

Research Article

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Structural assessment of St. Charles hyperbolic paraboloid roof**

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Abstract: At the time of completion in 1961, the roof of St. Charles Church became the largest unbalanced hyperbolic paraboloid structure in the United States and the only shell structure in Spokane, WA. Situated on an 8-acre site on the north side of the city, St. Charles is a modernist structure designed through partnership of Funk, Molander & Johnson engineers, architect William C. James and in consultation with Professor T.Y. Lin of the Structural Engineering Laboratory at the University of California, Berkeley. This asymmetric structure spans over 33.5 m (110 ft) and utilizes folded edge beams that taper from 1067 mm (42 in) at the base to a 76.2 mm (3 in) thickness at the topmost edge using regular strength reinforcing steel and concrete load carrying components. The novelty of the pre-stressed shell structure serves both architectural and structural design criteria by delivering a large, uninterrupted interior sanctuary space in materially and economically efficient manner.

This structural assessment summarizes the roof's historic design and construction according to the original construction documents, newspaper reports and historic photographs. The FEA is completed using UBC 1955 design loads and ACI 334 Concrete Shell Structures provisions.

Keywords: concrete design, thin shell, historic structure, case study, hyperbolic paraboloid

1 Introduction

The abundance of shell structures that were designed and constructed in the post-war period (circa-1950 to 1970) combined with the proliferation of analytical as well as subsequent, incipient finite element analysis (FEA) methods resulted in a high-water mark for the ubiquity of thin-shelled structures in the United States [1]. In subsequent years, engineers have been rediscovering the efficiency and elegance associated with thin shell structures that allow designers to deviate from linear systems while minimizing material, construction and maintenance costs. However, while modern FEA and optimization methods result in increasingly efficient designs, some of the earliest shells retain their importance both as benchmark structures and due to the analytical and modelling methods used to develop them [1–4]. More specifically, the St Charles roof provides a significant structural legacy in the development and evolution of these structures in the U.S. In light of these considerations, the purpose of this research is two-fold: 1) provide detailed historical information about the design and construction of a notable shell structure to allow others to study and recreate the design and 2) complete finite element structural analysis of the idealized structure from construction documents as a benchmark for ongoing studies of in-situ stresses of 3D scans of the actual built structure.

While shell structures were designed and constructed in Europe as early as the 1930's (as well as Gaudi's antecedent projects in the early 1900's), shell structures did not become more widespread among engineers in North America until the 1950's (note that Candela's first thin shell was built in Mexico City in 1951) [1]. For about a decade thereafter, the number of shell structures grew rapidly, culminating in the World Conference on Shell Structures hosted in San Francisco in 1962 [1, 5]. During this early period of shell structures in the United States, there was significant work by a series of well-known engineers (Billington, Bradshaw, Candela, Christiansen, Isler, Nervi, Tedesko, Torroja *et al.*) to develop analytical and modeling methods for design of a range of domes, folded plates, thin shells and hyperbolic paraboloids [2, 4, 6–10]. However, the closed form analytical solutions to these problems increase in com-

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plexity quickly as the surfaces deviate from simple geometry as described by Candela [7–9]. The resulting body of knowledge circa 1950 relied on simplifications of the problems (uniform loads, constant thickness shells and specific types of shell geometries) and scale models to verify designs prior to finite element analysis or finite difference methods which became the preferred method of solution in subsequent years to the present [2]. For these reasons, folded plates and “improper” shells were introduced both earlier and with greater adoption than proper shells [11, 12].

Hyperbolic Paraboloids are a specific class of laminar (*i.e.*, thin shell structures with thickness to radius of 1:50 or greater) structures with double curvature (*i.e.*, proper shells) which allows the structure to carry forces either solely or predominantly through membrane forces (uniform tension, compression and shear through the thickness of the shell) [7–9]. The roof of St. Charles Church is an asymmetric, hyperbolic paraboloid of varying shell thickness with thickened edge beams received into two abutments that are post-tensioned together through the floor slab as shown in Figure 1. This research presents 1) historical structural and construction assessment, 2) finite element analysis of the roof per construction documents 3) comparison to simplified analytical calculations for the hyperbolic paraboloid roof of St. Charles.

2 Methods

2.1 Historic assessment

The parish of St. Charles in Washington State was formed through a merger in 1950 and from the beginning of the church, the priest, Fr. Oakley O'Connor, felt that “one should build the best one can afford.” As a result, the 1950's were a decade of construction for the parish that culminated in the acceptance of a \$324,832 bid for a new sanctuary in July of 1959. The resulting hyperbolic paraboloid shell structure shown in Figure 1 is a 76.2 mm (3 in) thick shell that thickens to 1067 mm (42 in) at the base while spanning over 33.5 m (110 ft). Construction commenced the same month as the bid and was completed over the course of 24 months with the church dedication being held on October 25, 1961 [13–15].

