

Research Article

Qimao Liu*

Damping mathematical modelling and dynamic responses for FRP laminated composite plates with polymer matrix

<https://doi.org/10.1515/cls-2018-0003>

Received Nov 25, 2017; accepted Jan 26, 2018

Abstract: This paper proposes an assumption that the fibre is elastic material and polymer matrix is viscoelastic material so that the energy dissipation depends only on the polymer matrix in dynamic response process. The damping force vectors in frequency and time domains, of FRP (Fibre-Reinforced Polymer matrix) laminated composite plates, are derived based on this assumption. The governing equations of FRP laminated composite plates are formulated in both frequency and time domains. The direct inversion method and direct time integration method for nonviscously damped systems are employed to solve the governing equations and achieve the dynamic responses in frequency and time domains, respectively. The computational procedure is given in detail. Finally, dynamic responses (frequency responses with nonzero and zero initial conditions, free vibration, forced vibrations with nonzero and zero initial conditions) of a FRP laminated composite plate are computed using the proposed methodology. The proposed methodology in this paper is easy to be inserted into the commercial finite element analysis software. The proposed assumption, based on the theory of material mechanics, needs to be further proved by experiment technique in the future.

Keywords: FRP laminated composite plate; Viscoelastic damping; Dynamic response; Damping force; Polymer matrix

1 Introduction

The Fibre-Reinforced Polymer matrix (FRP) laminated composite plates are being widely used in the structural

applications because of their attractive performance characteristics, such as high strength-to-weight ratio, high stiffness-to-weight ratio, superior fatigue properties and high corrosion resistance [2, 3]. The polymer materials with long-chain molecules (for example, plastics, rubbers, acrylics, silicones, adhesives, and epoxies) exhibit viscoelastic damping behaviour [4]. The damping plays a very important role in structural dynamic behaviour, vibration and noise control, fatigue endurance, impact resistance. Chandra *et al.* reviewed the different composite damping mechanisms and analytical predictions of damping using macro/micromechanical and viscoelastic approaches [5]. Wei *et al.* developed a method for calculating the effective damping matrix on the laminate scale using the energy method [6]. Ohta *et al.* presented the damping analysis of fibre reinforced plastics laminated composite plates using the maximum strain and kinetic energies, which are evaluated analytically based on the three-dimensional theory of elasticity [7]. Berthelot described the damping modelling of unidirectional composites and laminates using Ritz method [8]. Berthelot *et al.* developed a synthesis method of damping analysis of laminate materials, laminates with interleaved viscoelastic layers and sandwich materials [9]. Tsai and Chang developed a 2-D analytical model for characterizing flexural damping responses of composite laminates [10]. Ghinet and Atalla modelled thick composite laminate and sandwich structures with linear viscoelastic damping using a discrete laminate model [11].

The purpose of this paper is to develop computational methods for dynamic responses of FRP laminated composite plates with polymer matrix in both frequency and time domains. An assumption “the fibre is elastic material and polymer matrix is viscoelastic material so that the energy dissipation only depends on the polymer matrix in dynamic response process.” is proposed in this paper. Based on this assumption, the damping force vectors in frequency and time domains are derived. The governing equations of FRP laminated composite plates are formulated in both frequency and time domains. The dynamic responses in frequency and time domains are computed

*Corresponding Author: Qimao Liu: ABB Corporate Research Center, Vasteras, SE-72226, Sweden; Department of Civil Engineering, Aalto University, Espoo, FI-00076 Aalto, Finland; Email: qimao.liu@se.abb.com; Tel.: +46 (0) 21345031

using the direct inversion method and direct time integration method for nonviscously damped systems proposed by Liu [1], respectively. The dynamic responses of FRP laminated composite plate, subjected to initial displacements or/and dynamic pressure on the surface of the plate, are studied to elucidate the proposed methods.

2 Governing equations

The governing equations of FRP laminated composite plate can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}(\dot{\mathbf{x}}(t)) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (1)$$

with initial conditions,

$$\begin{cases} \mathbf{x}(0) = \mathbf{x}_0 \\ \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0 \end{cases} \quad (2)$$

where \mathbf{M} is mass matrix and \mathbf{K} is stiffness matrix. The definition and computation of mass and stiffness matrices are given in author's previous work [19]. $\mathbf{D}(\dot{\mathbf{x}})$ is damping force vector. $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ are displacement, velocity and acceleration vectors, respectively. $\mathbf{f}(t)$ is loading vector.

Take the Laplace transform of Eq. (1) and let Laplace variable $s = i\omega$, we have

$$-\omega^2 \mathbf{M}\mathbf{x}(i\omega) + \mathbf{D}(i\omega) + \mathbf{K}\mathbf{x}(i\omega) = \mathbf{F}(i\omega) + i\omega \mathbf{M}\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0) \quad (3)$$

The modulus of elasticity of the fibre is assumed to be zero. However, Poisson's ratio and density of the fibre are not changed. Therefore, the polymer matrix can be treated as the isotropic viscoelastic material. The FRP laminated composite plate is equivalent to the isotropic viscoelastic plate. The governing equations of the isotropic viscoelastic material are

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \overline{\mathbf{K}}G(t)\mathbf{x}(0) + \overline{\mathbf{K}} \int_0^t G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau = \mathbf{f}(t) \quad (4)$$

where $G(t)$ is the relaxation function of viscoelastic material. $\overline{\mathbf{K}}$ is the viscoelastic stiffness term of polymer matrix.

