Y, concentrated on a neighbourhood of the union of orbit segments $T^Y[x,P_Y(x)]$, $x \in S_+$, such that $P_Z : S_+ \to \mathrm{int}\, S_-$ is well-defined and varies C^r-continuously with $Z \in U$.

For proof of this lemma see [3].

To isolate a given positive or negative semi-orbit from others, and control its behavior under perturbation, we construct a positive (resp. negative) tower, defined as a finite or infinite sequence of flowboxes $T = \{F_1, F_2, \dots\}$ (resp. $T = \{F_{-1}, F_{-2}, \dots\}$) satisfying, for each $i = \pm 1, \dots$

(i) $F_i \cap F_j = \emptyset$ unless $|i-j| \leq 1$

(ii) $F_i \cap F_{i+1} = S_+(i+1)$ (resp. $= S_-(i)$)

(iii) $S_+(i+1) \subset \operatorname{int} S_-(i)$ (resp. $S_-(i-1) \subset \operatorname{int} S_+(i)$)

(iv) T forms a locally finite family in R^2.

The floors of the tower are the transverse edges $S_\pm(i)$ and its height is the number h of flowboxes (finite or infinite). A positive semi-orbit can enter a positive tower T only via the bottom floor $S_+(1)$. An orbit can leave T via some set $S_-(i) - S_+(i+1)$, $i < h$, or else it crosses all floors of T before leaving T. Given a floor S of the positive tower T, we denote by $W(S,T,Y)$ the set of all points $x \in S$ which cross all subsequent floors of T before leaving T. Note that for any tower T and any floor S, the set $W(S,T,Y)$ is a nonempty closed interval (possibly a point) and when T has infinite height, every semi-orbit starting from $W(S,T,Y)$ escapes to infinity inside T. Note that if T is a tower for Y, it need not be a tower for vector fields Z near Y, since the edges of the flowboxes need not be Z-orbits. Nevertheless, if Z is near Y at points in T, we can still define the set $W(S,T,Z)$ as the set of points in S whose Z-semi-orbit crosses all subsequent floors of T in succession before leaving T. The following lemma is a persistence theorem for $W(S,T,Y)$. In (ii), $|J|$ denotes the length of the interval J. The result is formulated for positive towers, but analogous lemma is true for negative towers.

L e m m a 2. Suppose $T = \{F_1, F_2, \dots\}$ is a positive tower for Y, and S is a floor of T.

(i) There exists a strong C^0-neighbourhood U of Y (actually an intersection of compact-open neighbourhoods concentrated on any neighbourhood of the flowboxes of T) such that $W(S,T,Z) \neq \emptyset$ for $Z \in U$.

(ii) Given $\delta > 0$ there exists a strong C^1-neighbourhood U of Y (an intersection of compact-open C^1-neighbourhoods concentrated on any neighbourhood of the flowboxes of T) such that for every $Z \in U$ $(1-\delta)|W(S,T,Y)| \leqslant |W(S,T,Z)| \leqslant \leqslant (1+\delta)|W(S,T,Y)|$.

Proof. Note that $W(S,T,Y)$ is a nested intersection of intervals $W_i(S,T,Y) \subset S$, defined as the set of points whose semi-orbit crosses at least i succesive floors in T. The set in which these semi-orbits cross the i^{th} floor S_i is an interval $J_i \subset S_i$, and the subset of J_i corresponding to $W_{i+1}(S,T,Y)$ is the preimage by the Poincaré map of S_{i+1}. Note that this set is interior to J_i, by condition (iii) of the definition of tower.