Fr. O'Connor had been inspired by past designs for the World Expo and was convinced that the application of a thin shell would be ideal for a new church as it would eliminate interior columns, provide desirable acoustics and focus worship by the providing unobstructed views of the altar from all points in the sanctuary. The goal was to develop



(a)



(b)

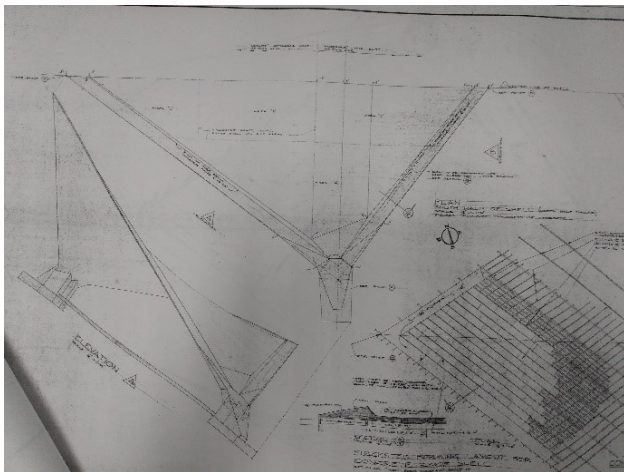
Figure 1: a) Hypar Roof in 1960 and b) Roof in 2020

a church that was “simple, very unique, and daring” [15]. From the inside of the finished design, a series of floating clerestory stained-glass windows are located just below the roofline emphasize the reality that the roof cantilevers over the pillarless interior while being supported at the two foundations on the north and south sides of the building as shown in Figure 2. The roof itself is a prestressed asymmetrical hyperbolic paraboloid shell with tapered edge beams as shown in Figure 3. The front (east) elevation overhangs a brick clad cylindrical form and is flanked by two full height structural masonry walls (shown below) that provide the shell with stability. Adjacent to the entry on each side are double doors with scenes on enameled panels, topped by tall, multi-light windows [15].

At the time, the design was innovative for bringing a novel structural system (hyperbolic paraboloid shell) that had been used in Russia, Europe and New Zealand to the Inland Northwest. In fact, while other shells had been de-



(a)



(b)

Figure 2: a) Finished shell showing 2 foundation connections and b) Isometric details of the shell

signed and were under construction, there was no other structure like this in the US at the time [15]. For context, Eero Saarinen's well-known TWA Building at Idlewild Airport (now JFK International Airport) in New York City, had begun construction only one month prior to St. Charles. Additionally, the only hyperbolic paraboloids that had been constructed at that time in the US were significantly smaller, including John Christiansen's upturned umbrella hyperbolic paraboloids for the nearby town of Wenatchee, WA, [1] which could be designed with relatively straightforward equations [16]. As a result, the concept of a thin shell concrete roof supported by buttresses, both for support and for pre-stressing, was so novel that the design team had to build a scale model for construction firms to be willing to bid the St. Charles project [15]. The load path for the shell structure consists of the prestressed shell and tapered

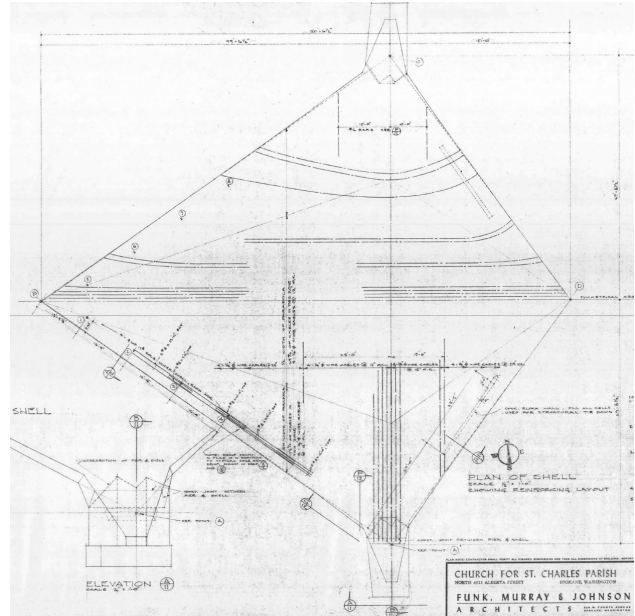
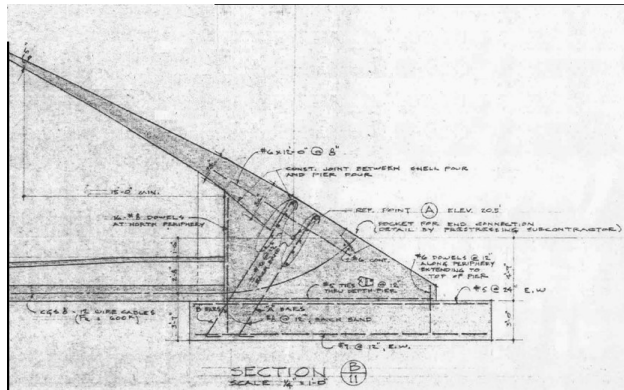