Take Laplace transform of Eq. (4) and let Laplace variable $s = i\omega$, we obtain

$$-\omega^2 \mathbf{M}\mathbf{x}(i\omega) + i\omega G(i\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega) = \mathbf{F}(i\omega) + i\omega \mathbf{M}\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0) \quad (5)$$

where the term $i\omega G(i\omega)$ is called the complex modulus of viscoelastic material.

$$i\omega G(i\omega) = G'(\omega) + iG''(\omega) \quad (6)$$

where $G'(\omega)$ is storage modulus and $G''(\omega)$ is loss modulus. The loss factor of viscoelastic material is

$$\eta(\omega) = \frac{G''(\omega)}{G'(\omega)} \quad (7)$$

By substituting Eq. (6) into Eq. (5), we have

$$-\omega^2 \mathbf{M}\mathbf{x}(i\omega) + G'(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega) + iG''(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega) = \mathbf{F}(i\omega) + i\omega \mathbf{M}\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0) \quad (8)$$

where $G'(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega)$ is elastic restoring forces (energy storage) and $iG''(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega)$ is damping forces (energy dissipation), in frequency domain.

An assumption: The fibre is elastic material and polymer matrix is viscoelastic material so that the energy dissipation only depends on the polymer matrix in dynamic response process. Therefore, the damping forces in frequency domain in Eq. (3) can be approximately represented by the damping forces (energy dissipation term) in Eq. (8), i.e.,

$$\mathbf{D}(i\omega) = iG''(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega) \quad (9)$$

By substituting Eq. (9) into Eq. (3), we have the governing equations of FRP laminated composite plate in frequency domain, i.e.,

$$-\omega^2 \mathbf{M}\mathbf{x}(i\omega) + iG''(\omega) \overline{\mathbf{K}}\mathbf{x}(i\omega) + \mathbf{K}\mathbf{x}(i\omega) = \mathbf{F}(i\omega) + i\omega \mathbf{M}\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0) \quad (10)$$

or

$$\mathbf{K}^*(i\omega) \mathbf{x}(i\omega) = \mathbf{F}^*(i\omega) \quad (11)$$

where

$$\mathbf{K}^*(i\omega) = -\omega^2 \mathbf{M} + iG''(\omega) \overline{\mathbf{K}} + \mathbf{K} \quad (12)$$

$$\mathbf{F}^*(i\omega) = \mathbf{F}(i\omega) + i\omega \mathbf{M}\mathbf{x}(0) + \mathbf{M}\dot{\mathbf{x}}(0) \quad (13)$$

For zero initial conditions, Eq. (11) is simplified as

$$\mathbf{K}^*(i\omega) \mathbf{x}(i\omega) = \mathbf{F}(i\omega) \quad (14)$$

Take the inverse Laplace transform of Eq. (10), we have the governing equations of FRP laminated composite plate in time domain, i.e.,

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}}(t) + \overline{\mathbf{K}}G(t)\mathbf{x}(0) \\ + \overline{\mathbf{K}} \int_0^t G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \end{aligned} \quad (15)$$

or

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \int_0^t \overline{\mathbf{K}}G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) - \overline{\mathbf{K}}G(t)\mathbf{x}(0) \quad (16)$$

where $\overline{\mathbf{K}}\overline{\mathbf{G}}(t - \tau)$ can be called symmetric matrix of the damping kernel functions. $\overline{\mathbf{G}}(t)$ can be called damping function.

$$\overline{\mathbf{G}}(t) = L^{-1} [iG''(\omega)|_{s=i\omega}] \quad (17)$$

The notation L^{-1} is the inverse Laplace transform in the previous equation (17). For zero initial conditions, Eq. (16) is simplified as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \int_0^t \overline{\mathbf{K}}\overline{\mathbf{G}}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (18)$$

It is noted that Eq. (11) can be solved by direct inverting matrix $\mathbf{K}^*(i\omega)$ at each frequency point. The solution of Eq. (16) can be obtained using direct time integration method for nonviscously damped structure systems [1].

3 Storage modulus $G'(\omega)$, loss modulus $G''(\omega)$ and damping function $\overline{\mathbf{G}}(t)$

The storage modulus, loss modulus and loss factor of viscoelastic material can be obtained by experimental test. The manufacturers often provide the values of storage modulus, loss modulus and loss factor of viscoelastic material at discrete frequency points for the users. According to the experimental results, many researchers have proposed different models to describe the frequency-dependent behaviour of the viscoelastic material. The complex modulus is often used to represent the properties of viscoelastic material in frequency domain. For example, Golla-Hughes-McTavish model (GHM) [12, 13], Anelastic Displacement model (AD) [14, 15] and Fractional Derivative Model (FD) [16, 17]. As an example, in this paper, the GHM model is used to describe the material properties of the polymer matrix (viscoelastic material). The GHM analytic model involves a series of mini-oscillator terms, i.e.,

$$G(s) = G^0 \left(1 + \sum_i \alpha_i \frac{s^2 + 2\zeta_i \omega_i s}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \right) \quad (19)$$

where α_i , ω_i and ζ_i are three independent parameters of the i th mini-oscillator. And s is Laplace variable. G^0 is equilibrium elasticity constant pertaining to the relaxation function of viscoelastic material, i.e.,

$$G^0 = \lim_{t \rightarrow \infty} G(t) \quad (20)$$

The storage modulus $G'(\omega)$ and loss modulus $G''(\omega)$ of the GHM model are given below.

