

Research Article

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An enhanced analytical and numerical thermal model of frictional clutch system using functionally graded materials

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Abstract: When a component was built from a functionally graded material (FGM), the temperature of contact was determined using an analytical model (FGM). The analytical solution is applied using the concept of equivalent properties. In order to fully examine the dynamic of the equivalent property principle, a numerical analysis was done. The FGM (silicon carbide and aluminum) friction liner was used. It focuses on a few characteristics that have an impact, such as load, velocity, friction material type, and slip time, which appear in dimensionless form. The results show that for loads and velocities, the ratio of the rate of dissipation to the rate of generation of heat transfer is one, whereas the behavior differs for friction material and slip time. Furthermore, this study's chosen FGM exhibits superior thermal behavior compared to Al and Sic, bolstering its viability as a friction liner in the clutch system.

Keywords: analytical solution, FEM, friction dry clutches, functionally graded materials, thermal problem

Nomenclature

A	Total area of clutch faces (m^2)
C	$1/[\sqrt{K_1\rho_1c_1} + \sqrt{K_2\rho_2c_2}]$
c	Specific heat ($\text{J kg}^{-1} \text{m}^{-1}$)
K	Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
I_1, I_2	Moment of inertia of two sides of the clutch
n	number of turns during minute (rpm)
q	Nominal value of the specific power of friction (W m^{-2})
Q	The total amount of heat produced during the slip (W)
t	Time (s)
T	Torque (Nm)
t_s	Slip time (s)
$\bar{T}$	Temperature (K)
$\bar{T}_0$	Initial temperature (K)

Greek Symbols

θ	Temperature rise (K)
θ_0	Temperature rise scaling factor (K)
ρ	Density of metals
τ	Dimensionless time
τ_s	Dimensionless slip time
α	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)

1 Introduction

A car's friction clutch is crucial because it significantly influences the speed of power transfer. The clutch connects

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and disconnects the motor from the gearbox and transfers torque from the engine to the wheels. Figure 1 depicts the three components of the system. When the clutch is engaged, the frictional heat produced by the shifting of the temperature is converted from mechanical energy into frictional contact between the surfaces of the clutch plates. The temperature field may increase to high levels during repeated connected engagements, leading to a significant decrease in the friction coefficient and ultimately leading to the inadequacy of clutch components [1–4]. Executing a thorough analysis of the temperature distribution strategy and heat generation is crucial to optimize the dry clutch transmission [5]. A new material science discovery that overcame geometrical and dimensional constraints sparked the evolution of the clutch framework toward an ideal design. During the last half of the century, discoveries in industrial and modern tribology were the driving force behind advancements in designs that compared and additionally loaded friction applications from various modern domains. For instance, efforts to replace asbestos had a significant impact on material progress. Fiber-based materials have expanded as a result of the discovery of flexible resins [6]. However, with the use of recently discovered friction materials and some effective machining techniques, equal development and material advancement allowed for further lubrication development [7]. New friction materials are constantly sought after, so it is important to focus on them from several angles. The first is thermal analysis. Different factors, such as sliding and

friction coefficient in the dry state, set boundaries for the dry clutch's operation, making other factors – such as speed, friction material, engagement frequency, face thickness, and roughness – all discernible. These factors have a significant impact on the total and distribution of heat produced by friction, which in turn affects the clutch's performance and lifespan. Thermal examination research is the primary concern of researchers and design engineers, and any effort to focus on it broadly and identify as many degrees as possible that achieve the ideal working conditions. As a result, the heat issue in the dry clutch was examined using a variety of techniques, including numerical analysis [8–16], experimental [16–20] and analytical [8,21–24].

In this work, first, a numerical model of the clutch system was established to obtain a thermal solution that would simulate the thermal problem during sliding time, which was used to make comparisons between the results of the normal model (layers) and the principle of equivalent properties. To determine whether the equivalent principle worked where functionally graded material (FGM) served as a friction lining. Secondly, a mathematical model for the clutch system was used to overcome the heat problem that arose during the sliding period, which depends on how you apply the equivalence principle to FGM. The effects of many parameters, such as torque, variety of friction materials, slip time, and velocity, have been examined. The novelty in this work is the development of the analytical solution to make it applicable to FGMs by using the principle of equivalent properties.

