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Research Article

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A study on the construction technology of the Seljuk minarets in Isfahan with focus on their geometric brick pattern

Abstract: Using decorative elements is an inseparable aspect of Iranian architecture. Architectural ornaments in many buildings, including the minarets, represent the architect's craftsmanship. As such, the minarets in Isfahan have different types of brickwork ornamentations, such as 90-degree herringbone *(Khofteh-Rasteh),* basket weave bond *(Hasiri),* and other complex types. Additionally, the highest minarets are usually constructed in a truncated conical shape to reduce their overall weight and ameliorate their stability against the wind, and lateral forces. Therefore, while the geometric integrity of brickwork patterns should be maintained, all the ornamentations are applied on a shrinking surface area. However, the practical solutions for the construction processes in these structures haven't been sufficiently investigated. Hence, this study aims to explore the methods of brickwork projection on the minarets and analyse the changes in girih patterns at different height levels. Accordingly, after surveying the selected single minarets in Isfahan, they were modeled using drafting software applications and then analysed.

Keywords: minarets of Isfahan, use of brick tile (girih) on curved façade, geometry in architecture, brickwork, geometric pattern

1 Introduction

The minaret is an architectural type with symbolic significance in the Middle East. The exterior surface is usually ornamented with brickwork patterns that are designed through mathematical and geometric calculations. Because of the minaret's overall shape, the brick patterns are constructed on a greater surface area at the bottom compared to the available surface at the upper parts. In other words, the cross-section diameter is gradually decreased during the construction; consequently, the surface area will be reduced as well. As a result, in projecting these brickworks on a shrinking surface, retaining the geometric integrity of these patterns could become very challenging. Nevertheless, even though the cross-section diameter is decreasing, the overall design should cover the exterior façade completely. Similarly, in implementing these patterns on the minaret, the pattern should be put together around the surface accurately and appropriately; that is, an implemented design without overlapping parts at different height levels. Besides, the integrity of geometric order in the implemented patterns should be successfully maintained.

Therefore, this study aims to answer these questions:

– How a two-dimensional brickwork pattern on the minaret's three-dimensional curved surface could be implemented without losing the integrity of geometric order?
– What practical solutions have been developed to maintain the geometric integrity on the minaret's cone-shaped surface?

1.1 Research background

Most previous studies about girih patterns are concerned with two-dimensional planes. So, this study is one of the earliest endeavors in analysing and addressing the challenges regarding the implementation of these patterns in three-dimensional structures and the practical solutions thereof. In previous studies about the application of girih in brickwork and the architectural geometry, traditional craftspeople (such as Lorzadeh, Maher-ul-Naghsh, and Sha'rbaf) and architects (such as Pirniya and Bozorgmehri) have discussed the implementation of girih patterns thoroughly [1]. In other studies, girih patterns have been seen concerning connections to mathematics [2]. Moreover, the similarity of

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geometry used in girih patterns and fractals has been discussed as well. Due to the repetitive and indefinite nature of the pattern, nature has been recognized as a source of inspiration in designing girih patterns [3]. Accordingly, different types of girih have been categorized into two groups based on their drawing method: those designed traditionally, or by modern means [4]. As such, some studies about Islamic geometric patterns discussed the method of designing girih patterns through angled lines and circles [5, 6]. In another study, apart from explaining the implementation of girih on different minarets, the brick-dimensions were mentioned [7]. In another study, the dynamic behavior of columns was analysed [8]. Even though exploring the diverse methods and their practical implementations have been in the focus of many studies, the practical solutions to design girih patterns in the minaret's brickwork as well as making essential changes in these structures are being addressed for the first time in the present study; correspondingly, the changes from the bottom of the minaret to its top (A to B) or from its exterior surface to the interior one (C to B) will be investigated in this article (Figure 1).
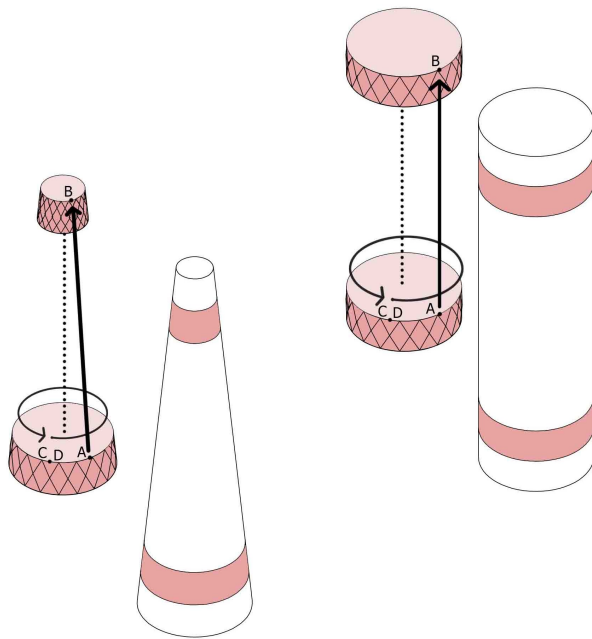

