Research Article

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Study of heat transfer through functionally graded material fins using analytical and numerical investigations

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Abstract: The present study used analytical and numerical methods to conduction works in functional graded material fin that is exposed to progressive boundary cooling and heating. The fin length was chosen as a direction to vary thermal characteristic in functionally graded material, and the physical properties were ascertained by the linear model. In heat sink, two approximation analytical techniques are employed to evaluated the thermal performance: one based on a mean value problem and the other strategy based on principle of the equivalent qualities. ANSYS APD's numerical solution was based on a stepwise modification of characteristics. The outcomes indicated that the thermal conductivity ratio significantly influences temperature levels, with the terminal temperature levels of the functionally graded fin varying exponentially in relation to the thermal conductivity ratio. The influence of material index begins with the length of functionally graded material-fin where the fin's terminus experiences the largest effect. A 6.756% is the maximum absolute error between numerical and analytical findings, which indicating a strong concordance between the two analysis methods.

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1 Introduction

Convection heat transmission from a surface to the adjacent fluid can be significantly improved by attaching thin metal strip to the surface, therefore augmenting the heat transfer area. Protrusions into the adjacent fluid are referred to as fins, frequently employed in diverse engineering applications to enhance the heat transmission. Finned surface are widely employed in refrigeration, air conditioning, steam power plants, cryogenics, cars, furnaces, and several industrial cooling systems, Consequently, it is entirely expected that numerous researchers have examined the thermal properties of these fins [1].

Functionally graded material (FGM) is comprised of ceramic and metal, which are graded in high and poor-temperature regions according to requirement applications, and was initially introduced by scientists in 1984 for developing work of thermal barriers subjected to significant temperature differentials. FGMs are considered a substitute for laminated composites owing to their exceptional thermal and mechanical properties [2]. FGMs offer numerous advantages that render them appealing for various applications. The ongoing spatial variation of thermophysical characteristics, including thermal conductivity and heat capacity, provides benefits unattainable with homogeneous materials. A functionally graded thermal fin can endure significant temperature gradient without incurring excessive thermal stress [3]. Research pertaining to the formulation of heating models for FGMs to ascertain temperature distribution in fin structures has garnered significant interest from numerous researchers. A lot of research that has been done on FGM fins lately has been on finding the local temperature field by using FGM distribution methods. Oguntala *et al.* [4] presented a novel hybrid methodology that integrates the Laplace transform with the Chebyshev Pseudospectral collocation technique. The proposed method is utilized to analyze the transient thermal performance of a convective-radiative fin composed of FGM subjected to Lorenz force effects. The finding shows that the transient thermal response of the fin is negligible under a linear function, but it is pronounced under an exponential function. The FGM fin temperatures, made by an exponential law, reached a stable state more swiftly than those made by linear and quadratic laws. The temperature gradient of the FGM fin is negligible at any magnetic field intensity and under both convective and radiative conditions, in comparison with both linear and exponential functions of the homogeneous structure fin. Babaelahi *et al.* [5] employed an analytical solution to evaluate the thermal fields in heat pipes utilizing a heat resistance approach, which they executed through FGM power-law fins. Decreasing the internal radius and increasing the external radius of the FGM fin according to a power-law relationship improved thermal performance, resulting in a reduction in the fin absolute temperature by up to 1 K. Increasing the fin's internal temperature and the ambient air adversely impacted the fin's performance by reducing cooling efficiency and convection fields.

The absolute temperature weakened by one and a half degrees Kelvin when the grading index and heat transfer coefficient were increased. On the other hand, decreasing the thickness style factor and conduction coefficients had a positive outcome on the dimensionless temperature of fin. Dogmaz *et al.* [6] conducted a numerical and experimental investigation to improve the thermal performance of annular FGM fins. The numerical calculations conducted to ascertain the optimal volume composition of FG fins were followed by the fabrication of FG fins using powder metallurgy method and the hot-pressing process. Empirical results suggest that the assessment of transfer convection is influenced by material composition, fin spacing, and the temperature differential between the base and the ambient environment. FG fin arrangement increases the heat transfer by 59%, while aluminum fin arrangement increases it by 33% in comparison to finless cylinders.

Furthermore, FG fins surpass aluminum fin arrangement by 40% in the convection values. Jemiseye *et al.* [7] demonstrated a hybrid computational study employing a combination of Laplace transformation and Legendre wavelet collocation methods for space- and time-dependent nonlinear thermal models of a longitudinal conductive-radiative functionally graded extended surface. The analysis results indicated that the inhomogeneity index is directly proportional to the FGM fin temperature. Nonetheless, the convective term is inversely related to the temperature under quadratic-law heat conductivity of the passive device demonstrating superior performance with reduced thermal distribution compared to linear- and exponential-law heat conductivity. Sobamowo *et al.* [8] used finite difference method to investigate the numerical of transient thermal performance of a convective-radiative fin with FGMs under the influence of magnetic field. The different variations in thermal conductivity were considered according to linear, quadratic, exponential, and power-law in the thermal models. The findings indicated that an increase in magnetic field parameters and radiative, along with the inhomogeneity index, enhances the thermal performance of the fin. Furthermore, it was determined that the transient responses of the FGM fin exhibiting power-law function and linear-law demonstrate the slowest and fastest thermal response, respectively. The predominant studies have modeled the thermal profile in FGM fins utilizing numerical techniques, specifically through the finite element method (FEM) and also it was focusing mainly on the analytical method.

Gaba *et al.* [9] provided an analytical solution for the examination of annular fins with a changeless weight obtained from FGM. A comprehensive second-order ordinary differential equation has been written to construe variations in thermal conductivity and thickness across a defined radii for gradations of material. Oguntala *et al.* [10] examined thermal models with the "Chebyshev-spectral-collocation-technique." This technique presumed that "the thermal characteristic of FGM adhered to a power-law- and linear function." This work examines the inhomogeneity index of FGM of porous heat sink with convective and radiative components in relation to the thermal conductivity. Khan and Aziz [11] came up with a way to figure out how well a steady-state fin works by using Legendre functions for the quadratic case and Bessel functions for linear and exponential thermal conductivity with longitudinal length. The material gradient of the FG-fin significantly affects the transient and steady-state performance of the fin. Hassanzadeh and Bilgili [12] investigated the thermal characteristics of materials employed in heat exchangers featuring longitudinal FGM fins. They computed the resultant equations by employing "Dirichlet and Neumann" boundary conditions (BCs). A new strategy and mean value theorem were used for solving the equations. The results proved that "the inhomogeneity index of FGM significantly affects the thermal energy characteristics in heat exchangers." To look at how the functionally graded rectangular fin reacts to heat, Jemiseye *et al.* [13] applied both numerical and analytical models, the analytical model applied "Laplace transformation-Legendre-wavelet-collocation-methods." When comparing quadratic heat conductivity to linear and exponential heat conductivities, the "passive device" showed better thermal dispersion and lower temperature. Sobamowo *et al.* [14] performed a nonlinear transient thermal analysis on a convective-radiative fin composed of FGM subjected to a magnetic field. They discovered that augmenting the "inhomogeneity-index" of radiative and magnetic field factors enhanced the fin's performance. Hassanzadeh and Bilgili [12] utilized a rough mathematical approach based on the "mean-value-theorem" to evaluate the performance of heat-sink. This study's findings indicate that replacing the fin's material with FGM significantly conserves energy as a result to improve heat transfer between the fin and the boundary. Ranjana and Mallick [15] employed analytical method and the "homotopy perturbation technique" to solve the nonlinear dimensionless problem for a conductive-convective-radiative annular fin made of FGM. The numerical results were used to validate the adopted numerical model. A comprehensive performance analysis is conducted, highlighting the significant impact of various parameters, including thermo-geometric parameters, conduction-radiation interactions, variations in (a) heat generation, (b) thermal conductivity, (c) the exponent of the heat transfer coefficient, and (d) surface emissivity on each of temperature-profile, thermal-efficiency, rate of heat transfer, and effectiveness. The differential transform method was used by George *et al.* [16] to look at how the material index of FGM, as well as convective and radiative factors, affected the porous heat sink performance. Kumar *et al.* [17] suggested a model that combines artificial-neural-networks and differential evolution methods to figure out how the porous, undulating fin will transfer heat. This work analyzed the impact of several factors on temperature distribution and the heat-transfer rate visually. Sobamowo *et al.* [18] carried out a finite element analysis to study the transient thermal response of convective-radiative FG-fin in response to a "temperature-varying-magnetic-field." The parametric analyses indicated that the temperature of the extended surface rises in accordance with the homogeneity index. As the radiation and magnetic field characteristics get better, the thermal distribution across the extended surface gets worse. This makes the heat transfer rate through the passive device faster.

