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SMOOTH MAPPINGS OF SPACES WITH THE SMOOTHNESS

In the present paper we consider the properties of smooth mappings in the category of spaces with smoothness in the Postnikov sense. The smooth mappings are the morphisms in this category.

Definition 1. Let (M, topM) be a certain topological space. A family $\mathcal{F}(M)$ of real functions defined on the set M and fulfilling the following conditions:

1. if $\omega : R^k \to R$ is the function of the class C^∞ and $\alpha_1,\dots,\alpha_k \in \mathcal{F}(M)$, then the function $\omega(\alpha_1(\cdot),\dots,\alpha_k(\cdot)):M \to R$ belongs to $\mathcal{F}(M)$;

2. any function coinciding in a certain neighbourhood of any point $p \in M$ with the function $\mathcal{F}(M)$ belongs to $\mathcal{F}(M)$;

will be called the smoothness in the Postnikov sense on the topological space (M, topM).

Definition 2. The triple $\mathcal{M} = (M, topM, \mathcal{F}(M))$ will be called the space with the smoothness in the Postnikov sense (briefly - the space with smoothness), where $\mathcal{F}(M)$ is the smoothness in the Postnikov sense on the topological space (M, topM).

Definition 3. Let $C \subset R^M$ be any family of functions defined on the set M. Then we denote by scC the set defined by the formula

$$scC = \left\{f : f = \varphi(\alpha_1(\cdot),\dots,\alpha_k(\cdot)),\ \alpha_1,\dots,\alpha_k \in C \text{ and } \varphi \in C^\infty(R^k)\right\}.$$

D e f i n i t i o n 4. Let $C \subset R^M$ and $(M, \mathrm{top}M)$ be any topology on the set M. A function β belong to $C_M(\mathrm{top}M)$ if and only if $\beta : M \to R$ and for every point $p \in M$ there exists $U \in \mathrm{top}M$ and function $\alpha \in C$ such that $p \in U$ and $\beta|U = \alpha|U$.

Let $\mathcal{M}$ and $\mathcal{N}$ be spaces with smoothness. A mapping f of the set M into the set N fulfilling the following conditions:

(a) f maps topM into topN continuously;

(b) $\beta \circ f \in \mathcal{F}(M)$ for every function $\beta \in \mathcal{F}(N)$,

is called a smooth mapping of the space $\mathcal{M}$ into the space $\mathcal{N}$. We will write

(1) $f : \mathcal{M} \to \mathcal{N}$ or $f : (M, \mathrm{top}M, \mathcal{F}(M)) \to (N, \mathrm{top}N, \mathcal{F}(N))$.

To verify the smoothness of mappings between spaces with smoothness we will often use the following lemma, which describes the smoothness in the operations sc and the localization.

L e m m a . Let $(M, \mathrm{top}M, \mathcal{F}(M))$ be a space with smoothness and $(N, \mathrm{top}N)$ be any topological space. If a map f of the topological space $(M, \mathrm{top}M)$ into the topological space $(N, \mathrm{top}N)$ is continuous and $\mathcal{F}(N)$ denotes a set of real functions defined on the set N, then from the condition

(i) if $\alpha \in \mathcal{F}(N)$, then $\alpha \circ f \in \mathcal{F}(M)$ for every $\alpha \in \mathcal{F}(N)$.

it follows that

(ii) if $\beta \in \mathrm{sc}\,\mathcal{F}(N)$, then $\beta \circ f \in \mathcal{F}(M)$,

(iii) if $\beta \in \mathcal{F}(N)_N(\mathrm{top}N)$, then $\beta \circ f \in \mathcal{F}(M)$ (see [3]).

P r o o f . Let β be an arbitrary function from the set $\mathrm{sc}\,\mathcal{F}(N)$. Then $\beta = \varphi(\alpha^1(\cdot), \dots, \alpha^s(\cdot))$, where $\alpha^1, \dots, \alpha^s \in \mathcal{F}(N)$ and $\varphi \in C^\infty(R^s)$. Therefore $\beta \circ f = \varphi(\alpha^1(\cdot), \dots, \alpha^s(\cdot)) \circ f = \varphi(\alpha^1 \circ f(\cdot), \dots, \alpha^s \circ f(\cdot))$, consequently $\beta \circ f \in \mathrm{sc}\,\mathcal{F}(M) = \mathcal{F}(M)$, because $\alpha^i \circ f \in \mathcal{F}(M)$ for $i = 1, \dots, s$ (by (i)).

