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M-Polynomials and Topological Indices of Dominating David Derived Networks

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Abstract: There is a strong relationship between the chemical characteristics of chemical compounds and their molecular structures. Topological indices are numerical values associated with the chemical molecular graphs that help to understand the physical features, chemical reactivity, and biological activity of chemical compound. Thus, the study of the topological indices is important. M-polynomial helps to recover many degree-based topological indices for example Zagreb indices, Randic index, symmetric division idex, inverse sum index etc. In this article we compute *M*-polynomials of dominating David derived networks of the first type, second type and third type of dimension *n* and find some topological properties by using these M-polynomials. The results are plotted using Maple to see the dependence of topological indices on the involved parameters.

Keywords: degree-based topological index; Zagreb index; general Randić index; symmetric division index; M-polynomial, Networks.

1 Introduction

The David derived and dominating David derived network of dimension n can be constructed as follows [1]: consider a n dimensional star of David network $SD(n)$ [4]. Insert a new vertex on each edge and split it into two parts, this gives the David derived network $DD(n)$ of dimension n.

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The dominating David derived network of the first type of dimension n which can be obtained by connecting vertices of degree 2 of $DDD(n)$ by an edge that are not in the boundary and is denoted by $D_1(n)$ [1].

The dominating David derived network of the second type of dimension n can be obtained by subdividing the new edges in $D_1(n)$ [1] and is denoted by $D_1(2)$.

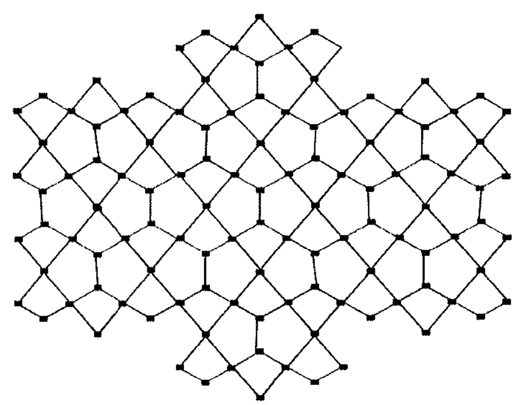

Figure 1: Dominating David derived network of the first type $D_1(2)$.

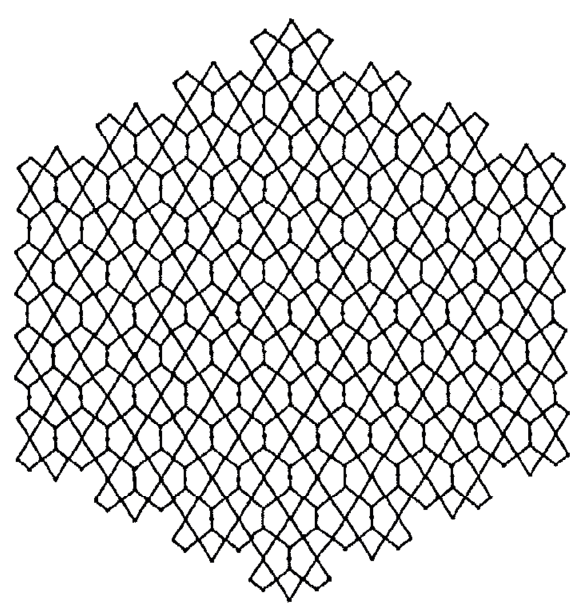

Figure 2: Dominating David derived network of the second type $D_2(4)$.

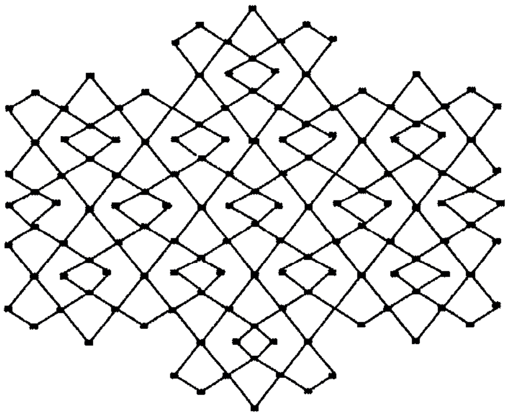

Figure 3: Dominating David derived network of the third type D_3 (2).

Table 1: Derivation of some degree-based topological indices from M-polynomial.

Topological Index	Derivation from $M(G;x,y)$
First Zagreb	$(D_x+D_y)(M(G;x,y))_{x=y=1}$
Second Zagreb	$(D_xD_y)(M(G;x,y))_{x=y=1}$
Second Modified Zagreb	$(S_xS_y)(M(G;x,y))_{x=y=1}$
Inverse Randić	$(D_x^\alpha D_y^\alpha)(M(G;x,y))_{x=y=1}$
General Randić	$(S_x^\alpha S_y^\alpha)(M(G;x,y))_{x=y=1}$
Symmetric Division Index	$(D_xS_y+S_xD_y)(M(G;x,y))_{x=y=1}$
Harmonic Index	$2S_xJ(M(G;x,y))_{x=1}$
Inverse sum Index	$S_xJD_xD_y(M(G;x,y))_{x=1}$
Augmented Zagreb Index	$S_x{}^3Q_{-2}JD_x{}^3D_y{}^3(M(G;x,y))_{x=1}$

where

$$D_x = x\frac{\partial(f(x,y))}{\partial x}, D_y = y\frac{\partial(f(x,y))}{\partial y}, S_x = \int_0^x \frac{f(t,y)}{t}dt, S_y = \int_0^y \frac{f(x,t)}{t}dt, J(f(x,y)) = f(x,x),$$

$$Q_\alpha(f(x,y)) = x^\alpha f(x,y).$$

The dominating David derived network of the second type of dimension n denoted by $D_3(n)$ can be obtained from $D_1(n)$ by introducing a parallel path of length 2 between the vertices of degree two that are not in the boundary [1,5].

In this report, M-polynomials of dominating David derived networks of the first type, second type and third type of dimension n, are computed. From these M-polynomials many degree-based topological indices are recovered. For example: first Zagreb index, second Zagreb index, modified second Zagreb index, Symmetric division index, generalized Randić index, generalized inverse Randić index, Augmented Zagreb index, etc for underlined networks. The results are plotted using maple 2015 software.

For basic definitions see [6-19].

The following table 1 relates some well-known degree-based topological indices to M-polynomial [2].

More results on the computation of these indices can refer to [20-27].

Ethical approval: The conducted research is not related to either human or animals use.

2 Main Results

2.1 M-Polynomial and topological indices of dominating David derived network of first type

Let $G = D_1(n)$ be the dominating David derived network of first type. From Figure 1, this gives vertex and edge partitions (see Table 2,3)

Theorem 1: Let $D_1(n)$ be the dominating David derived network of first type. Then the M-Polynomial of $D_1(n)$ is

$$M(D_1(n);x,y) = 4nx^2y^2 + (4n-4)x^2y^3 + (28n-16)x^2y^4 + (9n^2-3n+5)x^3y^3 + (36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4$$

Proof: Let $D_1(n)$ is the dominating David derived network of first type. It is easy to see form Figure 1 that

From Table 2, the vertex set of $D_1(n)$ have three partitions:

$$V_1(D_1(n)) = \{u \in V(D_1(n)): \quad d_u = 2\},$$

$$V_2(D_1(n)) = \{u \in V(D_1(n)): \quad d_u = 3\},$$

$$V_3(D_1(n)) = \{u \in V(D_1(n)): \quad d_u = 4\},$$

such that $|V_1(D_1(n))| = 20n-10$, $|V_2(D_1(n))| = 18n^2-26n+10$, and $|V_3(D_1(n))| = 27n^2-33n+12$.