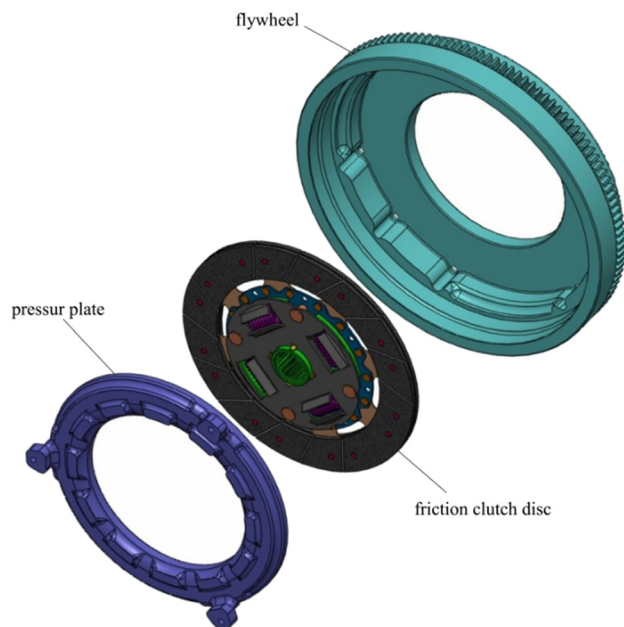


Figure 1: The main components of the clutch system.

2 The analytical model

2.1 Energy dissipation: A two-inertia system

In order to perform an analysis of the fundamental power transmission system depicted in Figure 2, the following assumptions must be met [25]:

1. It is assumed that torque T is always constant.
2. The friction coefficient remains constant.
3. Perfect rigidity is postulated for the system.

The dynamical relationships are as follows:

$$T = I \frac{d\omega}{dt} \xrightarrow{\text{side1}} T = I_1 \frac{d\omega_1}{dt} \xrightarrow{\text{side2}} T = I_2 \frac{d\omega_2}{dt}. \quad (1)$$

The Boundary Conditions:

1. at side 1: at $T = T_1$, $\omega_1 = \Omega_1$,
2. at side 2: at $T = T_2$, $\omega_2 = \Omega_2$.

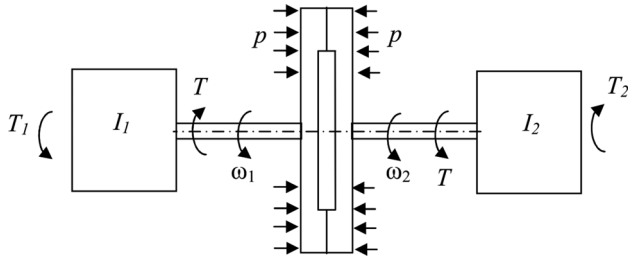


Figure 2: The basic system of power transmission.

When these equations are integrated, and the boundary conditions are substituted, give;

$$\omega_1 = \left(\frac{T_1 - T}{I_1} \right) t + \Omega_1 \quad \& \quad \omega_2 = \left(\frac{T - T_2}{I_2} \right) t + \Omega_2. \quad (2)$$

Thus, the rate at which heat is generated per unit area of the friction surface during a single clutch engagement is given by

$$q(t) = \frac{T}{A}(\omega_1 - \omega_2). \quad (3)$$

Figure 3 shows that the rate of energy loss during the sliding phase goes down in a straight line with time.

$$q(t) = q \left[1 - \frac{t}{t_s} \right] \text{ in the range } 0 \leq t \leq t_s. \quad (4)$$

By substituting Eq. (2) in Eq. (3) and equating with 4 obtain,

$$q(t) = \frac{T}{A}(\omega_1 - \omega_2) = \frac{T}{A}[\Omega_1 - \Omega_2 - t \left\{ \frac{(I_1 + I_2)T - (I_2 T_1 + I_1 T_2)}{I_1 I_2} \right\}] = q \left[1 - \frac{t}{t_s} \right], \quad (5)$$

$$t_s = \frac{I_1 I_2 (\Omega_1 - \Omega_2)}{(I_1 + I_2)T - (I_2 T_1 + I_1 T_2)}, \quad (6)$$

$$Q = \int_0^{t_s} A q(t) dt = \frac{T}{2} \frac{I_1 I_2 (\Omega_1 - \Omega_2)^2}{(I_1 + I_2)T - (I_2 T_1 + I_1 T_2)}. \quad (7)$$

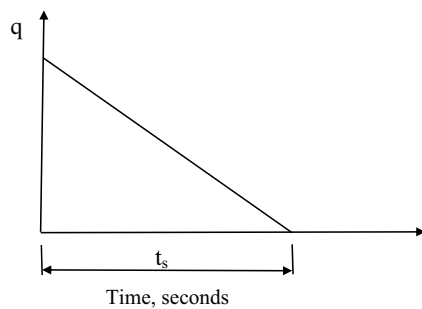


Figure 3: Behavior of heat flux per time during a clutch application.