The purpose of this work is to provide both an analytical and a numerical solution. In order to investigate the influence of FGM on the profile of fin's temperature, a one-dimensional, axial FGM fin is utilized. The thermal properties of the FG fins are characterized using a linear model. Additionally, theoretical model based on equivalent-property idea is equated with the linear FG-model, and a numerical analysis is conducted using ANSYS APDL. By comparing the numerical and analytical results, the study's scope is expanded and the reliability of the study is enhanced.

2 Problem description

A functionally graded fin has thickness (t), length (L), and width (W) and is subjected in media with environmental temperature (T_∞) and heat transfer coefficient (h). The left side temperature of FG-fin is constant (T_b) as illustrated in Figure 1.

FGM is a kind of composite materials and it is a new material used to design materials having certain distribution of material properties. The distribution of material qualities can be accomplished in one-dimensional (1D), two-dimensional, and three- dimensional FGMs. In 1D-FGMs), several functions or models, such as the power-law, exponential, and sigmoid models, are applied to delineate the arrangement of material attributes in graded structures [19]. This study used the 1D-FGM to examine the influence of material distribution on the temperature (value and profile) of FGM-fin, utilizing a linear formulation to characterize the thermal attributes of graded structures fin as demonstrated in relation (1).

$$P(x) = P_0 \times (1 + \gamma \times x), \tag{1}$$

where $P(x)$ is the attribute at any given place (x), γ is the linear equation slop $\left(\gamma = \frac{P(0)}{P(L)} - 1 = K_{\text{ratio}} - 1\right)$, and P_0 is the property located at the coordinates ($x = 0$).

The thermal properties along the length of FG-fin are displayed in Figure 2 when left materials of the FG-Fin is copper (Cu) with thermal conductivity (395 W/(m K)) (*i.e.*, $P(L)$ = 395 W/(m K)). From Figure 2a, the linear variation in

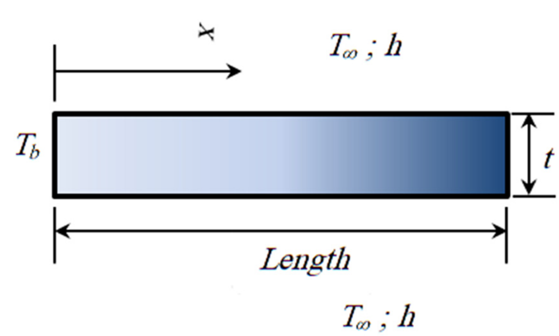

Figure 1: Dimensions and heat conditions of FG-fin.

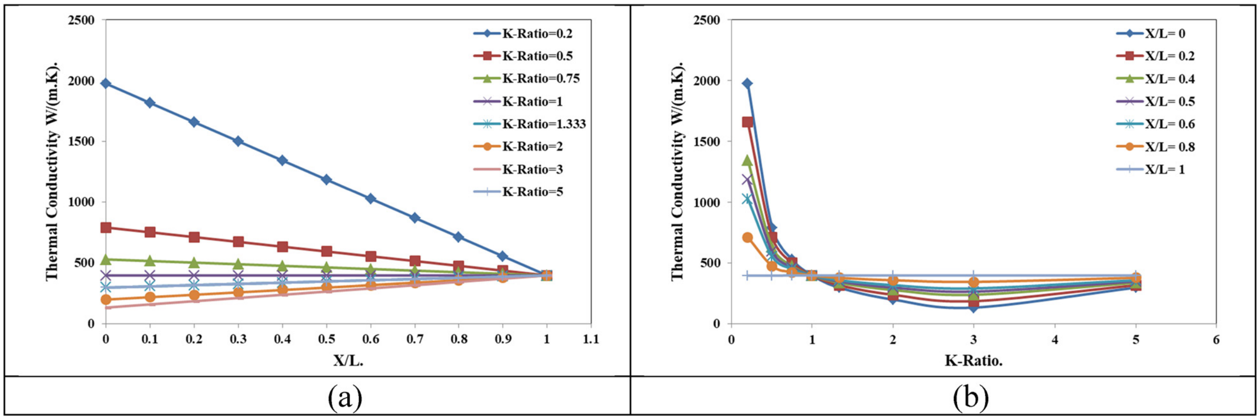

Figure 2: Variation in thermal conductivity along the length of FG-fin for different K-ratios with copper (Cu) as the left side material. (a) Effect of K-ratio. (b) Effect of (x/L).

thermal conductivity appears with different slops (γ). While Figure 2b shows the effect of K-ratio at different positions along the length of graded structure fin.

3 Mathematical models

For simulating the heat transfer behavior of FG-Fin, three new models are improved, one of these models is analytical model, the second one is numerical model using ANSYS software, while the final model depends on the equivalent properties of FG-Fin.

3.1 Analytical solution

As shown in Figure 2, a permanent temperature (T_b) at base and convection model occurring at both sides ($W \times L$) characterizes the thermal conditions of the FGM-fin. According to Figure 2, the heat conduction through the FGM-fin is equivalent to the heat convection from the FG-fin surface, and the governing equation is [20]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right) = \frac{h}{t}(T - T_\infty)\ 0 \le x \le L. \tag{2}$$

The boundary conditions (BCs) are

(i) $T = T_b \rightarrow$ at $x = 0$.

(ii) $\frac{\mathrm{d}T}{\mathrm{d}x} = 0 \rightarrow$ at $x = L$.