Figure 1: Pattern changes from the bottom to the top; the difference.

1.2 Research method

In this study, some of the minarets in Isfahan with a cylindrical or a truncated cone form were selected as case studies among all the existing single minarets in the former Seljuk capital where the perfection of brickwork was achieved. The data collection method has been through architectural surveying and field research, and the accuracy was achieved through a brick-by-brick architectural survey at different height levels in both minarets with simple brickworks and those with intricate girih patterns. All data were analysed and modelled through computer-aided design and drafting software applications: AutoCAD 2017 (Autodesk, Inc.) and Rhino v5.0, 2017 (Robert McNeel & Associates). After drawing the two-dimensional and three-dimensional models in AutoCAD, the construction procedure at different height levels was modeled in Rhino application and then analysed.

2 Girih

Girih, otherwise called *gereh* (meaning 'knot' in Parsi), consists of different Persianate geometric patterns made by a composition of oblique or orthographical interlacing lines that are aligned in a certain order [9]. Girih patterns have been the subject of great interest among many researchers and foreign tourists. Therefore, in quite a huge number of historic manuscripts and palimpsests, we encounter some information about these art-forms [10]. The designed patterns can be extended by connecting different girih patterns [4]. As an ornamentation style to cover the façade, girih patterns can be designed either within a fixed frame or could be developed further inside the existing girih pieces [11]. Girih could also be constructed with different materials in architecture and can be seen in wood, plasterwork, tilework, and brickwork decorations [12]. Based on the material used, the implementation of girih could either be flat or embossed; for instance, by the horizontal displacement of the brick, the girih pattern can be raised above the main surface [13]. So far, different design methods have been suggested for implementing elaborate girih patterns on flat surfaces [14]; but, the focus of this research remains on the implementation of girih on 3-D planes. In other words, the usage of girih in the brickwork ornamentations on minarets with truncated conical shapes will be analysed here.

2.1 Girih on a flat surface

Even though the simplest method for drawing a girih and implementing it on a surface is to create it on a flat 2-D plane, all design methods have one shared component that is called design grids. These grids are a set of lines usually

prepared in two different arrangements: the 'polar grid' and 'polygonal grid' subdivisions [15]. Even though there are many possible ways to draw girih patterns, the emphasis is mainly on the usage of either the central or axial symmetry; nevertheless, a combined method could be used as well [5]. As seen in Figure 2, new elaborate designs can be created by dividing the girih pieces and creating new girih patterns inside the previous ones [16].
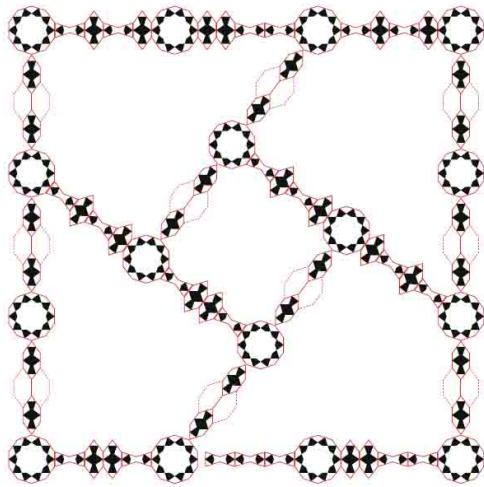

Figure 2: Example of new design subdivisions in *girih* [16].

