Research Article

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The triangle grid, the evolution of layered shells since the beginning of the 19th century**

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Abstract: This paper presents the evolution of and influence that the use of a triangular grid has had on the transformation of layered grid structures during the last two centuries. Historical facts, technological advances, economic and social crises among other factors influence the process of change and evolution of the grid systems in architecture to make construction processes more efficient by reducing the consumption of materials and qualified labor. Geometry and structure innovate styles and processes that are perfected over the years, achieving lighter and more stable structures. For the purpose of this paper historical data will be collected that allows the development of a theory that shows the influence of various periods based on technological advances. This research shows four historical periods in the evolution of layered grid structures. Being the triangular grid a widely used methodology in the construction of a diversity of geometric proposals and innovative construction systems for the solution of a diversity of buildings from the first domes built in the early 19th century to the free-form structures built from the late 20th century to the present day.

Keywords: grid structures, historic structures, shell structures, light structures, architectural geometry, digital architecture

1 Introduction

This paper will describe the transformation process of a methodology applied to the design and construction of light-layered grid structures for different uses, scales and materials, which will influence the conception of each constructed building. It also examines how some historical moments influence its development, generating an evolution of proposals adapted to the period in which it develops.

When talking about the triangle, the designer or mathematician imagines a three-sided polygon that can vary its typology according to the length of its sides or the size of the interior angles formed by its sides. However, for an architect it becomes not only an aesthetic tool of a building, it also becomes a way to stabilize a structure and at the same time make it lighter. The triangle is the only polygon that does not deform when a force is applied to it. Any other polygon to which a force is applied will not be stabilized until it is triangulated.

The structures that use the triangle as part of the design process can be seen from the works made by Marcus Vitruvius Pollio. However, the real transformation of the triangle in the use to stabilize or optimize a structure begins with the Industrial Revolution in England at the end of the 18th century, with the industrialization of the manufacturing processes and later, assembly on site.

In the case of layered grid structures, the transformation process starts in Paris with the construction of a metal grid dome using a single layer in the Halle aux Blés. At the beginning of the 20th century, the process of transformation continues from one layer to two or more layers trusses, mainly employed for the construction of bridges until the proposals of A. G. Bell for the fabrication of space structures.

“A single layer grid is, in fact, a two dimensional structure but is considered here because of its grid nature and certain characteristics which it has in common with double layer grids, particularly the ability to disperse heavy concentrated loads through all the members of the grid” [1].

The double layer grid structures are the most widely used typology in the construction of spatial structures, these structures consist in two layers interconnected by vertical and diagonal members forming a webbing [1, 2].

The structures with more than two layers are known as multilayered structures, examples are the pavilions built

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for the Montreal Universal Exposition in 1967 'Man the producer' and 'Pyramid and Volcano' [2].

The most outstanding transformation processes of grid structures has happened across almost 200 years of history of architecture and engineering. In the first stage, several processes influence their transformation and the development of new materials, which allow greater use of structures of larger scales and longer spans. In the second stage, the transformation of geometry is influenced by the lightness of the structures and therefore, by the structural system used to generate non-orthogonal geometries. In the third stage marked by historical moments like the Second World War, it allows the evolution to accelerate the transformation of geometry, the invention of new materials and the implementation of new construction processes. Finally, in the fourth stage, technological advances led by digital tools allow the optimization of design, manufacturing and construction processes.

Similar works have been published. In [3, 4], Makowski presents a compendium of research papers of different contributors, the papers are related to the history and development of braced domes and braced barrel vaults.

In another compendium [5], Makowski together with other authors that were authorities in the field present a series of papers related to the development and evolution of double-layer structures. This book presents methods of analysis, design and construction of double-layer grid structures.

In [1], a description of the most commonly types of space grids is presented, like domes, vaults, folded plates, tensegrity structures and other light structures. The study presents a review of structural systems, materials, analytical aspects, design and construction of spatial structures.

Also, in [6] it is describe the history, geometry, design and construction of space grid structures. As well, different constructions systems and materials that influence the development of shape are presented. Finally, examples of retractable, foldable and deployable space grid structures are described and analyzed.

In the same way, in [7] a description of design and optimization methods is presented for the generation of efficient shells forms and topologies. Academics and professional experts present techniques that are part of the history and new techniques using the power of computational design for creating elegant thin shell structures.

2 Stages of the transformation process

2.1 The first stage: The industrial revolution and the consolidation of the grid

The industrial revolution is marked by the improvement of James Watt's steam machine by developing the steam condenser, the invention of George Stephenson's locomotive and Robert Fulton's steamboat, among other inventions or improvements to the existing equipment in that time. In the construction industry, some of the advances of the time are the perfection of concrete mixtures and the emergence of steel as construction materials, optimizing industrialized processes (in some cases through prefabrication) and at that time, using modulation as a methodology of efficiency in the assembly of a structure.

The first grid structures proposed and built were those used in the construction of domes to cover large spans, built with metal structures [4]. An example is the Halle aux Blés, built in Paris, France in 1806 by Belanger and Brunet, where a ribbed iron dome of 40 m clear span was built, consisting of a radial dome grid for its structural solution [3, 8]. The Belanger's dome was designed with two alternate materials: wrought iron and cast iron. Eventually, the dome was built of cast iron that was cheaper than the wrought iron and less subject to atmospheric changes [9].

Other examples of a dome using a radial grid for its solution are those built in England: The Royal Pavilion in Bringhton (Figure 1) and the Greenhouse in Syon, both built in 1827 [10]. In the exterior of the Royal Pavilion, the domes and minarets stand out, built of cast iron frames that adapt the geometry of the structure to an arabesque style shape [3].

Also, a barrel vault of 19 m clear span was built in the early 19th century in Vienna for the Diana public baths, the structure consisted of a series of semicircular iron arches. Most of the structures of this stage were built with iron, where the rapid improvements in manufacturing techniques were of fundamental importance [3].

The invention of the locomotive and the evolution of the train in the Industrial Revolution promoted the construction of larger-scale structures. In view of the need to build the necessary infrastructure to be able to move trains for passengers and products, supplies and merchandise for the nascent industry, lattices were designed that would allow the construction of bridges and after train terminals, using materials such as wood, iron and steel. European countries such as England, France and Germany dominated at this time, as did the United States in America.

Figure 1: The Royal Pavilion [11].

The lattices arise as a structural methodology for the construction of bridges because of the need for cross-cliffs or rivers and later, to build the large coverings or roofs that require a greater distance between the supports to house the train terminals. Only in the United States in the 19th century, dozens of patents were registered for different patterns of possible trusses applied to bridges or any type of structure. The most notable are the trusses used for either a single deck beam or a bridge beam: Howe Truss, Pratt Truss, Baltimore Truss, Fink Truss, Warren Truss, Parker Truss, Warren Double Intersection Truss, Long Truss and K Truss between others (Figure 2). In [12], the truss system is defined as the arrangement of the rod forming a triangular configuration of a grid, because the triangular shape is a stable form.