Similar to Eq. (1), the thermal conductivity is

$$k(x) = k_0 \times (1 + \gamma \times x). \tag{3}$$

And Eq. (2) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k_0 \times (1 + \gamma \times x)\frac{\mathrm{d}T}{\mathrm{d}x}\right) = \frac{h}{t}(T - T_\infty). \tag{4}$$

To resolve the relation (4) suppose $\theta = \left(\frac{T - T_\infty}{T_b - T_\infty}\right)$, and $\eta = \frac{x}{L}$; therefore, $\mathrm{d}\theta = \frac{\mathrm{d}T}{T_b - T_\infty}$ and $\mathrm{d}\eta = \frac{\mathrm{d}x}{L}$. Utilizing the Chain rule, Eq. (4) and BCs are transformed into

$$(1 + \gamma\eta)\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} + \gamma\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = \frac{hL^2}{tk_0}\theta,$$

$$(1 + \gamma\eta)\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} + \gamma\frac{\mathrm{d}\theta}{\mathrm{d}\eta} - \Gamma^2\theta = 0. \tag{5}$$

So, $\Gamma^2 = \frac{hL^2}{tk_0}$,

where BCs are

(i) $\theta = 1 \rightarrow T = T_b$, at $\eta = 0$,

(ii) $\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = 0 \rightarrow \frac{\mathrm{d}T}{\mathrm{d}x} = 0$, at $\eta = 1$.

In relation (5), when replacement $\eta = 0.5$ [21]

$$(1 + 0.5\gamma)\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} + \gamma\frac{\mathrm{d}\theta}{\mathrm{d}\eta} - \Gamma^2\theta = 0. \tag{6}$$

The solving of above relation is

$$\theta(\eta) = c_1 e^{\mu_1\eta} + c_2 e^{\mu_2\eta}, \tag{7}$$

where $\mu_{1,2} = \frac{-(\gamma) \mp \sqrt[2]{(\gamma)^2 - 4(1 + 0.5\gamma)\Gamma^2}}{2 \times (1 + 0.5\gamma)}$.

To determine the values of A_1 and A_2, the BCs are implemented as follows:

$$A_1 = \frac{\frac{\mu_2}{\mu_1}e^{\mu_2 - \mu_1}}{\left(\frac{\mu_2}{\mu_1}e^{\mu_2 - \mu_1} - 1\right)} \text{ and } A_2 = \frac{1}{\left(1 - \frac{\mu_2}{\mu_1}e^{\mu_2 - \mu_1}\right)}$$

$$\therefore \theta(\eta) = \frac{\frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1}}{\left(\frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1} - 1\right)} e^{\mu_1 \eta} + \frac{1}{\left(1 - \frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1}\right)} e^{\mu_2 \eta}, \tag{8}$$

$$\therefore T(\eta) = T_\infty + \left[(T_b - T_\infty) \times \left(\frac{\frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1}}{\left(\frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1} - 1\right)} e^{\mu_1 \eta} + \frac{1}{\left(1 - \frac{\mu_2}{\mu_1} e^{\mu_2 - \mu_1}\right)} e^{\mu_2 \eta} \right) \right]. \tag{9}$$

For the homogenous fin, where $\gamma = 0$, the following results ensue:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} - \Gamma^2 \theta = 0, \tag{10}$$

Table 1: FG-fin dimensions and its component

Dimension	Value	Unit
Width	0.04	m
Length	0.1	m
Thickness	0.003	m
No. of divisions	10	—
Each part length	0.005	m
Each part width	0.04	m
Thickness of every FGM-fin piece	0.003	m

$$\theta(\eta) = c_1 e^{\mu_1 \eta} + c_2 e^{\mu_2 \eta}. \tag{11}$$

So, $\mu_{1,2} = \mp\Gamma$.

Also,

$A_1 = \frac{e^{-2\Gamma}}{(1 + e^{-2\Gamma})}$ and $A_2 = \frac{1}{(1 + e^{-2\Gamma})}$

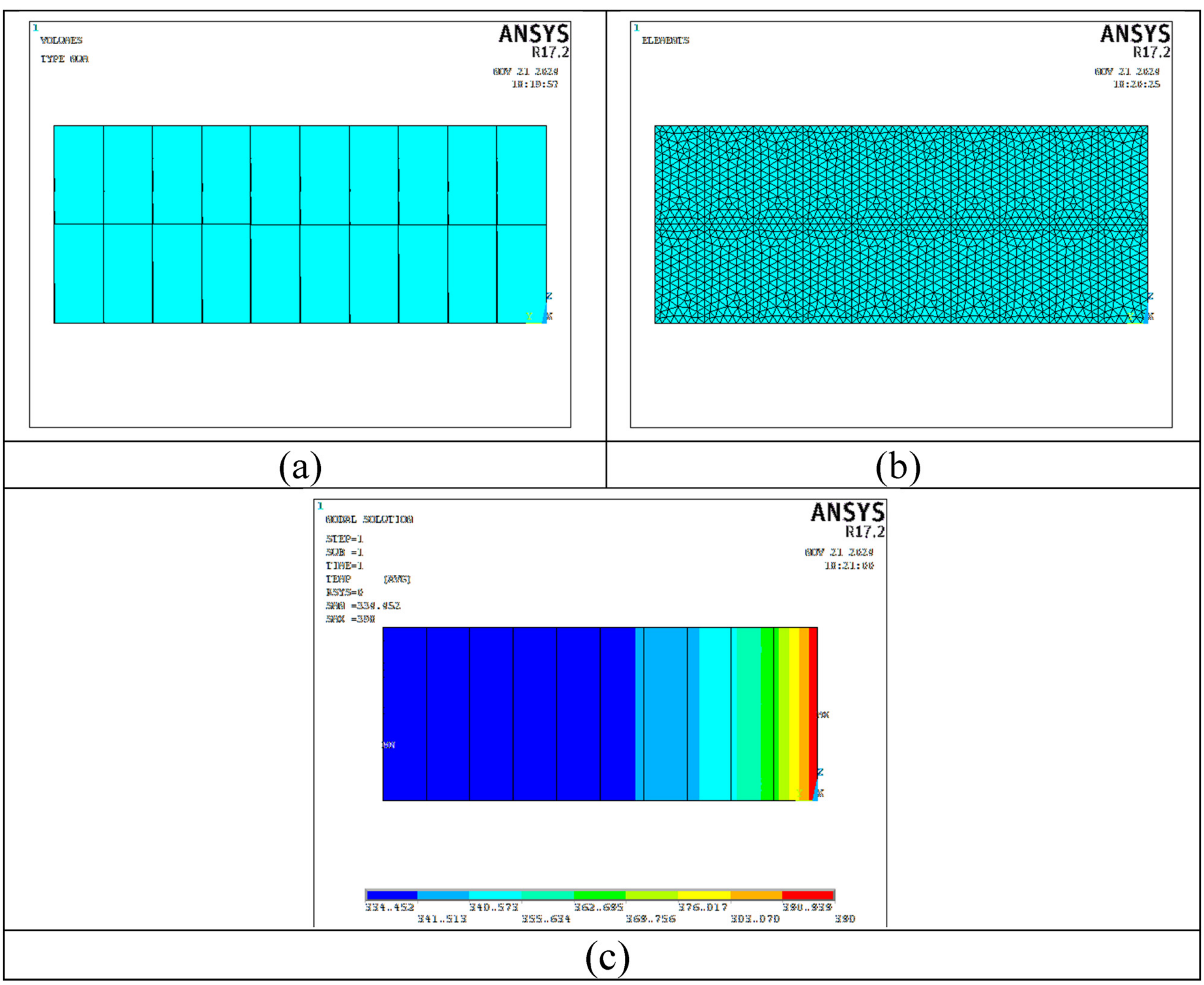

Figure 3: Procedure for finite element modeling utilizing Ansys APDL software. (a) The FGM-Fin structure. (b) A mesh drawing. (c) Results.

