Research Article **Open Access**

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Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique

DOI 10.1515/cls-2016-0007
Received Jan 08, 2016; accepted Jan 14, 2016

Abstract: In the present manuscript, free vibration response of circular cylindrical shells with functionally graded material (FGM) is investigated. The method of discrete singular convolution (DSC) is used for numerical solution of the related governing equation of motion of FGM cylindrical shell. The constitutive relations are based on the Love's first approximation shell theory. The material properties are graded in the thickness direction according to a volume fraction power law indexes. Frequency values are calculated for different types of boundary conditions, material and geometric parameters. In general, close agreement between the obtained results and those of other researchers has been found.

Keywords: Discrete singular convolution; free vibration; functionally graded material; cylindrical shells

1 Introduction

Circular cylindrical shells are generally used in different applications compared to the conical, spherical, shells of revolution and toroidal shells because of its simple and advantages geometry. Cylindrical shells are used in many engineering applications such as mechanical, civil and aerospace engineering. Thus, frequencies and mode shapes of such structures are important in the design of systems [1–5]. Many of researchers have spent great - efforts in order to analysis of the circular shells under different effects. As a consequence, a number of analytical and numerical methods have been also studied on the vibration analysis of circular cylindrical shells during the past fifty years [6–26].

By using the functionally graded materials in the structural components, FGM shells are gaining the considerable importance and find plenty of applications in high temperature applications in petro-chemical, civil and aerospace industries [27–30]. Nowadays, FGM based structures and devices such as beams, plates and shells have been widely used in aerospace, mechanical, automobile and civil engineering applications. Understanding of free vibration behavior of these structures is an important task for the successful applications. So, many studies have been made by researchers on this field [31–45].

Recently, the method of discrete singular convolution (DSC) proposed by Wei [46, 47] has been increasingly applied to solve many engineering and sciences problems such as mathematical physics, fluid and solid mechanics [48–52]. By this time, Ritz, Galerkin, finite differences, finite elements method, boundary element methods have widely used for vibration problem of shells. Recent years, the method of differential quadrature (DQ), discrete singular convolution (DSC) and meshless methods have become increasingly popular in the numerical solution of initial and boundary value problems in engineering applications. These methods can yield accurate solutions with relatively much fewer grid points. It has been also successfully employed for different solid and fluid mechanic problems. More recently, a detailed investigated and discussion of the strong formulation and differential quadrature methods has been presented by Tornabene *et al* [15].

In the present study, free vibration analysis of FGM circular cylindrical shells is performed on the basis of Love's shell theory. The related governing differential equation for vibration with corresponding boundary conditions is derived. Then, the method of DSC is used for numerical solution of the related differential equations. The effects of the power of the material property variation function, boundary conditions and mode numbers on frequency response of FGM shell are investigated. Some comparative results are also presented to show the convergence and accuracy of the results obtained by present DSC method. This is the first instance in which the DSC method has been adopted for free vibration analysis of FGM cylindrical shells.

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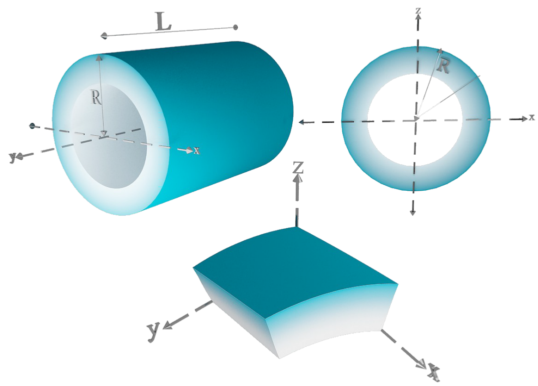