$$G'(\omega) = G^0 \left[1 + \omega^2 \sum_i \alpha_i \frac{(\omega^2 - \omega_i^2) + (2\zeta_i \omega_i)^2}{(\omega^2 - \omega_i^2)^2 + \omega^2 (2\zeta_i \omega_i)^2} \right] \quad (21)$$

$$G''(\omega) = G^0 \omega \sum_i \alpha_i \frac{(2\zeta_i \omega_i) \omega_i^2}{(\omega^2 - \omega_i^2)^2 + \omega^2 (2\zeta_i \omega_i)^2} \quad (22)$$

When the storage modulus $G'(\omega)$ and loss modulus $G''(\omega)$ are known already, then the loss factor $\eta(\omega)$ can be calculated using Eq. (7). The damping function $\overline{\mathbf{G}}(t)$ is calculated as follows.

$$\begin{aligned} \overline{\mathbf{G}}(t) &= L^{-1} [iG''(\omega)|_{s=i\omega}] \quad (23) \\ &= L^{-1} \left[G^0 i\omega \sum_i \alpha_i \frac{(2\zeta_i \omega_i) \omega_i^2}{(\omega^2 - \omega_i^2)^2 + \omega^2 (2\zeta_i \omega_i)^2} \Big|_{s=i\omega} \right] \\ &= L^{-1} \left[G^0 \sum_i \alpha_i \frac{(i\omega) (2\zeta_i \omega_i) \omega_i^2}{((i\omega)^2 + \omega_i^2)^2 - (i\omega)^2 (2\zeta_i \omega_i)^2} \Big|_{s=i\omega} \right] \\ &= L^{-1} \left[G^0 \sum_i \alpha_i \frac{s (2\zeta_i \omega_i) \omega_i^2}{(s^2 + \omega_i^2)^2 - s^2 (2\zeta_i \omega_i)^2} \right] \\ &= G^0 \sum_i \alpha_i \left[\frac{\omega_i \exp(t\omega_i \zeta_i) \sin(t\omega_i \sqrt{1 - \zeta_i^2})}{2\sqrt{1 - \zeta_i^2}} \right. \\ &\quad \left. - \frac{\alpha_i \omega_i \sin(t\omega_i \sqrt{1 - \zeta_i^2})}{2 \exp(\omega_i \zeta_i t) \sqrt{1 - \zeta_i^2}} \right] \end{aligned}$$

4 Lamina engineering constants

The modulus of elasticity of the fibre E_f , modulus of elasticity of the polymer matrix E_m , Poisson's ratio of the fibre ν_f , Poisson's ratio of the polymer matrix ν_m , density of the fibre ρ_f , density of the polymer matrix ρ_m and fibre volume fraction r_f are usually the knowns in the lamina. The lamina engineering constants will be computed in this section. If the lamina is reinforced by long circular cylindrical fibres which are equally spaced and aligned with the x_1 axis of the lamina, the explicit formulas of the lamina engineering constants are given in literature [18] as follows.

The modulus of elasticity of composite lamina,

$$E_1 = C_{11} - \frac{2C_{12}^2}{C_{22} + C_{23}} \quad (24)$$

$$E_2 = \frac{(2C_{11}C_{22} + 2C_{11}C_{23} - 4C_{12}^2)(C_{22} - C_{23} + 2C_{44})}{3C_{11}C_{22} + C_{11}C_{23} + 2C_{11}C_{44} - 4C_{12}^2}$$

Poisson's ratio of composite lamina,

$$\begin{aligned} v_{12} = v_{13} &= \frac{C_{12}}{C_{22} + C_{23}} \\ v_{23} &= \frac{C_{11}C_{22} + 3C_{11}C_{23} - 2C_{11}C_{44} - 4C_{12}^2}{3C_{11}C_{22} + C_{11}C_{23} + 2C_{11}C_{44} - 4C_{12}^2} \end{aligned} \quad (25)$$

Shear modulus of composite lamina,

$$\begin{aligned} G_{12} = G_{13} &= C_{66} \\ G_{23} &= \frac{E_2}{2(1 + v_{23})} \end{aligned} \quad (26)$$

Density of composite lamina,

$$\rho = r_f \rho_f + (1 - r_f) \rho_m \quad (27)$$

where density of the fibre ρ_f , density of the polymer matrix ρ_m and fibre volume fraction r_f are the knowns in the lamina.

