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# A NON-LINEAR BOUNDARY VALUE PROBLEM FOR AN INFINITE SYSTEM OF FOURTH-ORDER INTEGRO-DIFFERENTIAL EQUATIONS IN HALF-SPACE

## 1. Introduction

The aim of this paper is to prove the existence of solution of the non-linear boundary value problem (1)-(5) (see the formulas in the sequel). The problem is reduced to the infinite system of integral equations (22) and then Banach's fixed point theorem is applied. The reasoning is essentially based on the results of paper [7].

It is to be noted that Banach's fixed point theorem was for the first time applied to infinite systems by H.Adamczyk (see the system of functional equations in [1]) and then used in a similar way in the theory of singular integral equations: by J.Nazarowski [8] for a system containing the integrals of elliptic type and by A.Borzymowski and J.Brzeziński [6] for a system with the integrals of parabolic type examined previously in [4]. Also, a limit problem for an infinite system of parabolic integro-differential equations of the second order was solved by A.Borzymowski in [5] by using Tikhonov's fixed point theorem. To the best of my knowledge, infinite systems of integro-differential equations of an order higher than two have not been examined so far.

2. <u>The problem</u>

Let $E^n$ $(n \geqslant 2)$ be the euclidean $n$-space of the points $X(x_1, x_2, \ldots, x_n)$. Introduce in this space the following sets*)

$$E^+ \overset{df}{=} \left\{ X : X \in E^n \wedge x_n > 0 \right\},$$

$$E^0 \overset{df}{=} \left\{ X : X \in E^n \wedge x_n = 0 \right\}.$$

Also, denote

$$R \overset{df}{=} E^+ \otimes (0,T), \qquad R^0 \overset{df}{=} E^0 \otimes (0,T),$$

where $T$ is a finite positive number.

We consider the following boundary value problem:

Find a system of functions $u^l(A,t)$, $l = 1,2,\ldots$ satisfying in the domain $R$ the infinite system of integro-differential equations of the form

$$(1) \qquad \Omega^2\left[u^l(A,t)\right] = F_1^l\left[A,t; \left\{D^k u^l(A,t)\right\}, \left\{G^q(A,t)\right\}\right],$$

where

$$\Omega \overset{df}{=} \Delta - \frac{\partial}{\partial t} \overset{df}{=} \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2} - \frac{\partial}{\partial t},$$

$$D^k u^l \overset{df}{=} \frac{\partial^k u^l}{\partial x_1^{\alpha_1} \ldots \partial x_n^{\alpha_n}}; \quad \sum_{i=1}^{n} \alpha_i = k; \quad k = 0,1,2; \quad D^0 u^l = u^l,$$

$$G^q(A,t) = \int_0^t \int_{E^+} (t-\tau)^{-\frac{n}{2}+1} \exp\left[-\frac{|AB|^2}{4(t-\tau)}\right] F_6^q\left[B,\tau; \left\{D^k u^p(B,\tau)\right\}\right] dB d\tau$$

*) The notation is analogous to that in [3].

and fulfilling the initial conditions

(2) $$\lim_{t\to 0} u^1(A,t) = F_2^1(A),$$

(3) $$\lim_{t\to 0} \frac{\partial}{\partial t} u^1(A,t) = F_3^1(A)$$

$(A(x_1,\dots,x_n) \in E^+)$ and the boundary conditions

(4) $$\lim_{x_n\to 0} \frac{\partial^2}{\partial x_n^2} u^1(A,t) = F_4^1\left[P,t;\left\{D^k u^p(P,t)\right\}\right]$$

(5) $$\lim_{x_n\to 0} \frac{\partial^3}{\partial x_n^3} u^1(A,t) = F_5^1\left[P,t;\left\{D^k u^p(P,t)\right\}\right]$$

$(P \in E^0,\ t \in (0,T))$, where $p,q = 1,2,\dots;\ i,j=1,2,\dots,n$. Above, the symbol $\left\{u^p(A,t)\right\}$ denotes the infinite sequence $u^1(A,t),u^2(A,t),\dots$ and the remaining expressions in the brackets $\{\ \}$ should be understood analogously.

## 3. Assumptions

We make the following assumptions

$1^0$. The functions $F_1^1\left[A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\},\{v^q\}\right]$ are defined and continuous in the set

$$(A,t) \in R;\quad \omega^p,\dots,v^q \in (-\infty, +\infty)$$

and satisfy the conditions

(6) $$\left|F_1^1(A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\},\{v^q\})\right| \leq m_{F_1}^1\, t^{-\mu_{F_1}^{1,1}} \cdot \eta_1 + \bar{m}_{F_1}^1 \cdot \sum_{p=1}^{\infty}\Big(M_{F_1}^p|\omega^p| + M_{F_1}^{pi}|\omega_i^p| + \sum_{i,j=1}^{n} M_{F_1}^{pij}|\omega_{ij}^p|\Big) \cdot \eta_2 + \sum_{q=1}^{\infty} M_{F_1}^q|v^q|,$$

(where $\theta\in(0,1)$, $b>0$, $\bar{b}\in\left(0,\ \frac{1}{2}\ \frac{\theta b}{\theta+12Tb}\right)$),

$$(7)\quad \left|F_1^l\left(A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\},\{v^q\}\right)+\right.$$

$$\left.-F_1^l\left(A_1,t_1;\{\bar\omega^p\},\{\bar\omega_i^p\},\{\bar\omega_{ij}^p\},\{\bar v^q\}\right)\right|\leqslant$$

$$\leqslant k_{F_1}^l\, t^{-\mu_{F_1}^{l,1}}\left(|AA_1|^{\mu_{F_1}^{l,2}}+|t-t_1|^{\mu_{F_1}^{l,3}}\right)\exp\left[\frac{\theta(b-\bar b)|OA|^2}{\theta+12Tb}\right]+$$

$$+\bar k_{F_1}^l\sum_{p=1}^{\infty}\Big(K_{F_1}^p\left|\omega^p-\bar\omega^p\right|+\sum_{i=1}^{n}K_{F_1}^{pi}\left|\omega_i^p-\bar\omega_i^p\right|+$$

$$+\sum_{i,j=1}^{n}K_{F_1}^{pij}\left|\omega_{ij}^p-\bar\omega_{ij}^p\right|\Big)\cdot\eta_2+\sum_{q=1}^{\infty}K_{F_1}^q\left|v^q-\bar v^q\right|,$$

where $\eta_1=\exp\left[\frac{\theta(b-\bar b)|OA|^2}{\theta+12Tb}\right]$, $\eta_2=\exp\left[-\frac{(12Tb^2+\theta b)|OA|^2}{\theta+12Tb}\right]$.