$$\therefore T(\eta) = T_\infty + \left[(T_b - T_\infty) \times \left(\frac{e^{-2\Gamma}}{(1+e^{-2\Gamma})} e^{\mu_1 \eta} + \frac{1}{(1+e^{-2\Gamma})} e^{\mu_2 \eta}\right)\right]. \quad (12)$$

3.2 Numerical model

Depending on FEM, the numerical work is constructed utilizing APDL program (ANSYS software). The ten-node tetrahedral element (SOLID87) is used in this model and the brick element also can be used [22]. The convergent criteria of solution is adopted to choose the suitable size of a tetrahedral element. The following steps are used to build this model.

- The FG is segmented into ten pieces [23], with its measurements detailed in Table 1. As shown in Figure 3a, the measurements of every component are (w) in width, (L/10) in length, and (t) in thickness.
- Every piece of the graded material fin is a homogeneous compositional part and the thermal conductivity of each part can be calculated by

$$k_i = \frac{(k(i \times \mathrm{DL}) + k((i-1) \times \mathrm{DL}))}{2} \quad i = 1,\ 2,\ \ldots,\ 10, \quad (13)$$

where k_i is the thermal conductivity of the ith part of graded structure fin, and DL is the length of every piece of graded structure fin (as shown in Table 1). Therefore, ten values of thermal conductivity are inserted into the software ANSYS APDL.

- In this model, three BCs are used:
  (a) The first at the base or lower surface of FG-fin is constant temperature (T_b).
  (b) Insulated fin's tip (*i.e.*, $\frac{dT}{dx} = 0$).

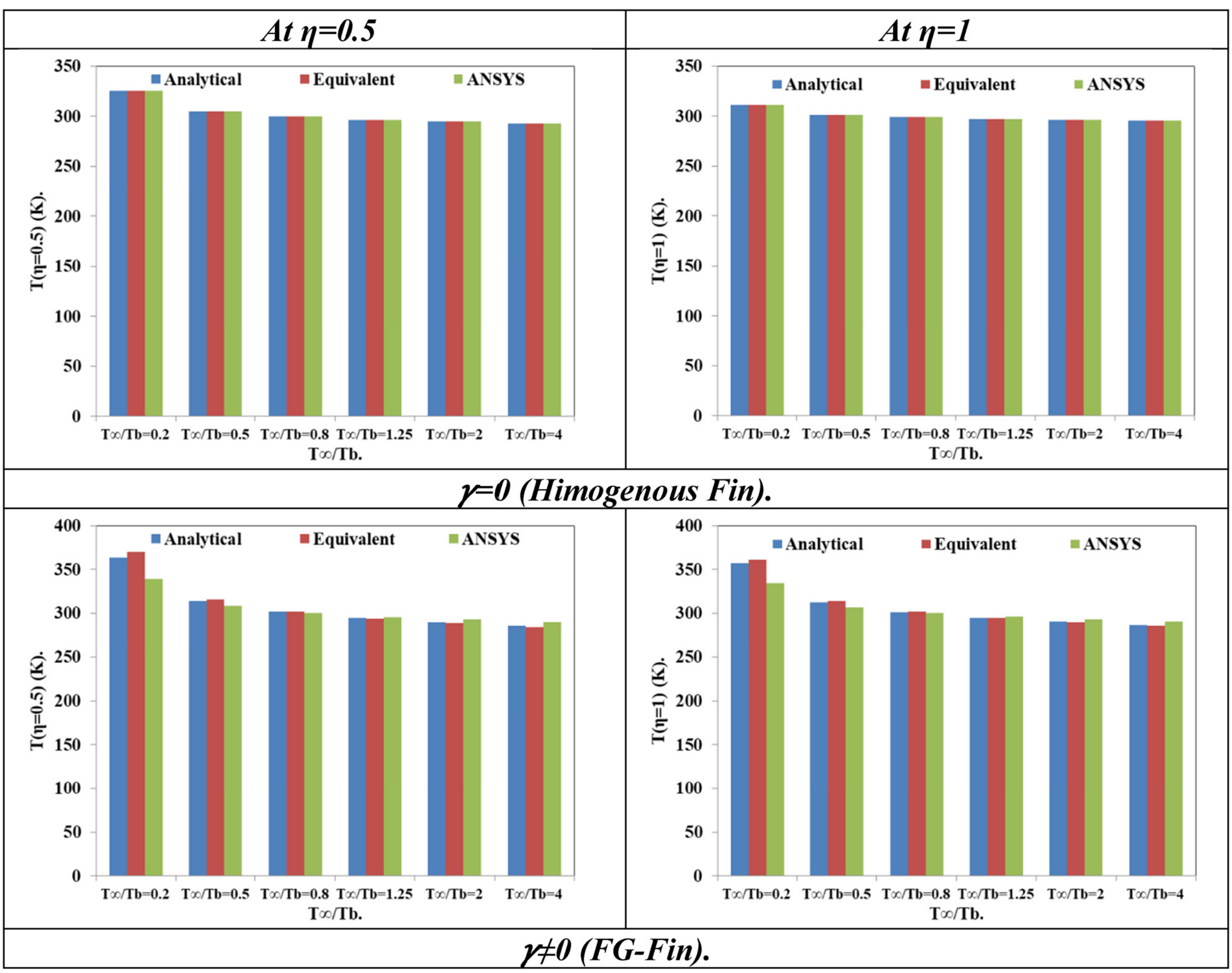

Figure 4: A comparison of temperature ascertained using numerical, equivalent property models, and analytical of temperature ratio at η = 1 and η = 0.5 when Al_2O_3 is the left-side material and Cu is the right-side material.

Table 2: Error rate of ANSYS and equivalent property outconmes with respect to analytical results for different property ratios (γ) at η = 0.5 and 1

Case no.	T_∞/T_b	Error percentage (%)			
		Equivalent property model		ANSYS model	
	T-ratio	η = 0.5	η = 1	η = 0.5	η = 1
1. γ = 0 (Homogenous fin)					
1.	0.20	0	0	+3.780	+2.837
2.	0.50	0	0	+1.008	+0.732
3.	0.80	0	0	+0.257	+0.184
4.	1.25	0	0	−0.206	−0.150
5.	2.00	0	0	−0.520	−0.374
6.	4.00	0	0	−0.786	−0.560
2. $\gamma \neq 0$ (FG-Fin)					
1.	0.20	−1.767	−1.292	+6.756	+6.320
2.	0.50	−0.511	−0.369	+1.953	+1.804
3.	0.80	−0.133	−0.096	+0.509	+0.467
4.	1.25	+0.109	+0.078	−0.416	−0.383
5.	2.00	+0.277	+0.198	−1.061	−0.969
6.	4.00	+0.422	+0.301	−1.611	−1.473

Table 3: Cases considered in present study

No. of cases	T_∞ (°C)	T_b (°C)	T_∞/T_b	Thermal conductivity ratio (K-ratio)
1	25	125	0.2	0.2, 0.5, 0.75, 1, 1.333, 2, and 5
2	25	50	0.5	
3	25	31.25	0.8	
4	25	20	1.25	
5	25	12.5	2	
6	25	6.25	4	

(c) Assuming a constant heat convection coefficient and a constant ambient temperature (T_∞). Free convection at two surfaces of a fin with an FGM.