Figure 1: Functionally graded circular shells

2 Formulations

2.1 Circular shell

Consider a cylindrical shell rotating about its symmetrical and horizontal axis at an angular velocity ω as shown in Figure 1. The thickness of the shell and length are denoted by h and L, respectively. The cylindrical shell is referred to a coordinate system (x, θ, z) as shown in Figure 1. The components of the deformation of the cylindrical shell with references to this coordinate system are denoted by u, v, w in the x, θ and z directions, respectively.
Based on Love's first approximation theory, the strain components of this vector are defined as linear functions of the normal (thickness) coordinate z, namely

$$\varepsilon_x = \varepsilon_1 + z\kappa_1, \tag{1a}$$

$$\varepsilon_\theta = \varepsilon_2 + z\kappa_2, \tag{1b}$$

$$\varepsilon_{x\theta} = \gamma + 2z\tau, \tag{1c}$$

where $\{\varepsilon\}^T = \{\varepsilon_1, \varepsilon_2, \gamma\}$ and $\{\kappa\}^T = \{\kappa_1, \kappa_2, 2\tau\}$ are respectively the strain and curvature vectors of the reference surface. They are defined by [1]

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \tag{2a}$$

$$\varepsilon_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \tag{2b}$$

$$\gamma = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}, \tag{2c}$$

$$\kappa_1 = -\frac{\partial^2 w}{\partial x^2}, \tag{3a}$$

$$\kappa_2 = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta}, \tag{3b}$$

$$\tau = -\frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta} + \frac{\partial v}{\partial x}. \tag{3c}$$

The force and moment resultants and can be obtained by

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} e \\ \kappa \end{Bmatrix}, \tag{4}$$

where A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending stiffnesses and calculated from the following equations:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q^{*}_{ij}\left(1, z, z^2\right) dz, \tag{5}$$

$i = 1, 2$ and $j = 3+i$. For an arbitrarily laminated composite shell, these stiffnesses can be given as [26]

$$\left(A_{ij}\right) = \sum_{k=1}^{N_L} Q^{(k)}_{ij}\left(h_k - h_{k-1}\right), \tag{6a}$$

$$\left(B_{ij}\right) = \frac{1}{2}\sum_{k=1}^{N_L} Q^{(k)}_{ij}\left(h_k^2 - h_{k-1}^2\right), \tag{6b}$$

$$\left(D_{ij}\right) = \frac{1}{3}\sum_{k=1}^{N_L} Q^{(k)}_{ij}\left(h_k^3 - h_{k-1}^3\right). \tag{6c}$$

Where N_L is the number of total layers of the laminated conical shell, $Q^{(k)}_{ij}$, the element of the transformed reduced stiffness matrix for the kth layer, and h_k and h_{k-1} denote distances from the shell reference surface to the outer and inner surfaces of the kth layer.
The transverse shear force resultants can be given from M_x, M_θ and $M_{x\theta}$ by

$$Q_x = \frac{1}{R(x)}\frac{\partial}{\partial x}\left[R(x)M_x\right] - \frac{M_\theta \sin\alpha}{R(x)} + \frac{1}{R(x)}\frac{\partial M_{x\theta}}{\partial \theta}, \tag{7}$$

$$Q_\theta = \frac{1}{R(x)}\frac{\partial}{\partial x}\left[R(x)M_{x\theta}\right] + \frac{M_{x\theta} \sin\alpha}{R(x)} + \frac{1}{R(x)}\frac{\partial M_\theta}{\partial \theta}, \tag{8}$$

where

$$\rho_t(x, \theta) = \frac{1}{h}\int_{-h/2}^{h/2} \rho(x, \theta, z) dz, \tag{9}$$

where ρ and ρ_t are, respectively, the density and density per unit area. Moment resultants and in-surface force can be obtained by

$$N = \left(N_x, N_\theta, N_{x\theta}\right)^T = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_\theta, \sigma_{x\theta}\right)^T dz, \tag{10a}$$

$$M = \left(M_x, M_\theta, M_{x\theta}\right)^T = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_\theta, \sigma_{x\theta}\right)^T z dz, \tag{10b}$$

where the stress vector field $(\sigma)^T = \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\}$. The stress vector of the kth layer for laminated composite conical shells in which each layer is orthotropic is

$$\{\sigma_k\} = \left[\overline{Q}_{ij}\right]\{\overline{\varepsilon}_k\}, \tag{11}$$

where $\{\overline{\varepsilon}_k\}^T = \{\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}\}$ is the strain vector. For a thin shell, the stresses defined in Equation (11) are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{\theta x} \end{Bmatrix}. \tag{12}$$