$2^o$. The functions $F_2^l(A)$ are of class $C^2(E^+)$ and satisfy the inequalities

$$(8)\qquad \left|\frac{\partial^k F_2^l(A)}{\partial x_1^{k_1}\dots\partial x_n^{k_n}}\right|\leqslant m_{F_2}^l\cdot\eta_1,\quad 0\leqslant\sum_{i=1}^{n}k_i=k\leqslant 2.$$

$3^o$. The functions $F_3^l(A)$ are defined and continuous in $E^+$ and fulfil the inequalities

$$(9)\qquad \left|F_3^l(A)\right|\leqslant m_{F_3}^l\cdot\eta_1$$

$4^\circ$. The functions $F_4^l\,(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})$ are defined and continuous in the set

$$(P,t)\in R^0;\ \omega^p,\omega_i^p,\omega_{ij}^p\in(-\infty,+\infty),$$

and satisfy, together with their first-order derivatives, the following conditions

$$(10)\quad \left|F_4^l\,(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})\right|\leqslant t^{\frac{1}{2}}\Big[m_{F_4}^l\, t^{-\mu_{F_4}^{l,1}}\cdot\eta_3+$$

$$+\bar{m}_{F_4}^l\sum_{p=1}^{\infty}(M_{F_4}^p\left|\omega^p\right|+\sum_{i=1}^{n}M_{F_4}^{pi}\left|\omega_i^p\right|+\sum_{i,j=1}^{n}M_{F_4}^{pij}\left|\omega_{ij}^p\right|)\cdot\eta_4\Big],$$

where $\eta_3=\exp\left[\dfrac{\theta(b-\bar{b})|OP|^2}{\theta+12Tb}\right]$, $\eta_4=\exp\left[-\dfrac{8Tb^2+4Tb\bar{b}+\bar{b}}{\theta+4T(2b+\bar{b})}|OP|^2\right]$,

$$(11)\quad \left|F_4^l(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})-F_4^l(P_1,t_1;\{\bar{\omega}^p\},\{\bar{\omega}_i^p\},\{\bar{\omega}_{ij}^p\})\right|\leqslant$$

$$\leqslant k_{F_4}^l\, t^{-\mu_{F_4}^{l,1}}\left((|PP_1|)^{\mu_{F_4}^{l,2}}+|t-t_1|^{\mu_{F_4}^{l,3}+\frac{1}{2}}\right)\cdot\eta_3\cdot\exp\left[-\bar{b}|OP|^2\right]+$$

$$+\bar{k}_{F_4}^l\cdot\sum_{p=1}^{\infty}(K_{F_4}^p\left|\omega^p-\bar{\omega}^p\right|+\sum_{i=1}^{n}K_{F_4}^{pi}\left|\omega_i^p-\bar{\omega}_i^p\right|+$$

$$+\sum_{i,j=1}^{n}K_{F_4}^{pij}\left|\omega_{ij}^p-\bar{\omega}_{ij}^p\right|)\cdot\eta_4\cdot\frac{|PP_1|^{\mu_{F_4}^{l,2}}+|t-t_1|^{\mu_{F_4}^{l,3}+\frac{1}{2}}}{|PP_1|^{\mu_{F_4}^{l,2}}+|t-t_1|^{\mu_{F_4}^{l,3}}}$$

and

$$(12)\quad \left|\frac{\partial}{\partial z}F_4^1(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\}) - \frac{\partial}{\partial z}F_4^1(P_1,t;\{\bar{\omega}^p\},\{\bar{\omega}_i^p\},\{\bar{\omega}_{ij}^p\})\right| \leqslant$$

$$\leqslant k_{F_4}^1\, t^{-\mu_{F_4}^{1,1}} \left|PP_1\right|^{\mu_{F_4}^{1,2}} \cdot \eta_3 \cdot \exp\left[-\bar{b}|OP|^2\right] +$$

$$+ \bar{k}_{F_4}^1 \sum_{p=1}^{\infty} \Big(K_{F_4}^p \left|\omega^p - \bar{\omega}^p\right| + \sum_{i=1}^{n} K_{F_4}^{pi}\left|\omega_i^p - \bar{\omega}_i^p\right| +$$

$$+ \sum_{i,j=1}^{n} K_{F_4}^{pij}\left|\omega_{ij}^p - \bar{\omega}_{ij}^p\right|\Big) \cdot \eta_4,$$

where $z = x_1,\dots,x_n,\ \omega^p,\ \omega_i^p,\ \omega_{ij}^p$ $(i,j=1,2,\dots,n;\ p=1,2,\dots)$.

$5^o$. The functions $F_5^1(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})$ are defined and continuous in the set

$$(P,t) \in R^o;\ \omega^p,\ \omega_i^p,\ \omega_{ij}^p \in (-\infty, +\infty)$$

and satisfy the conditions

$$(13)\quad \left|F_5^1(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})\right| \leqslant m_{F_5}^1\, t^{-\mu_{F_5}^{1,1}} \exp\left[\eta_5|OP|^2\right] +$$

$$+ \bar{m}_{F_5}^1 \cdot \eta_6 \cdot \sum_{p=1}^{\infty} \Big(M_{F_5}^p|\omega^p| + \sum_{i=1}^{n} M_{F_5}^{pi}|\omega_i^p| + \sum_{i,j=1}^{n} M_{F_5}^{pij}|\omega_{ij}^p|\Big),$$