- The steady-state heat transfer is addressed to resolve the present issue.
- To analyze the impact of varying material qualities of FG-fin on thermal behavior represented by temperature fields, the temperature profile at the middle of the FG-fin and along its length is illustrated.

3.3 Homogenous model

As shown in Figure 3a and using relation (13), the equivalent thermal conductivity of FGM-fins can be estimated by the following relation [24–29]:

$$\sum_{i=1}^{10}\left(\frac{\Delta L}{\lambda_i}\right) = \frac{L}{\lambda_{\text{eq}}},$$

$$\therefore \lambda_{\text{eq}} = \frac{L}{\sum_{i=1}^{10}\left(\frac{\Delta L}{\lambda_i}\right)}, \tag{14}$$

where (λ_{eq}) is the equivalent thermal conductivity and (λ_i) is the thermal conductivity of ith part.

4 Validation and accuracy of models

The validation and accuracy of finite element model and equivalent properties model compared with the proposed analytical model are studied. The alumina-copper FG-fin is considered assuming alumina (Al_2O_3) is the left side material and Cu is the right-side material. The comparisons between the temperature, that are calculated by the three models, are illustrated in Figure 4. Figure 4 shows the comparison among the temperature estimated utilizing three solutions (analytical solution, ANSYS model, and equivalent property model) for different (T_∞/T_b) at η = 1 and η = 0.5. The analytical model of homogenous material and equivalent property model are used together to estimate the temperature distribution of FG fins.

However, the variance between them increases once $\gamma \neq 0$. Also, temperatures calculated using ANSYS model is lower than that of analytical model at small (T_∞/T_b) ratio. Conversely, the numerical temperature rises when the (T_∞/T_b) ratio increases for η = 1. The largest percentage of absolute error between equivalent property and analytical result is 2.363% at (T_∞/T_b) = 0.3 and η = 0.5. The extreme percentage of absolute error between ANSYS and analytical results is 6.756% once (T_∞/T_b = 0.2) and (η = 0.5), also illustrated in Table 2. This study examines the influence of the slope of the linear equation (γ) and the temperature ratio (T_∞/T_b) on the temperature distribution over the length of the graded structure fin. The instances examined in current study are presented in Table 3.

5 Cases studied

The impact of thermal conductivity ratio (K-ratio) ($K_{\text{right}}/T_{\text{left}}$) and temperature ratio on temperature behavior on the FGM-Fins length direction are investigated in this work. The cases examined in this study are presented in Table 3.

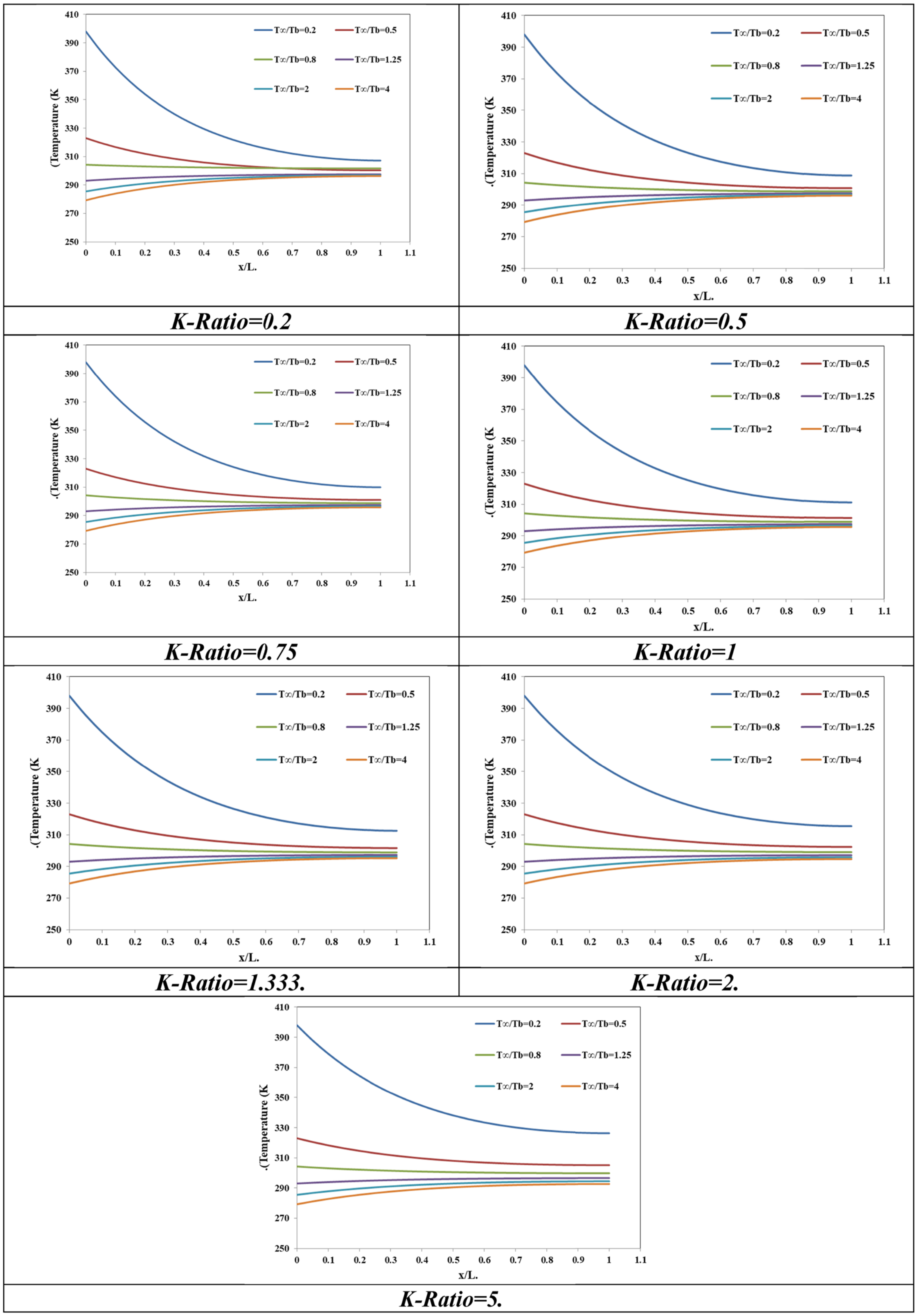

Figure 5: Temperature profiles of FG-fins for different ratio of (T_∞/T_b) and different K-ratios.