The six unique coefficients C_{ij} in Eqs. (24)–(26) can be calculated using the following formulae.

$$\begin{aligned} C_{11} &= \lambda_m + 2\mu_m - \frac{r_f}{D} \left(\frac{S_3^2}{\mu_m^2} - \frac{2S_6S_3}{\mu_m^2g} - \frac{aS_3}{\mu_m c} \right. \\ &\quad \left. + \frac{S_6^2 - S_7^2}{\mu_m^2g^2} + \frac{aS_6 + bS_7}{\mu_m gc} + \frac{a^2 - b^2}{4c^2} \right) \\ C_{12} &= \lambda_m + \frac{r_fb}{D} \left(\frac{S_3}{2c\mu_m} - \frac{S_6 - S_7}{2c\mu_m g} - \frac{a + b}{4c^2} \right) \\ C_{23} &= \lambda_m + \frac{r_f}{D} \left(\frac{aS_7}{2\mu_m gc} - \frac{ab + b^2}{4c^2} \right) \\ C_{22} &= \lambda_m + 2\mu_m - \frac{r_f}{D} \left(-\frac{aS_3}{2\mu_m c} + \frac{aS_6}{2\mu_m gc} + \frac{a^2 - b^2}{4c^2} \right) \\ C_{44} &= \mu_m - r_f \left(-\frac{2S_3}{\mu_m} + \frac{1}{\mu_m - \mu_f} + \frac{4S_7}{2\mu_m - 2\mu_m v_m} \right)^{-1} \\ C_{66} &= \mu_m - r_f \left(-\frac{S_3}{\mu_m} + \frac{1}{\mu_m - \mu_f} \right)^{-1} \end{aligned} \quad (28)$$

In order to obtain C_{ij} in Eq. (28), **first**, λ_m , μ_m and μ_f are calculated as follows.

$$\begin{aligned} \lambda_m &= \frac{E_m v_m}{(1 + v_m)(1 - 2v_m)} \\ \mu_m &= \frac{E_m}{2(1 + v_m)} \\ \mu_f &= \frac{E_f}{2(1 + v_f)} \end{aligned} \quad (29)$$

It should be noted that E_m , v_m , E_f and v_f are the knowns. **Second**, a , b , c and g can be computed using the following formulae.

$$\begin{aligned} a &= \mu_f - \mu_m - 2\mu_f v_m + 2\mu_m v_f \\ b &= -\mu_m v_m + \mu_f v_f + 2\mu_m v_m v_f - 2\mu_f v_m v_f \\ c &= (\mu_m - \mu_f) (-\mu_m + \mu_f - \mu_m v_m - 2\mu_f v_m) \end{aligned} \quad (30)$$

$$+ 2\mu_m v_f + \mu_f v_f + 2\mu_m v_m v_f - 2\mu_f v_m v_f)$$

$$g = 2 - 2v_m$$

and S_3 , S_6 , S_7 are given as

$$\begin{aligned} S_3 &= 0.49247 - 0.47603r_f - 0.02748r_f^2 \\ S_6 &= 0.36844 - 0.14944r_f - 0.27152r_f^2 \\ S_7 &= 0.12346 - 0.32035r_f + 0.23517r_f^2 \end{aligned} \quad (31)$$

Finally, we have

$$\begin{aligned} D &= \frac{aS_3^2}{2\mu_m c} - \frac{aS_6S_3}{\mu_m^2 gc} + \frac{a(S_6^2 - S_7^2)}{2\mu_m^2 g^2 c} + \frac{S_3(b^2 - a^2)}{2\mu_m c^2} \\ &\quad + \frac{S_6(a^2 - b^2) + S_7(ab + b^2)}{2\mu_m gc^2} + \frac{a^3 - 2b^3 - 3ab^2}{8c^3} \end{aligned} \quad (32)$$

By substituting λ_m , μ_m , μ_f , a , b , c , g , S_3 , S_6 , S_7 and D into Eq. (28), we obtain the values of the coefficients C_{11} , C_{12} , C_{23} , C_{22} , C_{44} and C_{66} .

5 Computational procedure

The computational procedure proposed for the dynamic responses of FRP laminated composite plates with polymer matrix is as follows.

1. The material properties E_f , E_m , v_f , v_m , ρ_f , ρ_m , and r_f are the knowns. Calculate the lamina engineering constants, i.e., the first row in Table 2. Compute the mass matrix \mathbf{M} and stiffness matrix \mathbf{K} .
2. Let $E_f = 0$, calculate the lamina engineering constants, i.e., the second row in Table 2. Compute the viscoelastic stiffness matrix $\bar{\mathbf{K}}$.
3. The parameters of viscoelastic material (polymer matrix of plates) model are the knowns. Calculate the loss modulus $G''(\omega)$ and damping function $\bar{G}(t)$.
4. Solving Eq. (11) gives the dynamic responses in frequency domain by direct inverting $\mathbf{K}^*(i\omega)$ at each frequency points.
5. Solving Eq. (16) gives the dynamic responses in time domain using direct time integration method for viscoelastic damped structure systems [1].