The element of the reduced stiffness are defined as

$$Q_{11} = \frac{E(z)}{1-\upsilon(z)^2},\quad Q_{12} = \frac{\upsilon(z)E(z)}{1-\upsilon^2}, \tag{13a}$$

$$Q_{22} = \frac{E(z)}{1-\upsilon(z)^2},\quad Q_{66} = \frac{E(z)}{2[1+\upsilon(z)]}. \tag{13b}$$

Following the Love's first approximation shell theory [31] governing equations for free vibration analysis of cylindrical shells can be given as [16, 17];

$$L_{11}u + L_{12}v + L_{13}w - \rho h\frac{\partial^2 u}{\partial t^2} = 0, \tag{14a}$$

$$L_{21}u + L_{22}v + L_{23}w - \rho h\frac{\partial^2 v}{\partial t^2} = 0, \tag{14b}$$

$$L_{31}u + L_{32}v + L_{33}w - \rho h\frac{\partial^2 w}{\partial t^2} = 0, \tag{14c}$$

where

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \left(\frac{A_{66}}{R^2}\right)\frac{\partial^2}{\partial\theta^2}, \tag{15}$$

$$L_{12} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x\partial\theta} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^2}{\partial x\partial\theta}, \tag{16}$$

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - \frac{B_{12}+2B_{66}}{R^2}\frac{\partial^3}{\partial x\partial\theta^2} - B_{11}\frac{\partial^3}{\partial x^3}, \tag{17}$$

$$L_{21} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x\partial\theta} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^2}{\partial x\partial\theta}, \tag{18}$$

$$\begin{aligned} L_{22} &= \left[A_{66} + \frac{3B_{66}}{R} - \frac{2D_{66}}{R^2}\right]\frac{\partial^2}{\partial x^2} + \\ &+ \left[\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4}\right]\frac{\partial^2}{\partial\theta^2}, \end{aligned} \tag{19}$$

$$\begin{aligned} L_{23} &= \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial\theta} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial\theta^3} - \\ &- \left[\frac{(B_{12}+2B_{66})}{R} + \frac{(D_{12}+2D_{66})}{R^2}\right]\frac{\partial^3}{\partial x^2\partial\theta}, \end{aligned} \tag{20}$$

$$L_{31} = B_{11}\frac{\partial^3}{\partial x^3} - A_{12}\frac{1}{R}\frac{\partial}{\partial x} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x\partial\theta^2}, \tag{21}$$

$$\begin{aligned} L_{32} &= \left[-A_{22}\frac{1}{R^2} + \frac{B_{22}}{R^3}\right]\frac{\partial}{\partial\theta} + \\ &+ \left[\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right]\frac{\partial^3}{\partial\theta^3} + \\ &+ \left[\frac{(B_{12}+2B_{66})}{R} + \frac{(D_{12}+4D_{66})}{R^2}\right]\frac{\partial^3}{\partial x^2\partial\theta}, \end{aligned} \tag{22}$$

$$\begin{aligned} L_{33} &= -A_{22}\frac{1}{R^2} - D_{11}\frac{\partial^4}{\partial x^4} - \\ &- \frac{2D_{12}+4D_{66}}{R^2}\frac{\partial^4}{\partial x^2\partial\theta^2} - \frac{D_{22}}{R^4}\frac{\partial^4}{\partial\theta^4} + \\ &+ \frac{2B_{12}}{R}\frac{\partial^2}{\partial x^2} + \left[\frac{2B_{22}}{R^3}\right]\frac{\partial^2}{\partial\theta^2}, \end{aligned} \tag{23}$$

where A_{ij}, B_{ij} and D_{ij} are the tensile, coupling and bending stiffness respectively, and defined above.