2.2 Girih on a curved surface

Any girih pattern with only one regular concave polygon (*Shamseh* or star) would be called 'singular-ground' or *'tak-zamineh'*; while those with multiple stars are 'multiple-ground' or 'provisional girih patterns', respectively called *'chand-zamineh'* or *'dast-gardan'* [4]. The former type is simple and regular, whereas the latter is complicated and irregular (see Figure 3). Notably, in the multiple-ground group, if the star polygons have a similar number of points, the geometric order can be maintained easily [17]. Kaplan, and Salesin, have stated that "as curvature decreases and we move from the sphere to the Euclidean plane to the hyperbolic plane, the same underlying pattern accommodates stars with ever-larger numbers of points" [18]. As illustrated in Figure 4–5, the repeating pattern in a hyperbolic plane (or the Euclidean plane or the sphere) includes congruent copies of a basic sub-pattern or motif [19].

These curved surfaces may have a simple curvature that tilts towards one direction; such as in the cylindrical or conical surfaces. Otherwise, they may have a double curvature that would lean towards two different directions,

like the curvature in the sphere or a saddle surface. This research, however, focuses on different types of girih patterns designed and implemented on surfaces with simple one-directional curvatures.
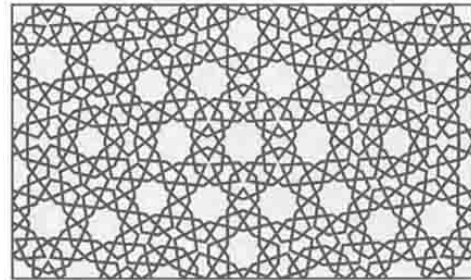

Figure 3: Example of *girih* patterns with 9-pointed, 10-pointed, 11-pointed, and 12-pointed stars [12].
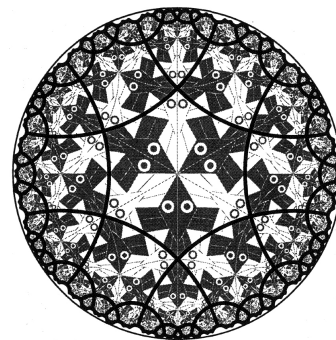

Figure 4: Repeating the main pattern on a hyperbolic plane [19].

2.3 Girih on a cylindrical surface

There are two different ways to draw a girih on a cylindrical volume:

a) Drawing on an unrolled 2-D cylinder: The unrolled surface of a cylinder has a rectangular shape. To draw a girih pattern on the cylindrical surface, it would be needed to model that on a rectangle with the length and width corresponding to the circumference of the cross-section circle and the height of the cylinder; then, this rectangular surface will be rolled back to form a cylinder again (Figure 6).

b) Drawing on a 3-D cylindrical volume: To draw the girih pattern in this method, first, a set of grid circles is drawn on the cylindrical shape. Subsequently, the girih will be drawn with the help of these grids (Figure 7). The role
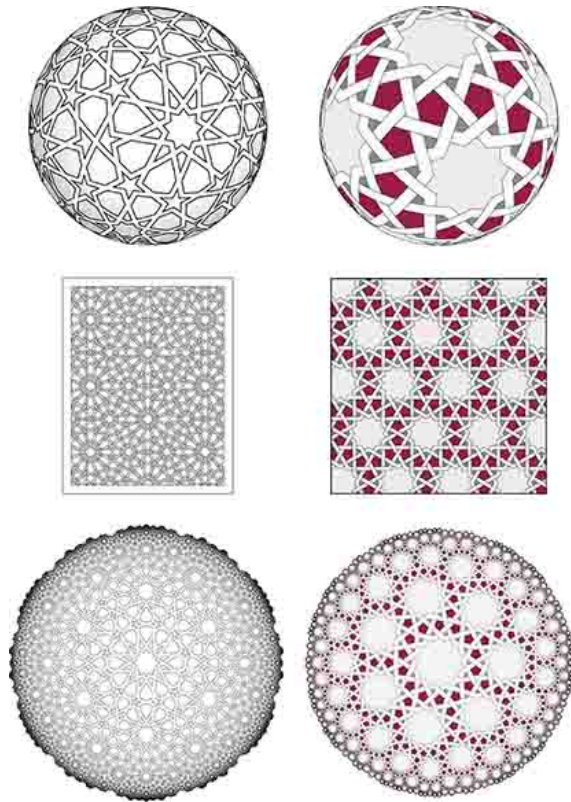
Figure 5: Example of projecting Islamic patterns on a sphere, Euclidean, and hyperbolic planes [18].