For a special case, at $T_1 = T_2 = 0$, when no external torques are acting on the system and it consists just of two inertias I_1 and I_2 rotating at different speeds [25],

$$q(t) = \frac{T}{A} \left[\Omega_1 - \Omega_2 - T \left(\frac{I_1 + I_2}{I_1 I_2} \right) t \right], \quad (8)$$

$$t_s = \frac{I_1 I_2 (\Omega_1 - \Omega_2)^2}{2 (I_1 + I_2) T}, \quad (9)$$

$$Q = \frac{I_1 I_2 (\Omega_1 - \Omega_2)^2}{2(I_1 + I_2)}. \quad (10)$$

2.2 The solution of the heat problem

Figure 4 shows a cross-section of a single-plate clutch operating under the influence of an axial clamping force P . The inner plate is made up of two friction plate materials riveted to thin spring steel segments. The two outer plates are made of steel and have various thicknesses, d_1 for the pressure plate and d_3 for the flywheel plate.

In this case, the rate of heat generation would be directly related to the tangential velocity and hence to the radial position on the plate surface, which would contradict the requirement that the pressure be uniform over the plate. The rate of energy loss at surfaces where rubbing occurs, in contrast, is solely a function of time. The rates of heat transfer between two surfaces in contact are not equal but rather dependent on their differing thermal characteristics [25].

Since only a small portion of the friction surface is heated during the slipping period (t_s) in typical clutch applications, the contacting bodies can be thought of as having an infinite thickness in this scenario. Odier [26] and

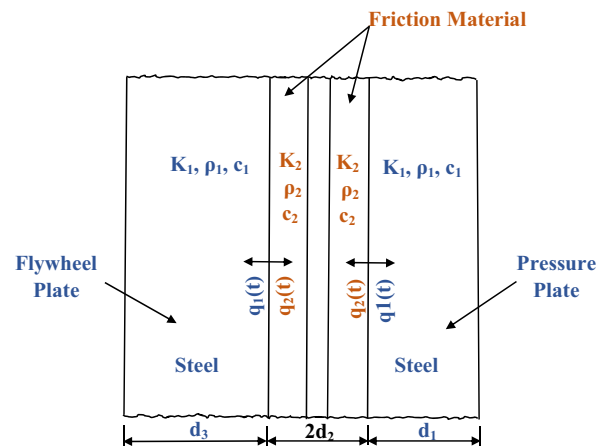


Figure 4: Diagrammatic sketch demonstrating the heat created at a single-plate friction clutch rubbing surfaces and its flow.

Hasselgruber [27] have thought about this heat conduction issue and have derived formulations for the growth in surface temperature that are as follows [25]:

$$\theta = \frac{2qCt^{1/2}}{\pi^{1/2}} \left[t^{1/2} \left(1 - \frac{2}{3} \frac{t}{t_s} \right) - (t - t_s)^{1/2} \frac{2}{3} \left(1 - \frac{t}{t_s} \right) \right]. \quad (11)$$

3 The selected materials

The metal chosen for the investigation is Sic-Al, which is functionally graded in terms of thickness and comes in three layers. The interface's first layer is made up of 57% aluminum and 43% silicon carbide, followed by a second layer that is made up of 19% silicon carbide and 81% aluminum, and a third layer that is entirely made up of aluminum (Table 1).