2.2 Functionally graded materials

Functionally graded materials are relatively new advanced composite material. After the invitation of the FGM, great deals of research have been made on fabrication and applications of this new material concept. Functionally graded materials are characterized by gradually changed physical properties.

$$p = p_0\left[1 + p_{-1}/T + p_1T + p_2T^2 + p_3T^3\right], \tag{24}$$

where p_i are the coefficients of temperature defined in Kelvin and them are unique to the constituent materials.

$$p = \sum_{j=1}^{k} p_j V_f, \tag{25}$$

where p_j and V_f are the material property and volume fraction of the constituent material j, respectively. The sum of volume fraction is defined as

$$\sum_{k=1}^{l} V_{fl} = 1. \tag{26}$$

For an uniform thickness shell, the volume fraction is defined by

$$V_f = \left(\frac{z}{h} + \frac{1}{2}\right)^N. \tag{27}$$

The power-law exponent is defined by N. The material properties for two-constituent FGM can be defined as [31]

$$E(z) = (E_1 - E_2)\left(\frac{z}{h} + \frac{1}{2}\right)^N + E_2 \tag{28}$$

$$\nu(z) = (\nu_1 - \nu_2)\left(\frac{z}{h} + \frac{1}{2}\right)^N + \nu_2 \tag{29}$$

$$\rho(z) = (\rho_1 - \rho_2)\left(\frac{z}{h} + \frac{1}{2}\right)^N + \rho_2 \tag{30}$$

3 Discrete Singular Convolution (DSC)

The method of discrete singular convolution (DSC) is based on the principles of the theory of wavelets and the theory of distributions [46]. Discrete singular convolutions algorithm was originally introduced by Wei [47] as a potential numerical method via some regularized kernels. Hilbert and delta transforms are generally used in this approach. Since then, applications of the DSC method to various science and engineering problems have been investigated and their successes have demonstrated the potential of the method as an attractive numerical analysis technique. In this paper, details of the DSC method are not given; interested readers may refer to the works of [46–59]. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [46]

$$F(t) = (T \star \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx, \tag{31}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [53–70] that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by [47]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin\left[(\pi/\Delta)(x-x_k)\right]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0, \tag{32}$$

where $\Delta = \pi/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., $\sigma = rh$. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x - x_M, x + x_M]$. This can be expressed as [48]

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x_i - x_k) f(x_k); \quad (n = 0, 1, 2, \dots,), \tag{33}$$

where superscript n denotes the nth-order derivative with respect to x. The displacement terms are taken as

$$u = U(x)\cdot\cos(n\theta)\cdot\cos(\varpi t), \tag{34a}$$

$$v = V(x)\cdot\sin(n\theta)\cdot\cos(\varpi t), \tag{34b}$$

$$w = W(x)\cdot\cos(n\theta)\cdot\cos(\varpi t), \tag{34c}$$

where ϖ is referred to as the circular frequency parameter. Substituting Equations (34) into Equations (14), the governing equations can be written as

$$\left[S_{ij}\right]\{D\} = 0. \tag{35}$$

In this study, the following two boundary conditions are considered. The letters S and C denote simply supported and clamped boundary conditions, respectively.

Simply supported edge (S)

$$V = 0,\ W = 0,\ N_x = 0,\ M_x = 0. \tag{36}$$

Clamped edge (C)

$$U = 0,\ V = 0,\ W = 0 \text{ and } \partial W/\partial x = 0. \tag{37}$$

DSC form of the boundary conditions can be easily written. For clamped edge, for example, given as

$$U_{i,j} = 0,\ V_{i,j} = 0,\ W_{i,j} = 0 \text{ and } \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta x) W_{i+k,j} = 0. \tag{38}$$