With the increase of the railway infrastructure, in the 19th century, the needs for lightweight structures were increasing which would be easy to manufacture and assemble, with wood and iron as the main building materials. This can be seen in most of the patents filed in the United States and England. One of the first trusses patented and applied to the construction of bridges is the one presented by S. H. Long (Figure 3) in the year 1839 with patent number US1,397, its origin being in 1835. It had a patent improvement in the year 1841 with number 34 [13, 14]. The lattice is characterized by the fact that vertical uprights braced diagonally in opposite directions joining the upper and lower chords.

The Howe truss (Figure 3) also appears as a solution in bridge construction, even though a patent is believed to exist in May 1940 [15]. It is observed in the patent of July 10th, 1940 with the number of patent US1,685, where the first proposal of its truss used in bridge construction is presented. In the W. Howe patent, he mentions that its elements can be highly stressed to prevent its tendency to vibrate and applicable to roof structures and in any type of structures [6].

The Pratt truss (Figure 3) was designed by Thomas Willis Pratt and Caleb Pratt in New England in 1943 and patented in 1944 under patent number US3,523 [7]. “One of the limitations of the Howe truss was the length between its supports, as the diagonal compression members were susceptible to bending as their length increased with increasing distance” [8]. Unlike the Howe truss, the Pratt truss elements will work in compression by reversing the direction of the diagonals.

In Europe, English engineers James Warren and Willoughby Monzani in 1848 under the name Construction of Bridges and Aqueducts with the patent number 12,242 patented the Warren truss. Two variants of the Warren beam are commonly used, the first using upper and lower chords joined with diagonals working in tension and compression. The second variant is added to the uprights at the points of intersection of the diagonals [16].

In view of the diversity of patents registered only in the United States and England, the patents are mentioned to understand the importance of their use in later structures and the simplicity with which they are adapted to the needs of each of the structures.

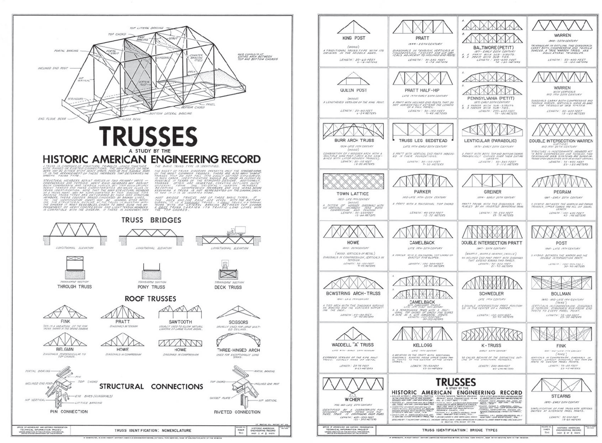

Figure 2: Lattice Trusses [17].

The transformation of the trusses, used for the building of bridges into vertical and horizontal structures for building envelopes, was consolidated with the construction of the Crystal Palace by the English landscape architect Joseph Paxton in London for the Universal Exhibition of 1851. A grid similar to a Long truss is used in the walls of the enclosure and a grid similar to Town truss is used in the arches of the barrel vault. The vault of the Crystal Palace was a great example of early-prefabricated barrel vaults using one layer [3]. It is important to mention that the methodology of prefabrication and modulation used in the construction of the Crystal Palace allows it to be assembled on site in approximately 6 months. The use of a prefabricated type node for the union of the bracings stands out, the first one is registered. The construction of the structure mainly used three materials: cast iron, riveted wrought iron and wood. The structure was completely designed to be free of massive surfaces such as walls and of sufficient transverse stiffness to the stresses [18].

The Crystal Palace had approximately 564 m long, 124 m wide and 33 m height. The covered area was approximately 74,000 m2. One square meter of the Palace construction eventually weighed approximately 5.35 kg [19].

The use of the grid on the roofs was consolidated with the construction of the first train stations, built mainly in Europe and the United States. The King's Cross station in London and the Gare de l'Est in Paris, both built in 1852, stand out. The King's Cross station old roof has 32 m arched scaffold span and two-bay hall measuring 244 m long [20, 21]. The arches were constructed from glued timber, in later years, the material changed to iron [21].

With the construction of the train stations, the spans of the structures increased. These transformations in manufacturing techniques were influenced by the development of iron quality and the appearance of steel.

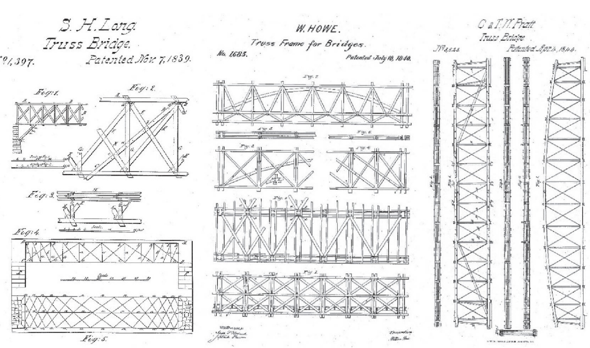

Figure 3: Left: S. H. Long Patent [14]; Center: W. Howe Patents [22, 23]. Right: C & T. W. Pratt Patent [24].

In this stage, the consolidation of the lattices influenced by the emergence of the industry at the end of the 18th century and its subsequent transformation dictated from the patents of trusses to the use of envelopes and roofs for buildings, which result from a view of the new needs that were emerging at that time. In [25, 26], the truss is presented as the origin of the evolution of lightweight grid structures like domes, slabs, space structures and saddles. This transformation is accompanied by the appearance of materials such as steel, the improvement of the quality and properties of concrete and iron, like the appearance of wrought iron that would improve resistance to tensile stress of soft iron. Also at this time, the improvement of the processes of mass production and subsequent assembly reduced time and improved quality.

2.1.1 The development of resistant materials

In this first stage characterized by the Industrial Revolution, various materials improve their mechanical properties or emerge as new structural options. Their properties make the industrialized processes more efficient through the prefabrication and modulation of their elements, thus reducing construction times and assembly costs. The cast iron, the wrought iron and steel were the most used materials in this stage. In the case of cast iron and wrought iron, their main difference is in how they are produced. Cast iron is liquefied and then shaped in a mold, typically cast iron contains between 2% and 4% of carbon. Wrought iron is brought to a malleable state with intense heat before being worked and shaped with tools to give it the final form, it has a very low carbon content, less than 0.08% [27, 28, 29].

In this stage, the gray cast iron was widely use. The gray cast iron can withstand high compressive stresses and has good resistance to corrosion but is relatively weak in tension and has an easier initiation of cracks once an item is finished. The use of cast iron in structural proposes began at the end of the 18th century [29, 30].

Wrought iron is less affected by rust and can better hold coatings and platings. Wrought iron, unlike cast iron, is resistant to tension, coming again to the fore in the railroad era [27, 29].