Figure 3: Plan drawing of the shell roof

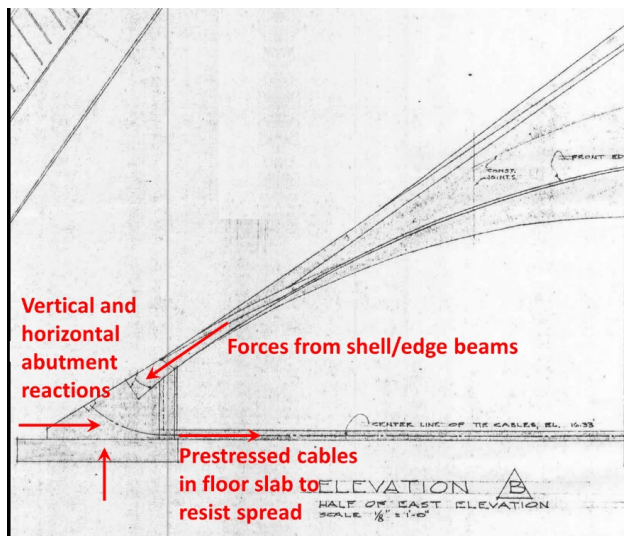
edge beams that were supported by two concrete abutments at the two low corners of the shell. An additional two full-height masonry walls on the short sides stabilized the structure. Figure 4 shows the two abutments were connected through the floor slab with 8 – 12 wire cables having $F_t = 2667 \text{ kN}$ (600 kip) that carried the outward thrust developed by the roof.

The prestress in the shell consisted of 4 – 6.35 mm (0.25 in) cables spaced from 305 mm (12 in) to 610 mm (24 in) on center in the field of the shell. The edge beams had additional prestressing 4 – 6.35 mm (0.25 in) cables as shown in Figure 5. Specific details and construction sequencing was provided by the contractors, including suggested methodology per construction notes.

The structural engineering for the church was completed by a Spokane firm Funk, Murray and Johnson with the structural consultant of Professor T.Y. Lin of the Structural Engineering Laboratory at the University of California, Berkeley. Lin is reported to have held that the mixture of reinforced steel with concrete, “when properly ‘folded’ made beams seven times stronger than normal.” This claim is likely associated with the pre-stressing used to balance the load [5] as well as a reference to the shell being curved with edge beams of varying thickness (although not technically a folded structure, [8, 9]). The roof is an asymmetric (1 side rises higher than the other) hyperbolic paraboloid with a folded edge beam. As typical with hyperbolic shell structures, the geometry provides most of the roof's strength through development of membrane stresses, not the amount or type of materials put into



(a)



(b)

Figure 4: a) Buttressed support detail and b) abutment and slab reactions

it. Consequently, the design used 84 m^3 (110 yd^3) 25 MPa (3500 psi) concrete and 228 MPa (33 ksi) reinforcing steel per ASTM A305 and 4 – 6.35 mm (0.25 in) wire with rupture strength, $F_r = 2667 \text{ kN}$ (600 kip) and tensile strength, $F_t = 1067 \text{ kN}$ (240 kip). However, note that the shell itself is prestressed and there are shear and moment forces present in the edge beams.

Based on available documentation, the initial design analysis was completed using simplified method treating the folded edge beams separately from the shell and looking at representative cuts of arches throughout the structure which provided the form diagram for the structure as shown in Figure 6 and 7, respectively.

The shear stresses were obtained using simplified projection methods for hyperbolic paraboloids bounded by straight edges and under uniform loads. The prestressing in the shell was determined using load balancing of the crit-