If $\beta \in \mathcal{F}(N)_N(\mathrm{top}N)$, then for every point $p \in N$ there exists a neighbourhood U open in $\mathrm{top}N$ and function $\alpha \in \mathcal{F}(N)$ such that $p \in U$ and $\beta|U = \alpha|U$. Hence we have $\beta|U \circ f = \alpha|U \circ f$ and $\beta \circ f|f^{-1}[U] = \alpha \circ f|f^{-1}[U]$. Because the set $f^{-1}[U]$ is open in topology $\mathrm{top}M$, so $\beta \circ f \in \mathcal{F}(M)_M(\mathrm{top}M) = \mathcal{F}(M)$.

Let M and N be the arbitrary sets and

(2) $$f : M \longrightarrow N$$

be an arbitrary mapping of the set M onto the set N. On the set R^N of all real functions defined on set N we define a function f^* as follows

(3) $$f^*(\beta) = \beta \circ f \quad \text{for} \quad \beta \in R^N.$$

D e f i n i t i o n 5. By $\mathrm{top}N_f$ we denote the topology on the set M such that a set A is open in topology $\mathrm{top}N_f$ iff there exists a set B open in $\mathrm{top}N$ and $A = f^{-1}[B]$. The topology $\mathrm{top}N_f$ will be called the topology induced on the set M from the topological space $(N, \mathrm{top}N)$ by the mapping f.

From the definition of the topology $\mathrm{top}N_f$ it follows that the mapping (2) is continuous with respect to $\mathrm{top}N_f$ and $\mathrm{top}N$.

T h e o r e m 1. If $\mathcal{N}$ is a space with smoothness, then:

(i) for every mapping (2)

(4) $$\mathcal{M}_f = (M, \mathrm{top}N_f, (f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f))$$

is the space with smoothness such that the mapping

(5) $$f : \mathcal{M}_f \longrightarrow \mathcal{N}$$

is smooth;

(ii) if $\mathrm{top}N_f \subset \mathrm{top}M$, then the set of functions $(f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f)$ is smallest in the sets $\mathcal{F}(M)$ such that $(M, \mathrm{top}M, \mathcal{F}(M))$ is the space with smoothness and the mapping (2) is smooth;

(iii) if the set $f[M]$ is open in topology $\mathrm{top}N$, then the mapping (5) is open.

P r o o f . (i) Let $\gamma \in \mathrm{scf}^*[\mathcal{F}(N)]$. Then $\gamma = \varphi(\alpha^1(\cdot),\ldots,\alpha^s(\cdot))$, where $\alpha^1,\ldots,\alpha^s \in f^*[\mathcal{F}(N)]$ and $\varphi \in C^\infty(R^s)$. Because $\alpha^i = \beta^i \circ f$ and $\beta^i \in \mathcal{F}(N)$ for $i = 1,\ldots,s$, so $\gamma = \varphi(\beta^1(\cdot),\ldots,\beta^s(\cdot)) \circ f$. But $\varphi(\beta^1(\cdot),\ldots,\beta^s(\cdot)) \in \mathrm{sc}\,\mathcal{F}(N) = \mathcal{F}(N)$, therefore $\gamma \in f^*[\mathcal{F}(N)]$, so $\mathrm{scf}^*[\mathcal{F}(N)] \subset f^*[\mathcal{F}(N)]$. From a property of the operation sc (see [3]) it follows that $\mathrm{scf}^*[\mathcal{F}(N)] = f^*[\mathcal{F}(N)]$ and we have $(\mathrm{scf}^*[\mathcal{F}(N)])_M(\mathrm{top}N_f) = (f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f)$. Because $\mathrm{sc}(C_M(\mathrm{top}M)) \subset (\mathrm{sc}C)_M(\mathrm{top}M)$ for an arbitrary family of functions $C \subset R^M$ and an arbitrary topology topM (see [2]), so

$$\mathrm{sc}((f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f)) \subset (\mathrm{scf}^*[\mathcal{F}(N)])_M(\mathrm{top}N_f).$$

Taking into consideration the property of the operation of localization (see [3]) we obtain that (4) is the space with smoothness.