From Table 3, the edge set of $D_1(n)$ have six partitions:

$$E_1(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 2, d_v = 2\},$$

$$E_2(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 2, d_v = 3\},$$

$$E_3(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 2, d_v = 4\},$$

$$E_4(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 3, d_v = 3\},$$

$$E_5(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 3, d_v = 4\},$$

$$E_6(D_1(n)) = \{e = uv \in E(D_1(n)): \quad d_u = 4, d_v = 4\}.$$

Table 2: The vertex partition of set of $D_1(n)$.

d_v	2	3	4
Number of vertices	$20n-10$	$18n^2-26n+10$	$27n^2-33n+12$

Table 3: The Edge partition of.

d_u	Number of edges
(2,2)	$4n$
(2,3)	$4n-4$
(2,4)	$28n-6$
(3,3)	$9n^2-13n+5$
(3,4)	$36n^2-56n+24$
(4,4)	$36n^2-52n+20$

By means of Figure 1, this gives

$$|E_1(D_1(n))| = 4n,$$

$$|E_2(D_1(n))| = 4n-4,$$

$$|E_3(D_1(n))| = 28n-16,$$

$$|E_4(D_1(n))| = 9n^2-13n+5,$$

$$|E_5(D_1(n))| = 36n^2-56n+24,$$

$$|E_6(D_1(n))| = 36n^2-52n+20.$$

Now according to the definition of the M-polynomial, this gives

$$M(D_1(n);x,y) = \sum_{i\le j} m_{ij}x^i y^j$$
$$= \sum_{uv\in E_1(D_1(n))} m_{22}x^2y^2 + \sum_{uv\in E_2(D_1(n))} m_{23}x^2y^3 + \sum_{uv\in E_3(D_1(n))} m_{24}x^2y^4$$
$$+ \sum_{uv\in E_4(D_1(n))} m_{33}x^3y^3 + \sum_{uv\in E_5(D_1(n))} m_{34}x^3y^4 + \sum_{uv\in E_6(D_1(n))} m_{44}x^4y^4$$
$$= |E_1(D_1(n))|x^2y^2 + |E_2(D_1(n))|x^2y^3 + |E_3(D_1(n))|x^2y^4 + |E_4(D_1(n))|x^3y^3$$
$$+ |E_5(D_1(n))|x^3y^4 + |E_6(D_1(n))|x^4y^4$$
$$= 4nx^2y^3 + (4n-4)x^2y^3 + (28n-16)x^2y^4 + (9n^2-13n+5)x^3y^3$$
$$+(36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4.$$

Figure 4 (shown above) is plotted by using Maple 15. This suggests that values obtained by M-polynomial show different behaviors corresponding to different parameters x and y. The values of M-polynomial can be controlled through these parameters. Clearly Figure 4 shows that along one side intercept is an upward opening parabola.

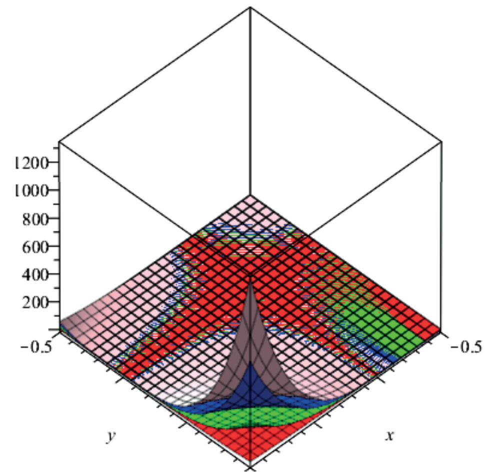

Figure 4: Plot of M-Polynomial of Dominating David Derived Network of first Type.

Next some degree-based topologcal indices of dominating David derived network of first type are computed from this M-polynomial.

Proposition 2. Let $D_1(n)$ be the dominating David derived network of first type

1. $M_1(D_1(n)) = 594n^2 - 682n + 242$
2. $M_2(D_1(n)) = 1089n^2 - 1357n + 501.$
3. ${}^mM_2(D_1(n)) = \frac{25}{4}n^2 - \frac{151}{36}n + \frac{41}{36}.$
4. $R_\alpha(D_1(n)) = 2^{2\alpha}4n + 6^\alpha(4n-4) + 8^\alpha(28n-16) + 3^{2\alpha}(9n^2-13n+5) + 12^\alpha(36n^2-56n+24) + 4^{2\alpha}(36n^2-52n+20).$
5. $RR_\alpha(D_1(n)) = \frac{1}{2^{2\alpha-2}}n + \frac{1}{6^\alpha}(4n-4) + \frac{1}{2^{3\alpha}}(28n-16) + \frac{1}{3^{2\alpha}}(9n^2-13n+5) + \frac{1}{12^\alpha}(36n^2-56n+20) + \frac{1}{2^{4\alpha}}(36n^2-52n+20).$
6. $SSD(D_1(n)) = 165n^2 - 160n + \frac{154}{3}.$
7. $H(D_1(n)) = \frac{156}{7}n^2 - \frac{102}{5}n + \frac{692}{105}.$
8. $I(D_1(n)) = \frac{2061}{14}n^2 + \frac{5201}{30}n + \frac{13127}{210}.$
9. $A(D_1(n)) = \frac{30788311}{24000}n^2 - \frac{1858527979}{216000}n + \frac{130461491}{216000}.$

Proof. Let

$$f(x,y) = 4nx^2y^2 + (4n-4)x^2y^3 + (28n-16)x^2y^4 + (9n^2-3n+5)x^3y^3$$
$$+ (36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4.$$

Then

$$D_x\left(f(x,y)\right)=8nx^2y^2+2(4n-4)x^2y^3+2(28n-16)x^2y^4+3(9n^2-13n+5)x^3y^3$$
$$+3(36n^2-52n+24)x^3y^4+4(36n^2-52n+20)x^4y^4.$$

$$D_y\left(f(x,y)\right)=8nx^2y^2+3\left(4n-4\right)x^2y^3+4(28n-16)x^2y^4+3(9n^2-13n+5)x^3y^3$$
$$+4(36n^2-52n+24)x^3y^4+4(36n^2-52n+20)x^4y^4.$$

$$(D_xD_y)\left(f(x,y)\right)=16nx^2y^2+6(4n-4)x^2y^3+8(28n-16)x^2y^4+9(9n^2-13n+5)x^3y^3$$
$$+12(36n^2-52n+24)x^3y^4+16(36n^2-52n+20)x^4y^4,$$

$$S_xS_y(f(x,y))=nx^2y^2+\frac{1}{6}(4n-4)x^2y^3+\frac{1}{8}(28n-16)x^2y^4+\frac{1}{9}(9n^2-13n+5)x^3y^3$$
$$+\frac{1}{12}(36n^2-52n+24)x^3y^4+\frac{1}{16}(36n^2-52n+20)x^4y^4$$

$$D_x^{\,a}D_y^{\,a}(f(x,y))=2^{2a}4nx^2y^2+6^a(4n-4)x^2y^3+8^a(28n-16)x^2y^4+3^{2a}(9n^2-13n+5)x^3y^3$$
$$+12^a(36n^2-52n+24)x^3y^4+4^{2a}(36n^2-52n+20)x^4y^4,$$

$$S_x^{\,a}S_y^{\,a}(f(x,y))=\frac{1}{2^{2a-2}}nx^2y^2+\frac{1}{6^a}(4n-4)x^2y^3+\frac{1}{8^a}(28n-16)x^2y^4+\frac{1}{3^{2a}}(9n^2-13n+5)$$
$$x^3y^3+\frac{1}{12^a}(36n^2-52n+24)x^3y^4+\frac{1}{4^{2a}}(36n^2-52n+20)x^4y^4,$$

$$D_xS_y(f(x,y))=4nx^2y^2+\frac{2}{3}(4n-4)x^2y^3+\frac{1}{2}(28n-16)x^2y^4+(9n^2-3n+5)x^3y^3$$
$$+\frac{3}{4}(36n^2-56n+24)x^3y^4+(36n^2-52n+20)x^4y^4,$$

