DEMONSTRATIO MATHEMATICA
Vol. XI No 4 1978

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A THEOREM SUPPLEMENTING A RESULT OF JAMES A. YORKE

In this paper a theorem will be proved showing that the assumptions of one of Yorke's theorems are never fulfilled in case $n = 1$.

Consider the system of differential equations of the first order

$$(1) \qquad x' = F(x),$$

where $x(t) = (x^1(t), x^2(t), \dots, x^n(x))$ is a sought vector-function for $t \in R$, R - the set of real numbers and $F(x)$ is a function fulfilling the Lipschitz condition

$$(2) \qquad \|F(x) - F(\bar{x})\| \leqslant L\|x - \bar{x}\| .$$

$\|\cdot\|$ denotes Euclidean norm in R^n.

The author of notes [1] and [2] considered periodic solutions of the system (1), i.e. solutions satisfying the equation

$$x(t + p) = x(t) \quad \text{for} \quad t \in R.$$

In the note [1] the following theorem has been proved.

T h e o r e m 1. If $x(t)$ is a periodic solution of the system (1) fulfilling condition (2), then $p \leqslant 2\pi/L$.

Under weaker assumptions about F one may prove for $n = 1$ the following

T h e o r e m 2. Let F be a continuous function. If the equation (1) possesses a solution $x(t) \in C^1$, $t \in R$ for $n = 1$, then it is not periodic.

P r o o f . Suppose that the solution $x(t)$ is periodic and non-constant. The function $x(t)$ assumes minimum at the point t_{min} and maximum at the point t_{max} ($t_{min} < t_{max} < t_{min} + p$). Consider the intervals $A = [t_{min}, t_{max}]$, $B = [t_{max}, t_{min} + p]$. In A there exists ξ_1 such that $x'(\xi_1) > 0$ and in B there exists ξ_2 such that $x(\xi_1) = x(\xi_2)$ (Darboux's property). For one of these numbers there must be $x'(\xi_2) \geqslant 0$. Hence $x'(\xi_1) > x'(\xi_2)$ and this together with $F(x(\xi_1)) = F(x(\xi_2))$ leads to a contradiction with (1).

From Theorem 2 it follows that in case $n = 1$ the assumptions of Theorem 1 are never fulfilled.

Theorem 2 is no longer true for $n \geqslant 2$ (see [1]).

REFERENCES

[1] J a m e s A. Y o r k e : The Lipschitz constant and the period of periodic solutions, Proc. Amer. Math. Soc. 22 (1969) 509-512.

[2] J a m e s A. Y o r k e : The period of periodic solutions and charged particles in magnetic fields, Lecture Notes in Mathematics 144 (1970) 267-268.

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Received July 29, 1976.