If $\beta \in \mathcal{F}(N)$, then $\beta \circ f \in f^*[\mathcal{F}(N)] \subset (f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f)$, that is the mapping (5) fulfils the condition (b), so it is smooth, because continuity follows from the definition of the topology $\mathrm{top}N_f$.

(ii) Now put $f: \mathcal{M} \to \mathcal{N}$ is smooth and $\mathrm{top}N_f \subset \mathrm{top}M$. Let $\alpha \in (f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f)$ and $p \in M$ be an arbitrary point of the set M. Then there exists a function $\alpha' \in f^*[\mathcal{F}(N)]$ and a set A open in $\mathrm{top}N_f$ such that $p \in A$ and $\alpha|A = \alpha'|A$. But $\alpha' = \beta \circ f$ for some function $\beta \in \mathcal{F}(N)$, so by (1) is $\alpha' \in \mathcal{F}(M)$. Because $A \in \mathrm{top}N_f \subset \mathrm{top}M$, so $\alpha \in \mathcal{F}(M)_M(\mathrm{top}M) = \mathcal{F}(M)$, i.e.

$$(f^*[\mathcal{F}(N)])_M(\mathrm{top}N_f) \subset \mathcal{F}(M).$$

(iii) From the definition of $\mathrm{top}N_f$ we get that $\mathrm{top}N_f = \{f^{-1}[B];\ B \in \mathrm{top}N\}$. Therefore $f[f^{-1}[B]] = B \cap f[M]$, so if $f[M] \in \mathrm{top}N$ then $B \cap f[M] \in \mathrm{top}N$ and the mapping (5) is open.

The space with smoothness $\mathcal{M}_f$ defined by (4) will be called the space with smoothness induced from the space $\mathcal{N}$ on the set M by the mapping (2).

T h e o r e m 2. If $\mathcal{N}$ is the space with smoothness, then for every mapping (2) there exists exactly one space with smoothness $\mathcal{M}$ such that the mapping (1) is smooth and for every space with smoothness $\mathcal{L}$ and for every mapping

(6) $$g : L \to M$$

the mapping

(7) $$g : \mathcal{L} \to \mathcal{M}$$

is smooth if and only if the mapping

(8) $$f \circ g : \mathcal{L} \to \mathcal{N}$$

is smooth.

P r o o f . Let $\mathcal{N}$ be a space with smoothness and let $\mathcal{M}$ be the space with smoothness satisfying the conditions of theorem. Let us put that there exists the smoothness $\hat{\mathcal{F}}(M)$ such that $\hat{\mathcal{M}} = (M, \mathrm{top}M, \hat{\mathcal{F}}(M))$ is the space with smoothness satisfying the conditions of theorem too. Then the mapping $f : \hat{\mathcal{M}} \to \mathcal{N}$ is smooth and for an arbitrary space with smoothness $\mathcal{L}$ and for any mapping (6) the mapping $g : \mathcal{L} \to \mathcal{M}$ is smooth iff the mapping (8) is smooth.

Consider the mapping $id_M : \mathcal{M} \to \hat{\mathcal{M}}$. From the assumption of the theorem we have that the mapping $f \circ id_M : \mathcal{M} \to \mathcal{N}$ is smooth, therefore $\beta \circ (f \circ id_M) \in \mathcal{F}(M)$ for every function $\beta \in \mathcal{F}(N)$ and the mapping $f \circ id_M : \mathrm{top}M \to \mathrm{top}N$ is continuous. Because $\beta \circ (f \circ id_M) = (\beta \circ f) \circ id_M$ and $\beta \circ f = \gamma \in \hat{\mathcal{F}}(M)$, so the mapping id_M is smooth. Continuity of the mapping $id_M : \mathrm{top}M \to \mathrm{top}\hat{M}$ is obvious. Hence for every function $\gamma \in \hat{\mathcal{F}}(M)$ we have $\gamma \circ id_M = \gamma \in \mathcal{F}(M)$, so $\hat{\mathcal{F}}(M) \subset \mathcal{F}(M)$.