(14) $\left| F_5^1(P,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\}) - F_5^1(P_1,t_1;\{\bar\omega^p\},\{\bar\omega_i^p\},\{\bar\omega_{ij}^p\})\right| \leqslant$

$$\leqslant k_{F_5}^1\, t^{-\mu_{F_5}^{1,1}} \left(|PP_1|^{\mu_{F_5}^{1,2}} + |t-t_1|^{\mu_{F_5}^{1,3}}\right) \eta_5 \exp\left[-b|OP|^2\right] +$$

$$+ \bar k_{F_5}^1 \sum_{p=1}^{\infty} K_{F_5}^p |\omega^p - \bar\omega^p| + \sum_{i=1}^{n} K_{F_5}^{pi} |\omega_i^p - \bar\omega_i^p| +$$

$$+ \sum_{i,j=1}^{n} K_{F_5}^{pij} |\omega_{ij}^p - \bar\omega_{ij}^p| \Big) \cdot \eta_6,$$

where $\eta_5 = \exp\left[\frac{\theta(b-\bar b)|OP|^2}{\theta+4T(b+2\bar b)}\right]$, $\eta_6 = \exp\left[-\frac{4Tb^2+8Tb\bar b+\theta\bar b}{\theta+4T(b+2\bar b)}|OP|^2\right]$.

$6^0$. The functions $F_6^1(A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})$ are defined and continuous in the set

$$(A,t) \in R;\ \omega^p,\ \omega_i^p,\ \omega_{ij}^p \in (-\infty, +\infty)$$

and fulfil the inequalitiès

(15) $\left| F_6^1(A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\})\right| \leqslant m_{F_6}^1\, t^{-\mu_{F_6}^{1,1}} \cdot \eta_7 +$

$$+ \bar m_{F_6}^1 \cdot \eta_8 \cdot \sum_{p=1}^{\infty} M_{F_6}^p |\omega^p| + \sum_{i=1}^{n} M_{F_6}^{pi} |\omega_i^p| + \sum_{i,j=1}^{n} M_{F_6}^{pij} |\omega_{ij}^p| ,$$

where $\eta_7 = \exp\left[\frac{\theta(b-\bar b)|OA|^2}{\theta+16Tb-4T\bar b}\right]$, $\eta_8 = \exp\left[\frac{-16Tb^2-4b\bar b+\theta\bar b}{\theta+16Tb-4T\bar b}|OA|^2\right]$.

$$(16)\quad \left|F_6^l(A,t;\{\omega^p\},\{\omega_i^p\},\{\omega_{ij}^p\}) - F_6^l(A_1,t_1;\{\bar\omega^p\},\{\bar\omega_i^p\},\{\bar\omega_{ij}^p\})\right| \leqslant$$

$$\leqslant k_{F_6}^l\left(|AA_1|^{\mu_{F_6}^{l,2}} \cdot \exp\left[-\frac{\bar b}{2}|OA|^2\right] + |t-t_1|^{\mu_{F_6}^{l,3}}\right) t^{-\mu_{T_6}^{l,1}} \cdot \eta_7 +$$

$$+ \bar k_{F_6}^l \sum_{p=1}^{\infty} K_{F_6}^p\left|\omega^p - \bar\omega^p\right| + \sum_{i=1}^{n} K_{F_6}^{pi}\left|\omega_i^p - \bar\omega_i^p\right| +$$

$$+ \sum_{i,j=1}^{n} K_{F_6}^{pij}\left|\omega_{ij}^p - \bar\omega_{ij}^p\right| \Big)\cdot \eta_8.$$

7°. The exponents appearing in the assumptions 1° - 6° satisfy the inequalities

$$(17)\quad 0 \leqslant \mu_{F_{\gamma_1}}^{l,1} < 1, \quad 0 < \mu_{F_{\gamma_1}}^{l,2} \leqslant 1, \quad 0 < \mu_{F_{\gamma_1}}^{l,3} \leqslant 1,$$

$(\gamma_1 = 1,4,5,6)$.

8°. The coefficients appearing in the aforesaid assumptions are positive numbers fulfilling the following conditions

$$(18)\quad \begin{cases} \max\limits_{\gamma_2} \limsup\limits_{l} m_{F_{\gamma_2}}^l = m_F < \infty, \\ \max\limits_{\gamma_1} \limsup\limits_{l} \bar m_{F_{\gamma_1}}^l = \bar m_F < \infty, \\ \max\limits_{\gamma_1} \limsup\limits_{l} k_{F_{\gamma_1}}^l = k_F < \infty, \\ \max\limits_{\gamma_1} \limsup\limits_{l} \bar k_{F_{\gamma_1}}^l = \bar k_F < \infty \end{cases}$$

$(\gamma_2 = 1,2,3,4,5,6)$.

$9^o$. We assume that all infinite number series of the appropriate coefficients appearing in the assumptions $1^o$ - $6^o$ are convergent and we denote the sums of these series by

$$(19)\quad \begin{cases} \sum_{p=1}^{\infty} M_{F_{\gamma_1}}^{p} = \bar{M}_{F_{\gamma_1}}, \quad \sum_{p=1}^{\infty}\sum_{i=1}^{n} M_{F_{\gamma_1}}^{pi} = \bar{\bar{M}}_{F_{\gamma_1}}, \\ \sum_{p=1}^{\infty}\sum_{i,j=1}^{n} M_{F_{\gamma_1}}^{pij} = \bar{\bar{\bar{M}}}_{F_{\gamma_1}}, \quad \sum_{p=1}^{\infty} K_{F_{\gamma_1}}^{p} = \bar{K}_{F_{\gamma_1}}, \\ \sum_{p=1}^{\infty}\sum_{i=1}^{n} K_{F_{\gamma_1}}^{pi} = \bar{\bar{K}}_{F_{\gamma_1}}, \quad \sum_{p=1}^{\infty}\sum_{i,j=1}^{n} K_{F_{\gamma_1}}^{pij} = \bar{\bar{\bar{K}}}_{F_{\gamma_1}}, \end{cases}$$

respectively.

Finally, for the sake of convenience, we introduce the following notation

$$(20)\quad \begin{cases} \max\,(\bar{M}_{F_{\gamma_1}}, \bar{\bar{M}}_{F_{\gamma_1}}, \bar{\bar{\bar{M}}}_{F_{\gamma_1}}) = M_F, \\ \max\,(\bar{K}_{F_{\gamma_1}}, \bar{\bar{K}}_{F_{\gamma_1}}, \bar{\bar{\bar{K}}}_{F_{\gamma_1}}) = K_F, \end{cases}$$

where $\gamma_1 = 1,4,5,6$.