By the DSC rule, the governing equations and the corresponding boundary conditions can be replaced by a system of simultaneously linear algebraic equations in terms of the displacements at all the sampling points. It is noted that for a well-posed problem the number of equations should be identical to the number of unknowns. A treatment commonly used in the open literature [46–70] is applied in this study. By rearranging the DSC form of the governing equations, one has the assembled form of the resulting equations as

$$[[S_{dd}]\,[S_{db}]]\left\{\begin{array}{c}\{U_d\}\\ \{U_b\}\end{array}\right\} - \Omega\,[[B_{dd}]\,[B_{db}]]\left\{\begin{array}{c}\{U_d\}\\ \{U_b\}\end{array}\right\} = \{0\}, \tag{39}$$

where $\{U_b\}$ represents the unknown boundary grid points values, whereas, $\{U_d\}$ represent the domain grid point unknowns. The subscript b represents the degree of freedom on the boundary and subscript d represents the degree of freedom on the domain. Substituting the DSC rule into the boundary conditions at the sampling points at two boundary points leads to

$$[[S_{bd}]\,[S_{bb}]]\left\{\begin{array}{c}\{U_d\}\\\{U_b\}\end{array}\right\} = \{0\}. \tag{40}$$

After rewriting Equation (40) as $U_b = -S_{bb}^{-1}S_{bd}U_d$ and then substituting the resulting equation into Equation (39), we obtain

$$SU = \Omega BU, \tag{41}$$

in which $S = S_{dd} - S_{db}S_{bb}^{-1}S_{bd}$, $B = B_{dd} - B_{db}S_{bb}^{-1}S_{bd}$ and U is the displacement vector on the domain. In this study, the numerical results are given by the dimensionless frequency parameter Ω, defined as [31]

$$\Omega = R\sqrt{\frac{\rho\left[1-\upsilon(z)^2\right]}{E(z)}}\varpi. \tag{42}$$

4 Numerical results

Table 1 lists the material properties for related three materials as T=300 K (room temperature). Three materials are denoted as Material I, Material II, and Material III for Stainless Steel, Zirconia and Nickel, respectively. By using these three materials, six different configurations (Type A, Type B, Type C, Type D, Type E, Type F) of FGM cylindrical shells are possible for inner and outer surfaces as listed in Table 2.

Firstly, a comparative study for vibration analysis of isotropic cylindrical shell and FG shell has been presented. In order to obtain a reasonable convergence for the frequency values, the number of required grid points in related directions of shells in the DSC solution should be determined.

To validate the analysis, obtained frequency values for FGM cylindrical shells are compared with the results given by Loy *et al.* [31] in the literature, as shown in Table 3 for different grid numbers. The comparison shows that the present DSC results agreed well with those in literatures. It is found from the results that, only 13 grid points can yield accurate results for frequency.

Table 4 shows the frequency values of FGM shell for six different cases and different index values. It is shown that the value of volume fraction exponent has less effect on frequency. For some types of materials (Type B, Type C, Type E) frequency values increased with the increasing value of volume fraction exponent. However, the frequencies decrease with p for the other type of material combinations (Type A, Type D, Type F). This situation is de-

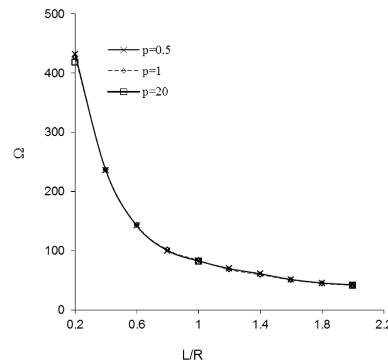

Figure 2: Variation of natural frequency with length-to-radius ratio of FGM cylindrical shell with SS edges (m=1; R/h=500; Type A)

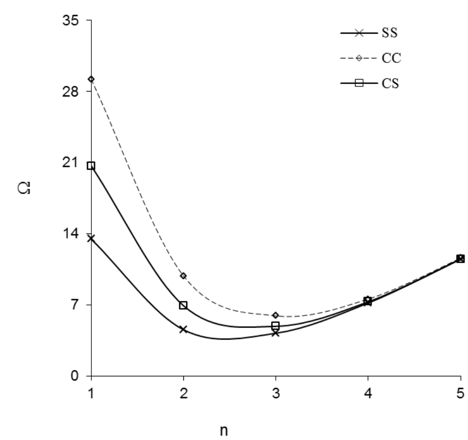

Figure 3: Variation of natural frequency with mode numbers of FGM cylindrical shell with different boundary conditions (m=1; R/h=500; Type A)