Now let us take the mapping $id_M : \hat{\mathcal{M}} \to \mathcal{M}$, and use analogously the properties of the space $\mathcal{M}$. Then we get that the mapping $f \circ id_M : \mathcal{M} \to \mathcal{N}$ is smooth, so $\beta \circ (f \circ id_M) \in \hat{\mathcal{F}}(M)$ for every function $\beta \in \mathcal{F}(N)$. Obviously the mapping $f \circ id_M :$ $: \mathrm{top}M \to \mathrm{top}N$ is continuous. Because $\beta \circ f \circ id_M = \gamma \circ id_M \in \hat{\mathcal{F}}(M)$, where $\gamma \in \mathcal{F}(M)$, so the mapping id_N is smooth. Because $\gamma \circ id_M = \gamma$ consequently $\mathcal{F}(M) \subset \hat{\mathcal{F}}(M)$ hence $\mathcal{F}(M) = \hat{\mathcal{F}}(M)$.

We have proved that the space with smoothness defined by (4) satisfies the conditions of the theorem. From Theorem 1 we have that the mapping (5) is smooth. Let (6) be any mapping such that the mapping (8) is smooth. Then $\beta \circ (f \circ g) \in \mathcal{F}(L)$ for every function $\beta \in \mathcal{F}(N)$ and the mapping $f \circ g : topL \to topN$ is continuous.

Let $\gamma \in f^*[\mathcal{F}(N)]$ be an arbitrary function. Because $\gamma = \beta \circ f$ and $\beta \in \mathcal{F}(N)$, so $\gamma \circ g = \beta \circ f \circ g \in \mathcal{F}(L)$ for every function $\gamma \in f^*[\mathcal{F}(N)]$, i.e. the mapping $g : \mathcal{L} \to \mathcal{M}_f$ is smooth (continuity of the mapping $g : topL \to topN_f$ follows from the fact that the mappings $f \circ g : topL \to topN$ and $f : topN_f \to topN$ are continuous). The proof is finished.

D e f i n i t i o n 6. Let $\mathcal{M}$ be a space with smoothness and $f : M \to N$ be any mapping of the set M onto the set N. By ${}_f topM$ we will denote the topology on the set N such that $B \in {}_f topM$ if and only if $f^{-1}[B] \in topM$. The topology ${}_f topM$ will be called the topology coinduced from the topological space $(M, topM)$ on the set N by the mapping f.

From the definition of the topology ${}_f topM$ is immediately follows that the mapping $f : topM \to {}_f topM$ is continuous.

T h e o r e m 3. If $\mathcal{M}$ is a space with smoothness, then for every mapping (2)

$$(9) \qquad {}_f\mathcal{N} = (N, {}_f topM, f^{*-1}[\mathcal{F}(M)])$$

is the space with smoothness such that the mapping

$$(10) \qquad f : \mathcal{M} \to {}_f\mathcal{N}$$

is smooth and the set $f^{*-1}[\mathcal{F}(M)]$ is greatest in the sets $\mathcal{F}(N)$ such that $\mathcal{N} = (N, topN, \mathcal{F}(N))$ is the space with smoothness and the mapping (1) is smooth.

P r o o f . Let $\beta \in f^{*-1}[\mathcal{F}(M)]$. Then $\beta = f^{*-1}(\alpha)$, where $\alpha \in \mathcal{F}(M)$ that is $\alpha = f^*(\beta) = \beta \circ f \in \mathcal{F}(M)$. Therefore if