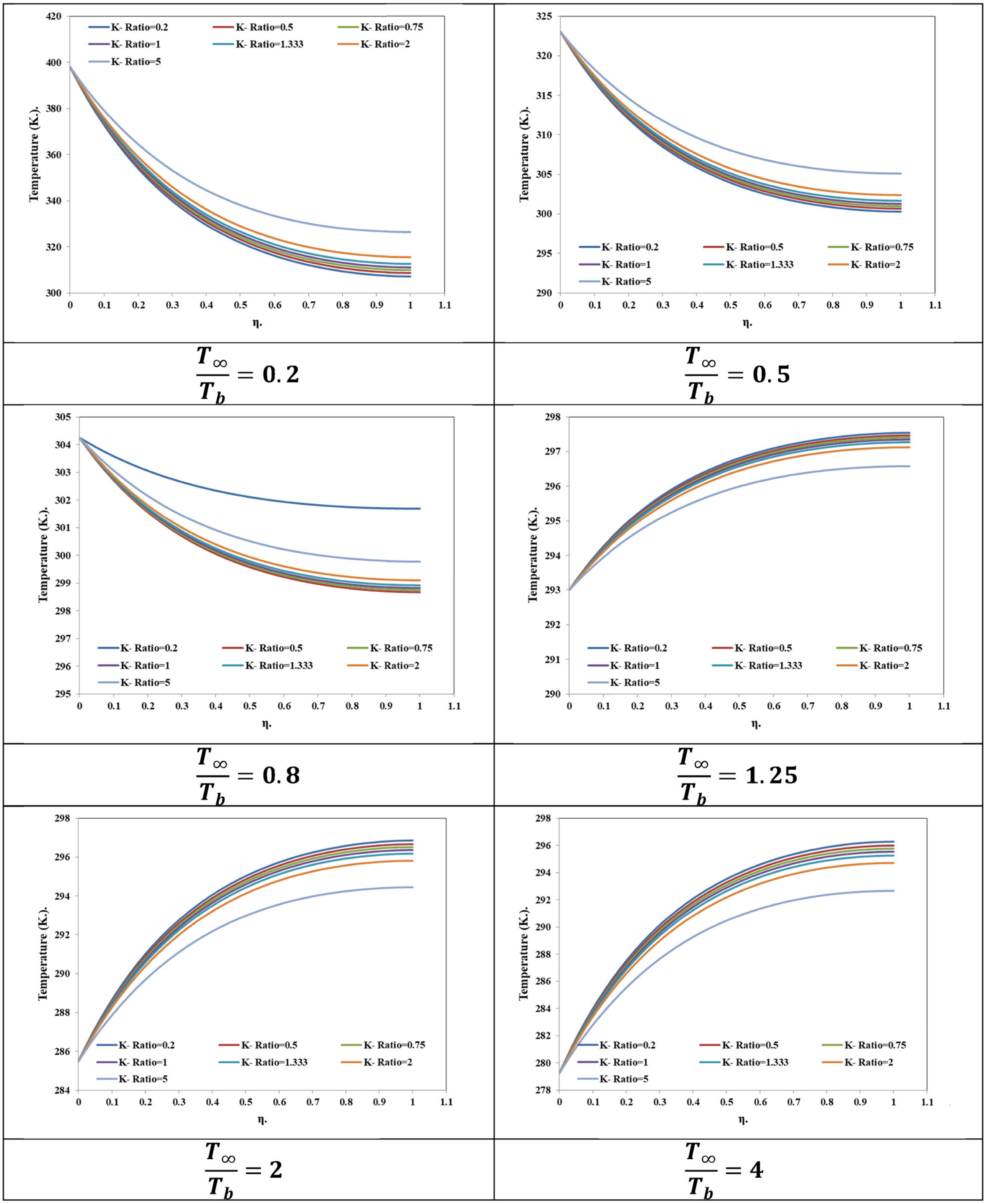

Figure 6: Temperature profiles over the length of FGM-fins for various K-ratio and different ratios of (T_∞/T_b).

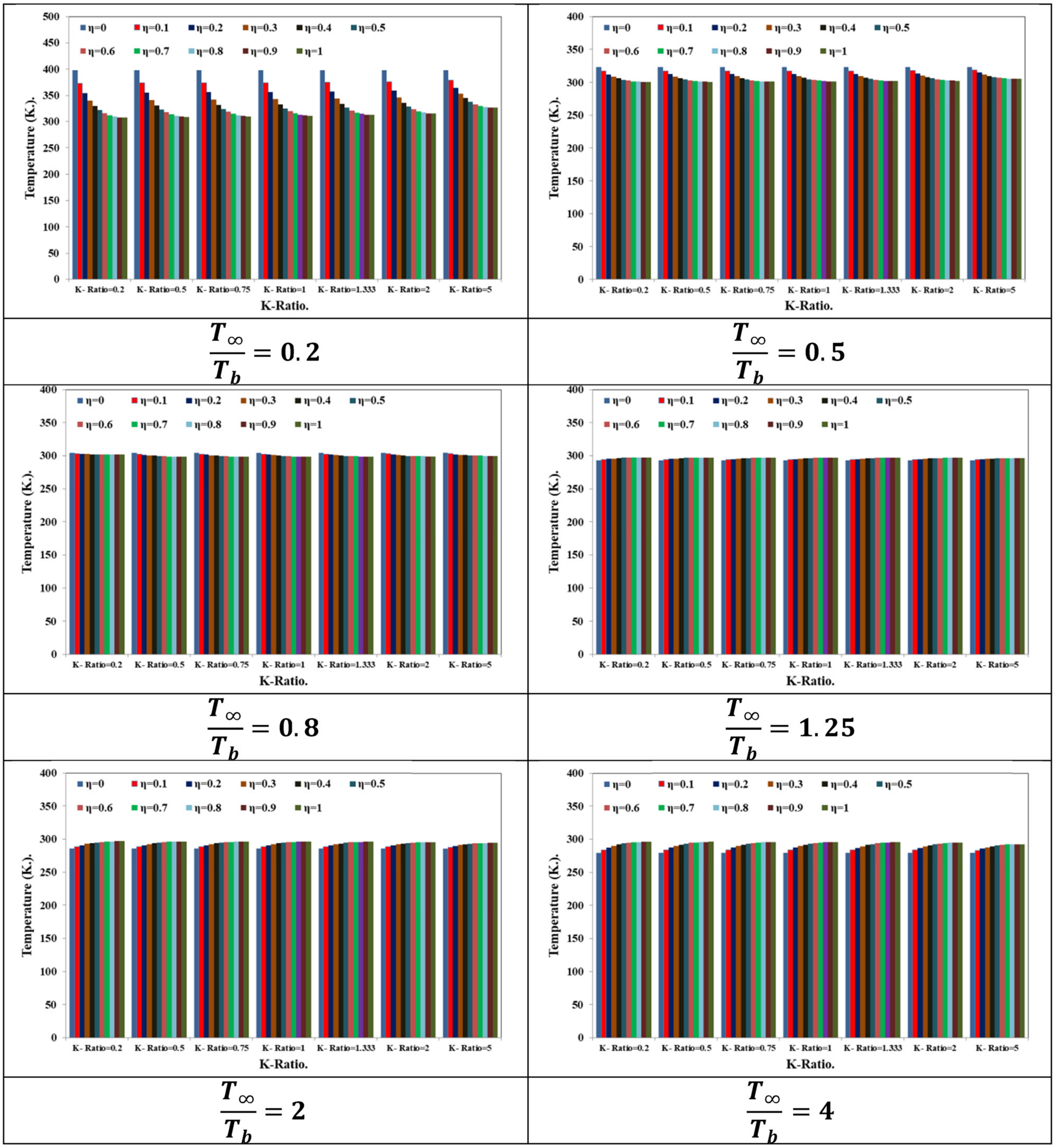

Figure 7: Temperature profiles over the length of FGM-fins for different ratios of (T_∞/T_b) and various K-ratios.