$$S_xD_y(f(x,y))=4nx^2y^2+\frac{3}{2}(4n-4)x^2y^3+2(28n-16)x^2y^4+(9n^2-3n+5)x^3y^3$$
$$+\frac{4}{3}(36n^2-56n+24)x^3y^4+(36n^2-52n+20)x^4y^4,$$

$$S_xJf(x,y)=nx^4+\frac{1}{5}(4n-4)x^5+\frac{1}{6}(28n-16)x^6+\frac{1}{6}(9n^2-3n+5)x^6$$
$$+\frac{1}{7}(36n^2-56n+24)x^7+\frac{1}{8}(36n^2-52n+20)x^8,$$

$$S_xJD_xD_y\left(f(x,y)\right)=4nx^4+\frac{6}{5}(4n-4)x^5+\frac{4}{3}(28n-16)x^6+\frac{3}{2}(9n^2-3n+5)x^6$$
$$+\frac{12}{7}(36n^2-56n+24)x^7+2(36n^2-52n+20)x^8,$$

$$S_x^{\,3}Q_{-2}JD_x^{\,3}D_y^{\,3}f(x,y)=32nx^2+8(4n-4)x^3+8(28n-16)x^4+\frac{729}{64}(9n^2-3n+5)x^4$$
$$+\frac{1728}{125}(36n^2-56n+24)x^5+\frac{512}{27}(36n^2-52n+20)x^6.$$

Now in view of Table 1, this gives:

1. $M_1(D_1(n))=(D_x+D_y)(f(x,y))\big|_{x=y=1}=594n^2-682n+242.$

2. $M_2(D_1(n))=D_xD_y(f(x,y))\big|_{x=y=1}=1089n^2-1357n+501.$

3. ${}^mM_2\left(D_1(n)\right)=S_xS_y(f(x,y))\big|_{x=y=1}=\frac{25}{4}n^2-\frac{151}{36}n+\frac{41}{36}.$

4. $R_a\left(D_1(n)\right)=D_x^aD_y^a\left(f(x,y)\right)\big|_{x=y=1}=2^{2a+2}n+6^a(4n-4)+8^a(28n-16)+$
$+3^{2a}(9n^2-13n+5)+12^a(36n^2-56n+24)+2^{4a}(36n^2-52n+20).$

5. $RR_a\left(D_1(n)\right)=S_x^aS_y^a\left(f(x,y)\right)\big|_{x=y=1}=\frac{1}{2^{2a-2}}n+\frac{1}{6^a}(4n-4)+\frac{1}{2^{3a}}(28n-16)+$
$+\frac{1}{3^{2a}}(9n^2-13n+5)+\frac{1}{12^a}(36n^2-56n+20)+\frac{1}{2^{4a}}(36n^2-52n+20).$

6. $SSD\left(D_1(n)\right)=\left(S_yD_x+S_xD_y\right)\left(f(x,y)\right)\big|_{x=y=1}=$
$=165n^2-160n+\frac{154}{3}.$

7. $H\left(D_1(n)\right)=2S_xJ\left(f(x,y)\right)\big|_{x=1}=\frac{156}{7}n^2-\frac{102}{5}n+\frac{692}{105}.$

8. $I\left(D_1(n)\right)=S_xJD_xD_y\left(f(x,y)\right)_{x=1}=\frac{2061}{14}n^2+\frac{5201}{30}n+\frac{13127}{210}.$

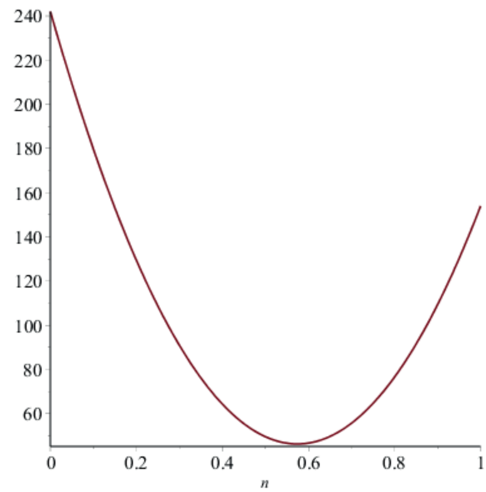

Figure 5: Plot of $M_1(D_1(n))$.

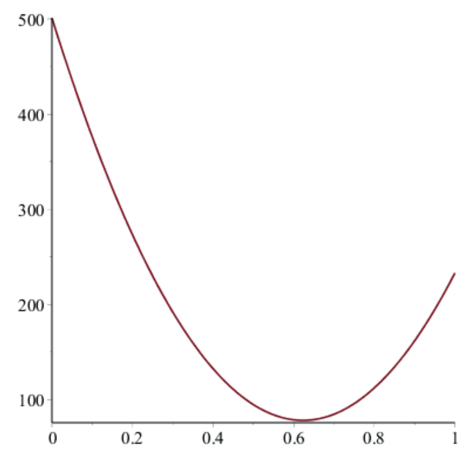

Figure 6: Plot of $M_2(D_1(n))$.

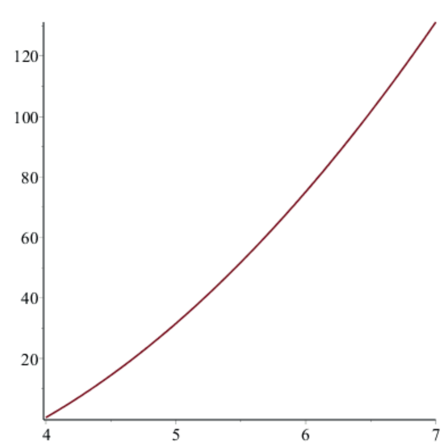

Figure 7: Plot of ${}^mM_2(D_1(n))$.

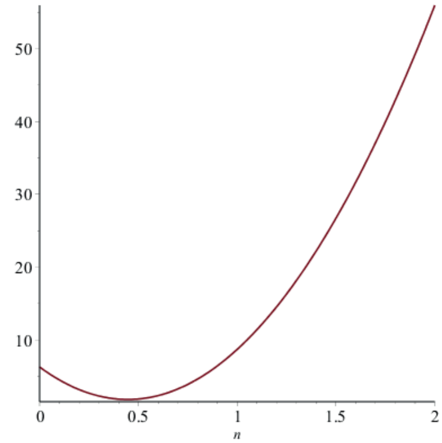

Figure 8: Plot of $R_{\frac{1}{2}}(D_1(n))$.

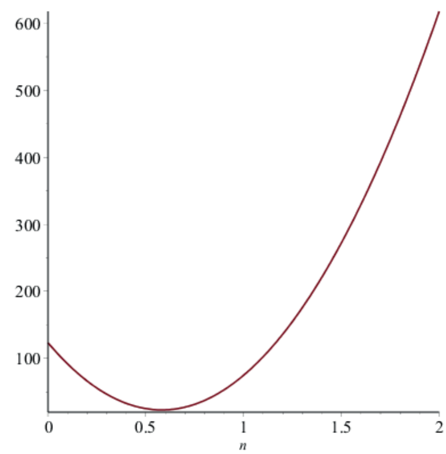

Figure 9: Plot of $RR_\alpha(D_1(n))$.

9. $A(D_1(n)) = S_x{}^3 Q_{-2} J D_x{}^3 D_y{}^3 (f(x,y))\big|_{x=1} =$
$$= \frac{30788311}{24000}n^2 - \frac{1858527979}{216000}n + \frac{130461491}{216000}.$$

2.2 M-polynomial and topological indices of Dominating David Derived network of second type

Let $D_2(n)$ be the dominating David derived network of the second type. From Figure 2, we infer the following vertex and edge partition (Table 4,5)

Theorem 3: Let $D_2(n)$ be the dominating David derived network of the second type. Then the M-Polynomial of $D_2(n)$ is
$$M((D_2(n);x,y) = 4nx^2y^2 + (18n^2-22n+6)x^2y^3 + (28n-16)x^2y^4 + (36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4.$$

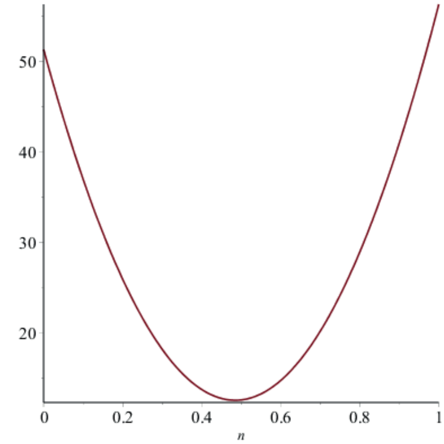

Figure 10: Plot of $SSD\,(D_1(n))$.