$\gamma \in scf^{*-1}[\mathcal{F}(M)]$ then (by Lemma) we get $\gamma \circ f \in \mathcal{F}(M)$, hence $\gamma \in f^{*-1}[\mathcal{F}(M)]$, i.e. $scf^{*-1}[\mathcal{F}(M)] \subset f^{*-1}[\mathcal{F}(M)]$. Now consider the function $\gamma \in (f^{*-1}[\mathcal{F}(M)])_N({}_f topM)$. Let $p \in M$ be an arbitrary point. Then $f(p) \in N$ and there exists a neibhgourhood $U \in {}_f topM$ and a function $\beta \in f^{*-1}[\mathcal{F}(M)]$ such that $f(p) \in U$ and $\gamma|U = \beta|U$. Thus we have $\gamma|U \circ f = \beta|U \circ f$, i.e. $\gamma \circ f|f^{-1}[U] = \beta \circ f|f^{-1}[U]$. Because $p \in f^{-1}[U] \in topM$ and $\beta \circ f \in \mathcal{F}(M)$, so $\gamma \circ f \in \mathcal{F}(M)_M(topM) = \mathcal{F}(M)$ thus $\gamma \in f^{*-1}[\mathcal{F}(M)]$ and $(f^{*-1}[\mathcal{F}(M)])_N({}_f topM) \subset f^{*-1}[\mathcal{F}(M)]$. The smoothness of the mapping (10) is obvious.

Let $\mathcal{N} = (N, topN, \mathcal{F}(N))$ be an arbitrary space such that the mapping (1) occurs there. Take an arbitrary function $\beta \in \mathcal{F}(N)$. Then $f^*(\beta) = \beta \circ f \in \mathcal{F}(M)$ hence $\beta \in f^{*-1}[\mathcal{F}(M)]$. It is easy to verify that from the conditions (a) and (b) and from the definition of the topology ${}_f topM$ it follows that $topN \subset {}_f topM$.

T h e o r e m 4. If $\mathcal{M}$ is a space with smoothness, then for every mapping (2) there exists exactly one space with smoothness $\mathcal{N}$ such that the mapping (1) is smooth and for every space with smoothness $\mathcal{L}$ and every mapping onto

$$h : N \to L \tag{11}$$

the mapping $h : \mathcal{N} \to \mathcal{L}$ is smooth iff the mapping

$$h \circ f : \mathcal{M} \to \mathcal{L} \tag{12}$$

is smooth.

P r o o f . Let $\mathcal{N}$ be a space satisfying the conditions of the theorem. Assume that there exists another space with smoothness $\hat{\mathcal{N}} = (N, top\hat{N}, \hat{\mathcal{F}}(N))$ also fulfilling the conditions of the theorem. Then the mapping $f : \mathcal{M} \to \hat{\mathcal{N}}$ is smooth and for every mapping (11) the mapping $h : \hat{\mathcal{N}} \to \mathcal{L}$ is smooth iff the mapping (12) is smooth. Consider the mapping $id_N : \hat{\mathcal{N}} \to \mathcal{N}$ and let the mapping $id_N \circ f : \mathcal{M} \to \mathcal{N}$ be smooth. Then $\beta \circ (id_N \circ f) \in \mathcal{F}(M)$ for every function $\beta \in \mathcal{F}(N)$. Because

$\beta \circ id_N = \beta$, so from $\beta \circ f \in \mathcal{F}(M)$ it follows that $\beta \in \hat{\mathcal{F}}(N)$. Hence $\mathcal{F}(N) \subset \hat{\mathcal{F}}(N)$ and the mapping $id_{\hat{N}}$ is smooth.

Consider now the mapping $id_N : \mathcal{N} \to \hat{\mathcal{N}}$. It is obvious that $id_N \circ f : \mathcal{M} \to \hat{\mathcal{N}}$ is smooth, hence $\beta \circ id_N \circ f \in \mathcal{F}(M)$ for every function $\beta \in \hat{\mathcal{F}}(N)$, i.e. $\beta \circ f \in \mathcal{F}(M)$ and $\beta \in \mathcal{F}(N)$. Thus $\hat{\mathcal{F}}(N) \subset \mathcal{F}(N)$.

Now we shall prove that the space with smoothness defined by (9) satisfies the conditions of theorem. From Theorem 3 it follows that the mapping (10) is smooth. Let the mapping (11) be such that the mapping (12) is smooth, i.e. $\beta \circ (h \circ f) \in \mathcal{F}(M)$ for every function $\beta \in \mathcal{F}(L)$ and the mapping $h \circ f : topM \to topL$ is continuous. Let $\gamma \in f^{*-1}[\mathcal{F}(M)]$. Then $\gamma \circ f \in \mathcal{F}(M)$. Therefore for every function $\beta \in \mathcal{F}(L)$ we have $\beta \circ h \circ f = \gamma \circ f \in \mathcal{F}(M)$, so $\beta \circ h \in f^{*-1}[\mathcal{F}(M)]$. Let $A \in topL$. Then $(h \circ f)^{-1}[A] \in topM$. Because $(h \circ f)^{-1}[A] = f^{-1}[h^{-1}[A]]$, so $h^{-1}[A]$ is a subset of the set N such that its image at the mapping f^{-1} is open in the topology topM. Therefore we get that $h^{-1}[A] \in_f topM$. In this way we have proved that the mapping h is smooth and the proof is finished.