"The ironmasters of the British and American Industrial Revolutions increased not only the knowledge of iron as a building material, but also expanded its use from tools and machines to buildings and bridges. The first of these buildings to display the use of cast iron as a building material was a cotton mill erected in 1801 in Manchester. This mill was the first successful display of cast iron beams as structural unit" [31].

The evolution of metallic materials for the manufacture of structures would be consolidated in the year 1855 with the patent of Sir Henry Bessemer, who would invent and patent the process to manufacture steel though at the beginning in other places, it would present an excess of oxygen and presence of phosphorus. Robert Mushet would solve the problem of excess oxygen by adding an alloy of iron, manganese and carbon to the cast [32].

Finally, at this stage, electric arc welding is used mainly for the manufacture of tools. Although welding was already in use in the early 19th century, builders did not consider its use as a necessity, because at that time they contemplated the use of rivets as a faster process for joining two or more elements. In 1924, the first all-welded buildings were constructed in the United States. [33].

2.1.2 Argument of the historical trajectory of the first stage

At this stage, the development and evolution of construction processes, manufacturing methodologies and subsequent construction methods were triggered by the Industrial Revolution. With the industrialization of construction, technological advances evolved faster, which normally had taken hundreds of years.

The first stage of the development of the triangular grid is considered the origin of bigger and more complex grid structures of different shapes and geometries for different applications during the first two decades of the 21st century. Architects and engineers explored new design and project development processes, making their structures more efficient since early stages of design, through the manufacturing processes and concluding with the final product. Compared to previous years, their material consumption was lower as shown by the construction of the Crystal Palace weighing 5.35 kg per m2. In addition, the labor force was reduced. In the United States and the United Kingdom during this stage the labor in construction industry decreased from 5.1% to 4.7% of the total labor force employed in all industries, such as agriculture, fishing and mining, where it increased [34].

At the beginning of the 19th century, the theory of structures was ready for rapid development. Elementary statics was applied to masonry arches and timber structures, and at this time the elastic theory of beam bending was determined. The theory is still in use [35].

2.2 The second stage: The geometric and scale transformation

In the last years of the 19th century, and with the appearance of new energy sources such as electricity and oil, the construction processes were improved. Structures became riskier in their geometry and mainly in their scales, which would later be transferred to the constructive processes and methodologies of manufacturing a grid structure.

2.2.1 Transformations of the grid in the second half of the 19th century

Independent of the construction of the first high buildings in Chicago in the United States in the second half of the 19th century, a grid high tower is proposed for the Universal Exposition of Philadelphia in 1874. The Centennial Tower designed by the engineers Clark and Reeves would not be built. The project of the tower contemplated a height of approximately 300 m, whose structure was designed with tubular supports and horizontal rings as transversal reinforcements, in the center was a steel core based on a tube for the installation of stairs [19].

On it comes the use of a grid similar to the Long trusses. The first great high tower would be built for the Universal Exhibition in Paris in 1889: the Eiffel Tower (300 m height) was designed by French architect Charles Léon Sthepen Sauvestre and built by French engineer Gustave Eiffel. “The tower was designed to be quite light for its size: approximately 10,600 tons, including 8,000 tons of iron and a large stone pedestal beneath each of its four feet” [36]. The Eiffel Tower uses trusses fabricated in two dimensions, where they were riveted together in the form of box trusses, so the way in which these trusses are used is close to making the tower structure perform like a three-dimensional design [37].

The consolidation of these grid high towers would take place in Russia in 1922, with the construction of the Shukhov Tower. At first, it was planned to be built with a similar height (350 m height) to the Eiffel Tower, but three times lighter because of the grid. With the beginning of the Russian Revolution and the shortage of materials, its height was reduced to 150 m height. The type and size of steel profiles used for its construction was designed to suit the actual load [38]. It stands out because of the use of the hyperboloids of revolution as the basic geometry of the structure, the double-curved surface strengths and save materials [38]. The Diagrid structural system applied by the architect Norman Foster for the competition of the Humana Headquarters Building in 1980 was similar to the used by V. Shukhov in his towers and grid shells, as shown in the design process presented in [39].

In 1866, the St. Pancras train station in England was built with a 78-meter span, the largest built up to that date in Europe, the roof consist of a single-bay train station hall that was built using twenty-five arches made of iron and partially covered by glass [21, 40].

In the year 1889, when the Eiffel Tower was built in the Universal Exposition, another great structure was being built. The Gallery of Machines designed by the French architect Charles Louis Ferdinand Dutert and the engineer

Victor Contamin stands out, the structure had 110,60 m span, the total length was 423 m, and a height to the ridge of 45 meters. Twenty frames made up the basic skeleton of the structure. For its construction, an ingenious solution called 'three-hinged arch' was used, which is a structural solution joining the frames at three points, at their two bases and at their ridge [19].

At the end of the century, the length of the spans increases, the proposals increase in scale and in consequence, the construction proposals change and evolve.

With the transformation of the scale, grids adapted to the new needs, mainly of laminar structures that appeared at the end of the 19th century. In Berlin, the engineer Johann Wilhelm Schwedler designed and built braced domes with his innovative grid, which would be named like his surname, Schwedler System, first published in 1866. In the same way, in 1892, August Föppl shows light barrel vaults, which consisted of several grid trusses that cover the length of the barrel, evolving to grid trusses that begin to be part of the barrel vault when they are braced, forming a laminar structure of one layer [4, 41].

Figure 4: Water tank by V. Shukhov [42].

At the beginning of the 20th century, the German architect Friedrich R. B. Zollinger developed a system of lightweight roofs that were easy to manufacture and which would be patented in 1921, forming Lamella grid units and creating a diagonally crossed structure based on the shape of the rhombus, constituting a cylindrical layer. The system was used for the construction of roofs for buildings using wood units that were screwed or nailed at their connection points. The length and thickness of the wooden planks depended on the span and the slope of the roof; however, as a general rule, the planks were two meters long and two and a half centimeters of thickness [43]. One of the advantages of the system is, that it was built with identical components. This was one of the reasons for the acceptance [3]. The system would later be used in its evolution to lamellar structures for different geometries and scales, such as domes and barrel vaults as used by Pier Luigi Nervi in the hangars of Orvieto and the Palazzetto dello Sport, among others.

With the transformation of the scale and the evolution of the grid, geometry is transformed into new proposals. The mentioned Shukhov Tower uses hyperboloids of revolution as a geometric basis for the stabilization of the high structure in combination with the grid. The origin of the geometry of the Russian engineer Vladimir Shukhov was first used in the construction of a water tank in Polibino, Russia in 1896 (Figure 4).

In 1897, a production hall (Figure 5) in Vyksa, Russia was built using a diamond grid similar to the Lamella grid, also designed by Russian engineer V. Shukhov. In the production hall, the evolution of the geometry from a barrel vault, a dome or a double-sloped roof was to a double-curved surface, using a diamond grid to stabilize the structure.

