Editorial

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Abstract: This short article serves as an epilog of the thirteen preceding papers in this special issue of CMAM. All contributions are authored by participants of the 7th Sino–German Workshop on Computational and Applied Mathematics at the Kiel University. The topics cover fourth-order problems, solvers and multilevel methods, a posteriori error control and adaptivity, and data science.

Keywords: Scientific Computing, Fourth-Order Problems, Multilevel Methods, Adaptive Algorithms, Data Science

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The 7th Sino–German Workshop on Computational and Applied Mathematics was held at the Kiel University, August 19–23, 2019. The symposium enhanced the mutual understanding of the state of the art on both sides and stimulated future bilateral cooperation on various topics of scientific computing. The bilateral workshop followed previous ones in Berlin (2005), Hangzhou (2007), Heidelberg (2009), Guangzhou (2011), Augsburg (2015), and Shanghai (2017).

The workshop in Kiel brought together invited speakers from Chinese and German universities as well as additional local attendees. The thirteen papers in this special issue are co-authored by leading numerical and applied mathematicians of both sides and demonstrate the remarkable advances in computational and applied mathematics. The papers cover a wide range of topics in contemporary applied mathematics and contribute to one or more of the four thematic blocks highlighted below.

Fourth-Order Problems

Three papers are concerned with the discretization of fourth-order problems. The paper by Wang, Li, and Zhang [9] introduces an $H(\mathrm{curl}^2)$-conforming spectral element method for some quad-curl problems. The paper by Carstensen and Hu [3] studies adaptive and multigrid algorithms for a modified version of the Argyris finite element, where nestedness of the finite element spaces is achieved through a relaxation of the C^2 continuity at vertices. The paper [7] by Tang and Xu investigates the nonconforming Morley finite element and a related multilevel scheme. Since these nonconforming spaces are non-nested, the authors employ a transfer operator by using the Powell–Sabin element as a conforming relative.

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PDE Solvers and Multilevel Methods

Another frequent topic focused on the solution of high-dimensional systems of linear equations resulting from the discretization of partial differential equations. The paper [6] proposes a new paradigm, which is useful in the design and find effective scalings of block-diagonal preconditioners for symmetric saddle-point problems. The approach is based on the natural physical units underlying the respective saddle-point problem and it suggests a natural combination of the intrinsic parameters. A dimensional consistency-based preconditioning is introduced as a systematic way to design parameter-robust preconditioners for saddle-point systems. The paper by Börm [2] investigates the improvements to solve the discrete problem from boundary element methods for the high-frequency Helmholtz equation. The improvement firstly makes the algorithm faster by taking advantage of the fact that the directional interpolation technique splits the kernel function into a plane wave and a smooth part, that can be approximated by interpolation. Another improvement is replacing the weight matrices with low-rank approximations, which significantly reduces the storage requirements.

Several articles discuss multilevel methods. Tang and Xu in [7] for the Morley element and Carstensen and Hu in [3] for the Argyris element, both for fourth order problems. The non-nestedness of the discrete spaces is treated in two different ways. While Tang and Xu manage grid transfer through an auxiliary element, the finite element space itself is enriched until nestedness in [3]. Both articles use their results to analyze local smoothers on adaptively refined meshes. In the paper [12], Witte, Arndt, and Kanschat adopt a more practical view and discuss the efficient implementation of subdomain smoothers.

A Posteriori Error Estimation and Adaptivity

The topic of adaptive mesh-refinement and a posteriori error estimation is also discussed in several contributions. Bertrand, Boffi, and Ma [1] consider the approximation of eigenvalues arising from the mixed Hellinger–Reissner elasticity problem by using a modification of the Hu–Zhang element. A residual-type a posteriori error estimator is proposed and the reliability of the error estimator is derived. The optimal convergence of the estimator and the contraction property of the adaptive finite element method (AFEM) are also established. Wick derives in [11] error estimates for phase-field models including domains with a slit. The error estimation of another generalization of the Cahn–Hilliard equation is investigated in the article [13], where the motility of the interface may be concentration dependent. Interface problems are also discussed in the paper [5] by Engwer and Westerheide, where partial differential equations in the bulk and on a surface are coupled through an unfitted discontinuous Galerkin method. The paper by Diening and Kreuzer [4] refers to an open question in the Dörfler marking: Is the threshold condition $\theta < \theta_\star$ for the Dörfler marking parameter necessary to obtain optimal algebraic rates of the AFEM? The authors first introduce a simplified version of the axioms of adaptivity with separate marking. Then a one-dimensional (non-PDE) example, which fits into the axioms of adaptivity, allows for exponential convergence. But the converge rate of the AFEM is arbitrarily small if $\theta > \theta_\star$ is chosen for the Dörfler marking.

Data Science

Wenzel, Nestler, Reuther, Simon, and Voigt develop in [10] reliable data analysis tools for the identification, classification, and tracking of defects in active nematics in both simulation and microscopy data. The methods are demonstrated on two kinds of models, an active nematodynamic model for microdubule bundels and a multi-phase field model for epithelia cell sheets. Tian and Wu investigate in [8] the problem of finite-rank approximation of Markov processes from dynamical samples. By quantifying the approximation error of Perron–Frobenius operators based on the kernel embedding of transition densities, a new variational approach is developed for seeking the optimal finite-rank approximation in a data-driven manner. This circumvents the limitations of the existing approaches and is applicable to more general dynamical systems including deterministic systems. The numerical experiments show that the proposed approach can achieve better compressibility.

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