Figure 6: Projecting patterns on an unrolled cylindrical surface [20].

of these grids is, thus, to make the drawings geometrically more accurate.

In both methods, the ratio between the dimensions of the girih pattern and the cylinder's radius should be considered [20]. The method selection depends on factors such as the designer's skill or the type of design. For instance, the second method is suitable for designing the patterns in circular grounds.

2.4 Girih on a truncated conical surface

In projecting a girih on a truncated cone-shaped minaret, it is important to retain the geometric order because of the diameter of the cross-section, and subsequently, the exterior surface area is gradually reduced in different cross-sections levels (see Figure 1).

3 Minaret

Minarets are mostly tall tower structures with a circular plan that are usually constructed near a mosque, or where

they would make the mosque's location discoverable from the distance [21]. The minarets near religious buildings are built either singularly or in pairs [22]. However, the number of minarets may vary in other places like Sultanieh Dome in Zanjan or Hagia Sophia in Istanbul. Moreover, each minaret consists of various structural elements such as the base, shaft, cap, and crown. In general, based on their formal structure, minarets can be divided into three different categories of cylindrical, truncated conical, or prismatic forms [23]. Also, they are usually embellished with various types of simple linear ornamentations or complicated and intricate girih patterns.

Minarets have unique characteristics in structural behavior, and various items affect minaret's dynamic responses, such as minaret cross-section diameter. The greater diameter in cross-section causes the less lateral dynamic response of the minaret [24]. Moreover, minarets need more weight in the lower cross-sections to have a more stable response subjected to lateral forces [25]. Hence, minarets are usually constructed in a truncated conical shape.

3.1 Brick minaret

Brick is one of the most commonly used materials in the construction of minarets. In Middle East, it has been used in most buildings due to the availability of this material as well as its adaptability to the regional climate [26]. In this region, particularly the Seljuk period, was the pinnacle of art and architecture, especially in developing stunning brickwork decorations [27]. The architecture of this era is marked with stunning brickwork on the building façades [28]; as such, the buildings that are attributed to this period are covered with bewildering brickwork patterns that prove the incredible skill of its artists and architect [29]. Additionally, the brick pieces that were used in the load-bearing
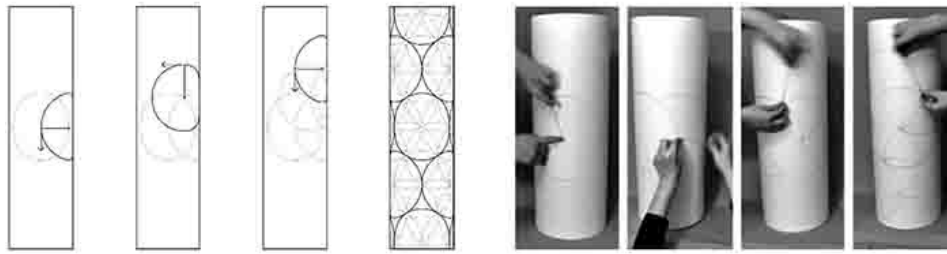
Figure 7: Method of drawing the patterns on a cylinder using a compass [20].

elements were usually molded with dimensions between 20–40 cm, and they were cut into smaller pieces to suit the architect's need for different height levels [26].

3.2 Girih in the truncated conical Seljuk minarets

Usually, higher minarets are constructed in a truncated cone-shaped form (Tables 1 and 2) to achieve higher stability against the wind force and seismic activities [23]. There are different types of brickwork on minarets. One of the main techniques of constructing brickwork is to make brick reliefs by horizontally displacing some bricks from the main surface [30]. In such minarets, the patterns should be changed due to the decrease of the circumference circle. As we move from the bottom to top, and the cross-section's diameter decreases, the pattern would inevitably and gradually become more compressed (Tables 1–5; also Figure 1); whether the girih is simple or intricate, this change may disturb the overall integrity of the geometric order in all types of brickwork. Therefore, to retain the geometric homogeneity, various solutions have been applied by the architects. These solutions mostly consist of either the change in the shape and size of the bricks or modifying the pattern type.