## 4. Solution of the problem

We seek the solution of the problem (1)-(5) in the form

$$(21)\quad u^l(A,t) = C\Big[H_1^l(A,t) + H_2^l(A,t) + V^l(A,t) - 2\,W_2^l(A,t) + 2W_3^l(A,t) + 4W_4^l(A,t)\Big], \qquad l = 1,2,3....$$

$(C = (2\sqrt{\pi})^{-n})$, where*)

*) The first four integrals were introduced in [2] and [3], and the remaining integrals in [7].

$$V^1(A,t) = \int_0^t \int_{E^+} (t-\tau)^{-\frac{n}{2}+1} \exp\left[-\frac{|AB|^2}{4(t-\tau)}\right] \Phi(B,\tau)\, dBd\tau,$$

$$H_1^1(A,t) = \int_{E^+} t^{-\frac{n}{2}} \exp\left[-\frac{|AB|^2}{4t}\right] \chi_1(B)dB,$$

$$H_2^1(A,t) = \int_{E^+} t^{-\frac{n}{2}+1} \exp\left[-\frac{|AB|^2}{4t}\right] \chi_2(B)dB,$$

$$W_0^1(A,t) = x_n \int_0^t \int_{E^0} (t-\tau)^{-\frac{n}{2}-1} \cdot \exp\left[-\frac{|AQ|^2}{4(t-\tau)}\right] \varphi(Q,\tau)dQd\tau,$$

$$W_1^1(A,t) = x_n^2 \int_0^t \int_{E^0} (t-\tau)^{-\frac{n}{2}-1} \exp\left[-\frac{|AQ|^2}{4(t-\tau)}\right] \psi(Q,\tau)dQd\tau,$$

$$W_2^1(A,t) = \int_{x_n}^{\infty} dz_2 \int_0^t \int_{E^0} (t-\tau)^{-\frac{n}{2}} \exp\left[-\frac{|A_2Q|^2}{4(t-\tau)}\right] \varphi(Q,\tau)dQd\tau,$$

$$W_3^1(A,t) = \int_{x_n}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \int_0^t \int_{E^0} (t-\tau)^{-\frac{n}{2}} \cdot \exp\left[-\frac{|A_2Q|^2}{4(t-\tau)}\right] \cdot$$

$$\cdot\ \psi(Q,\tau)dQd\tau,$$

$$W_4^1(A,t) = \int_0^t \int_{E^0} (t-\tau)^{-\frac{n}{2}+1} \exp\left[-\frac{|AQ|^2}{4(t-\tau)}\right] \psi(Q,\tau)dQd\tau$$

($B, A_2 \in E^+$, $Q \in E^0$ and $|AB|$ denotes the Euclidean distance of points A and B), with

$$(21')\quad \begin{cases} \Phi(A,t) = F_1^1\left[A,t;\left\{D^k u^1(A,t)\right\},\ \left\{G^q(A,t)\right\}\right], \\ \varphi(P,t) = F_4^1\left[P,t;\left\{D^k u^1(P,t)\right\}\right] + \\ \qquad -\dfrac{\partial^2}{\partial x_n^2}\left[H_1^1(P,t) + H_2^1(P,t) + V^1(P,t)\right], \\ \psi(P,t) = F_5^1\left[P,t;\left\{D^k u^1(P,t)\right\}\right] + \\ \qquad -\dfrac{\partial^3}{\partial x_n^3}\left[H_1^1(P,t) + H_2^1(P,t)+V^1(P,t)-2W_2^1(P,t)\right], \\ \chi_1(A) = F_2^1(A),\quad \chi_2(A) = F_3^1(A) - \Delta F_2^1(A). \end{cases}$$

It follows directly from Theorems 1-4 and Remarks 1 and 2 in [7], and from Theorems 5 and 7 in [3]*) that functions $u^1(A,t)$ satisfy the system (1) and the conditions (2)-(5). We are going to prove that the infinite system of integro-differential equations (21) possesses, in an appropriately choosen class of functions, a unique solution. To this purpose, let us consider an auxiliary system of the form

$$(22)\quad u_{rs}^1(A,t) = C\left[(H_1^1(A,t))_{rs} + (H_2^1(A,t))_{rs} + (V^1(A,t))_{rs} - 2\,(W_2^1(A,t))_{rs} + 2\,(W_3^1(A,t))_{rs} + 4\,(W_4^1(A,t))_{rs}\right],$$

*) In the sequel we prove (see formulas (35)-(40)) that the assumptions of the said theorems are satisfied.

where $r,s = 0,1,\dots,n$, and the symbol $(F)_{rs}$ denotes the derivative $\frac{\partial^2 F}{\partial x_r \partial x_s}$ when $r \neq 0$, $s \neq 0$; the derivative $\frac{\partial F}{\partial x_r}$ or $\frac{\partial F}{\partial x_s}$ when $r \neq 0$, $s = 0$ or $r = 0$, $s \neq 0$ respectively and the function $F$ when $r = s = 0$.

The densities of the integrals $H_1^l, \dots, W_4^l$ in (22) are given by formula (21'), with the replacement of the arguments $u^l(A,t)$, $\frac{\partial u^l(A,t)}{\partial x_i}$ and $\frac{\partial^2 u^l(A,t)}{\partial x_i \partial x_j}$ by the arguments $u_{oo}^l(A,t)$, and $u_{ij}^l(A,t)$ respectively. System (22) is an infinite system of non-linear integral equations whose unknowns are functions $u_{rs}^l$ $(r,s = 0,1,\dots,n;\ l = 1,2,\dots)$. Based on Banach's fixed point theorem (c.f. [9], p.14), we shall prove the existence and the uniqueness of solution of the system.