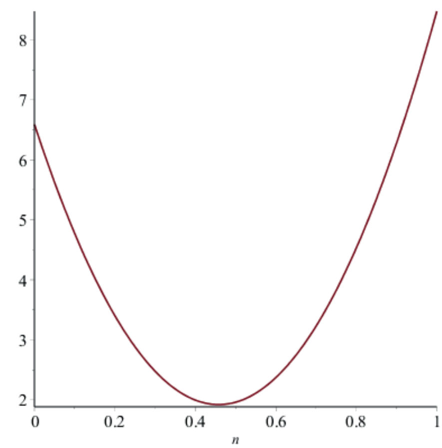

Figure 11: Plot of H $(D_1(n))$.

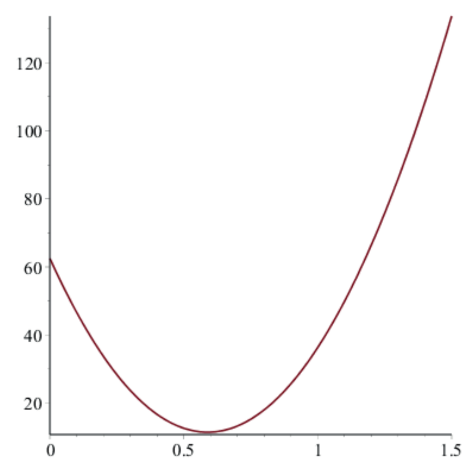

Figure 12: Plot of $I\,(D_1(n))$.

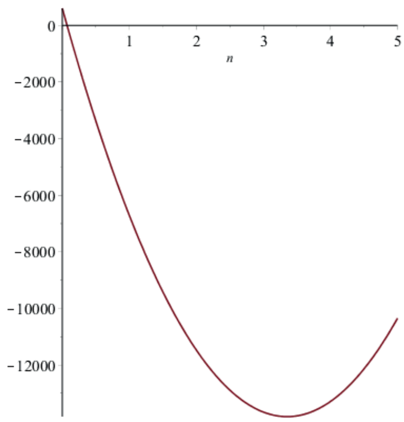

Figure 13: Plot of $A(D_1(n))$.

Proof: Let $D_2(n)$ is the dominating David derived network of second type. It is easy to see form figure 2 that there are three type of vertices in the vertex set of $D_2(n)$:

$$V_1(D_2(n)) = \{u \in V(D_2(n)):\quad d_u = 2\},$$
$$V_2(D_2(n)) = \{u \in V(D_2(n)):\quad d_u = 3\},$$
$$V_3(D_2(n)) = \{u \in V(D_2(n)):\quad d_u = 4\},$$

with

$|V_1(D_2(n))| = 9n^2 + 7n - 5$,

$|V_2(D_2(n))| = 18n^2 - 26n + 10$

and

$|V_3(D_2(n))| = 27n^2 - 33n + 12$.

Also the edge set of $D_2(n)$ has five type of edges:

$$E_1(D_2(n)) = \{e = uv \in E(D_2(n)):\quad d_u = 2, d_v = 2\},$$
$$E_2(D_2(n)) = \{e = uv \in E(D_2(n)):\quad d_u = 2, d_v = 3\},$$
$$E_3(D_2(n)) = \{e = uv \in E(D_2(n)):\quad d_u = 2, d_v = 4\},$$
$$E_4(D_2(n)) = \{e = uv \in E(D_2(n)):\quad d_u = 3, d_v = 4\},$$
$$E_5(D_2(n)) = \{e = uv \in E(D_2(n)):\quad d_u = 4, d_v = 4\}$$

such that $|E_1(D_2(n))| = 4n$, $|E_2(D_2(n))| = 18n^2 - 22n + 6$, $|E_3(D_2(n))| = 28n - 16$, $|E_4(D_2(n))| = 36n^2 - 56n + 24$, and $|E_5(D_2(n))| = 36n^2 - 52n + 20$.

In light of the definition of the M-polynomial, it is deduced that

$$M((D_2(n)); x, y) = \sum_{i \le j} m_{ij} x^i y^j$$
$$= \sum_{uv \in E_1(D_2(n))} m_{22} x^2 y^2 \sum_{uv \in E_2(D_2(n))} m_{23} x^2 y^3 + \sum_{uv \in E_3(D_2(n))} m_{24} x^2 y^4$$
$$+ \sum_{uv \in E_4(D_2(n))} m_{34} x^3 y^4 + \sum_{uv \in E_5(D_2(n))} m_{44} x^4 y^4$$

Table 4: The vertex partition of $D_2(n)$.

d_v	2	3	4
Number of vertices	$9n^2 + 7n - 5$	$18n^2 - 26n + 10$	$27n^2 - 33n + 12$

Table 5: Edge partition of $D_2(n)$.

d_u	***Number of edges***
(2,2)	$4n$
(2,3)	$18n^2 - 22n + 6$
(2,4)	$28n - 16$
(3,4)	$36n^2 - 56n + 24$
(4,4)	$36n^2 - 52n + 20$

$$= |E_1(D_2(n))| x^2 y^2 + |E_2(D_2(n))| x^2 y^3 + |E_3(D_2(n))| x^2 y^4$$
$$+ |E_4(D_2(n))| x^3 y^4 + |E_5(D_2(n))| x^4 y^4$$
$$= 4nx^2 y^3 + (18n^2 - 22n + 6)x^2 y^3 + (28n - 16)x^2 y^4 + (36n^2 -$$
$$- 56n + 24)x^3 y^4 + (36n^2 - 52n + 20)x^4 y^4.$$

Figure 14 is plotted using Maple 15. This suggests that values obtained by M-polynomial show different behaviors corresponding to different parameters x and y. We can control these values through these parameters.

Now some degree-based topologcal indices of the dominating David derived network of the second type are computed from this M-polynomial.

Proposition 4. Let $D_2(n)$ be the dominating David derived network of the second type

1. $M_1(D_2(n)) = 630n^2 - 734n + 262.$
2. $M_2(D_2(n)) = 1116n^2 - 1396n + 516.$
3. $^{m}M_2(D_2(n)) = \frac{33}{4}n^2 - \frac{85}{12}n + \frac{9}{4}.$
4. $R_\alpha(D_2(n)) = (6^\alpha 18 + 12^\alpha 36 + 12^{4\alpha} 36)n^2 + (2^{2\alpha+2} - 6^\alpha 22 + 2^{3\alpha} 28$ $- 12^\alpha 56 - 2^{4\alpha} 52 + 6^\alpha 6 - 2^{3\alpha} 16 + 12^\alpha 24 + 2^{4\alpha} 20.$
5. $RR_\alpha(D_2(n)) = \frac{1}{2^{2\alpha}} 4n + \frac{1}{6^\alpha}(18n^2 - 22n + 6) + \frac{1}{2^{3\alpha}}(28n - 16) +$ $+ \frac{1}{12^\alpha}(36n^2 - 56n + 24) + \frac{1}{2^{4\alpha}}(36n^2 - 52n + 20).$
6. $SSD(D_2(n)) = 186n^2 - \frac{1285}{6}n + \frac{254}{3}.$
7. $H(D_2(n)) = \frac{927}{35}n^2 - \frac{412}{15}n + \frac{1147}{105}.$
8. $I(D_2(n)) = \frac{5436}{35}n^2 - \frac{2776}{15}n + \frac{7036}{105}.$