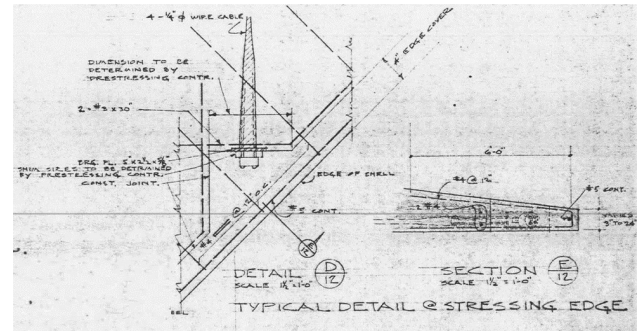


Figure 5: Typical Detail at Stressing Edge

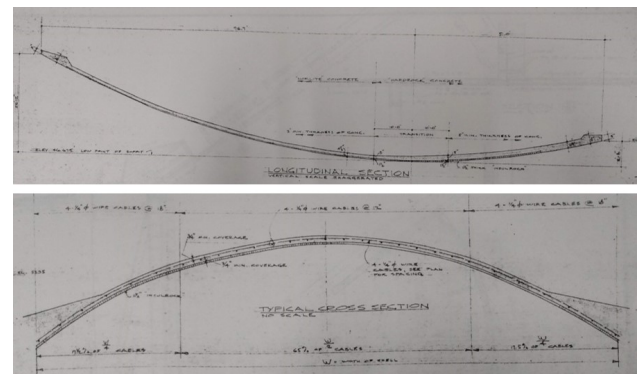


Figure 6: Orthogonal longitudinal shell sections designed as arches

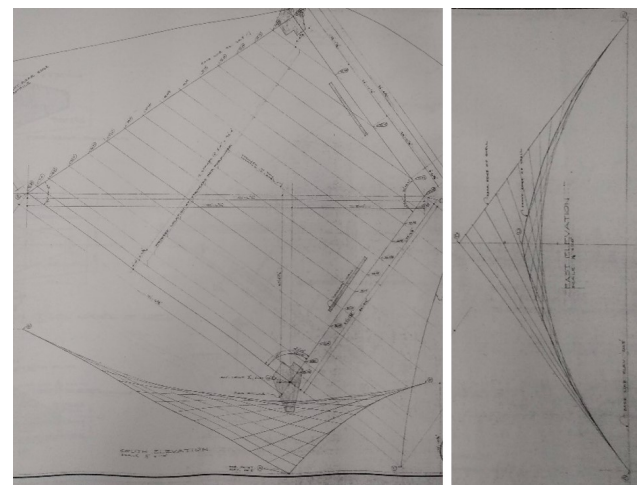


Figure 7: Shell form diagrams

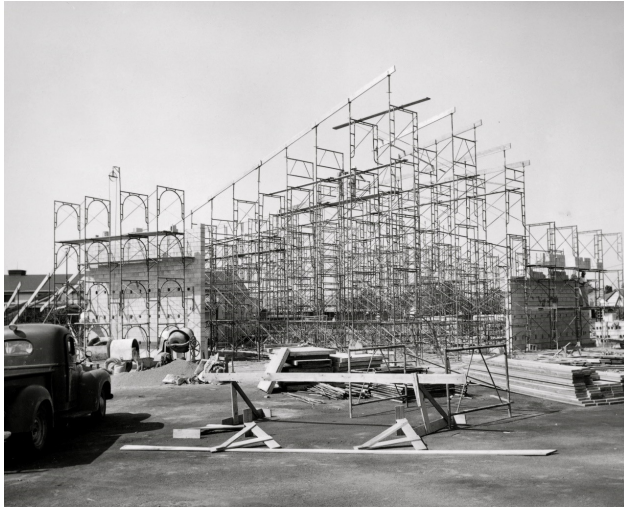
ical (i.e., dead) load. The governing loads are determined per UBC 1955 and the SEAW historical snow loads for concrete shell design [17–20].

The construction began in summer of 1959 with excavation of the basement followed by basement footings, walls and slabs with construction moving on to the superstructure by late summer. The phasing for the shell consisted of 1) pour foundation pad and embed the prestressing cables,

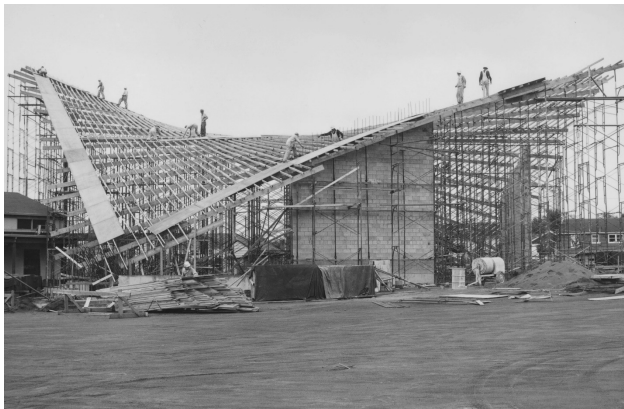
2) construct the 2 full height masonry walls 3) erect scaffolding and formwork for the shell, 4) construct shell and

prestress before removal of formwork, 5) pour abutment piers, 6) stress foundation cables and, finally, 7) remove formwork for the roof structure as shown in Figure 8.

The completed structure was also proof loaded in a whimsical albeit unsafe fashion as shown in Figure 9.



(a)



(b)



(c)

Figure 8: a) Scaffolding b) Formwork and c) Concrete shell



Figure 9: Scaffolding and progression of construction

2.2 Analytical method

Around the time of St. Charles design development, in 1959, there were limited analytical methods available for the design of hyperbolic paraboloid thin shells, notably, the hyperbolic paraboloid roofs shell designs by Aïmond [21] as well as the methods of Candela [7] and Martin [22]. However, both of these methods are based on a pure membrane theory have the following limitations with respect to the St. Charles roof as they both: 1) ignore the impact of the edge beams, choosing instead to assume that forces are resolved via shear stresses, 2) apply uniform loads over the rectangular horizontal projection, 3) are intended for hyperbolic paraboloids that are linearly bounded. Obviously, the edge beams, prestressed shell, asymmetric geometry, and curved hyperbolic paraboloid geometry found in the St. Charles roof defy application of these simplified methods. In fact, these limitations have been previously addressed in the literature as Addensens *et al.*, assert that “behavior of complex curved shells cannot be accurately represented by one unifying membrane theory”. Even more recent attempts at developing closed-form analytical models of these types of complicated shells have been unfruitful [23, 24].

2.3 Finite element analysis method

The finite element analysis follows a similar procedure as that in Henriksson *et al.* [25].

Based on the drawing set, the geometry is computationally modelled and analyzed in finite element software SOFiSTiK FEA [26]. Starting with the 4 corner points (A through D in Figure 10 below), the geometry has been found by laying out straight tendons through specified structural points on the vertical planes through edge A-B or B-C. The geometrical properties of hyperbolic paraboloids (Hypar) surfaces can be described by two parabolas in opposite direction. The arch forces are resolved by an edge beam at the boundary, which for Hypar roofs are typically straight. However, two of the edges at St. Charles roof (A-D and C-D) are curved.