6 Example

A square plate (2 m×2 m) in Figure 1 is laminated in a $[0/90/\pm 45]_S$ symmetric laminate configuration, as shown in Figure 2. The thickness of the plate is 10 mm and the thickness of each lamina is 1.25 mm. The material properties of fibre and polymer matrix are listed in Table 1. The

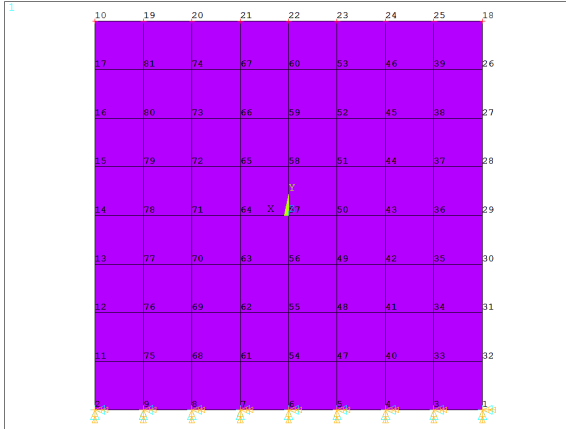


Figure 1: FRP laminated composite plate and node number (Element type: SHELL181).

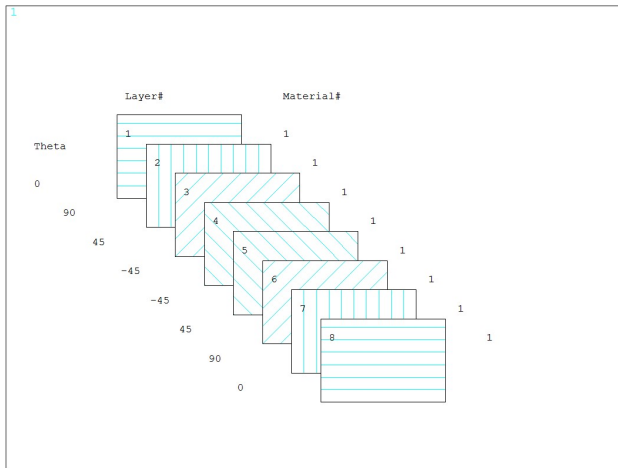


Figure 2: Layers of FRP laminated composite plate in $[0/90/\pm 45]_S$.

Table 1: Material properties of fibre and polymer matrix.

	Young's modulus	Poisson's ratio	Density
Fibre	$E_f = 294000 \text{ MPa}$	$\nu_f = 0.2$	$\rho_f = 1810 \text{ kg/m}^3$
Matrix	$E_m = 4200 \text{ MPa}$	$\nu_m = 0.3$	$\rho_m = 1240 \text{ kg/m}^3$

fibre volume fraction is 0.6. The lamina engineering constants shown in Table 2 are calculated using the formulae in previous Section. The first row in Table 2 is the lamina engineering constants when the fibre Young's modulus $E_f = 294000 \text{ MPa}$ (real). These engineering constants are used to produce stiffness matrix \mathbf{K} and mass matrix \mathbf{M} of the FRP laminated composite plate. The second row in Table 2 is the lamina engineering constants when the fibre Young's modulus $E_f = 0 \text{ MPa}$ (assumption). These constants will be used to produce stiffness matrix $\bar{\mathbf{K}}$ (viscoelastic stiffness term of polymer matrix). The two-term

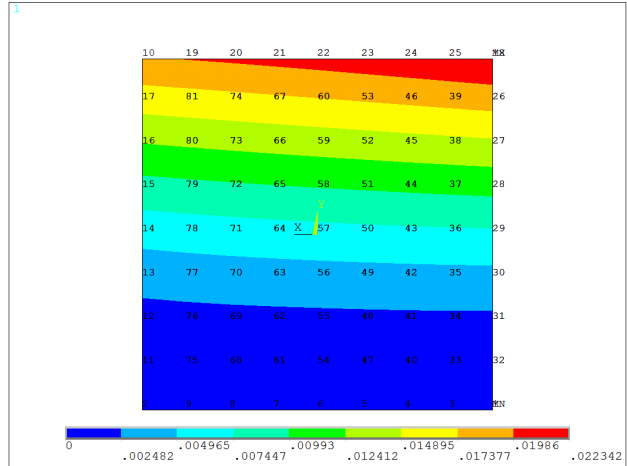


Figure 3: Initial displacements (z direction).

GHM model parameters of viscoelastic material (polymer matrix) are listed in Table 3. The FRP laminated composite plate is divided into 64 elements using element SHELL181 of ANSYS. The nodes 1-10 are fixed, as shown in Figure 1.

The loading case : given the initial displacements, i.e., the displacements of the plate when the nodes 10 and 18-25 are subjected to the concentrated forces, 10 N, along the axis. The initial displacements of z direction are shown in Figure 3. The loading case : given dynamic pressure on the surface of the plate. The pressure in time domain is

$$p(t) = p_0 \sin\left(\frac{\pi}{2}t\right) \text{ N/m}^2 \quad (33)$$

where $p_0 = 10 \text{ N}$. The pressure in frequency domain, using Laplace transform, is

$$P(\omega) = L[p(t)]|_{s=i\omega} = p_0 \frac{1}{1 - \omega^2} \quad (34)$$

where the notation L is Laplace transform and s is Laplace variable.

6.1 Dynamic response in frequency domain

The dynamic responses in frequency domain are obtained by solving Eq. (11) using direct inverting the matrix $\mathbf{K}^*(i\omega)$ method. The amplitude and phase of the z direction displacement of node 10 of the plate, subjected to the dynamic pressure on the surface of the plate (i.e., loading case (2)), are shown in Figure 4. The amplitude and phase of the z direction displacement of node 10 of the plate, subjected to both the initial conditions (i.e., loading case (1)) and dynamic pressure on the surface of the plate (i.e., loading case (2)), are shown in Figure 5.