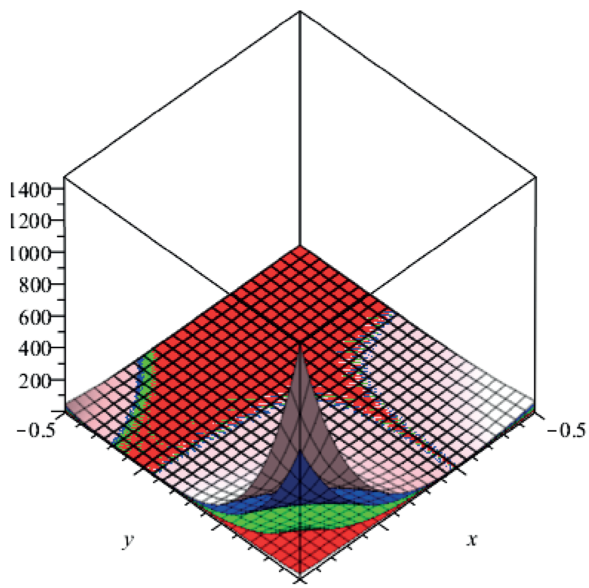

Figure 14: Plot of M-Polynomial of Dominating David Derived Network of second Type.

9. $A(D_2(n)) = \frac{496624}{375}n^2 - \frac{5670736}{3375}n + \frac{2129744}{3375}.$

Proof. Let $M(D_2(n);x,y) = f(x,y) = 4nx^2y^3 + (18n^2-22n+6)x^2y^3 + +(28n-16)x^2y^4 + (36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4$.

Then, this yields

$$D_x(f(x,y)) = 8nx^2y^2 + 2(18n^2-22n+6)x^2y^3 + 2(28n-16)x^2y^4 +3(36n^2-52n+24)x^3y^4 + 4(36n^2-52n+20)x^4y^4,$$

$$D_y(f(x,y)) = 8nx^2y^2 + 3(18n^2-22n+6)x^2y^3 + 4(28n-16)x^2y^4 +4(36n^2-52n+24)x^3y^4 + 4(36n^2-52n+20)x^4y^4,$$

$$(D_xD_y)(f(x,y)) = 16nx^2y^2 + 6(18n^2-22n+6)x^2y^3 + 8(28n-16)x^2y^4 +12(36n^2-52n+24)x^3y^4 + 16(36n^2-52n+20)x^4y^4,$$

$$S_xS_y(f(x,y)) = nx^2y^2 + \frac{1}{6}(18n^2-22n+6)x^2y^3 + \frac{1}{8}(28n-16)x^2y^4 + \frac{1}{12}(36n^2-52n+24)x^3y^4 + \frac{1}{16}(36n^2-52n+20)x^4y^4,$$

$$D_x^{\alpha}D_y^{\alpha}(f(x,y)) = 2^{2\alpha}4nx^2y^2 + 6^{\alpha}(18n^2-22n+6)x^2y^3 + 8^{\alpha}(28n-16)x^2y^4 + 12^{\alpha}(36n^2-52n+24)x^3y^4 + 4^{2\alpha}(36n^2-52n+20)x^4y^4,$$

$$S_x^{\alpha}S_y^{\alpha}(f(x,y)) = \frac{1}{2^{2\alpha}}4nx^2y^2 + \frac{1}{6^{\alpha}}(18n^2-22n+6)x^2y^3 + \frac{1}{8^{\alpha}}(28n-16)x^2y^4 + \frac{1}{12^{\alpha}}(36n^2-52n+24)x^3y^4 + \frac{1}{4^{2\alpha}}(36n^2-52n+20)x^4y^4,$$

$$D_xS_y(f(x,y)) = 4nx^2y^2 + \frac{2}{3}(18n^2-22n+6)x^2y^3 + \frac{1}{2}(28n-16)x^2y^4 + \frac{3}{4}(36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4,$$

$$S_xD_y(f(x,y)) = 4nx^2y^2 + \frac{3}{2}(18n^2-22n+6)x^2y^3 + 2(28n-16)x^2y^4 + \frac{4}{3}(36n^2-56n+24)x^3y^4 + (36n^2-52n+20)x^4y^4,$$

$$S_xJf(x,y) = nx^4 + \frac{1}{5}(18n^2-22n+6)x^5 + \frac{1}{6}(28n-16)x^6 + \frac{1}{7}(36n^2-56n+24)x^7 + \frac{1}{8}(36n^2-52n+20)x^8,$$

$$S_xJD_xD_y(f(x,y)) = 4nx^4 + \frac{6}{5}(18n^2-22n+6)x^5 + \frac{4}{3}(28n-16)x^6 + \frac{12}{7}(36n^2-56n+24)x^7 + 2(36n^2-52n+20)x^8,$$

$$S_x^3Q_{-2}JD_x^3D_y^3f(x,y) = 32nx^2 + 8(18n^2-22n+6)x^3 + 8(28n-16)x^4 + \frac{1728}{125}(36n^2-56n+24)x^5 + \frac{512}{27}(36n^2-52n+20)x^6,$$

Now from table 1

1. $M_1(D_2(n)) = (D_x+D_y)(f(x,y))\big|_{x=y=1} =$
   $= 630n^2 - 734n + 262.$
2. $M_2(D_2(n)) = D_xD_y(f(x,y))\big|_{x=y=1} = 1116n^2 - 1396n + 516.$
3. ${}^{m}M_2(D_2(n)) = S_xS_y(f(x,y))\big|_{x=y=1} = \frac{33}{4}n^2 - \frac{85}{12}n + \frac{9}{4}.$
4. $R_{\alpha}(D_2(n)) = D_x^{\alpha}D_y^{\alpha}(f(x,y))\big|_{x=y=1} = (6^{\alpha}18 - 12^{\alpha}36 - -12^{4\alpha}36)n^2 + (2^{2\alpha+2} - 6^{\alpha}22 + 2^{3\alpha}28 -12^{\alpha}56 - 2^{4\alpha}52)n + 6^{\alpha}6 - 2^{3\alpha}16 + 12^{\alpha}24 + 2^{4\alpha}20.$
5. $RR_{\alpha}(D_2(n)) = S_x^{\alpha}S_y^{\alpha}(f(x,y))\big|_{x=y=1} =$
   $= \frac{1}{2^{\alpha}}4n + \frac{1}{6^{\alpha}}(18n^2-22n+6) + \frac{1}{2^{3\alpha}}(28n-16) + \frac{1}{12^{\alpha}}(36n^2-56n+24) + \frac{1}{2^{4\alpha}}(36n^2-52n+20)$
6. $SSD(D_2(n)) = (S_yD_x + S_xD_y)(f(x,y))\big|_{x=y=1} =$
   $= 186n^2 - \frac{1285}{6}n + \frac{254}{3}$
7. $H(D_2(n)) = 2S_xJ(f(x,y))\big|_{x=1} = \frac{927}{35}n^2 - \frac{412}{15}n + \frac{1147}{105}.$
8. $I(D_2(n)) = S_xJD_xD_y(f(x,y))_{x=1} = \frac{5436}{35}n^2 - \frac{2776}{15}n + \frac{7036}{105}$
9. $A(D_2(n)) = S_x^3Q_{-2}JD_x^3D_y^3(f(x,y))\big|_{x=1} = \frac{496624}{375}n^2 - -\frac{5670736}{3375}n + \frac{2129744}{3375}$

2.3 M-polynomial and topological indices of Dominating David Derived networks of type three

Let $G = D_1(n)$ be the dominating David derived network of third type. By means of Figure 3, it is given that:

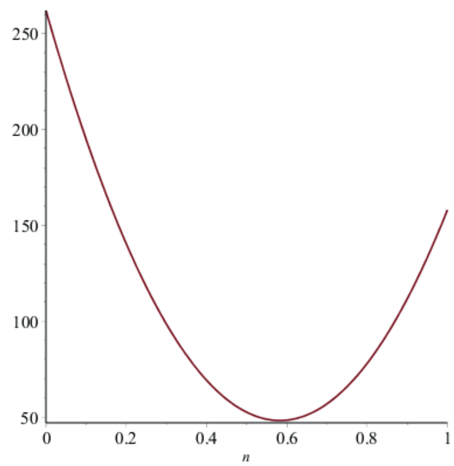

Figure 15: Plot of $M_1(D_2(n))$.