Figure 5: Production Hall in Vyksa, Rusia by V. Shukhov. Source: Alamy.

2.2.2 Transformations of the grid at the beginning of the 20th century

Proposed by the inventor Alexander G. Bell, the space structure was one of the greatest evolutions in the reticular structures. Although they were mainly used in the search for artifacts that could make man fly, the space structures were used principally for the construction of kites, were capable of lifting a person and also for the construction of tower-type structures, for example, the Beinn Bhreagh Observation Tower [18, 44]. These proposals, besides innovating the construction of space structures, use prefabricated modular elements and prefabricated nodes of union, very similar to those that later Max Mengeringhausen would put on sale in the year 1942, the MERO node.

The Mero node would open an international market for prefabricated systems for the construction of space structures, patented in 1936. The first structure would be built in 1940 in Berlin [6, 45]. The largest project built at the time with the MERO node was the West Berlin Interbau Exposition during the 1950's (the golden age of the development of grid structures), with dimensions of approximately 52 m × 102 m [46].

In Asia, Dr. Fujio Matsushita of Tomoegumi Iron Works Ltd (now TOMOE Corporation) would carry out the most important transformations. In 1933, the Japanese company manufactured one of the first prefabricated connection systems called Diamond-Truss for the construction of lightweight grid structures.

Prior to the beginning of World War II, Albert Kahn took up grid trusses for the construction of structures, used mainly in the automotive industry and later for the manufacture of armaments with the beginning of the war. The need to have bigger hangars in view of the growth of the scale of the airplanes, as well as the increase of the production of different types of armament like tanks of war, consolidated the use of lightweight structures. Albert Kahn managed to build long spans to make the processes of manufacture and storage of the big airplanes that being built more efficient.

2.2.3 The importance of transformations in the second stage

The structures and grids built in the late 19th century and early 20th century, complemented by advances in construction and structural systems using materials such as iron, steel and wood, would constitute a significant advance in the transformation of future architectural proposals. The structures and grids evolved in geometry and processes, giving rise to the future complex geometries built by Eero Sarinen or Felix Candela. V. Shukhov would become one of the precursors of the hyperbolic paraboloids and later free form geometries.

When thinking about constructive and structural systems, A. G. Bell's proposals would be one of the most important transformations of the 20th century, with the construction of its spatial structures with modular prefabricated elements, which would facilitate the assembly process. These transformations would continue to evolve until the 21st century. The proposals of the prefabricated elements adapted to any structural proposal but was mainly used for the construction of structures of long spans because of the needs of military transportation like airplanes and ships.

2.2.4 Arguments of the historical trajectory of the second stage

During this period, the influence and development of buildings was determined by the beginning of prefabricated processes of easy assembly and the emergence or evolution of new materials. The result were new geometries that would emerge and the increase of structural scales in conjunction with the distance between the structural supports. The beginning of the oil industry influenced the development of structures.

This stage influenced the third stage affected by a war. Decreasing energy and material consumption became a necessity in a world affected economically and socially. A solution in the construction industry was the evolution from the plane to space proposed by A.G. Bell. Bell, which allowed the processes to be more efficient and to give a solution to the new needs that arose after the war, construction processes that have even continued to this day.

In the United States and the United Kingdom during this stage the labor in construction industry increased at the end of the 19^{th} century from 4.7% to 6.5%. But at the beginning of the 20th century, the labor force used in construction decreased from 6.5% to 3% of the total employed in all industries, such as agriculture, fishing and mining where it increased [34].

In the second half of the 19th century, various methods of structural analysis have emerged. With the availability of a wide variety of wrought iron rolled sections, the construction of trusses and girders became widespread, especially for railway bridges. Statically indeterminate method appeared for practical convenience. "Innovation in the theory of structures towards the end of the century also included adaptation of the principles discovered to the analysis of forces and moments in rigidly jointed frameworks" [35].

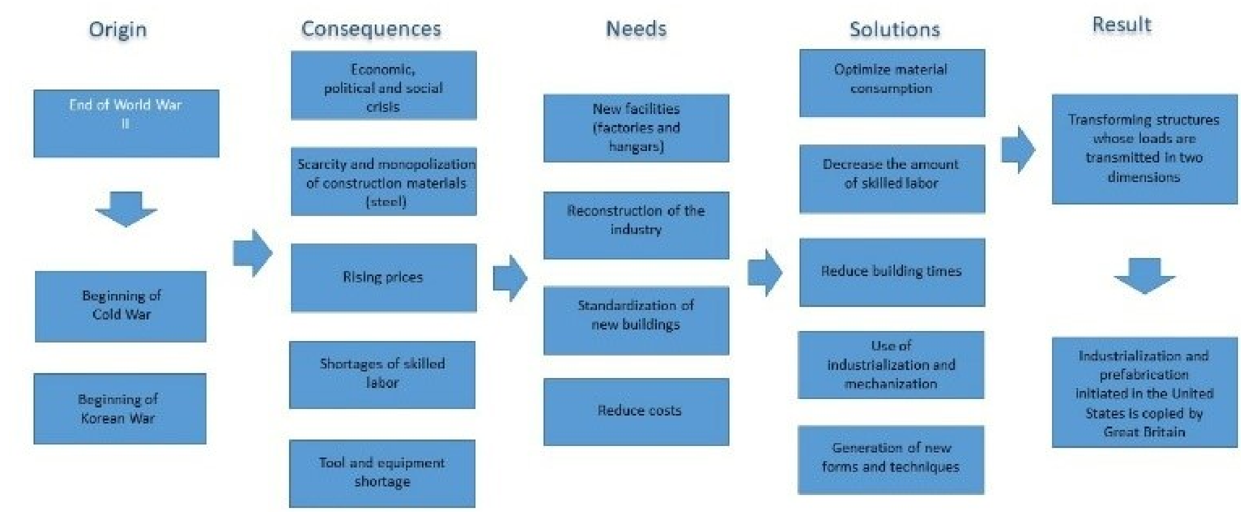

Figure 6: The building industry after Second World War.

2.3 The third stage: The crisis of World War II, the emergence of new materials and processes that are more efficient

The end of World War II would bring different types of crises. The economic crisis in which the destroyed European countries would be plunged stands out. Where most of the money would be invested in reconstruction. Another crisis would be compounded by the lack of building materials, equipment, tools and machinery, as well as the labor needed to carry out the work.

The post-war period is not only characterized by crises that must be solved for the resurgence of the economies of the destroyed countries. However, the shortage of materials would be an obstacle caused by the practically inexistent production of the steel industry in the different countries that participated in the war. Added to this, the crisis of materials started in the beginning of the Cold War and the Korean War, which would mean that the little steel that was produced in most countries would once again be monopolized by the armaments industry and the available steel for the construction industry would be very little.

2.3.1 The new materials

The shortage of materials such as steel for the construction of the structures needed in the process of rebuilding the post-war world, as well as the start of the two new wars, led to the emergence of new materials to replace steel. This induced the birth of a new architecture.