4 Equivalent principle of FGM properties

Consider steady-state heat transfer in the context of a unit-length multi-structure design with n layers of varying thickness t_i and any property λ_i . Because the flux led *via* layers is similar in each layer of the discrete structure and should be similar in homogenized material, if it is first assumed the heat flow is one-dimensional and perpendicular to the layer direction, it can be written [28]:

$$\lambda_{sy} = \frac{\sum_{i=1}^n (t_i)}{\sum_{i=1}^n \left(\frac{t_i}{\lambda_i} \right)}. \quad (12)$$

The above equation is used to compute thermal conductivity, where λ_{sy} is the effective characteristics of the FGM in the y direction, also known as the thickness direction. While employing thermal energy balance for specific heat [29],

$$c_T = \frac{(m_1 c_1 + m_2 c_2 + m_3 c_3 + m_4 c_4 + \dots)}{M}, \quad (13)$$

where c_T is the sample's specific heat capacity, M is the sample's mass, $c_{1,2,3,4}$ is the specific heat capacity of the appropriate layer, and $m_{1,2,3,4}$ is the sample's mass of the respective layer. And density is calculated as follows [30]:

$$\rho_{th} = \frac{\rho_1 + \rho_2 + \rho_3 + \rho_4 + \dots}{n_i}, \quad (14)$$

where $\rho_{1,2,3,4}$ is the density of each layer and n_i is the total number of layers.

Table 1: Displays all of the problem's parameters

Parameter	Value
Inner radius of clutch disc and axial cushion [m]	0.06
Outer radius clutch disc and axial cushion [m]	0.09
Frictional facing thickness [m]	0.003
Inner radius of the pressure plate	0.064
Outer radius of pressure plate [m]	0.094
Inner radius of flywheel [m]	0.031
Outer radius of the flywheel [m]	0.125
Pressure plate thickness d_1 [m]	0.005
Flywheel thickness of d_3 [m]	0.01
Coefficient of friction (μ) of material Sic and Al	0.6, 0.47
Angular sliding velocity (n) [rpm]	5,300
Density of materials: Sic and Al [kg m^{-3}]	3,100, 2,770
Specific heat of materials: Sic and Al [$\text{J kg}^{-1} \text{K}^{-1}$]	750, 923.5
Thermal conductivity of material Sic and Al [$\text{W m}^{-1} \text{K}^{-1}$]	120, 160
Density [kg m^{-3}], specific heat [$\text{J kg}^{-1} \text{K}^{-1}$], and thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$] of the flywheel, pressure plate, and axial cushion	7,200, 400, 56
Slipping time (t_s) [s]	0.4
Torque [N m]	126
Initial temperature [K]	300

A used numerical model was examined to demonstrate the capability of the method for computing the effective properties offered (as per Eqs. (2)–(4)), where it used the model boundaries that were established in 2012 by Abdullah and Schlattmann [30]. The details of a model are illustrated in Figures 5 and 6, and the numerical technique steps (FE) to solve the transient thermal problem are depicted as a flow-chart in Figure 7. Two FGM numerical models were compared, one of which was built using the standard method or, at the very least, is made up of several layers, and the other whose properties were determined by the equations. The results depicted in Figure 8 indicated very good agreement between the two models, confirming the validity of the suggested approach.

5 Validation of the analytical processing

A validation test was carried out to make sure the produced solution was error-free. As shown in Figure 9, a comparison analytical model was run using only the numerical portion of Sabri *et al.*'s research [5]. Table 2 and Figure 7 show the data and boundary conditions used in the comparison. The largest inaccuracy discovered was 3.3310^{-4} .

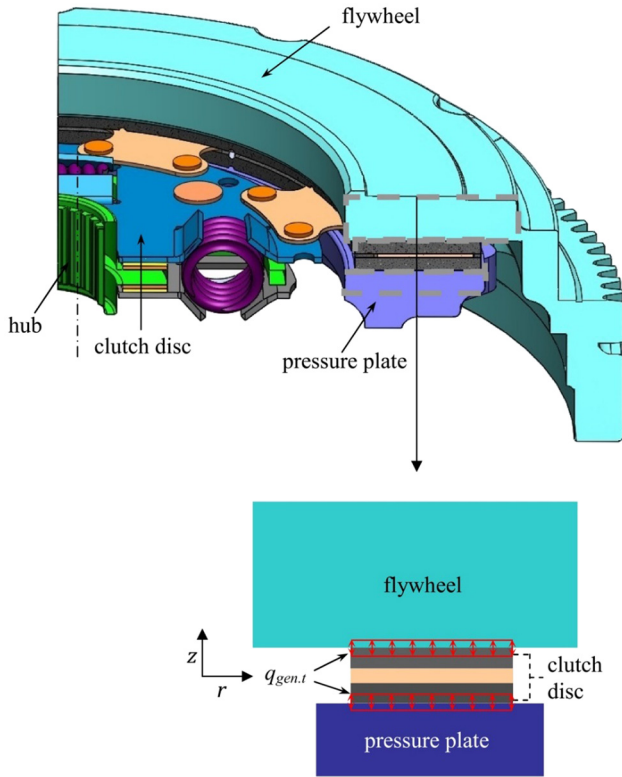


Figure 5: Axisymmetric model of a clutch system with boundary conditions.