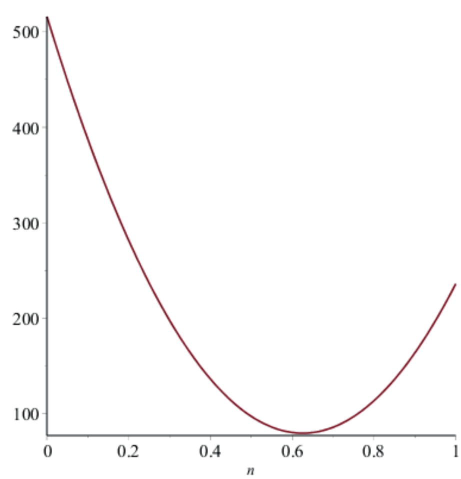

Figure 16: Plot of $M_2(D_2(n))$.

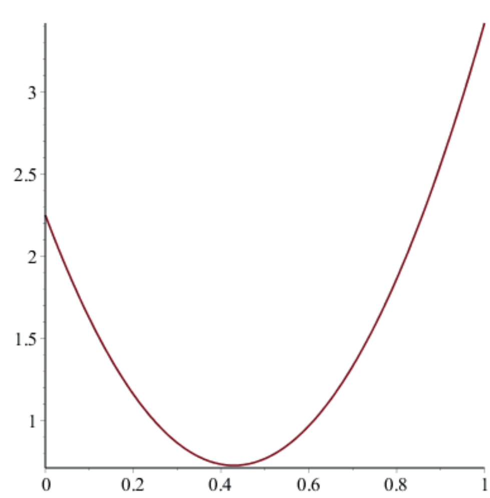

Figure 17: Plot of $^{m}M_2(D_2(n))$.

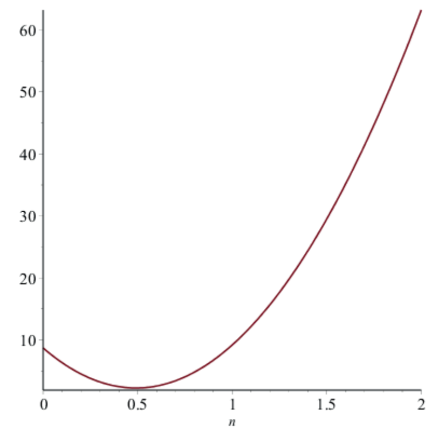

Figure 18: Plot of $R_{-1/2}(D_2(n))$.

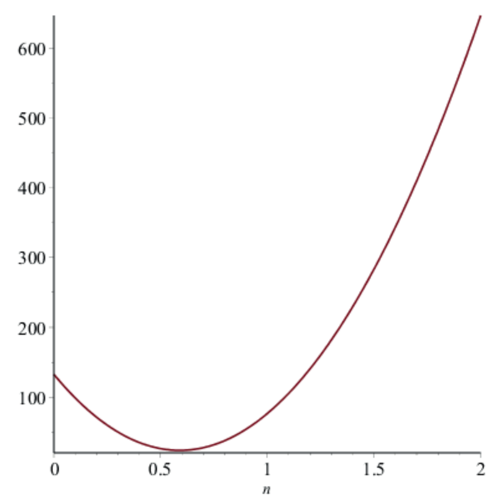

Figure 19: Plot of $RR_{-1/2}(D_2(n))$.

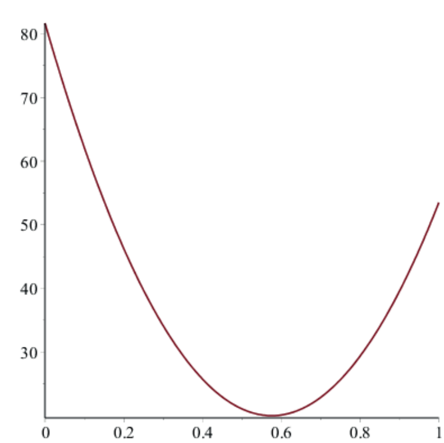

Figure 20: Plot of $SSD(D_2(n))$.

Theorem 5: Let $D_3(n)$ be the dominating David derived network of the third type. Then the M-Polynomial of $D_3(n)$ is
$M(D_3(n);x,y) = 4nx^2y^2 + (36n^2-20n)x^2y^4 + (72n^2-108n+44)x^4y^4$.

Proof: Let $D_3(n)$ is the dominating David derived network of the third type. In view of Table 6, the vertex set of $D_3(n)$ has two partitions:

$$V_1(D_3(n)) = \{u \in V(D_3(n)): \quad d_u = 2\},$$

$$V_2(D_3(n)) = \{u \in V(D_3(n)): \quad d_u = 4\},$$

such that $|V_1(D_3(n))| = 18n^2-6n$ and $|V_2(D_3(n))| = 45n^2-59n+22$.

Using Table 7, the edge set of $D_3(n)$ has three partitions:

$$E_1(D_3(n)) = \{e = uv \in E(D_3(n)): \quad d_u = 2, d_v = 2\},$$
$$E_2(D_3(n)) = \{e = uv \in E(D_3(n)): \quad d_u = 2, d_v = 4\},$$
$$E_3(D_3(n)) = \{e = uv \in E(D_3(n)): \quad d_u = 4, d_v = 4\},$$

which satisfy $|E_1(D_3(n))| = 4n$, $|E_2(D_3(n))| = 36n^2-20n$ and $|E_3(D_3(n))| = 72n^2-108n+44$.
Followed from the definition of the M-polynomial, we get

$$M(D_3(n);x,y) = \sum_{i \le j} m_{ij}x^iy^j$$
$$= \sum_{uv \in E_1(D_3(n))} m_{22}x^2y^2 + \sum_{uv \in E_2(D_3(n))} m_{24}x^2y^4 + \sum_{uv \in E_3(D_3(n))} m_{44}x^4y^4$$
$$= |E_1(D_1(n))|x^2y^2 + |E_2(D_3(n))|x^2y^4 + |E_3(D_3(n))|x^4y^4$$
$$= 4nx^2y^2 + \left(36n^2-20n\right)x^2y^4 + \left(72n^2-108n+44\right)x^4y^4.$$

Figure 24 is plotted by using Maple 15. This suggests that values obtained by M-polynomial show different behaviors corresponding to different parameters x and y. We can control values of M-polynomial through these parameters.

Now some degree-based topologcal indices of Dominating David Derived Network of the third type are computed from this M-polynomial.

Proposition 6. Let $D_3(n)$ be the Dominating David Derived Network of the third type

1. $M_1(D_3(n)) = 792n^2 - 968n + 352.$
2. $M_2(D_3(n)) = 1440n^2 - 1872n + 704.$
3. $^{m}M_2(D_3(n)) = 9n^2 - \frac{33}{4}n + \frac{11}{4}.$
4. $R_\alpha(D_3(n)) = 9(2^{3\alpha+2} + 2^{4\alpha+3})n^2 + (2^{2\alpha+2} - 5\cdot 3^{\alpha+2} + +27\cdot 2^{4\alpha+2})n + 11\cdot 2^{4\alpha+2}.$
5. $RR_\alpha(D_3(n)) = (\frac{1}{2^{4\alpha-3}} + \frac{1}{2^{3\alpha-2}})9n^2 + (\frac{1}{2^{2\alpha-2}} - \frac{5}{2^{3\alpha-2}} - -\frac{27}{2^{4\alpha-2}})n + \frac{11}{2^{4\alpha-2}}.$

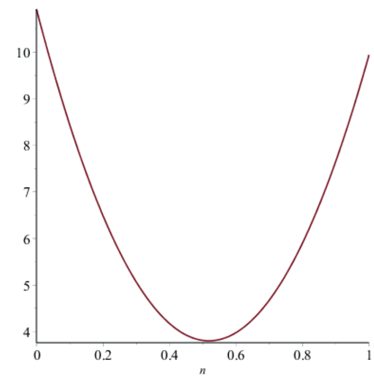

Figure 21: Plot of $H\,D_2(n)$).