Some new materials emerged as prototypes or solutions for the necessary structures and buildings. The ones that stand out are a new aluminum alloys, plywood, plastics and even cardboard.

2.3.1.1 Wood

Wood is one of the oldest structural materials which is used to solve a structure working by tension. It is also a light material unlike other old structural materials such as stone or mud. In the post-war world, the production processes of wood change, its mechanical properties improved, and a variety of products emerged that can be used in the construction of structures. The cellulose obtained from organic materials begins to become raw material for the production of new materials that at the same time minimize maintenance costs. Plywood and wood laminates are materials that have evolved in this stage and are still used today. The mixtures of resins with wood produce a material that improves the tension properties like casein glue and phenol-formaldehyde resin glue. The manufacturing of Plywood and wood laminates cost was 35% cheaper than solid wood [47, 48]. Another advantage of lamination was that it provided greater freedom in converting trees into structural elements without limiting the size [48].

The advantages of building the structures with wood after the end of the war are varied. It is easy to obtain. As it is a renewable material, its production is easy with decreased use of labor for manufacture, assembly of the structures and less construction times, among others. Thus, in the second half of the 20th century, it became one of the favorite materials for the construction of any type of structure, especially lightweight structures.

The grid structures made of wood in the second half of the 20th century show greater flexibility in the geometric proposals and diversity of grids and meshes. More complex forms, and in some cases, structures with large spans are proposed in this way. The use of wood for the building of large-span structures would be limited. Their construction were mainly in countries such as the United States and Japan. An example is the dome for the sports field of the Montana State College of Wilson and Berg for 12,500 spectators in the year 1957. The roof had 28 m of height and a diameter of 90 m, wood arch ribs with a section of 7 in by 16 ¼ in were used for its construction [49].

2.3.1.2 Aluminum

Aluminum was apparently first produced in the mid-19th century, although its mechanical properties were not the best. With the development of the duralumin alloy in the first half of the 20th century, a new material was obtained with the necessary properties to be used in the manufacture of structures. The duralumin had almost the same strength in tension and compression as steel [48].

In this stage, aluminum proved to be an expensive material to produce but cheap to manufacture for commercial use. Aluminum alloys showed good resistance to corrosion. Also, a good structure design with aluminum beams saved up to 50% in material weight compared to steel. The wholesale price index in 1950 of aluminum was one-third the cost of structural steel [50].

Several projects are built at this stage around the world. Those built by R. B. Fuller stand out in the United States, with the geodesic dome for the Ford Rotunda Building, being the first geodesic dome to be built in a building. The diameter of the dome was 28 m, the aluminum was the material used for its construction, weighting 8.5 tons in total. This proposal was close to 6.5 kg per square meter, compared to the 195 kg per square meter that steel structures averaged [10, 51].

In Europe, the Dome of Discovery (Figure 7) becomes an outstanding example of the use of aluminum as a structural material through extruded sections and later, the use of sheets as a roof cover. The structure diameter was 105 m, weighting 322 tons in total. In terms of structural efficiency, the Dome of Discovery was four times as efficient as the best dome of that time in the United States, 31.24 kg vs 122.06 kg per square meter [52, 53].

Figure 7: Dome of Discovery and The Skylon [54].

2.3.1.3 Steel

Steel is a material with an important shortage for the construction of structures when being monopolized by the war industry before the beginning of two new wars. Even though its use was limited in the construction of structures, its employment succeeded to obtain efficient products with equal or better mechanical properties.

Fowler and Baker [10] used the first metallic tubulars in the building of the Forth Bridge in Scotland in 1890. Aber in the second half of the 20th century, the prefabrication and modulation of the materials would influence the emergence of new sections of prefabricated and modulated tubulars in the United States and Europe.

During the 1950’s, France and England led the market in manufacturing using tubulars due to the high cost of steel in Europe, mainly with a circular section. From the 1960’s, due to the lag that the United States showed with Europe in the manufacture of materials that were easy to produce and handle, the US Steel’s National Tube Division offered a variety of rectangular and square structural tubes sections [46]. The tubes would bring advantages, such as the reduction of the weight of the structures between 30% and 40% lighter and more efficient designs, A7 and A36 carbon steel was used for their fabrication [55].

2.3.1.4 Other materials

Concrete is another material that is used in the construction of grid structures, looking to reduce the weight of the structures. There are several examples of its use in construction, such as those built in the Orvieto Hangars during World War II by Pier Luigi Nervi and the slabs of the Yale University Art Gallery by L. Kahn.

The plastic materials were shown as an option for the construction of lightweight grid structures at the end of the war and with the emergence of new techniques during post-war industrialization. R. B. Fuller experimented in the construction of geodesic domes with a diameter between 2.75 m and 6.7 m. The cost was 85 cents of dollar per square meter. Pimpl and Stenzel use the geodesic dome to build housing prototypes with plastic materials [56, 57]. In [58], Z. S. Makowski mentioned some advantages such as its high strength-to-weight ratio, even higher than steel, its high resistance to corrosion and its easy handling to generate shapes, making it adaptable to production lines. However, with the oil crisis in the 1970's, the manufacture and production of plastics increased its costs, causing plastics to be no longer considered as a structural material, among other disadvantages.

Cardboard and paper became structural materials for the construction of grid proposals. Once again, R. B. Fuller experimented with this material for the building of his geodesic domes. Even the first geodesic dome built by R. B. Fuller was manufactured with cardboard for the 10th Triennale in Milan in 1954, where he presented the geodesic dome to the world for the first time [59]. In the 1950's, Fuller presented a proposal for a geodesic dome of 5.5 meters in diameter to reach up to 33 meters in diameter to function as a marine shelter. The prototype was made of cardboard and covered with polyester, its weight was one-thirtieth the weight of a steel structure [60]. Today, the architect Shigeru Ban has built a variety of structures using this material.

With Frei Otto, textiles became an option in the construction of membrane-type and mesh structures. With the construction of the Music Pavilion of the Federal Garden Exhibition in Kassel, Germany in 1955 [10], Frei Otto began the construction of tensile structures using double-curved geometries that would later evolve with the study of organic forms. Another example of the use of textiles as a structural material would be Peter Testa's Carbon Tower at the beginning of the 21st century, where he uses textiles reinforced with carbon fibers to solve the structure of his proposal.

2.3.2 The new structural systems

Faced with the need to optimize processes and make the consumption of materials more efficient, new structural systems that are totally industrialized, prefabricated and modulated have emerged around the world, finding their main use in spatial and laminar structures, using different grids, principally the triangular grid. A large number of patents for nodes would arise mainly in Japan, United States and various European countries.