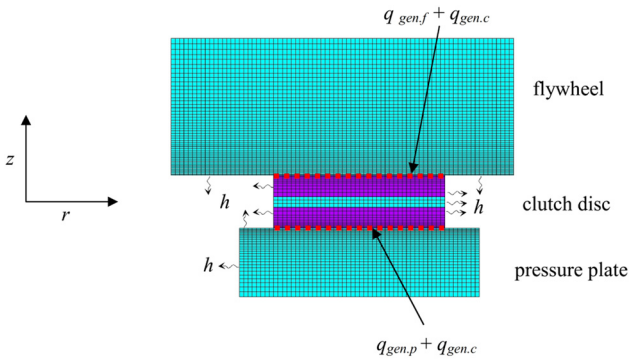


Figure 6: FE model with boundary conditions of a single-disc clutch system (no. of elements = 6,488).

6 Results and discussions

In order to obtain the range of temperatures under some parametric analysis, this study includes an analytical solution. The boundaries and factors without dimensions were displayed [31]:

$$\tau = \frac{at}{\delta^2}, \quad \tau_s = \frac{at_s}{\delta^2}, \quad \theta = \frac{\bar{T} - \bar{T}_0}{\theta_0}, \quad \theta_0 = \frac{q\delta}{K}$$

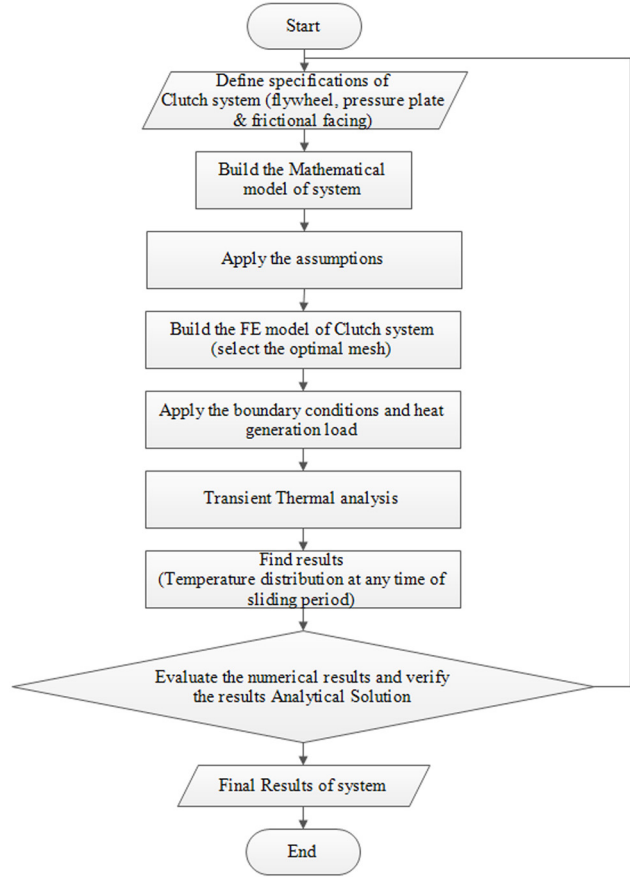


Figure 7: The flowchart of the developed numerical analysis using FEM.

$$\delta = \sqrt{3at_s}, \quad \alpha = K/\rho c$$

In all figures, the vertical axis shows the relation between energy dissipation and energy generation, while the horizontal axis represents the total time to slip time.

6.1 Effect of the friction material on the clutch's thermal behavior

The type of friction material used has a significant impact on the disc clutch case's thermal and tribological conductivity. The type of friction material used has a significant impact on the disc clutch case's thermal and tribological conductivity. This section examines the thermal behavior of three different disc clutch lining materials. Aluminum Al, FGM, and silicon carbide Sic were used as the materials. Figure 10 shows that the temperature of the Sic-Al liner during the sliding phase is significantly lower than that of the other two Sic and alumina lining types. The Sic-Al liner is the best option from a thermal perspective.