For the first practical solution (type I), brick shapes are converted from rectangular pieces to trapezoid ones at the upper levels of the minaret by cutting the bricks one by one; thus, the geometric order will be retained as the cross-section diameter is reduced. Moreover, a more stable shape will be constructed as well (Tables 1–5).

In the second solution (type II), the architect would change the design at the higher levels, so that the brick cutting would not become necessary. Hence, using this method involves a simpler practical implementation.

As illustrated in Tables 1 and 2, the change process in the geometric patterns is shown, for a cylinder, a truncated cone (resembling the minaret's form), and a cone. Analysing these pictures indicates that in the upper parts of

these models, the patterns are compressed. To further investigate this process, those minarets in the Isfahan Province that were attributed to the Seljuk period were chosen as case studies so their patterns could be photographed and analysed for this research.

3.2.1 Girih in the Chehel-Dokhtaran minaret

The Chehel-Dokhtaran minaret is known as the first minaret in Isfahan that contains an inscription dating back to the 11th century AD (Figures 8a and 8b). The surface of this truncated cone-shaped minaret is covered with brickwork decorations.

However, since the difference between the cross-section diameters at the top and bottom, respectively, 2.30 m and 2.90 m, is negligible [26], the minaret's outlook resembles the minarets with a cylindrical shape. In Table 3 the changes of patterns on this minaret are analyzed.

In this minaret, both types I and II solutions are applied to retain the geometric integrity in the patterns. The belt course between the two frames containing different girih patterns illustrates the usage of the second solution. Nevertheless, due to the considerable height of this minaret, this solution has not been sufficient. Therefore, in the structure's upper levels, brick conversion (type I solution) is observed

3.2.2 Girih in the minarets of Masjed-Ali and Sareban in Isfahan

The minarets of *Masjed-Ali* and *Sareban* (Figures 9a and 9b) are both attributed to the Seljuk era. The minaret of *Ali* mosque or *Masjed-Ali* minaret is constructed near the *Jameh* mosque in Isfahan. The *Sareban* minaret is situated close to the northern parts of the *Jubareh* district in the city of Isfahan. In both case studies, the usage of type I solution

Table 1: The change process in simple brickwork pattern modeled on different 3-D shapes (cylinder, truncated cone, and cone).
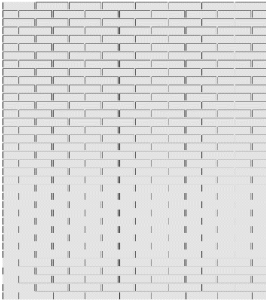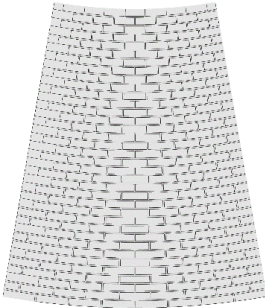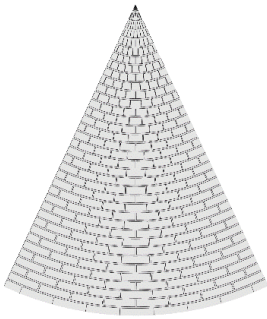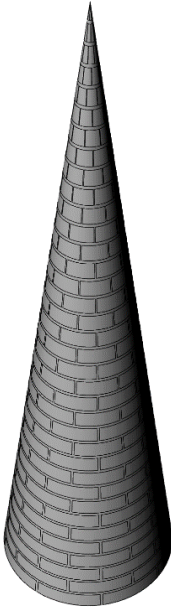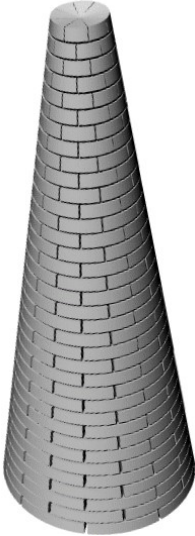
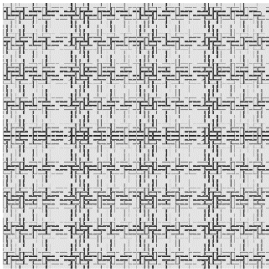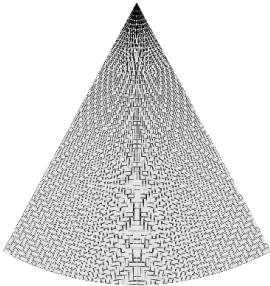| No	Title	Cylinder	Truncated cone	Cone
1	Unrolled 2-D model			
2	2-D model projection to 3-D			
3	Brick structure			