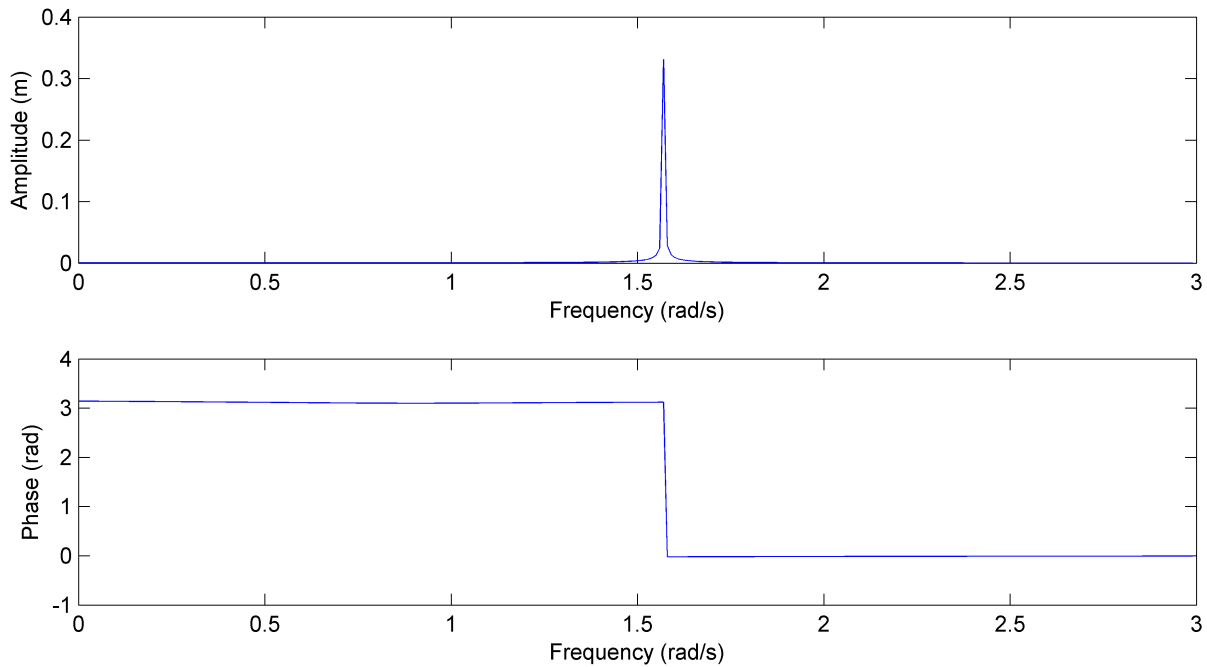


Figure 4: Dynamic response in frequency domain (zero initial conditions).

Table 2: Lamina elastic properties (fibre volume fraction: $r_f = 0.6$).

Fibre Young's modulus	Young's moduli (MPa)	Shear moduli (MPa)	Poisson's ratio	Density (kg/m ³)
$E_f = 294000$ MPa	$E_1 = 178093$ $E_2 = E_3 = 15565$	$G_{12} = G_{13} = 6224$ $G_{23} = 5782$	$\nu_{12} = \nu_{13} = 0.23$ $\nu_{23} = 0.35$	$\rho = 1582$
$E_f = 0$ MPa (assumption)	$E_1 = 1680$ $E_2 = E_3 = 785$	$G_{12} = G_{13} = 408$ $G_{23} = 296$	$\nu_{12} = \nu_{13} = 0.30$ $\nu_{23} = 0.33$	$\rho = 1582$

Table 3: Two-term GHM parameters.

Parameter	G^0	α_1	ζ_1	ω_1	α_2	ζ_2	ω_2
Value	1	1	0.8	2	2	0.5	1

6.2 Dynamic response in time domain

The dynamic responses in time domain can be achieved by solving Eq. (16) using direct time integration method for nonviscously damped structure systems [1]. The parameter of the direct time integration method is $\delta = 1$ in this paper. The free vibration of node 10 along the z axis, caused by the initial conditions (loading case (1)), is shown in Figure 6. The forced vibration of node 10 along the z axis, caused by the dynamic pressure on the surface of the plate (loading case (2)), is shown in Figure 7. The forced vibration of node 10 along the z axis, caused by both the initial conditions (loading case (1)) and dynamic pressure on the surface of the plate (loading case (2)), is shown in Figure 8.