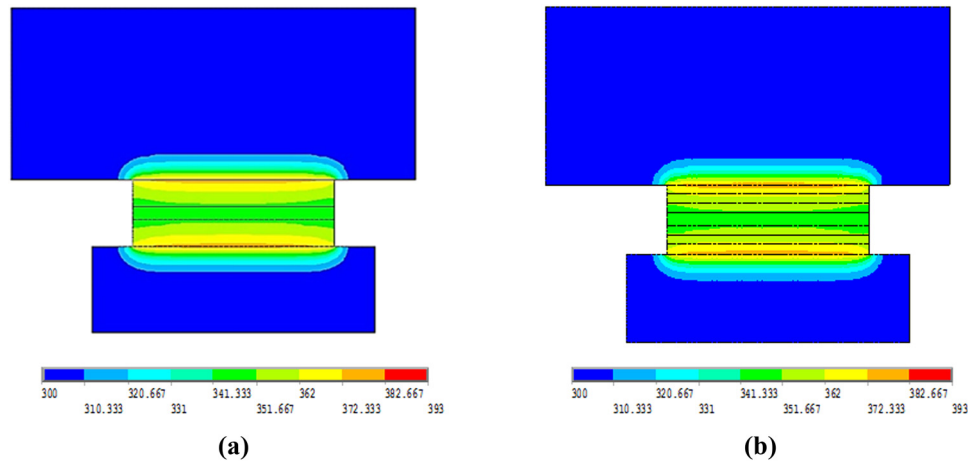


Figure 8: The numerical examination of equivalent properties approach (temperatures in Kelvin): (a) FGM_equivalent properties, (b) FGM_3 Layers.

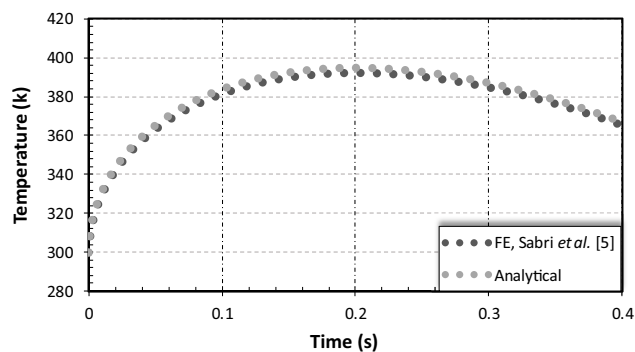


Figure 9: The comparison between the analytical and numerical solutions.

Table 2: The important data about the working conditions and material used for comparison [5]

Parameter	Value
Frictional facing thickness [m]	0.004
Angular sliding velocity (ω) [rad s^{-1}]	295
Density of friction material [kg m^{-3}]	1,000
Specific heat of friction materials [$\text{J kg}^{-1} \text{K}^{-1}$]	1,000
Thermal conductivity of friction material [$\text{W m}^{-1} \text{K}^{-1}$]	0.75
Slipping time (t_s) [s]	0.4
Applied pressure [MPa]	1
Initial temperature [K]	300

6.2 Influence of the speed of rotation

The friction torque and thermal streams entering and subsequently affecting the thermal behavior of the clutch plate are impacted by the rotating speed. Figure 11 depicts the progression of the maximum clutch plate temperature as a function of slip time at various speed rates. The dimensionless

figure shows that the rate of dissipation to generation energy is constant for varying velocity values at constant torque and slip time.

6.3 Effect of loads on the disc temperature

Figure 12 depicts how applying pressure to the clutch full plate's two contact faces affects how the clutch behaves thermally as a function of the slip time. The magnitudes of the chosen pressure for the study were (1, 0.5, 0.125, and 0.62) MPa [12]. The clutch plate's temperature increases in direct proportion to the amount of stress it is under. This happens as a result of the incoming heat stream being directly affected by the pressure being applied to the plate. When other factors are constant, the dimensionless structure responds to changing pressure values in the same form, where it is under constant effect with dimensionless time.