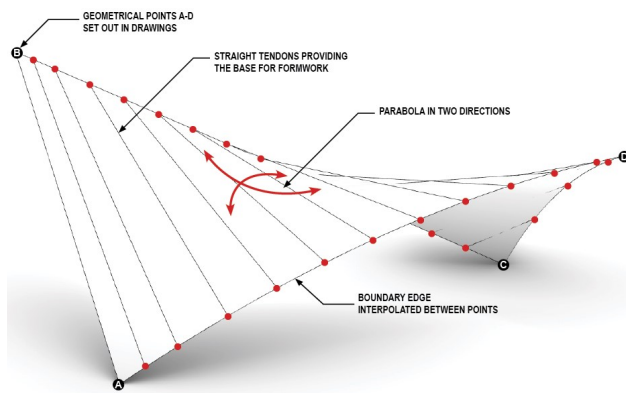


Figure 10: Overview geometry of Hyperbolic Paraboloid shell

The geometry of the shell is an important part of the analysis and the coordinates for the key points are included in Table 1 for reference. Corner points (A, B, C, D) are highlighted and the points are numbered in a counter-clockwise direction, along edges A-D, D-C and C-B.

In the finite element analysis, the concrete material properties assume a self-weight of 25.0 kN/m^3 and Youngs Modulus of 31.5 MPa (C25/30). B220 steel reinforcement with yield strength of 220 MPa has been assumed. The pre-stress of the reinforcing steel value is 1138 MPa (165 ksi), which accounts for a 103 MPa (15 ksi) loss from initial stress. This results in 470 kN/m prestressing force of the shell in direction B-D (points) and 370 kN/m in A-C direction.

Non-linear analysis with quadrilateral elements have been used to incorporate pre-stressing.

As a first step to evaluate the geometry, three different cases are studied and described in Table 2 and shown in Figure 11. All three models described below are studied as single layer mesh, representing the middle of the shell, with uniform thickness (FE1 and FE2) as well as varying thickness based on the structural drawings (FE3).

Furthermore, a model (FE4) is a studied as a refinement of FE3 but with the addition of supports at the masonry walls at the back-span of the roof as well as pre-stress as shown in Figure 12.

The thickness distribution of model FE3 and FE4 is approximated based on the structural drawings and described in Figure 13.

Table 1: Coordinates of shell

Point	X (mm)	Y (mm)	Z (mm)	Point	X (mm)	Y (mm)	Z (mm)
A	0	0	6248	C	0	42491	6248
1	720	984	6979	16	-2426	40792	7524
2	1631	2230	7862	17	-4552	39304	8641
3	3072	4198	9168	18	-6678	37815	9759
4	4512	6166	10371	19	-8803	36327	10876
5	5952	8135	11476	20	-10929	34839	11994
6	7392	10103	12486	21	-13055	33350	13111
7	8832	12071	13397	22	-15180	31862	14228
8	10272	14040	14210	23	-17306	30373	15346
9	11712	16008	14919	24	-19431	28885	16463
10	13153	17976	15519	25	-21557	27397	17581
11	14593	19945	16004	26	-23683	25908	18698
D	15544	21246	16261	27	-25808	24420	19815
12	14030	23316	15828	28	-27934	22932	20933
13	9564	29419	13823	29	-29279	21990	21640
14	5098	35523	10833	B	-30342	21246	22199

Table 2: Description of FEA models

Model	Description
Model FE1	Conceptualized asymmetric Hypar shell with straight edges and 150 mm constant thickness and supports at Hypar low points.
Model FE2	Constant 150 mm thickness shell with curved edge (based on construction drawings)
Model FE3	Model FE2 with varied thickness of shell (600-80 mm) with edge beam (average 200 mm)
Model FE4	Development of Model FE3, with tie down supports at masonry walls and concrete pre-stress.

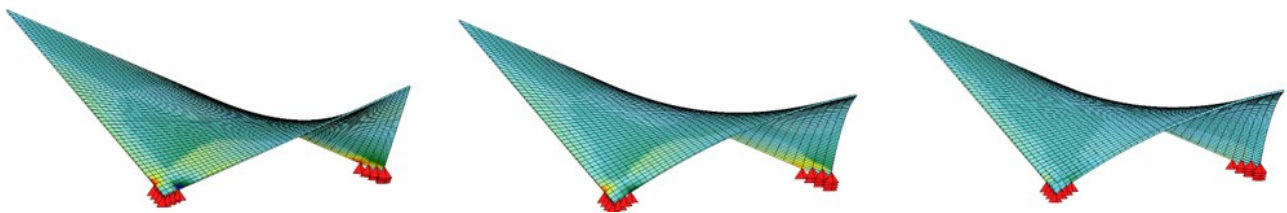


Figure 11: Overview of Finite element models FE1 (left), FE2 (middle) and FE3 (right)

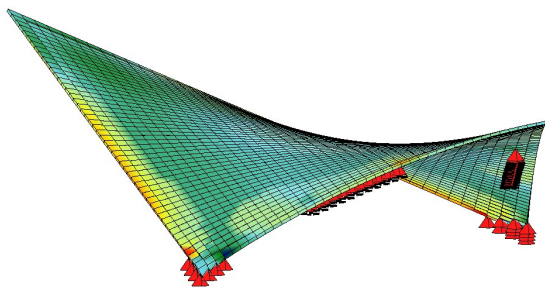


Figure 12: Finite element model FE4, with support points at masonry walls

Reinforcement, acting in the middle surface of shell, is assumed to resist principal tensile stresses. Uniform snow load of 1.44 kN/m² (30 psf) and self-weight combination is assumed to be the governing case for the shell roof, hence this case has been the focus of this study. The pre-stress is applied in two directions, according to structural drawings. In this study, time-dependent factors of the pre-stress has not been considered such as creep and shrinkage which should be studied in further research.