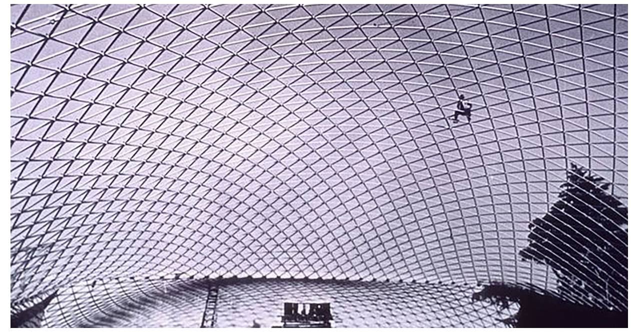

Figure 8: Free-form grid shell by Francisco Castaño. Source: Geometrica.

The prefabricated nodes combined with the mentioned new tubular sections would complement each other since World War II until today. This symbiosis would be the most important transformation of this stage, mainly in the construction of covers and roofs of diverse geometries. An example of the geometric evolution with this transformation is the construction of the first free form grid shell by Francisco Castaño (Figure 8) for an auditorium in Toluca, Mexico in 1968. F. Castaño used the Triodetic system, manufactured in aluminum and patented by A. Fentiman in Canada.

However, the first prefabricated node marketed in the world is the MERO node, manufactured by M. Mengeringhausen in 1942. This system would open the market in the prefabrication of space structures. At present, the company has a diversity of alternatives of the node that allows giving solutions to a variety of structures and geometries.

Diverse prefabricated systems would result from the commercialization of the Mero node in 1942. These include the Mobilar system by K. Wachsmann, the Unistrut system by C. W. Attwood, the Octet Truss system by R. B. Fuller and the SDC system by Stéphane du Château, among others [6, 18]. The diversity of prefabricated node systems would allow for the solution of practically any challenge posed by the designer.

In addition, in the 1950's, high strength bolts would emerge, replacing rivets and normal strength bolts. The factors that influenced their development were varied. First, there was a scarcity of machinery in countries in crisis such as riveting machines in addition to the difficulty of finding skilled labor that could handle them. So it was sought and achieved to develop a system of greater resistance that could be mounted without the need for skilled labor, as were the high resistance bolts. Finally, another factor would be the time, since the time needed to install a rivet is greater than the time needed to install a bolt, for example, in this stage a two men team could assemble 400 bolts in one day and a four men team could assemble 300 rivets in one day.

Although the cost of high-strength bolts was 5 times higher than rivets, the savings in labor, equipment and time resulted in a savings of $4 per ton at this time. On the other hand, the strength of high-strength bolts was three times that of ordinary bolts; a high-strength bolt was manufactured from medium carbon 1038 steel and SAE Grade 5, achieving a tensile strength of approximately 125,000 psi with bolt sizes above 5/8 in. [61, 62].

At the same time, the introduction of welding applied by means of an electric arc would decrease time and costs caused by a development that suffers this joining methodology, besides the greater use of steel for the construction of structures of different dimensions. With the introduction of the new methodology of welding application, a tendency towards the manufacture of lighter elements begins to develop and to have less quantity of material capable of resisting the forces involved in the structures more efficiently. Steel saving in welded structures could be around 20% to 30% as compared with conventional joining [48].

2.3.3 High-rise buildings and large span roofs

With the culmination of World War II and the emergence of a new world order and the need for the emergence of an organization that tries to guarantee peace, the United Nations grew the need to have its own facilities. A high-rise building was being built again after it was stopped in the 1930's because of the Great Depression.

Therefore, the grid systems were also used in the construction of high buildings by showing greater efficiency in the consumption of materials and greater structural safety, mainly against the impacts of nature. Examples of high-rise buildings are those designed and built by SOM at the Alcoa Building in San Francisco and the John Hancock Center in Chicago; consolidating the use of the diagonal grid in high-rise buildings.

The John Hancock Center used for the first time the braced structural tube, built in steel and reaching a height of 330 meters. The truncated pyramid shape of the tower lightens the structure and together with the structural system achieves a weight of no more than 135 kg per square meter, most tall buildings of the time required between 200 and 225 kg per square meter [63, 64].

At the same time, the need arose to cover the great sports buildings with laminar and/or spatial grid structures. Events such as the Olympics or the Soccer World Cup became a technological showcase for the construction industry, as were the Universal Exhibitions since the construction of the Crystal Palace in 1851 in London. The Sports Palace in Mexico City designed by Candela, Peyrí and Castañeda is an example of the importance of the grid to solve long spans. The dome of the Sports Palace reaches a maximum span of 193 m. The structure is made up of 22 trusses with a height of 5 meters each one [65].

2.3.4 Arguments of the historical trajectory of the third stage

This stage includes various technological advances, new materials, and new constructive and structural processes. However, with the appearance of great architects and engineers, innovative designs emerged, ahead of their time and in some cases because of their risky proposals without the fear of not being recognized.

With the tubular construction and the new welding techniques that emerged, result in structural weight savings of up to 60%. Another example, is the invention of the German pipe cutting machine, savings of up to 90% in the cost of cutting process. [46].

Scales are increased, geometries are transformed and processes are made more efficient. Challenges appeared in various historical moments that modified the construction industry, mainly due to the crises that would arise. By analyzing each of the built grid proposals and their builders or designers, a better understanding of the challenges that emerged and how they were solved is achieved.

At this stage, structural analysis took a big step forward with the introduction of computers for mathematical calculations and stress analysis. An example is the one carried out in the Sport Palace of Mexico City, where Eng. Julio Damy uses software with a series of subroutines for the calculations (TRANS, SUMAR, MULT, MATINV) with the interest that they could be used for arrays of any dimension [66].

2.4 The fourth stage: The digital revolution

At the end of the 20th century, a process of transformation began. A geometrically risky architectural style started on a par with the technological advances corresponding to the digital tools used for its design. Deconstructivism and the digital revolution in architectural design would revolutionize the way a building is designed and built. Although it is important to take into account that the development of AutoCAD software in 1981 and ArchiCAD in 1987 (software widely used for architectural design until today), it is in the last decade of the 20th century when a process of transformation began.

In 1992, Frank Gehry designed and built The Fish in the Olympic Village in Barcelona, a proposal designed, manufactured and built digitally for the first time in a software that was not designed for the use of architectural projects, CATIA software, which showed a gap in the digital tools used in architectural design [67].

The greatest complexity of this methodology and the new emerging architectural style would occur when Frank Gehry himself would design the Walt Disney Concert Hall in Los Angeles, California in the same year. The free forms that Gehry designed in the proposal for the building raised the challenge of its construction. The problem caused it to be at risk of not being built.

The solution would be presented in the same year (1992) with its culmination in 1997. The Guggenheim Museum of Bilbao would provide answers to the problems posed by the construction of free forms in the construction of a building. F. Gehry and his team use the CATIA software (used mainly for the design and construction of ships) and 3D scanning as methods to complement their methodology. The steel grid framing structure provides strength and lightness for the complex geometry. "The system has the ability to span long column-free distances due to the overall depth of the structure and low self-weight of the frame structure itself; while at the same time having significant stiffness against lateral loads due to the ever-present curvature of the various geometries" [68]. As with The Fish, F. Gehry designed, manufactured, and built the building with the support of digital tools [67].