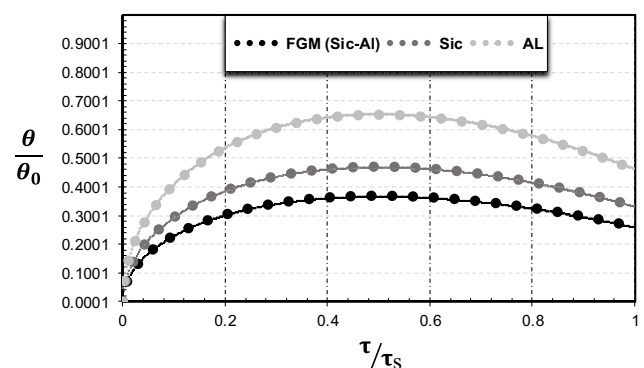


Figure 10: Temperature distribution at the dry clutch's component interfaces for various materials when the torque is 126 Nm, and the speed is 5,300 rpm.

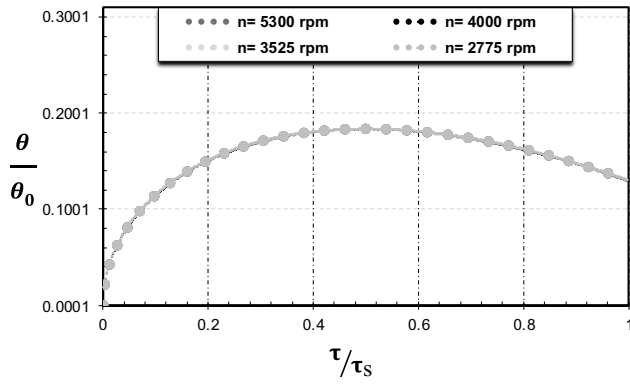


Figure 11: Temperature distribution at the dry clutch's component interface for various speeds, torque of 126 Nm, and slip time of 0.4 s.

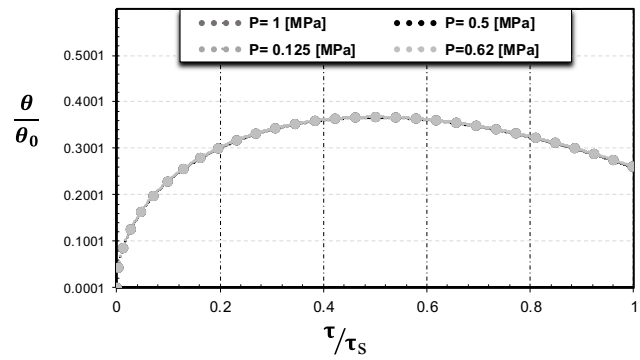


Figure 12: Temperature distribution at the dry clutch's component interface for various loads at 5,300 rpm and 0.4 s of slip.

6.4 Effect of sliding time on the disc temperature

Figure 13 depicts the analytical results of temperature fluctuations in the contacting surfaces during various slipping periods. The magnitudes of the study's chosen slip time were 0.25, 7.75, and 12.75 s [30]. The curves that address the thermal pattern of behavior of the clutch system can

be seen to have a few important points. First and foremost, $\bar{T}_0 = 300$ K at the starting point ($t_s = 0$). On the other hand, the final point deals with the conclusion of the slipping time period. Additionally, the peak occurred in the middle of the slipping time period.

The quantity of heat generated is a function of time, beginning at zero at the beginning of the slip, increasing until it reaches its greatest value at the halfway point, and then decreasing from its most extreme value to zero as the