For definiteness, denote by P_Y^{ij} $(i > j)$ the inverse Poincaré map of Y, from S_i to S_j. Thus

$$P_Y^{ij} = P_Y^{i\ i-1} \circ P_Y^{i-1\ i-2} \circ \dots \circ P_Y^{j+1j}.$$

Now, by Lemma 1 there are C^0 estimates concentrated on any neighbourhood of $F_1 \cup \dots \cup F_j$ which guarantee that Z near Y defines a corresponding Poincaré map $P_Z^{ij} : S_i \rightarrow S_j$ which is C^0 near P_Y^{ij}, and C^r near if Z is C^r-near Y on these sets. In particular, it is easy to see that by estimates on the first i flowboxes we can insure that each Poincaré map P_Z^{j+1j} maps S_{j+1} into the interior of S_j, for all $j \leqslant i$. This guarantees in particular that

$$J_i(Z) = P_Z^{10} \circ P_Z^{21} \circ \dots \circ P_Z^{i\ i-1}(S_i)$$

is nonempty compact interval interior to $J_{i-1}(Z)$, and hence the finite intersection property gives us conclusion (i).

To prove conclusion (ii), we note that for any monotone C^1 map f between intervals the length of the image of a subinterval I is

$$|f(I)| = \int_I |f'(x)|\,dx.$$

Thus, if two maps $f,g : J \to J'$ satisfy an estimate of the form

(a) $|(1-\alpha)|f'(p)| \leqslant |g'(p)| \leqslant (1+\mathcal{L})|f'(p)|$ $\forall\, p \in J$ then any interval $I \subseteq J$ satisfies

(b) $(1-\mathcal{L})|f(I)| \leqslant |g(I)| \leqslant (1+\mathcal{L})|f(I)|$.

Lemma 1 tells us that for any $\mathcal{L} < 1$ we can obtain (a) for $f = P_Y^{ii-1}$, $g = P_Z^{ii-1}$ by controlling the C^1 distance between Y and Z near F_i. Thus, given $\delta > 0$, pick $\mathcal{L}_i > 0$ such that $(1-\delta) < \prod_{i=1} (1-\mathcal{L}_i)$, $(1+\delta) > \prod_{i=1} (1+\mathcal{L}_i)$ and then make C^1 estimates on $Y|F_i$ which insure that (a) (hence (b)) holds for each i with $\mathcal{L} = \mathcal{L}_i$, $f = P_Y^{ii-1}$, $g = P_Z^{ii-1}$. By induction, we obtain the analogue of conclusion (ii) for each set $W_i(S,T,Y)$ and $W_i(S,T,Z)$, $i \leqslant h$, and hence (ii).

C o r o l l a r y . If semi-orbit $T_+^Y(x)$ escapes to infinity, there exists an infinite positive tower T with $x \in S$, such that $W(S_1,T,Y) = \{x\}$, and hence for any Z whose restriction to T is sufficiently C^r-near $Y|T$, there exists a unique Z-semi-orbit which escapes to infinity inside T.

T h e o r e m 2. Let S be a transverse section to $Y \in G_1^r(R^2)$. For any neighbourhood $U \subset G_1^r(R^2)$ of Y there exists an open set $V \subset U$ such that if $Z \in V$, then Z has no oscillation regions whose boundary intersect int S.

P r o o f . Let U be a given neighbourhood of Y, P, Q denote ends of S. Suppose that E is an oscillation region of Y and $\mathrm{Fr}E \cap \mathrm{int}\, S \neq \emptyset$. Let $a \in \mathrm{Fr}E \cap \mathrm{int}\, S$. It follows from the definition of oscillation regions that $\mathrm{Fr}E \cap \mathrm{int}\, S = \{a\}$ and $T^Y(a)$ escapes to infinity either for $t \to +\infty$ or for $t \to -\infty$. Assume that $T^Y(a)$ escapes to infinity for $t \to -\infty$. Let $Q \in E$, $p \neq q$, $p,q \in T_+^Y(a)$, S_1, S_2 be small enough transverse sections at p, q such that