6.3 Discussion

The mass matrix \mathbf{M} and stiffness matrix \mathbf{K} of the FRP laminated composite plates are often accurate enough for engineering applications. However, it is difficult to evaluate the damping force vector \mathbf{D} because the classic damping model doesn't fit the viscoelastic damping (since polymer matrix is viscoelastic material). The methods in this paper to evaluate the viscoelastic damping force vector are based on the linear viscoelastic theory and a proposed assumption that "the fibre is elastic material and polymer matrix is viscoelastic material so that the energy dissipation depends only on the polymer matrix in dynamic response process." The dynamic response in frequency domain is obtained by direct inverting the matrix $\mathbf{K}^*(i\omega)$ at each frequency point. The dynamic response in time domain is achieved using direct time integration method for nonviscously damped structure systems [1], in this paper, the parameter $\delta = 1$. The numerical results (Figures 4–8) show that the proposed methods can successfully compute the

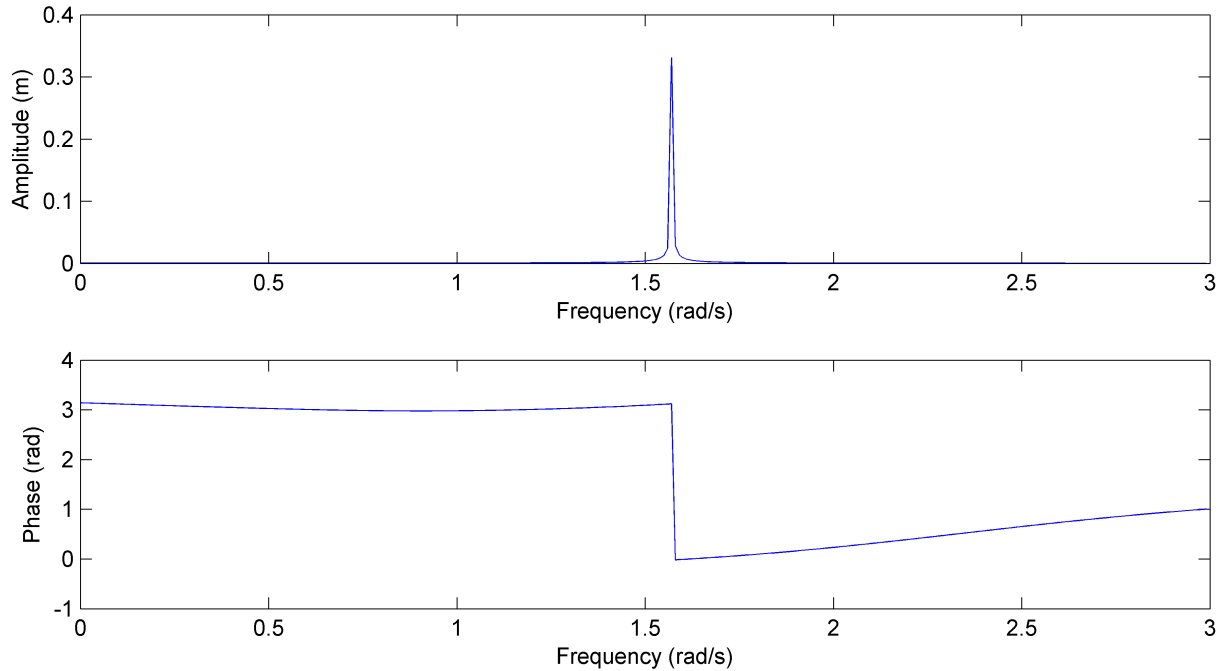


Figure 5: Dynamic response in frequency domain (nonzero initial conditions).

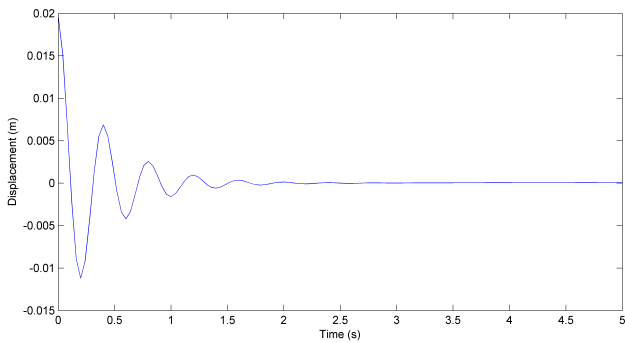


Figure 6: Free vibration.

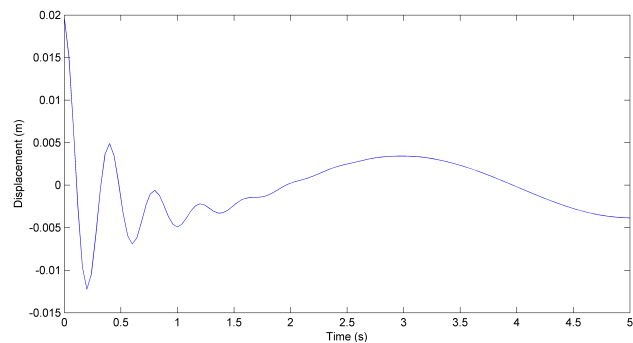


Figure 8: Forced vibration with nonzero initial conditions.

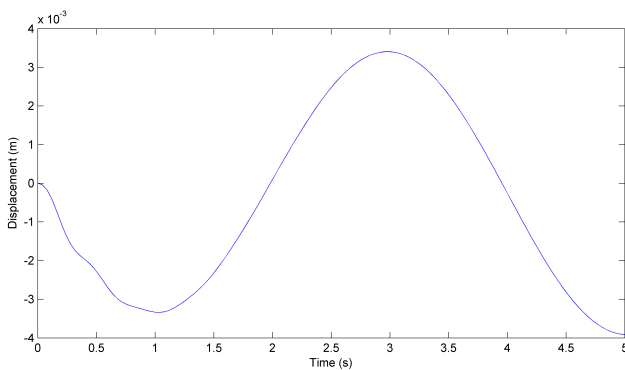


Figure 7: Forced vibration with zero initial conditions.

dynamic responses in both frequency and time domains. Dynamic responses in frequency, as shown in Figures 4

and 5, indicate that the initial displacements have influence on the phase of displacement response, but not on the amplitude of displacement response. The dynamic response in time domain, as shown in Figure 6, indicates that the free vibration decays quickly in this example.