Let us consider the space $\Lambda$ that consists of all infinite sequences $U = \{u_{rs}^l\}$ of functions defined and of class $C^1$ on the set $R \cup R^o$, satisfying the condition

$$\sum_{l=1}^{\infty} \sum_{r,s=0}^{n} a_{rs}^l \left(S\left[u_{rs}^l(A,t)\right] + H\left[u_{rs}^l(A,t)\right]\right) < \infty,$$

where

$$(23) \qquad a_{rs}^l = \max\left(K_{\gamma_1}^l, K_{\gamma_1}^{lr}, K_{\gamma_1}^{lrs}\right), \qquad (\gamma_1 = 1,4,5,6),$$

$$(24) \qquad S\left[f_{rs}^l(A,t)\right] = \sup_{(A,t)\in R} \left(\left|f_{rs}^l(A,t)\right| t^{\mu_1^l} \exp\left[-b|OA|^2\right]\right),$$

$$(25) \qquad H\left[f_{rs}^l(A,t)\right] =$$

$$= \sup_{(A,A_1)\in E^+;\, t,t_1\in(0,T)} \left(\frac{\left|f_{rs}^l(A,t) - f_{rs}^l(A_1,t_1)\right|}{|AA_1|^{\mu_2^l} + |t-t_1|^{\mu_3^l}} t^{\mu_1^l} \exp\left[-b|OA|^2\right]\right),$$

with

$$(26)\qquad \mu_1^l = \max_{\gamma_1} \mu_{F_{\gamma_1}}^{l,1}, \quad \mu_2^l = \min_{\gamma_1} \mu_{F_{\gamma_1}}^{l,2}, \quad \mu_3^l = \min_{\gamma_1} \mu_{F_{\gamma_1}}^{l,3}.$$

We define the distance of points $U \in \Lambda$ and $\bar{U} \in \Lambda$ by the formula

$$(27)\qquad d(U,\bar{U}) = \sum_{l=1}^{\infty} \sum_{r,s=0}^{n} a_{rs}^l \Big( S\left[u_{rs}^l(A,t) - \bar{u}_{rs}^l(A,t)\right] + H\left[u_{rs}^l(A,t) - \bar{u}_{rs}^l(A,t)\right] \Big).$$

It can be easily proved that $\Lambda$ is a complete metric space.

Let $Z$ be a set of all points of the space $\Lambda$ satisfying the conditions

$$(28)\qquad \begin{cases} \left|u_{rs}^l(A,t)\right| \leqslant \varrho_1 t^{-\mu_1^l} \exp\left[b\,|OA|^2\right], \\ \left|u_{rs}^l(A,t) - u_{rs}^l(A_1,t_1)\right| \leqslant \\ \qquad \leqslant \varrho_2\, t^{-\mu_1^l}\left( |AA_1|^{\mu_2^l} + |t-t_1|^{\mu_3^l}\right) \exp\left[b|OA|^2\right], \end{cases}$$

where $\varrho_1$ and $\varrho_2$ are certain positive constants that will be appropriately chosen later on, $b$ is a positive constant, and the exponents $\mu_1^l$, $\mu_2^l$, $\mu_3^l$ are given by formula (26).

In view of the system (22), we map the set $Z$ by the transformation

$$(29)\qquad v_{rs}^l(A,t) = \hat{\Xi}[U](A,t) \overset{df}{=} C\Big[(H_1^l(A,t))_{rs} + (H_2^l(A,t))_{rs} + (V^l(A,t))_{rs} - 2(W_2^l(A,t))_{rs} + 2(W_3^l(A,t))_{rs} + 4(W_4^l(A,t))_{rs}\Big],$$

where the right-side member is identical with that in (22).

Denote by $Z'$ the image of $Z$ in transformation (29). We shall find sufficient conditions for the inclusion $Z' \subset Z$. First of all we shall obtain some auxiliary estimates. From (15), (28) and (17)-(20) it follows that

$$(30) \qquad \left|F_6^1(A,t;\left\{u_{rs}^p\right\})\right| \le (m_{F_6}^1 + \varrho_1 \bar{m}_{F_6}^1 \cdot M_{F_6})\, t^{-\mu_1^1} \cdot \eta_7 \le$$

$$\le (m_F + \varrho_1 \bar{m}_F M_F)\, t^{-\mu_1^1} \cdot \eta_7$$

holds true, and using (16)-(20), (28) and the inequality

$$(31) \quad |AA_1|^{\alpha} \exp\left[-\bar{c}|OA|^2\right] \le \text{const}^{*)}, \quad (\alpha > 0, \bar{c}\text{-a positive constant})$$

we get

$$(32) \qquad \left|F_6(A,t;\left\{u_{rs}^p\right\}) - F_6(A_1,t_1;\left\{\bar{u}_{rs}^p\right\})\right| \le$$

$$\le \text{const}\ (k_F + \varrho_2 \bar{k}_F K_F)\cdot\left(|A_1A|^{\mu_2^1} + |t-t_1|^{\mu_3^1}\right) \cdot \eta_7 .$$

By (30), (31) and Theorem 2 in [7] we can conclude that the integral

$$J^1(A,t) = \int_0^t \int_{E^+} (t-\tau)^{-\frac{n}{2}+1} \exp\left[-\frac{|AB|^2}{4(t-\tau)}\right] F_6^1(B,\tau;\left\{u_{rs}^p\right\})\,dB\,d\tau$$

satisfies

$$(33) \qquad \left|J^1(A,t)\right| \le \text{const}\ (m_F + \varrho_1 \bar{m}_F M_F) \cdot \eta_1,$$

*) Const is a positive constant depending on n, T and b.

$$(34)\quad \left|J^1(A,t) - J^1(A_1,t_1)\right| \leqslant \text{const}\ (m_F+\varrho_1\bar{m}_F M_F)\left(|AA_1|^{\mu_3^1} + |t-t_1|^{\mu_2^1}\right) t^{-\mu_1^1}\cdot \eta_1.$$

Note that in virtue of (6), (28), (33) and (17)-(20) we obtain

$$(35)\qquad \left|F_1^1(A,t;\{u_{rs}^p\},\{v^q\})\right| \leqslant C_1 t^{-\mu_1^1}\cdot \eta_1,$$

where $C_1 = \text{const}\ (m_F + \varrho_1\bar{m}_F M_F)(1+M_F)$, and from (7), (28), (34), (31) and (17)-(20) it follows that

$$(36)\quad \left|F_1^1(A,t);\{u_{rs}^p\},\{v^q\}, - F_1^1(A_1,t_1;\{\bar{u}_{rs}^p\},\{\bar{v}^q\})\right| \leqslant$$

$$\leqslant \text{const}\ (k_F+\varrho_2\bar{k}_F K_F)(1+K_F)\ t^{-\mu_1^1}\left(|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}\right)\cdot \eta_1$$

is valid. Also, by (10), (28) and (17)-(20) we have

$$(37)\qquad \left|F_4^1(P,t;\{u_{rs}^1\})\right| \leqslant C_1\ t^{-\mu_1^1+\frac{1}{2}}\cdot \eta_3.$$