$S_2 \subset T_+(S_1)$, $C = \overline{T_+^Y(S_1) \cap T_-^Y(S_2)}$. Because $p,q \in \mathrm{Fr}E$ then there exists a sequence of points $q_n \in S_2 \cap E$, $n = 1,2,\dots$, such that $q_n \to q$, q_{n+1} lies between q_n and q, $T^Y(q_n,t_n)$ is the first point in which $T_+^Y(q_n)$ meets $S_1 \cap E$, $T^Y(q_n,t_n) \to p$ and $t_n \to +\infty$. Moreover there exists a sequence s_n such that $0 < s_n < t_n$, $s_n \to +\infty$ and $T^Y(q_n,s_n)$ is the first common point of $T_+^Y(q_n)$ and int S. It is clear that $T^Y(q_n,s_n) \to a$. Let Y' be a vector field with a support C. Y' is transverse to Y and Y' leads in the direction of E. There exist $t > 0$ and $q_n \in S_2 \cap E$ such that $X = Y + tY' \in U$ and $q_n \in T_+^X(a)$. Because $X = Y$ outside C then $T^X(q_n,t) = T^Y(q_n,t)$ for $0 \leqslant t \leqslant t_n$. Thus $T_+^X(a)$ intersects int S at a and at $b = T^X(q_n,s_n)$. There exists a neighbourhood $V \subset U$ of X with the property that if $Z \in V$ then there exist $a^Z, b^Z \in$ int S such that the arc $T^Z[a^Z,b^Z]$ lies in the ε-neighbourhood of $T^X[a,b]$. By Lemma 2 and Corollary we can choose V in such way that $T_-^Z(a^Z)$ escapes to infinity for $t \to -\infty$. It follows from the properties of $T^Z(a^Z)$ that (a^Z,Q) is contained in a closed region G_1 bounded by $T^Z[a^Z,b^Z]$ and $[a^Z,b^Z] \subset S$. Hence if $x \in [a^Z,Q]$ then $T_+^Z(x) \subset \overline{G}_1$, so $T_+^Z(x)$ is bounded for $Z \in V$. This implies that the boundary of oscillation regions of $Z \in V$ cannot intersect $[a^Z,Q]$ (recall that each component of the boundary of oscillation regions is an unbounded, invariant set which separates R^2). Suppose that V does not satisfy thesis of Th.2. Therefore there exists $Y \in V$ which has an oscillation region E such that $\mathrm{Fr}E \cap \mathrm{int}\, S \neq \emptyset$ (i.e. $\mathrm{Fr}E \cap (P,a^Y) \neq \emptyset$). Let $c = \mathrm{Fr}E \cap (P,a^Y)$. By Def.2 $T^Y(c)$ escapes to infinity for $t \to +\infty$ or for $t \to -\infty$. Assume that $T^Y(c)$ escapes to infinity for $t \to +\infty$. We repeat the construction made for previous Y. Let $p,q \in T_-^Y(c)$, $p \neq q$. S_1,S_2,C,Y' are defined analogously. We change only the direction of Y'. Now Y' leads in the opposite direction to E. This construction together with Lemma 2 and

Corollary imply that there exists an open set $V_1 \subset V$ with property if $Z \in V_1$ then there exist $c^Z, d^Z \in (P, a^Z)$, $c^Z \neq d^Z$, such that $T^Z_+(c^Z)$ escapes to infinity for $t \to +\infty$ and $T^Z_-(c^Z)$ intersects int S at $d^Z \in (P, c^Z)$. Thus a segment (P, c^Z) is contained in a closed region G_2 bounded by $T^Z[d^Z, c^Z]$ and $[d^Z, c^Z]$. So, if $x \in [P, c^Z]$ then $T^Z_-(x) \subset \bar{G}_2$ for $Z \in V_1$. Hence the boundary of oscillation region of $Z \in V_1$ cannot intersect $[P, c^Z] \cup [a^Z, Q]$. It remains to prove that $(c^Z, a^Z) \cap \mathrm{Fr}E = \Phi$ for any oscillation region E of $Z \in V_1$. Suppose that for some $Y \in V_1$ $\mathrm{Fr}E \cap (c^Y, a^Y) \neq \emptyset$. Hence $P \in E$ or $Q \in E$. If $P \in E$ then there exists a sequence of arcs $a_n = \{T^Y(x_n, t) : 0 \leq t \leq t_n\} \subset E$ such that $x_n \in \mathrm{int}\, S$, $T^Y(x_n, t_n) \in \mathrm{int}\, S$ and $x_n \to x$, $T^Y(x_n, t_n) \to x$ (by x we denote the unique common point of FrE and (c^Y, a^Y)). Therefore $a^Y \in (x_k, P) \subset A_k \subset E$ for some k. Because $T^Y_-(a^Y)$ escapes to infinity for $t \to -\infty$, so $T^Y_-(a^Y)$ intersects the arc $a_k = \{T^Y(x_k, t) : 0 \leq t \leq t_k\} \subset \mathrm{Fr}A_k$ which is impossible. By the same arguments Q cannot belong to E. This implies that V_1 satisfies all necessary conditions.