It is easy to implement the proposed methods by the secondary development of the commercial finite element analysis software, *i.e.*, ANSYS, Abaqus, etc. The mass matrix \mathbf{M} , stiffness matrix \mathbf{K} and viscoelastic stiffness term $\bar{\mathbf{K}}$ are computed and extracted using APDL Math of ANSYS in this paper. Therefore, the proposed methods in this paper can be employed directly in the real engineering practice. It is noted that the proposed methods are limited to the linear viscoelastic response analysis.

7 Conclusion

The dynamic response analysis methods in both frequency and time domains for the FRP laminated composite plates are proposed in this paper. The assumption, *i.e.*, “The fibre is elastic material and polymer matrix is viscoelastic material so that the energy dissipation depends only on the polymer matrix in dynamic response process.”, is used to derive the damping force vector. The dynamic responses in frequency and time domains are achieved using direct inversion method at each frequency point and direct time integration method for nonviscously damped structure systems [1], respectively. From the viewpoint of numerical computation, the proposed methods can successfully obtain the dynamic responses of the FRP laminated composite plates in both frequency and time domains. And the proposed methodology in this paper is also easy to be inserted into the commercial finite element analysis software. The proposed methodology takes GHM analytic model as an example. However, it may be extended to other viscoelastic models, *i.e.*, Anelastic Displacement model (AD) and Fractional Derivative Model (FD). Please also be noted that the assumption, based on the theory of material mechanics, needs to be further proved by experiment technique in the future.

References

- [1] Liu Q. Computational method of the dynamic response for non-viscously damped structure systems. *Journal of Engineering Mechanics* 2014; DOI: 10.1061/(ASCE)EM.1943-7889.0000806.
- [2] Reddy JN. *Mechanics of laminated composite plates and shells: theory and analysis*, 2nd edition. Washington (DC): CRC Press, 2003.
- [3] Decolon C. *Analysis of composite structures*. London: Hermes Penton Ltd., 2002.
- [4] Rao MD. Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes. *Journal of Sound and Vibration* 2003; 262(3): 457-74.
- [5] Chandra R, Singh SP, Gupta K. Damping studies in fiber-reinforced composites - a review. *Composite Structures* 1999; 46: 41-51.
- [6] Wei YT, Gui LJ, Yang TQ. Prediction of the 3-D effective damping matrix and energy dissipation of viscoelastic fibre composites. *Composite Structures* 2001; 54: 49-55.
- [7] Ohta Y, Narita Y, Nagasaki K. On the damping analysis of FRP laminated composite plates. *Composite Structures* 2002; 57: 169-175.
- [8] Berthelot JM. Damping analysis of laminated beams and plates using the Ritz method. *Composite Structures* 2006; 74: 186-201.
- [9] Berthelot JM, Assarar M, Sefrani Y, Mahi AE. Damping analysis of composite materials and structures. *Composite Structures* 2008; 85: 189-204.
- [10] Tsai JL, Chang NR. 2-D analytical model for characterizing flexural damping responses of composite laminates. *Composite Structures* 2009; 89: 443-447.
- [11] Ghinet S, Atalla N. Modeling thick composite laminate and sandwich structures with linear viscoelastic damping. *Computers and Structures* 2011; 89: 1547-1561.
- [12] Golla DF, Hughes PC. Dynamics of viscoelastic structures—a time domain finite element formulation. *Journal of Applied Mechanics* 1985; 52(4): 897-906.
- [13] McTavish DJ, Hughes PC. Modeling of linear viscoelastic space structures. *Journal of Vibration and Acoustics* 1993; 115(1): 103-110.
- [14] Lesieutre GA., Mingori DL. Finite element modeling of frequency-dependent material properties using augmented thermodynamic fields. *Journal of Guidance and Control Dynamics* 1990; 13(6): 1040-1050.
- [15] Lesieutre GA, Bianchini E. Time domain modeling of linear viscoelasticity using anelastic displacement using anelastic displacement fields. *Journal of Vibration and Acoustics* 1995; 117(4): 424-430.
- [16] Bagley RL, Torvik PJ. Fractional calculus—a different approach to the analysis of viscoelastically damped structures. *AIAA Journal* 1983; 21(5): 741-748.
- [17] Torvik PJ, Bagley RL. “Fractional derivatives in the description of damping: materials and phenomena.” in “The Role of Damping in Vibration and Noise Control.”-ASME 1987; 5(DE): 125-135.
- [18] Luciano R, Barbero EJ. Formulas for the stiffness of composites with periodic Microstructure. *International Journal of Solids and Structures* 1995; 31(21): 2933-2944.
- [19] Liu Q. Analytical sensitivity analysis of frequencies and modes for composite laminated structures. *International Journal of Mechanical Sciences* 2015; 90: 258-277.