With the Digital Revolution in architectural design, the transformations in methodologies and processes are emerging as the years go by. The transformations that stand out are:

NURBS (Non-uniform rational B-Splines), genetics and free forms. The complexity of emerging forms without any geometric order represents the development of new methodologies and design techniques. OpenNURBS software emerged in the last decade of the 20th century as a design solution, later changing its name to Rhinoceros.

Parameters, algorithms and performance optimization. With the evolution of geometries and their complexity to design and build them, the digital tools also have to be transformed to obtain the highest possible efficiency in the solution methodologies and their subsequent construction. With the design using parameters and its optimization, the projects begin to be more responsible with their environment and the users. The Grasshopper add-on of Rhinoceros was released in 2007, whit the software the era of scripting begins.

Materiality is transformed with the birth of new geometries and therefore more risky structures and the need for new materials and construction processes begins to emerge.

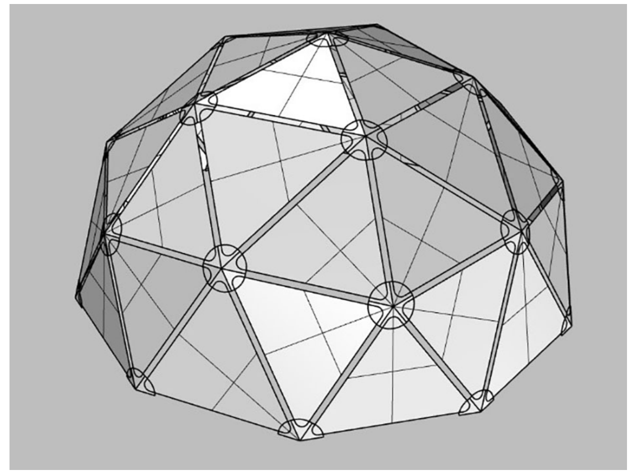

Figure 9: 3D model of a geodesic dome using parametric tools.

2.4.1 Free-form, digital fabrication and the grid

At the end of the 20th century and the beginning of the 21st century, with the deconstructivist movement led by F. Gehry, Z. Hadid, M. Fuksas and D. Libeskind among others, complex geometries and free forms emerge and the complexity to bring them to reality increases. In most cases, the grid becomes an ideal solution in the transformation process by triangulating the surfaces, an example of which is the constructive and structural system used in the building of the Guggenheim Museum in Bilbao, where a triangular grid is used for the solution of the structure and the complex surfaces that were generated.

Given the complexity of the geometries and their design process, the problem is transferred to their manufacture and subsequent assembly. The solution is digital manufacturing, which allows for greater efficiency and accuracy in the production of the elements. The accuracy in the manufacture of the elements becomes a necessity given the complexity.

With the new challenges that arise, the grid becomes a solution in this transformation process. The geometries can be deployed in the plane, which allows greater simplicity in their manufacture. The most used grids are the triangular, hexagonal and orthogonal ones in different formats of structural behavior, either laminar or spatial.

Examples of this process of using the grid are very varied in the structural solutions. From those already mentioned are by F. Gehry, The Guggenheim Museum in Bilbao and the Walt Disney Concert Hall as well as the envelopes or roofs with complex geometries or free forms designed by Z. Hadid such as the Heidar Aliyev Centre and those designed by M. Fuksas at My Zeil in Frankfurt and the New Trade Centre in Milan. Finally, there are N. Foster's high-rise buildings such as the Building 30 St Mary Axe in London and the Hearst Tower in Chicago.

The structural solution of the complex geometry of the Heidar Aliyev Centre stands out. For its construction employed a classic method first used in 1942 and presented in section 2.3.2, the MERO STK node is chosen for its flexibility to connect tubular elements in different directions, developing a double-layered grid structure. An optimization strategy was applied to find the ideal geometry for its fabrication and subsequent construction [69].

2.4.2 Parametric Design and the use of algorithm

Regardless of the debate among experts determining if Parametricism is a design methodology or a new architectural movement as mentioned by its creator P. Schumacher, Schumacher bases his statements on the variety of geometries that can be proposed for various uses. Whether architectural, fashion design, furniture, etc., the great benefits of its use is to solve various design projects and the possibility of complementing the methodology or style with other platforms.

Even though parametric design is born in this century, it can be considered as having its origin in the works of the architect A. Gaudi. Nevertheless, its main evolution is during the 21st century, mainly with the release of Grasshopper and the beginning of scripting in architectural design. The move to a new design methodology would pose a stage of adaptation by the designer and constant updating.

There are several advantages of the use of scripting in design, but one of the most applied in the form-finding and structural optimization, which allows through generative algorithms to obtain the desired geometry by parameters optimization. In the structures or reticular envelopes, the optimization of the structural behavior is to make the consumption of materials, decrease of the deformations or diminishing the axial loads more efficient by allowing obtaining the form that adapts to a necessity by the use of the grid.

Although it is important to mention that the use of parametric design will not be exclusive to complex geometries, form finding or optimization, its use is currently to solve or make more efficient any design process.

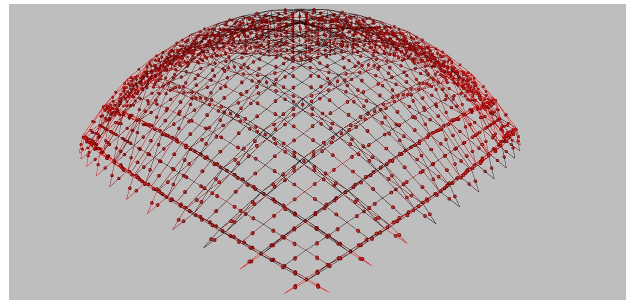

Figure 10: 3D model of the structure of the Sports Palace of Mexico City using parametric tools.

2.4.3 Materiality and construction processes

With the evolution of geometry, transformation processes in the construction of new buildings became necessary. The adaptation of the grid to the new challenges was achieved through the processes and techniques of construction. One of them was already mentioned. The Digital Revolution was consolidated by the use of digital platforms for the design, manufacture and assembly, but another emerged from the optimization of existing methodologies such as the use of grids or prefabricated nodes which were adapted to the new needs, some were even developed only for the specific project.

Materials evolve by obtaining better mechanical properties to support the stresses to which the grids are used in free forms or complex geometries that are subjected. Steel is an example of moving into a new era by optimizing the performance that offers to structures. Aluminum, although its costs are high, remains a light and resistant material that allows the generation of any geometry. Wood improves the properties of resins, obtaining better performance in composites that are manufactured as fire resistance or bigger sections.

2.4.4 Argument of the historical trajectory of the fourth stage

The fourth stage is marked by the introduction of digital tools to architecture and the building industry, from the emergence of new digital tools for 3D modeling, manufacturing and subsequent construction, to the application of artificial intelligence in the project design process. The design and manufacture of elements that use the grid as an architectural element is not different, but its use is mainly focused on the modeling of structures.