L e m m a 3. For any compact set $K \subset R^2$ and open set $U \subset G^r_1(R^2)$ there exist open set $V \subset U$, sets $S_1, \dots, S_k$ and the points $p^Y_1, \dots, p^Y_n \in \mathrm{int}\, K$ such that:

(i) $S_1, \dots, S_k$ are transverse sections to any $Y \in V$,

(ii) $p^Y_1, \dots, p^Y_n$ are critical hyperbolic fixed points of $Y \in V$

(iii) if $x \in K - \{p^Y_1, \dots, p^Y_n\}$, $Y \in V$ then there exists S_i such that $T^Y(x) \cap \mathrm{int}\, S_i \neq \emptyset$.

Proof of this lemma is very easy, so we leave it to the reader.

T h e o r e m 3. For any compact set $K \subset R^2$ there exists a dense and open set $V \subset G^r_1(R^2)$ such that if $Y \in V$ then Y has no oscillation regions with boundary intersecting K.

P r o o f . Let us be given $K \subset R^2$ and open set $U \subset G^r_1(R^2)$. We start with showing that we may choose

$x \in \mathrm{Fr}E \cap K$ to be regular. If $x \in \mathrm{Fr}E \cap K$ and $Y(x) = 0$ then x is a saddle. Because $x \in \mathrm{int}\, K$ then there exists $y \in W^u_Y(x)$ such that $y \in \mathrm{Fr}E \cap K$. Thus we can assume that $Y(x) \neq 0$. It follows from Lemma 3 (iii) that $T^Y(x) \cap \mathrm{int}\, S_i \neq \emptyset$ for some $1 \leqslant i \leqslant k$.

Suppose that $i = 1$. Using Th.2 we get an open set $V_1 \subset V$ with the property that if $Y \in V_1$ then Y has no oscillation regions whose boundary intersects $\mathrm{int}\, S_1$. Now if we apply again Th.2 we get open sets $V_i \subset V_{i-1} \subset V_1 \subset U$ which satisfy the thesis for $\mathrm{int}\, S_1 \cup \ldots \cup \mathrm{int}\, S_i$, $1 \leqslant i \leqslant k$. Thus by Lemma 3 (iii) the set V_k satisfies Theorem 3.

T h e o r e m 4. The set $\{Y \in H^r(R^2) : \Omega(Y) = \mathrm{Per}\, Y\}$ is residual $(r \geqslant 1)$.

P r o o f . Let K_m be a sequence of compact subsets of R^2 such that $\bigcup_{m=1}^{\infty} K_m = R^2$. Denote by $C(K_m)$ the subset of $G^r_1(R^2)$ with the property that if $Y \in C(K_m)$ then Y has no oscillation regions whose boundary intersect K_m. By Th.3 $C(K_m)$ contains an open and dense subset for any $m \in N$. Thus $\bigcap_{m=1}^{\infty} C(K_m)$ contains a residual subset. It is not difficult to see that if $Y \in \bigcap_{m=1}^{\infty} C(K_m)$ then Y has no oscillation regions. This sentence and Th.1 imply that $\Omega(Y) = \mathrm{Per}\, Y$ for any $Y \in F^r(R^2) = \bigcap_{m=1}^{\infty} C(K_m) \cap G^r_2(R^2)$. Because $F^r(R^2)$ is a residual subset of $G^r_1(R^2)$, $G^r_1(R^2)$ is an open and dense in $H^r(R^2)$, so $F^r(R^2)$ is a residual subset of $H^r(R^2)$.

C o r o l l a r y . If $Y \in H^r(R^2)$ is Ω-stable or structurally stable then $\Omega(Y) = \mathrm{Per}\, Y$.

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Received November 21, 1979.