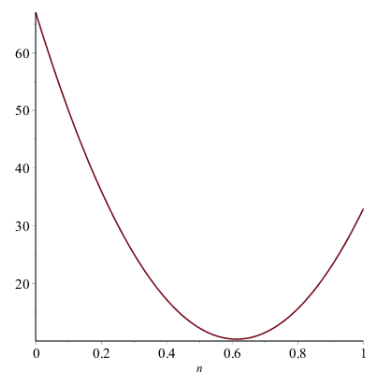

Figure 22: Plot of $I\,D_2(n)$).

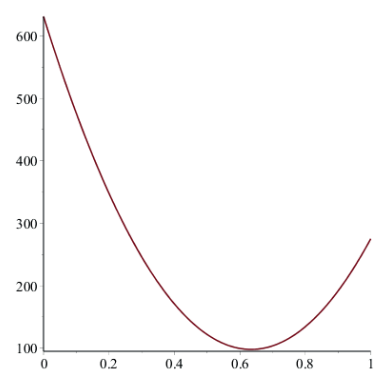

Figure 23: Plot of $A\,D_2(n)$).

Table 6: The vertex partition of $D_3(n)$.

d_v	2	4
Number of vertices	$18n^2-6n$	$45n^2-59n+22$

Table 7: Edge partition of $D_3(n)$.

d_u	*Number of edges*
(2,2)	$4n$
(2,4)	$36n^2-20n$
(4,4)	$72n^2-108n+44$

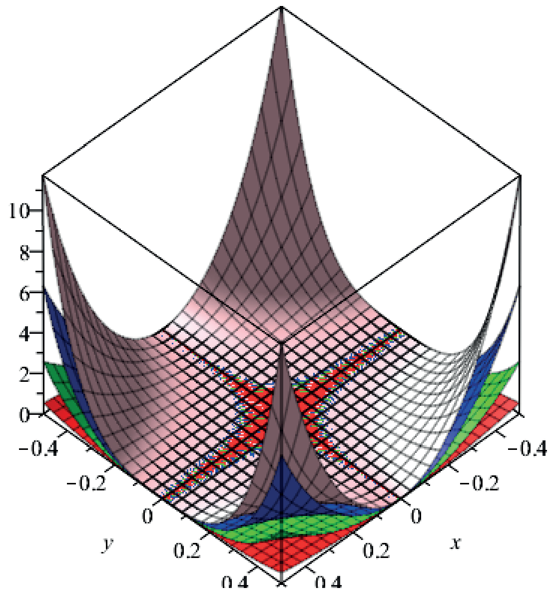

Figure 24: Plot of M-Polynomial of Dominating David Derived Network of third Type.

6. $SDD(D_3(n)) = 234n^2 - 258n + 88.$
7. $H\left(D_3(n)\right) = 30n^2 - \frac{95}{3}n + 11.$
8. $I(D_3(n)) = 192n^2 - \frac{716}{3}n + 88.$
9. $A(D_3(n)) = \frac{4960}{3}n^2 - 2176n + \frac{22528}{27}.$

Proof. Let

$$M(D_3(n);x,y) = f(x,y) = 4nx^2y^2 + \left(36n^2 - 20n\right)x^2y^4 + \\ + \left(72n^2 - 108n + 44\right)x^4y^4$$

Then, derived from this

$$D_x(f(x,y)) = 8nx^2y^2 + 2(36n^2 - 20n)x^2y^4 + 4(72n^2 - \\ -108n + 44)x^4y^4.$$

$$D_y(f(x,y)) = 8nx^2y^2 + 4(36n^2 - 20n)x^2y^4 + 4(72n^2 - \\ -108n + 44)x^4y^4$$

$$(D_xD_y)(f(x,y)) = 16nx^2y^2 + 8(36n^2 - 20n)x^2y^4 + 16(72n^2 - \\ -108n + 44)x^4y^4.$$

$$S_xS_y(f(x,y)) = nx^2y^2 + \frac{1}{8}(36n^2 - 20n)x^2y^4 + \frac{1}{16}(72n^2 - \\ -108n + 44)x^4y^4.$$

$$D_x^{\alpha}D_y^{\alpha}(f(x,y)) = 2^{2\alpha+2}nx^2y^2 + 2^{3\alpha}(36n^2 - 20n)x^2y^4 + \\ + 2^{4\alpha}(72n^2 - 108n + 44)x^4y^4.$$

$$S_x^{\alpha}S_y^{\alpha}(f(x,y)) = \frac{1}{2^{2\alpha-2}}nx^2y^2 + \frac{1}{2^{3\alpha}}(36n^2 - 20n)x^2y^4 + \\ + \frac{1}{2^{4\alpha}}(72n^2 - 108n + 44)x^4y^4.$$

$$D_xS_y(f(x,y)) = 4nx^2y^2 + \frac{1}{2}(36n^2 - 20n)x^2y^4 + (72n^2 - \\ -108n + 44)x^4y^4.$$

$$S_xD_y(f(x,y)) = 4nx^2y^2 + 2(36n^2 - 20n)x^2y^4 + (72n^2 - \\ -108n + 44)x^4y^4.$$

$$S_xJf(x,y) = nx^4 + \frac{1}{6}(36n^2 - 20n)x^6 + \frac{1}{8}(72n^2 - 108n + 44)x^8.$$

$$S_xJD_xD_y(f(x,y)) = 4nx^4 + \frac{4}{3}(36n^2 - 20n)x^6 + 2(72n^2 - \\ -108n + 44)x^8.$$

$$S_x^3Q_{-2}JD_x^3D_y^3f(x,y) = \frac{256}{2^3}nx^2 + \frac{512}{4^3}(36n^2 - 20n)x^4 + \\ + \frac{4096}{6^3}(72n^2 - 108n + 44)x^6.$$

Now from table 1

1. $M_1(D_3(n)) = (D_x + D_y)(f(x,y))\big|_{x=y=1} = 792n^2 - 968n + 352.$
2. $M_2(D_3(n)) = D_xD_y(f(x,y))\big|_{x=y=1} = 1728n^2 - 2032n + 704.$
3. ${}^{m}M_2\left(D_3(n)\right) = S_xS_y(f(x,y))\big|_{x=y=1} = 9n^2 - \frac{33}{4}n + \frac{11}{4}$
4. $R_{\alpha}\left(D_3(n)\right) = D_x^{\alpha}D_y^{\alpha}(f(x,y))\big|_{x=y=1} = 9(2^{3\alpha+2} + \\ + 2^{4\alpha+3})n^2 + (2^{2\alpha+2} - 5\cdot 3^{\alpha+2} + 27\cdot 2^{4\alpha+2})n + 11\cdot 2^{4\alpha+2}.$
5. $RR_{a}(D_3(n)) = S_x^{a}S_y^{a}(f(x,y))\big|_{x=y=1} = \\ = \left(\frac{1}{2^{4a-3}} + \frac{1}{2^{3a-2}}\right)9n^2 + \left(\frac{1}{2^{2a-2}} - \frac{5}{2^{3a-2}} - \frac{27}{2^{4a-2}}\right)n + \frac{11}{2^{4a-2}}.$
6. $SDD(D_3(n)) = (S_yD_x + S_xD_y)(f(x,y))\big|_{x=y=1} = \\ = 234n^2 - 258n + 88.$
7. $H(D_3(n)) = 2S_xJ(f(x,y))\,|_{x=1} = 30n^2 - \frac{95}{3}n + 11.$
8. $I(D_3(n)) = S_xJD_xD_y(f(x,y))_{x=1} = 192n^2 - \frac{716}{3}n + 88.$
9. $A(D_3(n)) = S_x^3Q_{-2}JD_x^3D_y^3(f(x,y))\big|_{x=1} = \frac{4960}{3}n^2 - \\ -2176n + \frac{22528}{27}.$