The application of algorithms has been the greatest advance in the simulation and optimization of structures,

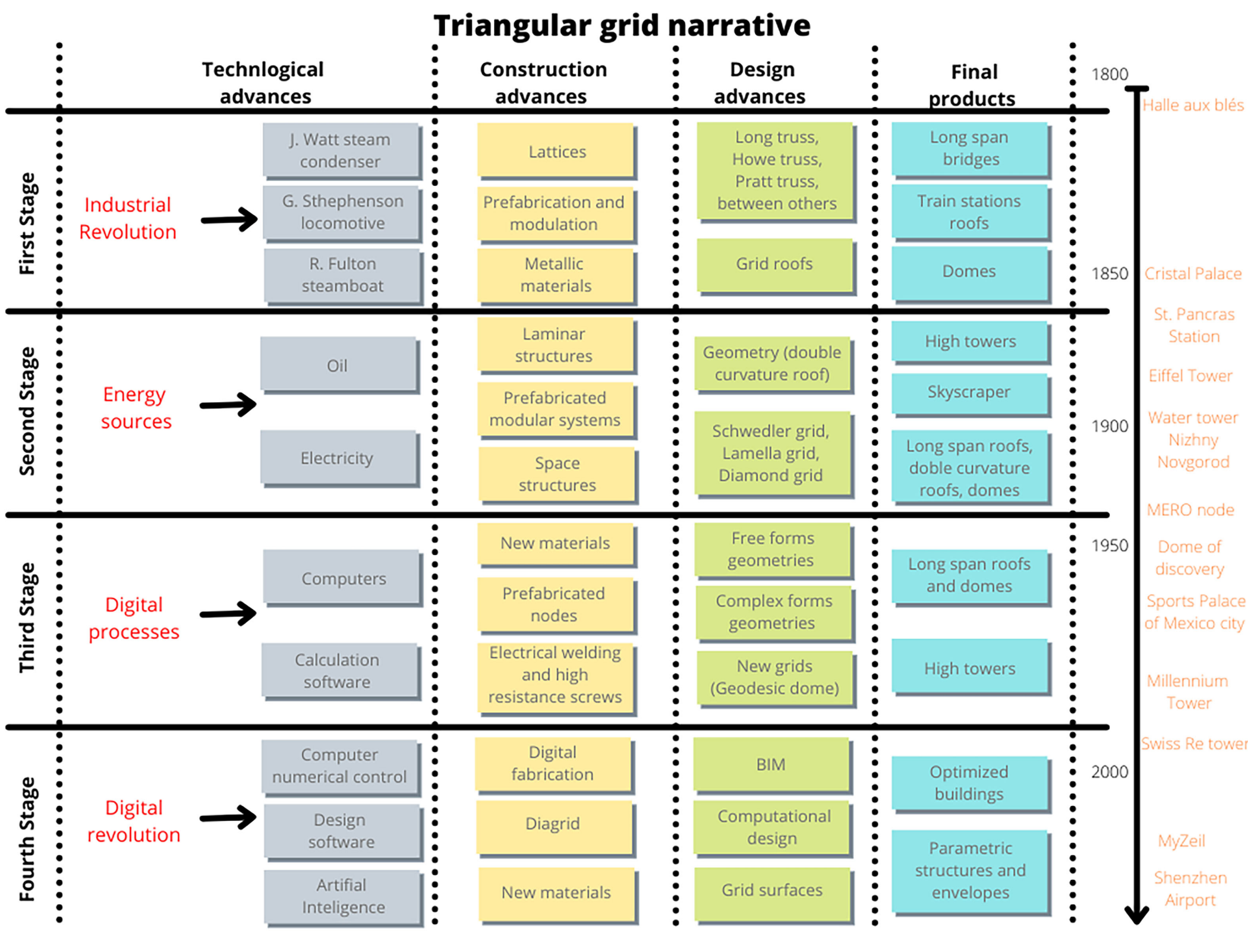

Figure 11: 3D model of the structure of the Sports Palace of Mexico City using parametric tools.

being a powerful methodology for the creation of free forms of easy construction and strong.

In this stage, digital structural analysis tools evolve from the previous stage. The application of form-finding methodologies through machine learning using optimization algorithms such as Galapagos or Wallacei X in Grasshopper. Parametric tools such as Karamba 3D, Millipede or Alpaca 4D among others are based on finite element methodologies [70].

The main objective of applying form-finding in structures is to design a standardized, economical, lightweight and structurally stable object, where the result is to find an optimal solution to the pre-defined parameters [70].

The future of the triangular grid will continue to be promising. The new digital tools have allowed designers to risk more in their proposals. These are unlimited where the only limitation will be the imagination of the designer and the cleverness of the builder.

3 The grid and the transformation of the building.

The grid has suffered a process of transformation in buildings during the last 200 years, the most notable being the typology of the grid, geometry and construction processes. These transformations allowed us to observe how structures evolve from linear elements to laminar structures, to finally, become spatial structures in different types of buildings, both horizontal and vertical.

The diverse transformations of the grid in different parameters have allowed corroborating that it is a necessary methodology for the solution of structures and envelopes of buildings. In the beginning of the 19th century, with the Industrial Revolution, the development of bridge grid trusses is characterized as the origin of the transformations until the 21st century. At the end of the 19th century, the engineer V. Shukhov characterized his structures by a geometric transformation and a lightness in their construction, with the curve being an essential tool for stabilization.

In the first years of the 20th century, A. G. Bell transformed the structures from the plane to the space, through an innovative modular and prefabricated constructive process, its consolidation being in the post-war world. At this time, Eng. Felix Samuely, the designer of the structure The Skylon (Figure 6) for the British exhibition in 1951, mentioned the importance of the transformation from the plane to the space as one of the transcendental transformations of the century [71]. Finally, the last transformation would be given by the so-called Digital Revolution and the use of new technologies to optimize the processes from its design to its construction, achieving the goal where any geometry can be built.

4 Conclusion

The triangular grid has been shown for over 200 years as a tool, as a methodology and as a process to provide solutions to any type and morphology of a building, where each of the transformation processes has been linked to a historical moment that influences its evolution and optimization process through the emergence of new materials, construction processes and technologies.

Various projects with different geometries were solved with the use of the grid, such as Euclidean and non-Euclidean geometries, using materials with different mechanical properties such as cardboard and even the most resistant ones such as steel, from the use of handmade and industrialized manufacturing methodologies to digital manufacturing technologies.

The past and present show a future without time limits for the use of the grid in architecture. Its use can be considered timeless as the only true limitation in this case would be the architect's mind. However, the historical moments of humanity will be those that will really dictate the future of the grid in architecture.

The understanding of the grid from its origin allows students to understand the generation of diverse forms and their structural support from a geometric perspective. Thus, the grid becomes a tool for the design of any building applied to a design workshop. The diversity of constructive and structural systems that apply the grid facilitate their application of architectural proposals.

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