By a similar argument, taking into account relations (11), (28), (17)-(20) and (31), we obtain

$$(38)\quad \left|F_4^1(P,t;\{u_{rs}^p\}) - F_4^1(P_1,t_1;\{\bar{u}_{rs}^p\})\right| \leqslant$$

$$\leqslant \text{const}(k_F+\varrho_2\bar{k}_F K_F)\cdot\left(|PP_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1+\frac{1}{2}}\right) t^{-\mu_1^1}\cdot \eta_3,$$

and using (12), (28) and (17)-(20) we can conclude that

(39) $$\left|\frac{\partial}{\partial z} F_4^1(P,t;\{u_{rs}^p\}) - \frac{\partial}{\partial z} F_4^1(P_1,t;\{\bar{u}_{rs}^p\})\right| \leqslant$$

$$\leqslant \text{const}\ (k_F + \varrho_1 \bar{k}_F K_F)\ |PP_1|^{\mu_2^1}\ t^{-\mu_1^1} \cdot \eta_3$$

holds.

Relations (13), (28) and (17)-(20) yield

(40) $$\left|F_5^1(P,t;\{u_{rs}^p\})\right| \leqslant C_1 t^{-\mu_1^1} \cdot \eta_5,$$

and from (14), (28), (31) and (17)-(20) it follows that the inequality

(41) $$\left|F_5^1(P,t;\{u_{rs}^p\}) - F_5^1(P_1,t_1;\{\bar{u}_{rs}^p\})\right| \leqslant$$

$$\leqslant \text{const}\ (k_F+\varrho_2 \bar{k}_F K_F)\ t^{-\mu_1^1}\ (|PP_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}) \cdot \eta_5.$$

is fulfilled.

Furthermore, let us observe that basing on (31) and using notation (24), (25) we can write the estimates (31)-(36) and (47)-(52) of paper [7] in the following form

(42) $$S[V(A,t)] \leqslant \text{const}\ S[\Phi(A,t)],$$

(43) $$H[V(A,t)] \leqslant \text{const}\ S[\Phi(A,t)],$$

(44) $$S\left[\frac{\partial^2}{\partial x_n^2} V(A,t)\right] \leqslant \text{const}\ S[\Phi(A,t)],$$

(45) $$H\left[\frac{\partial^2}{\partial x_n^2} V(A,t)\right] \leqslant \text{const}\ S[\Phi(A,t)],$$

(46) $$S\left[\frac{\partial^3}{\partial x_n^2\,\partial x_1} V(A,t)\right] \leqslant \text{const } S\left[\Phi(A,t)\right],$$

(47) $$H\left[\frac{\partial^3}{\partial x_n^2\,\partial x_1} V(A,t)\right] \leqslant \text{const } S\left[\Phi(A,t)\right],$$

(48) $$S\left[W_2(A,t)\right] \leqslant \text{const } S\left[\varphi(P,t)\right],$$

(49) $$H\left[W_2(A,t)\right] \leqslant \text{const } S\left[\varphi(P,t)\right],$$

(50) $$S\left[W_3(A,t)\right] \leqslant \text{const } S\left[\psi(P,t)\right],$$

(51) $$H\left[W_3(A,t)\right] \leqslant \text{const } S\left[\psi(P,t)\right],$$

(52) $$S\left[W_4(A,t)\right] \leqslant \text{const } S\left[\psi(P,t)\right],$$

(53) $$H\left[W_4(A,t)\right] \leqslant \text{const } S\left[\psi(P,t)\right].$$

respectively. Finally, basing on formula (27) in [7], we have

$$\frac{\partial^3}{\partial x_1\,\partial x_n^2}\left[-2\,W_2(A,t)\right] = \frac{\partial}{\partial x_1} W_0(A,t),$$

whence, applying Theorem 2 in [7], we obtain

(54) $$S\left[\frac{\partial^3}{\partial x_n^3} W_2(A,t)\right] \leqslant \text{const } S\left[\varphi(P,t)\right],$$

(55) $$H\left[\frac{\partial^3}{\partial x_n^3} W_2(A,t)\right] \leqslant \text{const } S\left[\varphi(P,t)\right].$$

Now, we are proceeding to the inclusion $Z' \subset Z$.

Note that by assumptions (8) and (18) in this paper and Theorem 4 in [7], we obtain*)

(56) $$\left|(H_1^1(A,t))_{rs}\right| \leqslant \text{const } m_F\, t^{-\mu_1^1} \cdot \eta_3,$$

(57) $$\left|(H_1^1(A,t))_{rs} - (H_1^1(A_1,t_1))_{rs}\right| \leqslant$$
$$\leqslant \text{const } m_F t^{-\mu_1^1} (|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}) \cdot \eta_3.$$

Similarly, in virtue of assumptions (9) and (18) above and Theorem 3 in [7] we have

(58) $$\left|(H_2^1(A,t))_{rs}\right| \leqslant \text{const } m_F t^{-\mu_1^1} \cdot \eta_3,$$

(59) $$\left|(H_2^1(A,t))_{rs} - (H_2^1(A_1,t_1))_{rs}\right| \leqslant$$
$$\leqslant \text{const } m_F t^{-\mu_1^1} (|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1})\, \eta_3.$$

From (35) and (18) in this paper and from Theorem 2 in [7] it follows that

(60) $$\left|(V^1(A,t))_{rs}\right| \leqslant \text{const } C_1\, t^{-\mu_1^1} \cdot \eta_3$$

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*) It is easily seen that all inequalities in the theses of Theorems 2-5 in [7] remain valid if the differentiation with respect to $x_n$ is replaced by the differentiation with respect to an arbitrary independent variable $x_i$ $(i=1,2,\dots,n-1)$.