T h e o r e m 5. For every mapping (2) the following conditions are equivalent:

(i) the mapping (1) is smooth;

(ii) $\mathcal{F}(M)$ is the smoothness containing the smoothness $(f^*[\mathcal{F}(N)])_M(topN_f)$ and $topN_f \subset topM$;

(iii) $\mathcal{F}(N)$ is the smoothness contained in the smoothness $f^{*-1}[\mathcal{F}(M)]$ and $topN \subset_f topM$.

P r o o f . The implication (i) $\Rightarrow$ (ii) immediately follows from Theorem 1 and similarly, the implication (i) $\Rightarrow$ (iii) - from Theorem 3. We will prove the implication (ii) $\Rightarrow$ (i). Let $\beta \in \mathcal{F}(N)$. Then $\beta \circ f \in (f^*[\mathcal{F}(N)])_M(topN_f) \subset \mathcal{F}(M)$. Also if $A \in topN$ then $f^{-1}[A] \in topN_f \subset topM$. The proof of implication (iii) $\Rightarrow$ (i) is similar.

T h e o r e m 6. If the mapping (2) is one-to-one and onto, then the following conditions are equivalent:

(i) the mapping (1) is a diffeomorphism;

(ii) the smoothness $\mathcal{F}(M)$ coincides with the smoothness $f^*[\mathcal{F}(N)]$, $topN_f \subset topM$ and the mapping f is open;

(iii) the smoothness $\mathcal{F}(N)$ coincides with the smoothness $f^{*-1}[\mathcal{F}(M)]$, $topN \subset_f topM$ and the mapping f is open.

P r o o f . We shall prove the equivalence of conditions (i) and (ii). The proof of equivalence of conditions (i) and (iii) is similar to the above. Let us put that the condition (i) holds. Let $\alpha \in f^*[\mathcal{F}(N)]$. Then $\alpha = \beta \circ f$ and $\beta \in \mathcal{F}(N)$. Because $\beta \circ f \in \mathcal{F}(M)$ so $\alpha \in \mathcal{F}(M)$ and $f^*[\mathcal{F}(N)] \subset \subset \mathcal{F}(M)$. If $\alpha \in \mathcal{F}(M)$, then $\alpha \circ f^{-1} \in \mathcal{F}(N)$ so $(\alpha \circ f^{-1}) \circ f \in \in f^*[\mathcal{F}(N)]$, i.e. $\alpha \in f^*[\mathcal{F}(N)]$, thus $\mathcal{F}(M) \subset f^*[\mathcal{F}(N)]$ and we get coincidence. The remaining two assertions of the condition (ii) immediately follows from the condition (i).

Now let the condition (ii) hold. Because the mapping f is one-to-one and onto, it is sufficient to prove, that the mappings f and f^{-1} are smooth. Let $\beta \in \mathcal{F}(N)$. Then $\beta \circ f \in \in (f^*[\mathcal{F}(N)])_M(topN_f)$ and (by Theorem 1 (ii)) $\beta \circ f \in \mathcal{F}(M)$. If $A \in topN$, then $f^{-1}[A] \in topN_f \subset topM$, so the mapping $f : topM \longrightarrow topN$ is continuous. Thus the mapping f is smooth. If $\alpha \in \mathcal{F}(M)$ then $\alpha \in f^*[\mathcal{F}(N)]$ whether $\alpha = \beta \circ f$ and $\beta \in \mathcal{F}(N)$. Hence $\alpha \circ f^{-1} = \beta \circ f \circ f^{-1} = \beta \in \mathcal{F}(N)$. Because the mapping f is open, the mapping f^{-1} is continuous, consequently the mapping f^{-1} is smooth. The proof is completed.

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