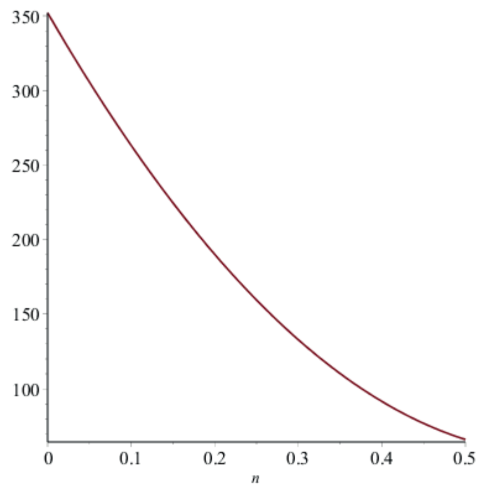

Figure 25: Plot of $M_1(D_3(n))$.

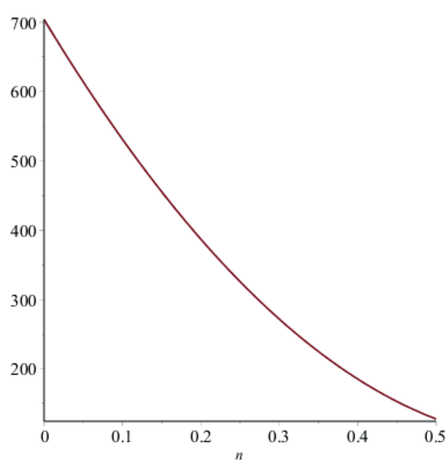

Figure 26: Plot of $M_2(D_3(n))$.

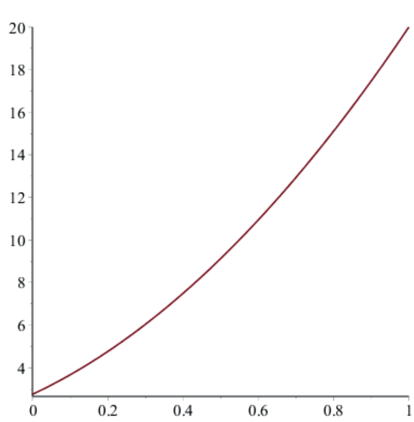

Figure 27: Plot of ${}^{m}M_2(D_3(n))$.

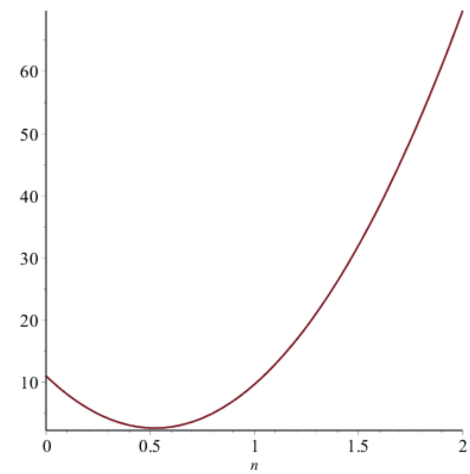

Figure 28: Plot of $R_{-1/2}(D_3(n))$.

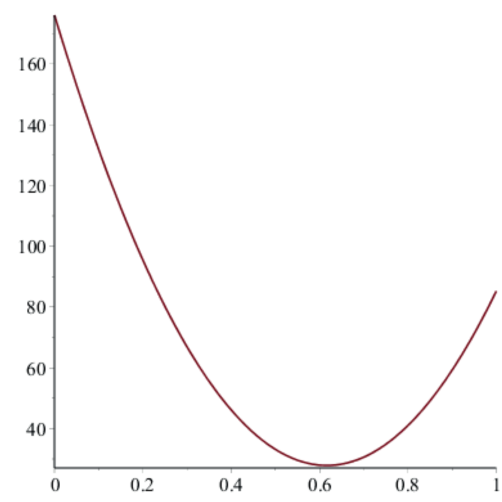

Figure 29: Plot of $RR_{-1/2}(D_3(n))$.

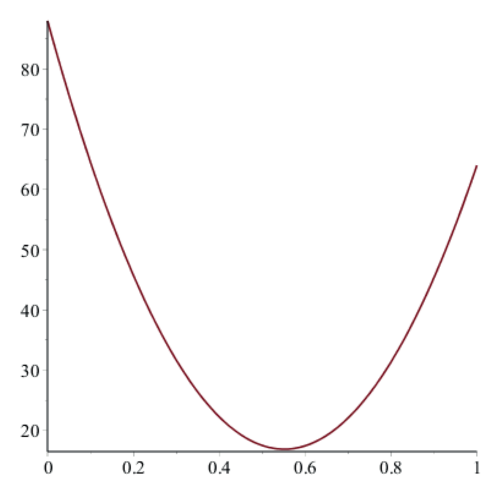

Figure 30: Plot of $SDD(D_3(n))$.

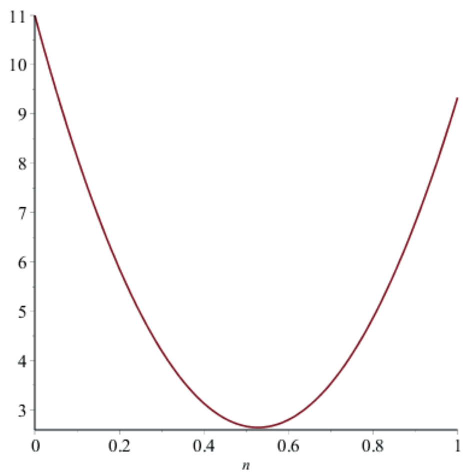

Figure 31: Plot of $H(D_3(n))$.

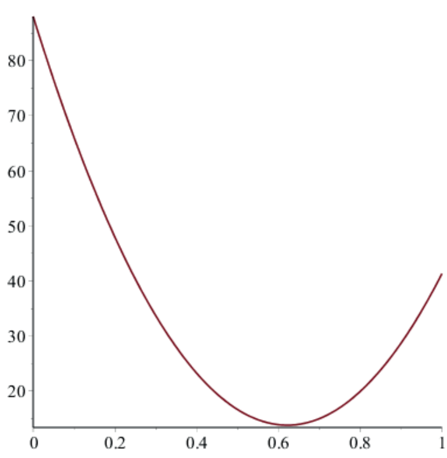

Figure 32: Plot of $I(D_3(n))$.

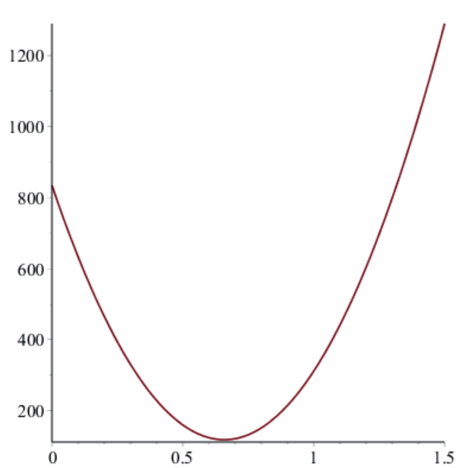

Figure 33: Plot of $A(D_3(n))$.

3 Conclusion

M-polynomials of dominating David Derived networks of the first, second and third type were computed. Many degree-based topological indices of these networks form have been recovered from their *M*-polynomials. Note that first Zagreb index and some particular cases of Randić index was calculated directly in [1].

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