Table 2: The change process in 90-degree herringbone brickwork pattern modeled on different 3-D shapes (cylinder, truncated cone, and cone).
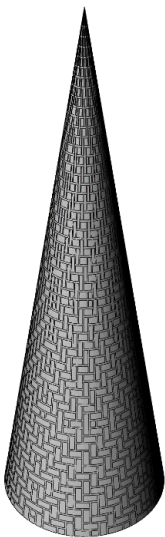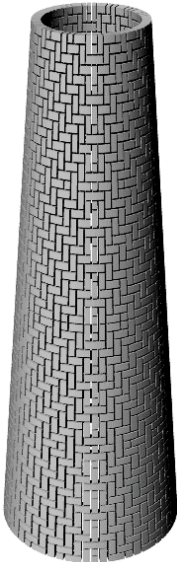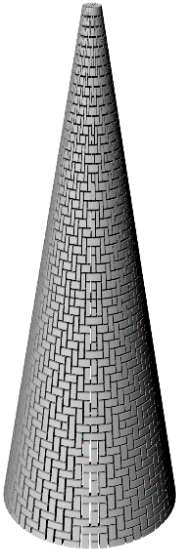
No	Title	Cylinder	Truncated cone	Cone
1	Unrolled 2-D model			
2	2-D model projection to 3-D			
3	Brick structure			

Table 3: The change process in *girih* pattern of the brickwork in the Chehel-Dokhtaran minaret modeled on different 3-D shapes (cylinder, truncated cone, and cone).
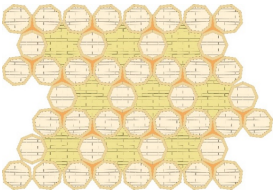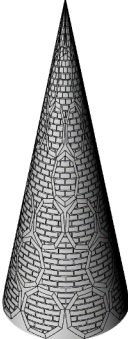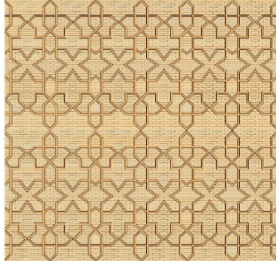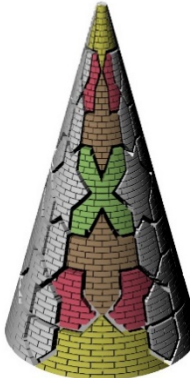
No	Title	Cylinder	Truncated cone	Cone
1	Unrolled 2-D model			
2	2-D model projection to 3-D			

Table 4: The change process in *girih* pattern of the brickwork in the Sareban and Masjed-Ali minarets modeled on different 3-D shapes (cylinder, truncated cone, and cone).

No	Title	Cylinder	Truncated cone	Cone
1	Unrolled 2-D model			
2	Brick structure			

(a)

(b)

Figure 8: The brickwork ornamentations on Chehel-Dokhtaran Minaret a) overall view; b) detailed view.

(a)   **(b)**

Figure 9: Minarets in the city of Isfahan; a) *Sareban* b) *Masjed-Ali*.
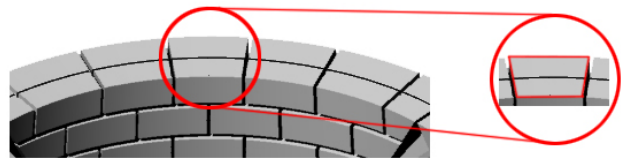

Figure 10: The conversion of bricks from a rectangular shape to a trapezoid one.

3.2.3 Girih in the Golpayegan minaret

Golpayegan is a town in Isfahan Province, located at the North-West of Isfahan city. The brick minaret in Golpayegan is also attributed to the Seljuk period (Figure 11) and has considerable brickwork decorations, among which two Kufic inscription panels are spectacular.

The minaret consists of two main parts: a prismatic shape at the bottom, and a portion resembling a truncated cone in which all types of girih adjustments can be observed (Table 5).