6 Results and discussion

In this section, the results of the parametric case studied described in Section 5 are displayed considering the effect of Temperature Ratio considering the effect of (T_∞/T_b) ratio and thermal conductivity ratio (K-ratio).

6.1 Influence of temperature ratio

Figure 5 illustrates the temperature distribution affected by the (T_∞/T_b) ratio on the length of FGM-Fin for various K-ratios. Typically, when the (T_∞/T_b) ratios are less than 1 (during the cooling process), the temperature diminishes

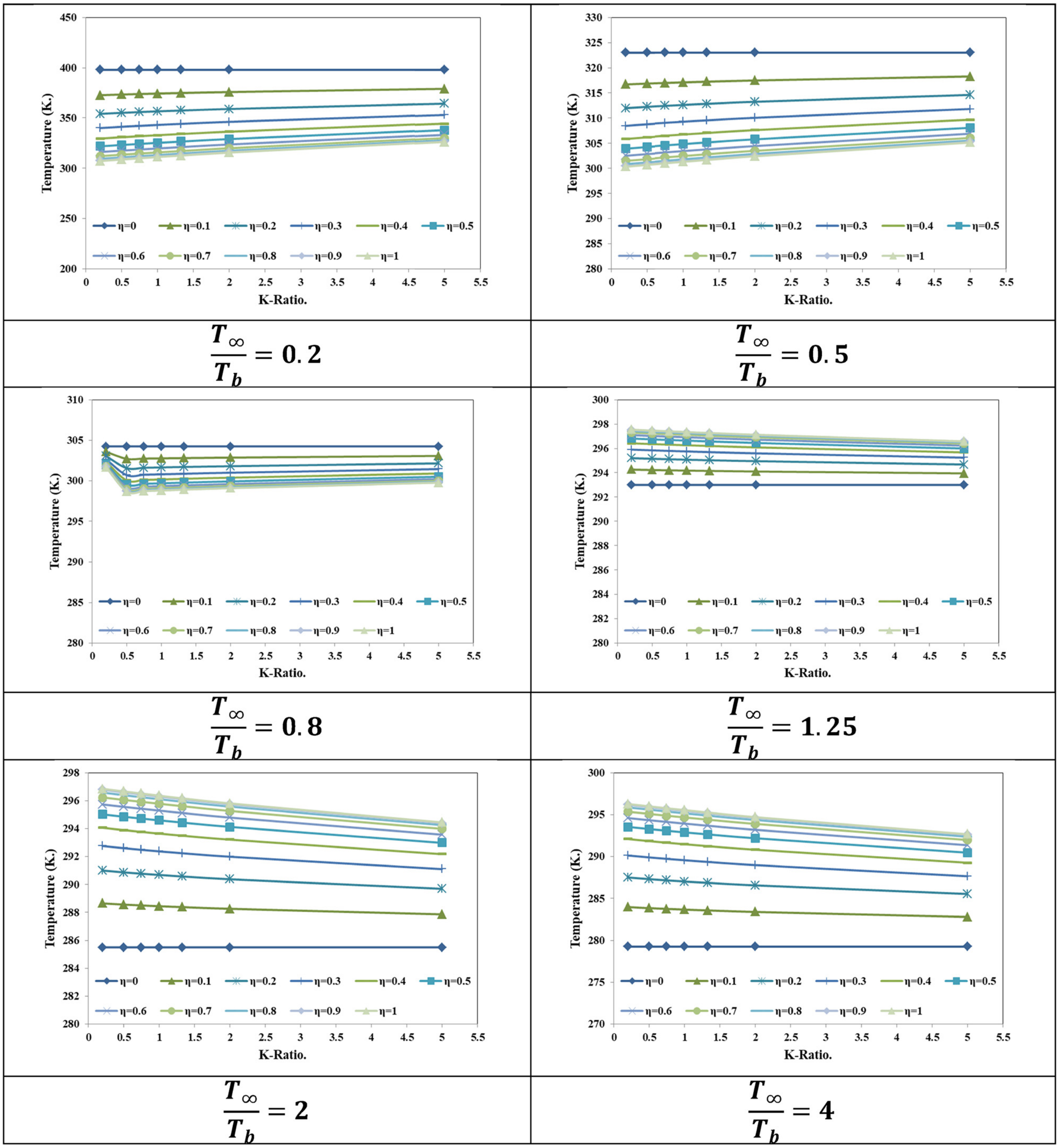

Figure 8: Temperature of FG-fins for different K-ratios and different ratios of ambient to base temperatures.

over the length of the FGM-fin. If the (T_∞/T_b) ratio exceeds 1 (as heating procedure), the temperature rises along the length of the FGM-fin. At the FGM-fin lower surface or base at ($\eta = 0$), the input temperature diminishes as the (T_∞/T_b) ratio increases; under these conditions, the effect of FG material is negligible. However, when ($\eta > 0$), the impact of FG material is similar to a peak effect occurring at $\eta = 1$. This fact is also depicted in Figure 6.

6.2 Effect of K-ratio

Figure 7 shows temperatures at different points through the length of FG-fins due to increase in the K-ratio and at different ratios of (T_∞/T_b). It can be seen that the decrease or increase in temperature at any point is affected slightly by the K-ratio at a constant ratio of (T_∞/T_b). For example, the temperatures at (η = 1) when K-ratio = 0.2 and K-ratio = 4 is slightly increased at constant (T_∞/T_b) ratio and this is illustrates sharply in Figure 8. At (η = 1), the temperature increases with increasing K-ratio when (T_∞/T_b) is less than 0.8. On the other hand, the temperature decreases with increasing K-ratio when (T_∞/T_b) is greater than unity.

7 Conclusion and prospective directions

This work has provided analytical and numerical analyses to examine the thermic action of FG-fins. The thermal characteristics of the fin are expected to vary linearly along the length of FG-fin. The current study has also taken into account the equivalent material properties of the FG-fin. The current analysis has also taken into account the equivalent material properties of the FG-fin.