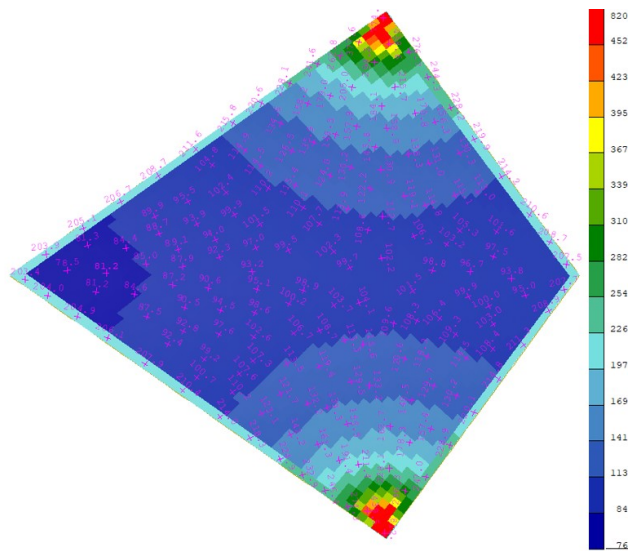


Figure 13: Thickness distribution of shell elements in model FE3 and FE4

3 Results

Studying the deflections of the three initial models under dead load, the long cantilever tip of model FE1 experience the greatest deflections, up to 600 mm. A significant decrease in deflections are noted for the curved edge options, with 330 mm and 224 mm max deflection at the tip for model FE2 and FE3, show in Figure 14 respectively.

Due to the double curvature nature of the shell, stresses are studied in the two principal directions in the top and bottom surface, shown in Figure 15. As shown in the diagrams in Figure 11, the geometry and shell thickness have a significant impact on the stress distribution. Additionally, model FE3 has a significantly increased performance in both tensile (positive values) and compressive stress (negative values). Any local stress concentrations, in particular close to the supports have been ignored as these are 1) in part associated with FEA modelling constraints compared to in-situ constructed structure and 2) to be locally resolved via the construction details as evidenced by the nearly 60-year lifespan of the structure.

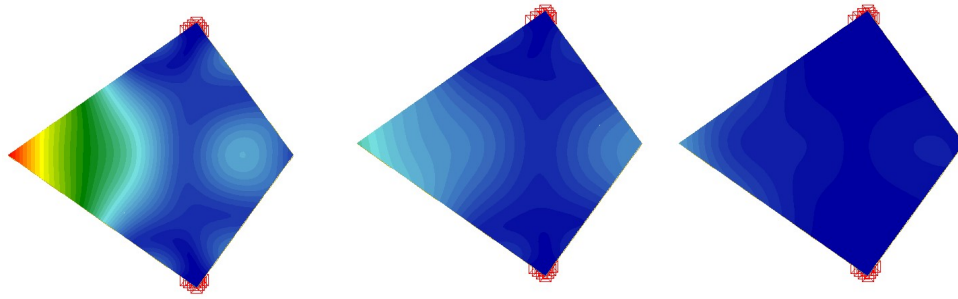


Figure 14: Deflections of three studied models Model FE1 (Left) Model FE2 (middle) and Model FE3 (right)

