DEMONSTRATIO MATHEMATICA
Vol. IX No 1 1976

Marian Jaszczak

ON SOME GENERALIZATION OF THE NOTION OF DECOMPOSABLE TENSOR IN A LOCALLY DECOMPOSABLE RIEMANNIAN SPACE

Let M_n be an n-dimensional locally decomposable Riemannian manifold of class C^∞. Hence by assumption M_n is a differentiable manifold endowed with

1) a tensor field $F : BM \to BM$ satisfying, in the domain of an arbitrary chart x from the atlas of M_n, the following identities:

$$F_j^i F_i^h = A_j^h \tag{1}$$

and

2) a field of a metric tensor g satisyfing, in the domain of the chart x, the identities

$$F_j^t F_i^s g_{ts} = g_{ji}. \tag{2}$$

Moreover, the following identities hold

$$\nabla_j F_i^h = 0, \quad \nabla_j F_{ih} = 0, \tag{3}$$

where BM is a module of smooth vector fields on M_n and ∇ denotes a Levi-Civita connection on M_n ([1] p. 221).

Let $f : BM^p \to FM$ be a tensor field of type (p,o) on M_n. Here $BM^p = BM \times BM \times \ldots \times BM$, and FM is the ring of smooth functions on M_n. We assume the following definition.

D e f i n i t i o n . A tensor field $f : BM^p \to FM$ of type (p,o) will be called 1/p-decomposable field provided that in the domain of an arbitrary chart $x \in \mathrm{atl}\, M_n$ the following identities hold

$$F^r_j \nabla_r f_{ii_2 \ldots i_p} = F^r_i \nabla_j f_{ri_2 \ldots i_p}. \tag{4}$$

It is easy to see that if we assume that the tensor f is a pure tensor, i.e. if in the domain of an arbitrary chart $x \in \mathrm{atl}\, M$ the following identity hold

$$\bigwedge_{i_1 \leq i_k, i_h \leq i_p} F^r_{i_k} f_{i_1 \ldots r \ldots i_p} = F^r_{i_h} f_{i_1 \ldots r \ldots i_p}$$

then the tensor f is decomposable in the usual sense ([1] p. 225).

If we assume that the indices $i_2, \ldots, i_p$ denote successive covariant derivatives of the vector $\hat{f}_i$, i.e. if we assume that $\hat{f} : BM \to FM$ and

$$f_{ii_2 \ldots i_p} = \nabla_{i_p} \nabla_{i_{p-1}} \ldots \nabla_{i_2} \hat{f}_i$$

then our definition becomes the definition of p-decomposable vector in the sense given in [2].

Hence the definition given above is a generalization of the definition of decomposable tensor and the definition of p-decomposable tensor.

It is easy to see that analogously as in [1] p. 226 and [2] we can prove the following theorem.

T h e o r e m . A necessary and sufficient condition in order that in a compact orientable locally decomposable Riemannian space M_n a tensor of type (p,o) be 1/p-decomposable is that in the domain of an arbitrary chart $x \in \mathrm{atl}\, M_n$ the following identities hold

$$g^{ji} \nabla_j \nabla_i f_{i_1 \ldots i_p} - F^{ji} \nabla_j \nabla_i f_{i_1 \ldots i_{p-1} r} F^r_{i_p} = 0.$$

BIBLIOGRAPHY

[1] K. Yano: Differential geometry on complex and almost complex spaces. Pergamon Press 1965, p.221-226.

[2] K.D. Singh, A. Nigam: On generalizations of product-conformal Killing vectors, Demonstratio Math. 7 (1974) 411-422.

INSTITUTE OF MATHEMATICS, TECHNICAL UNIVERSITY OF WARSAW

Received February 14, 1975.