As heat conductivity increases along the length of the FG-fin when the K-ratio approaches unity, the outcomes for analogous attributes resemble those of the analytical solution. The disparity between them increases as the K-ratio diverges from unity. In the scenario where k (x = L) exceeds k (x = 0), when (T_∞/T_b) < 1 (indicating cooling process), the numerical findings are inferior to the analytical results. Conversely, when the (T_∞/T_b) ratio exceeds 1 (indicating a heating process), the numerical results surpass the analytical results. The largest error between the two solutions is 6.756% at η = 0.5 and T_∞/T_b = 0.2, whereas the maximum error between the numerical and equivalent property results is 2.363% at η = 0.5 and T_∞/T_b = 0.2. When k (x = L) > k (x = 0) (or K-ratio < 1), a cooling process occurs as the T_∞/T_b ratio is less than 1, resulting in a modest reduction in the fin temperature. Conversely, during heating, the T_∞/T_b ratio exceeds 1. FGM serves as a deterrent to heat stress. The reduction in thermal conductivity along the length of the FG-fin adversely impacts heat transmission by conduction along the fin.

In conclusion, the thermal performance of FG-fin surpasses that of convectional fins.

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References

[1] Cheng-Hung H, Chien-Tsuen L. Inverse method in estimating the base heat flux for irregular fins. J Thermophys Heat Transf. 2014;28(2):320–6.

[2] Masoud A, Mehdi A. Transient thermal stresses in two-dimensional functionally graded thick hollow cylinder with finite length. Arch Appl Mech. 2010;80:353–76.

[3] Zhong Z, Wu LZ, Chen WQ. Progress in the study on mechanics problems of functionally graded materials and structures. Adv. Mech. 2010;40(5):528–541.

[4] Oguntala G, Sobamowo G, Abd-Alhameed R. A new hybrid approach for transient heat transfer analysis of convective radiative fin of functionally graded material under Lorentz force. Therm Sci Eng Prog. 2020;16:1–9.

[5] Babaelahi M, Babazadeh MA, Saadatfar M. New design for the cold part of heat pipes using functionally graded material in heat sink with variable thickness fins: An analytical approach. J Therm Eng. 2024;10(5):1323–34.

[6] Dogmaz MA, Safak I, Gunes S, Reddy JN. An investigation of the thermal performance of functionally graded annular fins on a horizontal cylinder under natural convection. Exp Heat Transf. 2024;1–23. doi: 10.1080/08916152.2024.2396337.

[7] Jemiseye AE, Sobamowo GM, Mayowa OO. Transient thermal cooling of electronics systems using functional graded fins: hybrid

computational analysis. J Eng Exact Sci. 2023;9(4):15810–01e. doi: 10.18540/jcecvl9iss4pp15810-01e.

[8] Sobamowo MG, Oguntal GA, Yinus AA, Adedibu AO. Analysis of transient heat transfer in a longitudinal fin with functionally graded material in the presence of magnetic field using finite difference method. World Sci N. 2019;137:166–87.

[9] Gaba VK, Tiwari AK, Bhowmick S. Performance of functionally graded exponential annular fins of constant weight. Adv Functionally Graded Mater Struct. 2016. doi: 10.5772/63100.

[10] Oguntala GA, Sobamowo GM, Abd-Alhameed RA, Noras JM. Numerical study of performance of porous fin heat sink of functionally graded material for improved thermal management of consumer electronics. IEEE Trans Comp Packag Manuf Technol. 2019;9(7):1271–83.

[11] Khan WA, Aziz A. Transient heat transfer in a functionally graded convecting longitudinal fin. Heat Mass Transf. 2012;48:1745–53.

[12] Hassanzadeh R, Bilgili M. Improvement of thermal efficiency in computer heat sink using functionally graded materials. Commun Adv Comput Sci Appl. 2014;2014:1–13.

[13] Jemiseye AE, Gbeminiyi MS, Olayiwola OM. Transient thermal cooling of electronics systems using functional graded fins: hybrid computational analysis. J Eng Exact Sci. 2023;9(4):1–15.

[14] Sobamowo MG, Oguntala GA, Yinusa AA. Nonlinear transient thermal modeling and analysis of a convective-radiative fin with functionally graded material in a magnetic environment. Model Simul Eng Volume. 2019;2019:1–16.

[15] Ranjana R, Mallick A. An efficient unified approach for performance analysis of functionally graded annular fin with multiple variable parameters. Therm Eng. 2018;65(9):614–26.

[16] George O, Isah D, Gbeminiyi S, Raed A, Jim N. Nonlinear thermal analysis of a convective-radiative longitudinal porous fin of functionally graded material for efficient cooling of consumer electronics. Int J Ambient Energy. 2019;43(1):385–99.

[17] Kumar C, Nimmy P, Nagaraja KV, Kumar RSV, Verma A, Alkarni S, et al. Analysis of heat transfer behavior of porous wavy fin with radiation and convection by using a machine learning technique. Symmetry. 2023;15(1601):1–21.

[18] Sobamowo MG, Salami MO, Yinusa AO, Abubakar EH. Finite Element Analysis of transient thermal behavior of radiative-convective fin with functionally graded material under the influence of magnetic field. J Complex Flow. 2023;5(1):31–8.

[19] Raghad AN, Ameen AN, Luay SA. Modeling and analyzing the free vibration of simply supported functionally graded beam. J Aerosp Technol Manag. 2022;14:e1522. https://www.jatm.com.br/jatm/article/view/1264/938.

[20] Incropera FP, DeWitt DP. Introduction to heat transfer. 6th edn. UK: Wiley; 2011.

[21] Acosta-Iborra A, Campo A. Approximate analytic temperature distribution and efficiency for annular fins of uniform thickness. Int J Therm Sci. 2009;48:773–80. doi: 10.1016/j.ijthermalsci.2008.05.012.

[22] Lawrence KL. Ansys Workbench Tutorial Release 2024, SDC Publications; 2024. p. 1–280.

[23] Walaa MH, Luay SA, Mohanad A, Hassan MR. Investigating static deflection of non-prismatic axially functionally graded beam. Mater Des Process Commun. 2022;2022(1):7436024.

[24] Turant J, Radaszewska E. Thermal properties of functionally graded fibre material. Fibres Text East Europe. 2016;24(4):68–73.

[25] Nasr AJ, Ishan YH, Muhsin JJ, Aljibori HSS, Mohammed MN, Oday IA. An enhanced analytical and numerical thermal model of frictional clutch system using functionally graded materials. Curved Layer Struct. 2024;11(1):1–9.

[26] Nasr AJ, Ihsan YH, Oday IA, Mohammed MN. Finite element analysis of the thermal and thermo-mechanical coupling problems in the dry friction clutches using functionally graded material. Curved Layer Struct. 2024;11(1):1–13.

[27] Giunta G, Crisafulli D, Belouettar S, Carrera E. A thermo-mechanical analysis of functionally graded beams via hierarchical modelling. Compos Struct. 2013;95:676–90.

[28] Qian G, Yi W, Haowen W, Jun T. Characterization and prediction of tensile properties of carbon fiber-reinforced thermoplastics composed of hybrid short carbon fiber/PA6 fiber nonwoven mats. Compos Struct. 2024;334:117996.

[29] Brischetto S, Leetsch R, Carrera E, Wallmersperger T, Kröplin B. Thermo-mechanical bending of functionally graded plates. J Therm Stresses. 2008;31(3):286–308. doi: 10.1080/01495730701876775.