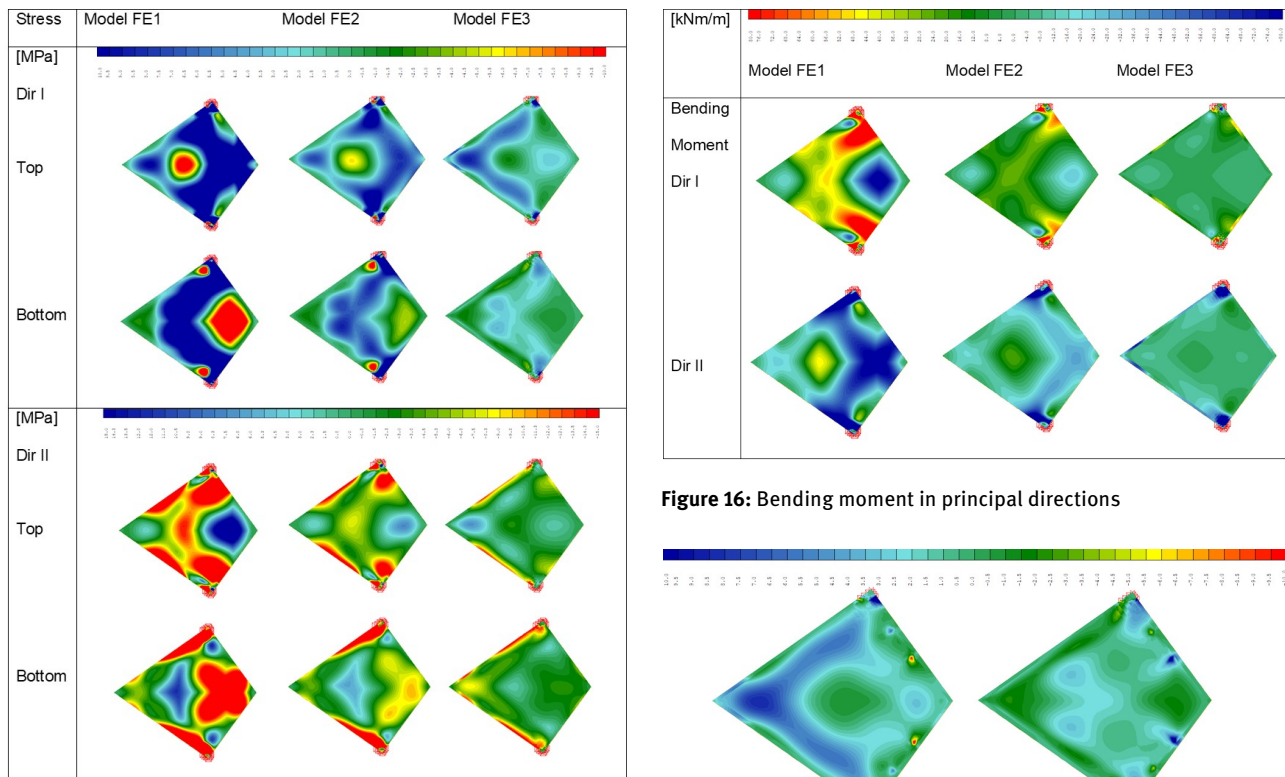


Figure 15: Principal stress in two directions, top and bottom surface

The difference in stress of top and bottom surface indicates that the shell is subject to a combination of membrane forces and bending moment, which is shown in the Figure 16.

Model FE4 is the closest representation of the actual shell roof included in this study. The stress distribution, shown in Figure 17 and 18 are similar to the stress in model FE3 in both magnitude and distribution, however with the difference that stress concentrations are evident at the tie-down supports.

The maximum principal compression found in the shell is 15 MPa (around edge beam and supports) which is well within limits of the concrete strength. Maximum princi-