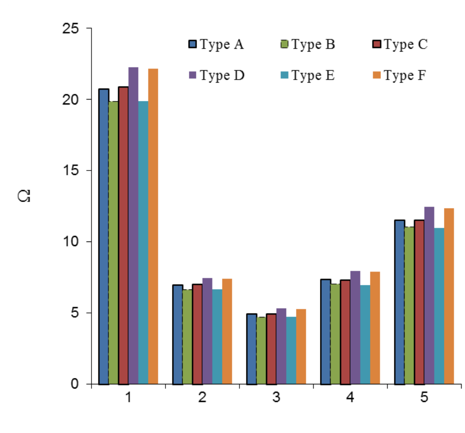

Figure 4: Frequency values for different FGM configurations of C-S shell

Table 1: Material properties of FG materials for circular cylindrical shell

Material Types	Material Constants	Coefficients				
		p_0	p	p_1	p_2	p_3
Material I	υ	0.3262	0	-2.002×10^{-4}	-3.797×10^{-7}	0
(Stainless Steel)	$\rho(kg/m^3)$	8166	0	0	0	0
	$E(N/m^2)$	201.04×10^{9}	0	3.079×10^{-4}	-6.534×10^{-7}	0
Material II	υ	0.2882	0	1.133×10^{-4}	0	0
(Zirconia)	ρ (kg/m^3)	5700	0	0	0	0
	$E(N/m^2)$	244.27×10^{9}	0	-0.1371×10^{-4}	1.133×10^{-4}	-3.681×10^{-10}
Material III	υ	0.310	0	0	0	0
(Nickel)	$\rho(kg/m^3)$	8900	0	0	0	0
	$E(N/m^2)$	223.95×10^{9}	0	2.794×10^{-4}	-3.998×10^{-9}	0

Table 2: Types of FGM circular shell for each layer (surface)

Types of FGM shells	Inner Surface	Outer Surface
Type A	Nickel	Stainless Steel
Type B	Stainless Steel	Nickel
Type C	Stainless Steel	Zirconia
Type D	Zirconia	Stainless Steel
Type E	Nickel	Zirconia
Type F	Zirconia	Nickel

pendent on the values of material parameter for two constituent materials configuration for FGM shell. Namely, the ratio of modulus of elasticity and ratio of Poisson's are significant effect on this change (decrease or increases). This phenomenon detailed investigated by Iqbal *et al.* [33]. Natural frequency of FG circular cylindrical shell with different boundary conditions is listed for material D in Table 5. It is shown that the natural frequencies initially decrease gradually and then increase with n. The frequency versus mode numbers trend is very similar to homogenous shell. Also, the frequencies values in C-C boundary conditions are greater than that for C-S and S-S boundary conditions.

Variation of natural frequencies with length-to-radius ratio (L/R) of FGM cylindrical shell with SS (both ends simply supported) edges are depicted in Figure 2 for the value of m=1; R/h=500. The material configuration is the Nickel at the inner surface and stainless steel at the outer surface. It is seen that the frequency values of FGM shells are decreased rapidly with the inceaseing value of the length-to-radius ratio. Becasue, the stiffness of the shell decrase with the increasing of L/R ratio. Also, the effect of volume fraction index on frequency is very small. The effect of boundary conditions on frequency with mode numbers is

Table 3: Comparative study of natural frequency of FG circular cylindrical shell with simply supported edges (m=1;h/R=0.002;L/R=20; Type A; p=2)

n	Ref. 31	Present DSC results			
		N=9	N=13	N=15	N=17
1	13.103	12.9982	13.1088	13.1088	13.1088
2	4.4435	4.44967	4.45013	4.45013	4.45013
3	4.1235	4.11034	4.12365	4.12365	4.12365
4	6.9820	6.98285	6.98271	6.98271	6.98271
5	11.151	11.1498	11.1516	11.1516	11.1516
6	16.323	16.3301	16.3241	16.3241	16.3241
7	22.454	22.4638	22.4550	22.4550	22.4550
8	29.533	29.6013	29.5346	29.5346	29.5346
9	37.559	37.5749	37.5602	37.5602	37.5602