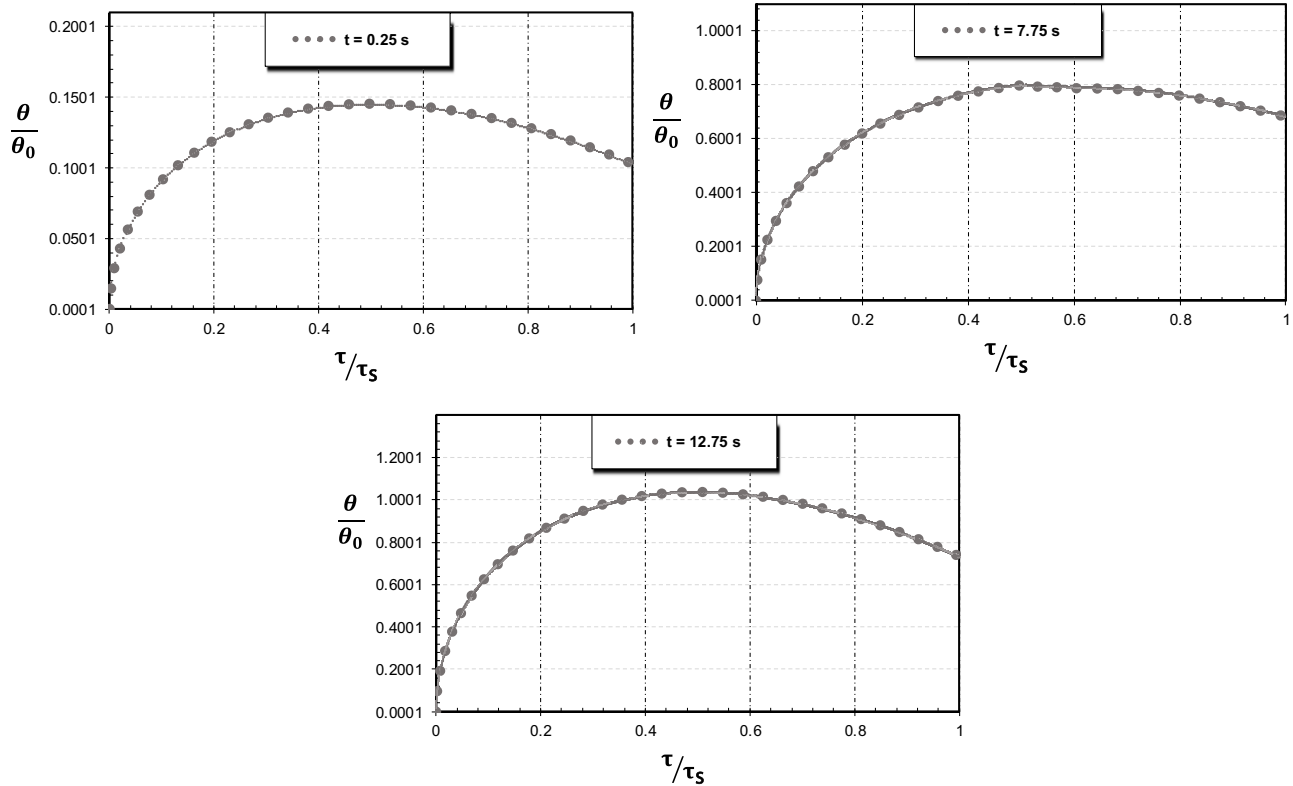


Figure 13: Temperature distribution at the dry clutch's component interface for various slip periods, with a velocity of 5,300 rpm and a constant torque of 126 Nm.

clutch is fully engaged. variety through the engagement duration, and sliding velocity. This is due to the start of the engaging stage, after with contacts. Numerous characteristics, including torque variation across the engagement length and sliding velocity, were affected by the temperatures that were produced during the sliding period. This is because after the clutch and flywheel make contact, the torque of the transmission begins to steadily increase. Transmission torque is now lower than motor torque because it is insufficient to overcome the negative torque generated by the wheels [32]. At the halfway point of the engagement stage, the vehicle starts to move, the clutch velocity closes in on the motor velocity, and the transmitted torque overcomes the negative torque. As a result of the increased speed and torque, a greater load will be required to overcome the beginning or negative rates, which causes the rate of heat generation to increase at a noticeably faster rate.

7 Conclusions and remarks

The transient temperature field in a friction material, in which a pad is constructed of a FGM, was calculated using a mathematical model. It was anticipated that as one moved away from the contact surface, an FGM's thermal conductivity would grow exponentially. For the FGM (Sic + Al), a numerical study was done to compare the normal and equivalent characteristics. Following the applications of the dimensionality formula to the results, it was discovered that some parameters, such as velocity and load, exhibited the same behavior regarding the relationship between temperature and time, indicating that the rate of heat generation and dissipation is constant independent of changes in the parameter values. The sliding time and the variety of friction materials showed a difference in thermal behavior with time because the generation of heat is subject to different characteristics in each case, which indicates the great importance of these two factors on the heat generated and then on the work efficiency and life of the clutch system. The FGM also demonstrated that, when compared to the other materials of which it is composed, it has good thermal behavior, increasing the likelihood that it can be used in this application. On the other hand, the work system necessitates good mechanical properties, which these kinds of materials have.

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