Figure 16: Bending moment in principal directions

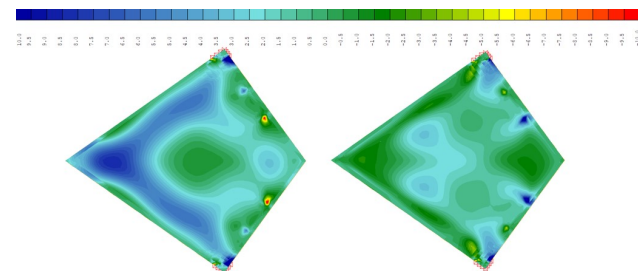


Figure 17: Principal stress in direction I of model FE4, top and bottom surface

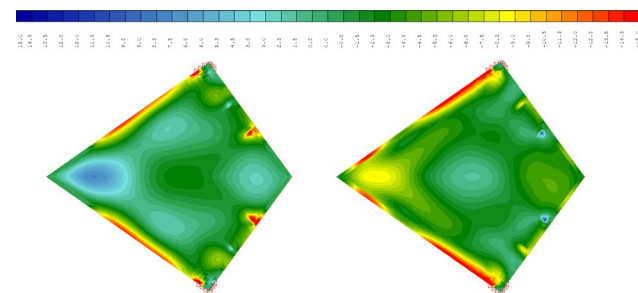


Figure 18: Principal stress in direction II of model FE4, top and bottom surface

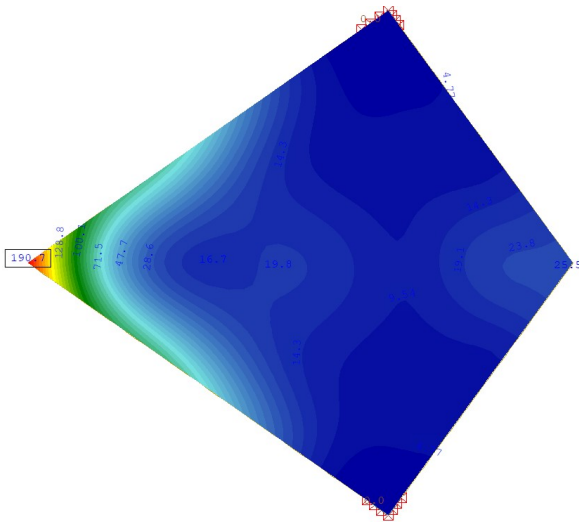


Figure 19: Deflection of FE4 model

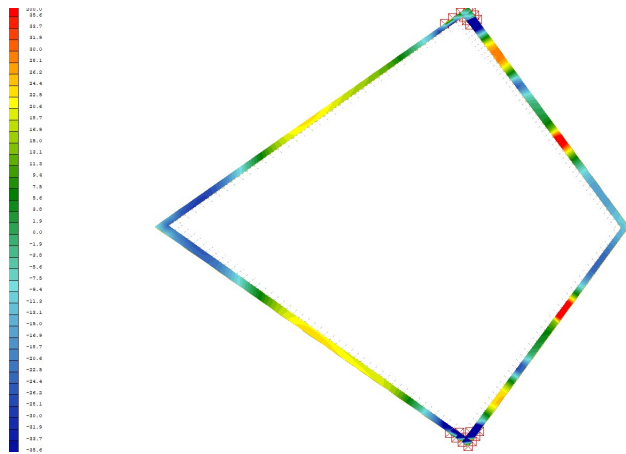


Figure 20: Edge beam bending moment, $M_{max} = 36$ kN/m (red) and $M_{min} = -26$ kN/m (blue)

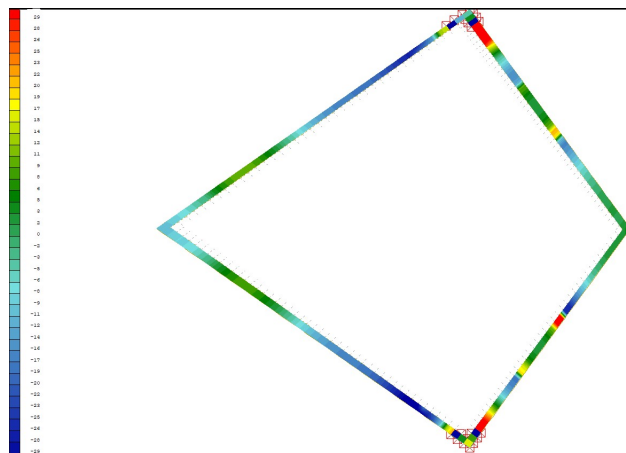


Figure 21: Edge beam bending shear force, $V_{max} = 30$ kN (red) and $V_{min} = -30$ kN (blue)

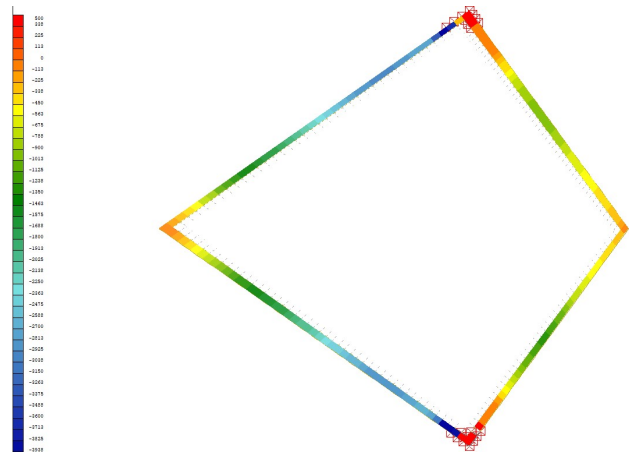


Figure 22: Edge beam bending axial force, $P_{max} = 500$ kN/m (red) and $V_{min} = -2930$ kN/m (blue)

pal tensile stress in most areas low, however reaches up to 10 MPa around the hypar tip (close to point B) the top surface. However, the reinforcement in this area is sufficient to handle this peak stress.

Figure 19 shows the deflections of model FE4 is similar to FE3 model, with maximum deflection of 190 mm at the tip (point B) of shell roof.

Since the edge beam is of note in this design, the axial, shear and moment forces are provided in Figure 20 through Figure 22.

4 Discussion and future work

The St. Charles modified hypar roof located in Washington, United States is a significant shell structure for a multiplicity of reasons including its role as: 1) the earliest constructed asymmetric shell in the United States, 2) a relatively long-span (at time of construction) hypar shell bounded by curvilinear edges, 3) a modified hypar shell that has prestress in two directions, 4) one of the few shells designed by T.Y. Lin and 5) a rare case of a historic hypar shell that has construction documentation (pictures and drawings) extant and is used for comparison to FEA analysis. This paper addresses the current dearth of information about the project by presenting the historic context, construction process and design documentation and comparing that information to contemporary FEA and results. Key details, including the plan dimensions, material properties and shell key points are provided in this paper to facilitate further study of this structure and establish the St. Charles hypar roof as a case-study for modern engineers. Since most designers of the shell structures from the 1920-1960's are

no longer around, there is a need to both preserve and re-discover many of the design and construction techniques that were once common knowledge. Through comparison of principal stresses from several different FEA models, the impact of design parameters (variable shell thickness, straight vs curved edges, folded edge beam) is shown to result in the ideal structure as designed and constructed. This study presents areas for additional research (*i.e.*, the folded edge beam) as well as establishing a baseline for ongoing studies. This ongoing work includes development of a full 3D model of the structure using repeat real-time kinematic (RTK) positioning surveys and repeat structure-from-motion aerial photogrammetry analyses. The resulting analytical model will be imported into FEA software for determination of in-situ shell stresses as part of ongoing preservation work. In-situ data will be used to calibrate the FEA and inform decisions about the roof's long-term structural performance and further understanding of this complex structure.

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