and

$$(61) \quad \left|(V^1(A,t))_{rs} - (V^1(A_1,t_1))_{rs}\right| \leq$$

$$\leq \text{const } C_1 t^{-\mu_1^1} \left(|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}\right) \cdot \eta_3.$$

are valid. Also, relations (8), (9) and (35), Theorems 2-4 in [7], and estimate (37) yield for the functions $u_{rs}^p \in Z$ the following inequality

$$(62) \quad \left| F_4^1(P,t;\{u_{rs}^p\}) - \frac{\partial^2}{\partial x_n^2}\left[V^1(P,t) + H_1^1(P,t) + H_2^1(P,t)\right] \right| \leq$$

$$\leq C_2 \, t^{-\mu_1^1+\frac{1}{2}} \cdot \eta_3$$

with $C_2 = \text{const}\left[m_F+(1+M_F)(m_F+\varrho_1\bar{m}_F)M_F\right]$, whence, in virtue of Theorem 5 and inequality (25) in [7] and Theorem 3 in [3], it follows that

$$(63) \quad \left|(W_2^1(A,t))_{rs}\right| \leq \text{const } C_2 t^{-\mu_1^1} \cdot \eta_5,$$

$$(64) \quad \left|(W_2^1(A,t))_{rs} - (W_2^1(A_1,t_1))_{rs}\right| \leq$$

$$\leq \text{const } C_2 t^{-\mu_1^1} \left(|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}\right) \cdot \eta_5$$

hold. By basing on relations (40), (9), (10), (35) and (62) given above and Theorems 2-4 in [7], and by subsequent use of the equality

$$\frac{\partial^3}{\partial x_n^3}\left[-2\, W_3(A,t)\right] = \frac{\partial}{\partial x_n} W_o(A,t)$$

and of Theorem 3 in [3], we obtain the following estimate

(65) $$\left|F_5^1(A,t;\{u_{rs}^p\}) - \frac{\partial^3}{\partial x_n^3}\left[H_1^1(A,t) + H_2^1(A,t) + V^1(A,t) - 2W_3^1(A,t)\right]\right| \leqslant$$
$$\leqslant C_3 t^{-\mu_1^1} \cdot \eta_5,$$

where $C_3 = \text{const}\ (2+M_F)(m_F+\varrho_1\bar{m}_F M_F)$, whence, by Theorem 5 in [7], relation (24) in [7] and Theorem 3 in [3], we can conclude that inequalities

(66) $$\left|(W_3^1(A,t))_{rs}\right| \leqslant \text{const}\ C_3 t^{-\mu_1^1} \cdot \eta_1,$$

(67) $$\left|(W_4^1(A,t))_{rs}\right| \leqslant \text{const}\ C_3 t^{-\mu_1^1} \cdot \eta_1,$$

(68) $$\left|(W_3^1(A,t))_{rs} - (W_3^1(A_1,t_1))_{rs}\right| \leqslant \text{const}\ C_3 t^{-\mu_1^1}(|AA_1|^{\mu_2^1} +$$
$$+ |t-t_1|^{\mu_3^1}) \cdot \eta_1,$$

(69) $$\left|(W_4^1(A,t))_{rs} - (W_4^1(A_1,t_1))_{rs}\right| \leqslant$$
$$\leqslant \text{const}\ C_3\ t^{-\mu_1^1}\ (|AA_1|^{\mu_2^1} + |t-t_1|^{\mu_3^1}) \cdot \eta_1.$$

hold good. From (56), (58), (60), (63), (66), (67) and (29) it follows that

$$\left|v_{rs}^1(A,t)\right| \leqslant \text{const}\ (3+M_F)(m_F+\varrho_1\bar{m}_F M_F)\ t^{-\mu_1^1} \exp\left[b|OA|^2\right],$$

is valid, and (57), (59), (61), (64), (68) and (69) yield

$$\left| v_{rs}^{1}(A,t) - v_{rs}^{1}(A_1,t_1) \right| \leq$$

$$\leq \mathrm{const}(3+M_F)(m_F+\varrho_1\bar{m}_F M_F)\ t^{-\mu_1^1} \exp\left[ b|OA|^2 \right].$$

Basing on the last two estimates, on the definition of the set $Z$ (see (28)) and on the properties of the integrals appearing in (29), we can conclude that the transformation (29) maps $Z$ into itself if the following system of inequalities

$$(70)\qquad \begin{cases} \mathrm{const}\ (3 + M_F)(m_F + \varrho_1\bar{m}_F M_F) \leq \varrho_1, \\ \mathrm{const}\ (3 + M_F)(m_F + \varrho_1\bar{m}_F M_F) \leq \varrho_2 \end{cases}$$

is satisfied, where const denotes a positive constant depending only on $n,T$ and $b$. It is readily seen that the inequalities (70) hold true if the coefficient $m_F$ (see (18)) is sufficiently small and the parameters $\varrho_1$ and $\varrho_2$ are chosen sufficiently large.

Now, we shall find sufficient conditions for the operation (29) to be a contraction. Let $U = \left\{ u_{rs}^{p} \right\}$ and $\bar{U} = \left\{ \bar{u}_{rs}^{p} \right\}$ be two arbitrarily fixed points of the set $Z$ (see (28)). In virtue of (7), (16), (33), (17)-(20), (23) and (27) we can write

$$(71)\qquad S\left[ F_1^1(A,t;\left\{u_{rs}^p\right\},\left\{v^q\right\}) - F_1^1(A_1,t_1;\left\{\bar{u}_{rs}^p\right\},\left\{\bar{v}^q\right\}) \right] \leq$$

$$\leq \bar{k}_F(1+K_F)d(U,\bar{U}),$$

whence, by (42)-(45), we obtain*)

*) The symbol $\bar{V}^1(A,t)$ denotes the integral $V^1(A,t)$ (see (21)) whose density $\Phi(A,t)$, is equal to $F_1^1(A,t;\{u_{rs}^p\},\{v^q\}) - F_1^1(A,t;\{\bar{u}_{rs}^p\},\{\bar{v}^q\})$. The symbols $\bar{W}_2^1(A,t),\bar{W}_3^1(A,t),\bar{W}_4^1(A,t)$ are understood in a similar way.