In this minaret, type II solution has been applied; that is, to project this pattern, not only the brick shape is changed at different height levels, but also in the upper parts of this minaret, the joint width between the brick blocks is reduced as well.

is seen. The geometric patterns on the upper side of the minaret do not seem to change.

The results of the field survey in this study revealed that the brick blocks were most probably cut into the desired shapes and smaller sizes; so that, despite the gradual decrease of cross-section dimensions, the geometric pattern retains its overall integrity; the cuts convert the bricks from a rectangular shape to a trapezoid form (Figure 10).

Also, the larger dimensions of girih pieces *(Mohreh)* could be another reason as to why the architect did not change the patterns in the upper parts (Table 4).

Table 5: The change process in *girih* pattern of the brickwork in the Sareban and Masjed-Ali minarets modeled on different 3-D shapes (cylinder, truncated cone, and cone).
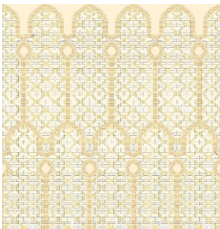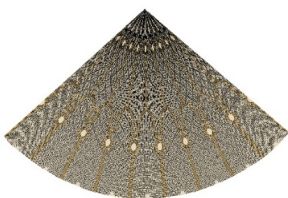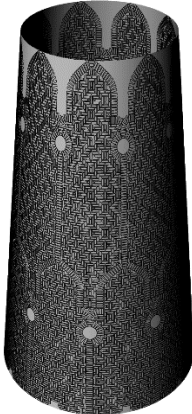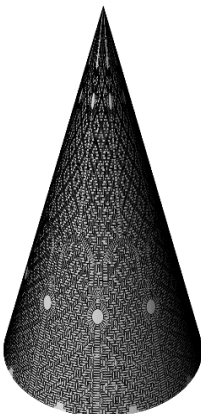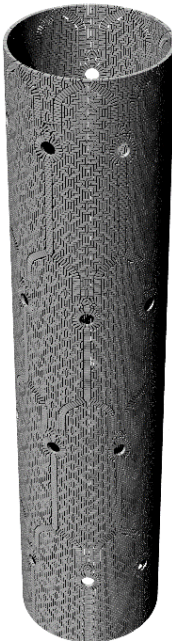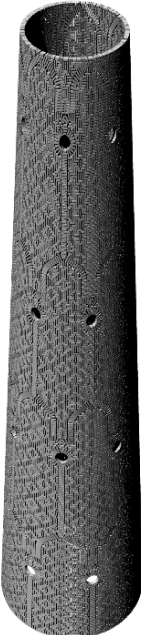
No	Title	Cylinder	Truncated cone	Cone
1	Unrolled 2-D model			
2	2-D model projection to 3-D			
3	Brick structure			

Figure 11: The *Golpayegan* minaret.

4 Conclusion

The truncated cones have a smaller diameter at the top compared to the bottom of the shape. To implement geometric patterns on these shapes, if the outer cross-section diameter at the top has little difference from the diameter at the bottom, simple solutions were applied. Therefore, reducing the distance between the horizontal brick rows in higher levels may keep the geometric integrity in the façade. Yet, in some minarets, the difference of cross-section diameters at the top and bottom of the minaret is not negligible. Hence, the implementation of girih may have required other solutions.

Architects and skilled craftspeople applied two main practical solutions to keep the geometric order in the minaret's surface consistent and integrated. In some complex examples, a combination of both methods was used. In the first and simpler practical solution, one or several belt courses divided the frames containing different girih patterns. Whereas in the second method the architect required more skills because the brick dimensions and shapes were to be modified. The bricks should have been cut to form trapezoid shapes.

Present findings on analysing brick minarets have also revealed that using girih patterns with larger pieces could make the lack of geometric coherence less pronounced. In these case studies, reducing the distance between the bricks seems to have helped the architects in retaining the geometric integrity to a great extent. Notably, since the spectator observes the minaret from the ground level, any disintegration and lack of perfect geometric order remain almost unnoticeable.

All in all, this study has provided new explanations regarding the practical solutions of implementing girih patterns on the minaret's curved surface. Apart from these findings, it is suggested to use the method and process of this analysis in similar studies on other minarets and structures with elements having two-directional curved planes, such as domes.

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