shown in Figure 3. According to the figure, frequency values in C-C boundary condition is greater than that for C-S and S-S type of boundary conditions. With the increase of the mode number, teh frequency value decreases when n< 3. Then the frequency valus increase slowly to a stable value. Frequency values for different FGM configurations of C-S shell is drawn in Figure 4 for p=15; h/R=0.002 and L/R=20 values. For some types of materials (Type B, Type C, Type E) frequency values increased with the increasing value of volume fraction exponent. However, the frequencies decrease with p for the other type of material combinations (Type A, Type D, Type F). This situation is dependent on the values of material parameter for two constituent materials configuration for FGM shell. Namely, the ratio of modulus of elasticity and ratio of Poisson's are significant effect on this change (decrease or increases). This phenomenon detailed investigated by Iqbal *et al.* [33]. Vari-

Table 4: Frequency values of simply supported (S-S) FGM circular cylindrical shell with different types and modes

p	n	FGM types for inner and outer material					
		Type A	Type B	Type C	Type D	Type E	Type F
	1	13.3895	13.0387	14.2828	13.7341	14.0703	13.1821
0.3	2	4.5301	4.4160	4.8351	4.6601	4.7640	4.4671
	3	4.2120	6.9392	4.4830	4.3073	4.4150	4.1261
	1	12.9504	13.4834	13.6185	14.4623	13.003	14.3708
10	2	4.3896	4.5681	4.6082	4.9076	4.4001	4.8742
	3	4.0715	4.2369	4.2411	4.5783	4.0458	4.5425
	1	12.9334	13.5051	13.5960	14.5004	12.9682	14.4361
15	2	4.3831	4.5755	4.6020	4.9185	4.3904	4.8951
	3	4.0654	4.2451	4.2431	4.5813	4.0431	4.5562

Table 5: Natural frequencies of FG circular cylindrical shell with different boundary conditions (m=1;h/R=0.002;L/R=20; Type D; p=15)

n	SS	CC	CS
1	14.5012	31.4103	22.2627
2	4.9185	10.6005	7.4646
3	4.5813	6.41363	5.3103
4	7.6503	8.2011	7.9245
5	12.4026	12.5338	5.3087

ation of frequency values with mode number of SS shell for different constituent of material is given in Figure 5 for S-S shell. Freqeuncies values are firstly decrased (for n=1,2,3) and increasing rapidly with the mode numbers, as similar to the isotropic one.

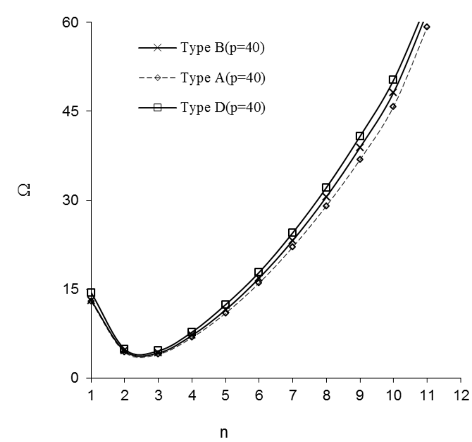

Figure 5: Variation of frequency values with mode number of SS shell for different types of FGM

5 Conclusions

By using the efficient numerical method, free vibration analysis of functionally graded cylindrical shells has been investigated. It is possible to say that the method of DSC provides a controllable numerical accuracy by using the suitable bandwidth. This is more important for large scale computations. The implementation of boundary conditions, programming and formulation procedures are found to be straightforward and simple. Being a non-iterative method, the method DSC is relatively less computationally intensive. Also the method of DSC gives reasonably accurate values for frequencies. The required computing time is very small. Although not provided here, the method is also produces more accurate results for classical single material shell.

6 Acknowledgements

The financial support of the Scientific Research Projects Unit of Akdeniz University is gratefully acknowledged.

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