(72) $S\left[(\bar{V}^1(A,t))_{rs}\right] + H\left[(\bar{V}^1(A,t))_{rs}\right] \leqslant \text{const } \bar{k}_F(1+K_F)d(U,\bar{U}).$

Furthermore, let us note that relations (11), (71), (44), (17)-(20), (23) and (27) yield

(73) $$S\left[F_4^1(P,t;\{u_{rs}^p\}) - F_4^1(P,t;\{\bar{u}_{rs}^p\}) + \frac{\partial^2}{\partial x_n^2}\left[\bar{V}^1(A,t)\right]\right] \leqslant$$

$$\leqslant \bar{k}_F\left[K_F + \text{const } (1+K_F)\right]d(U,\bar{U}),$$

so that, using (48), (49), equality (25) in [7] and Theorem 3 in [3], we have

(74) $$S\left[(\bar{W}_2^1(A,t))_{rs}\right] + H\left[(\bar{W}_2^1(A,t))_{rs}\right] \leqslant$$

$$\leqslant \text{const } \bar{k}_F\left[K_F + \text{const}(1 + K_F)\right] d(U,\bar{U}).$$

Since by (14), (17)-(20), (71), (73), (46), (54), and (23) and (27) the inequality

$$S\left[F_5^1(P,t;\{u_{rs}^p\}) - F_5^1(P,t;\{\bar{u}_{rs}^p\}) + \frac{\partial^3}{\partial x_n^3}\left[\bar{V}^1(A,t) - 2\bar{W}_2^1(A,t)\right]\right] \leqslant$$

$$\leqslant \text{const } \bar{k}_F\ (1+K_F)d(U,\bar{U}),$$

holds good, we can conclude, based on (50)-(53) above, relation (24) in [7] and Theorem 3 in [3], that the following estimates

(75) $S\left[(\bar{W}_3^1(A,t))_{rs}\right] + H\left[(\bar{W}_3^1(A,t))_{rs}\right] \leqslant \text{const } \bar{k}_F(1+K_F)d(U,\bar{U}),$

(76) $S\left[(\overline{W}_4^l(A,t))_{rs}\right] + H\left[(\overline{W}_4^l(A,t))_{rs}\right] \leqslant \text{const}\ \overline{k}_F(1+K_F)d(U,\overline{U})$

are valid. Now, let us consider the points $\hat{\Xi}\,U$ and $\hat{\Xi}\,\overline{U}$ being the images (in transformation (29)) of two arbitrarily fixed points $U$ and $\overline{U}$ of $Z$. Using the definition (27), we have the equality

$$d(\hat{\Xi}U, \hat{\Xi}\overline{U}) = \sum_{l=1}^{\infty}\sum_{r,s=0}^{n} a_{rs}^l \Big(S\Big[(\overline{V}^l(A,t) - 2\overline{W}_2^l(A,t) + 4\overline{W}_4^l(A,t) + 2\overline{W}_3^l(A,t))_{rs}\Big] + H\Big[(\overline{V}^l(A,t) - 2\overline{W}_2^l(A,t) + 4\overline{W}_4^l(A,t) + 2\overline{W}_3^l(A,t))_{rs}\Big]\Big),$$

which, together with (72) and (74)-(76), yields

$$d(\hat{\Xi}U, \hat{\Xi}\overline{U}) \leqslant C_o\,\overline{k}_F\,K_F\,(1+K_F)d(U,\overline{U}),$$

where $\overline{k}_F$ and $K_F$ are given by (18) and (20) respectively, and $C_o$ is a positive constant depending only on $n,T$ and $b$. Hence, operation (29) is a contraction if the coefficient $\overline{k}_F$ is so small that the inequality

(77) $$C_o\,\overline{k}_F\,K_F\,(1 + K_F) < 1$$

holds true.

On joining the above-obtained results and on using Banach's fixed point theorem (see e.g. [9], p.14), we can conclude that the following lemma is valid.

L e m m a 1. If assumptions $1^o$-$9^o$ are satisfied and if inequalities (70) and (77) hold true, then the system of integral equations (22) possesses one and only one solution $U^* = \{\overset{*}{u}{}_{ij}^l\}$, $U^* \in \Lambda$.

Let us note that from (30), (32), (35), (36), (8), (9), (37)-(41) and from the properties of the integrals appearing in system (22) (see Theorems 1-5 and 7 in [3], Remarks 1 and 2 and Theorems 1-5 in [7]) the following relations

$$\overset{*}{u}{}_{i0}^{1}(A,t) = \frac{\partial \overset{*}{u}{}_{00}^{1}(A,t)}{\partial x_i}, \qquad \overset{*}{u}{}_{ij}^{1}(A,t) = \frac{\partial^2 u_{00}^{1}(A,t)}{\partial x_i \, \partial x_j}$$

$((A,t) \in R)$ result, whence, and by Lemma 1 above, we can assert that the infinite sequence $\left\{\overset{*}{u}{}_{00}^{1}(A,t)\right\}$ is the unique solution of the system of integro-differential equations (21). Moreover, it follows directly from assumptions (8) and (9), from inequalities (35)-(40) above and from Remark 1 in [7] (see the proof of Theorem 6 in [7]) that the elements of the sequence $\left\{\overset{*}{u}{}_{00}^{1}\right\}$ satisfy system (1) and conditions (2)-(5) and hence this sequence is a solution of the considered boundary value problem (1)-(5).

Basing on the results of this section we can conclude that the following theorem is valid.

T h e o r e m 1. If assumptions $1^{\circ}$ - $9^{\circ}$ and inequalities (70) and (77) are satisfied, then the boundary value problem (1) - (5) possesses a solution being the limit of the sequence of successive approximations defined by

$$U_0 = \left\{u_0^1(A,t)\right\}, \qquad U_{m+1} = \hat{T}\left[U_m\right]$$

$(m=0,1,\dots)$, where $\hat{T}$ is the operator appearing on the right-hand side of equality (21). The error of the m-th approximation $U_m$ satisfies the inequality

$$d(U_m,\overset{*}{U}) \leqslant \frac{\left[\text{const}\ \bar{k}_F\, K_F(1 + K_F)\right]^m}{1 - C_0\, \bar{k}_F\, K_F(1+K_F)}\, d(U_0,U_1).$$

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