The Extent to which Contingent Convertible Leasing Protects Bank Deposits:A Barrier Option Approach

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This paper proposes an alternative solution to the problem related to the risk that banks incur in the protection of deposits. This solution lies in the use by banks of contingent convertible leasing contracts to face financial distress situations by solidifying their own funds and thus improving the quality of deposit protection. Convertible contingent leases are instruments that are automatically convertible into shares when the bank reaches a level of financial distress. They allow a limited bailout of the bank in times of generalized crisis when they are not able to issue sufficient levels of new equity.

Keywords: banking, convertible contingent leases, default probability, barrier option, capital structure

1. Introduction

One of the most advanced solutions proposed for the problem of inadequate capital in adverse times is the use of contingent convertible bonds. Under IAS 17 (International Accounting Standard 17), in almost all cases, the lease would be accounted for as an operating lease and therefore off balance sheet. Banks may use leasing for their own purposes and in this case, accounting standards today require lessees to capitalize all leases (i.e. a right of use and a lease liability must be recognized on the balance sheet, see IFRS 16).

However, in terms of IFRS 16, when a bank is a lessee, although it recognizes both a lease asset and a lease liability on its balance sheet, the impact on the liquidity coverage ratio is significantly variable. The corresponding lease liabilities will increase the bank's total liabilities, resulting in a reserve requirement. In this article, we propose an alternative solution to the problem related to the risk incurred by banks in protecting deposits. This solution lies in the use by banks of contingent convertible leases to address situations of financial distress by strengthening their capital base and thus

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indirectly improving the quality of deposit protection while complying with prudential regulations for banks.

Contingent convertible leases are instruments that are automatically convertible into equity when the bank reaches a level of financial distress. They allow for a limited bailout of the bank in times of generalized crisis when it is unable to issue sufficient new capital. From the perspective of protecting bank deposits, the leasing model proposed in this article is an alternative way for the bank to increase revenues and indirectly limit deposit risk. The idea is that to meet the financing needs of its customers, the bank proceeds by purchasing an asset from a leasing company under a contingent convertible lease and selling it to the company under a standard lease. Thus, the bank acts as lessee in the case of a contingent convertible lease and as lessor through its subsidiaries. By doing so, the bank shares the risk with the leasing company and reduces the excessive deployment of deposits. The bank capital structure with a convertible contingent leasing contract is modeled to indirectly improve the quality of deposit protection while complying with prudential regulations.

We analyze a type of contingent convertible leasing. Indeed, this type of contract is automatically converted into equity based on a predetermined conversion rate in the event of financial deterioration. We present a closed form solution for the price of CoCo-Leasing, then we use this model to analyze the effect of the design of contingent convertible leasing on the stability of financial institutions in particular on the protection of bank deposits indirectly. We start by introducing a contingent convertible leasing contract in a simple bank capital structure that also includes deposits and equity. In our case, conversion occurs once the value of the asset reaches a predetermined threshold in times of financial distress, while default occurs (the bank is seized) as soon as the value of the asset falls below a threshold linked to the amount of deposits.

In this article, the valuation of the various liabilities is based on the standard option price developed by Black and Scholes (1973), Merton (1974), and Black and Cox (1976). The instability of banking, particularly in times of financial distress, is a concern that has been at the forefront of the debate surrounding the introduction of contingent convertible leasing.

In the analytical framework, we consider the stabilizing effect that contingent convertible leasing can have. Indeed, we compare a bank capital structure that includes a CoCo-Leasing that converts to equity in distress to one without CoCo-Leasing. Therefore, we show that the inclusion of contingent convertible leasing and its design can have important effects on financial stability. We focus on two channels. First, we quantify the reduction in the probability of default with a CoCo-Leasing. Second, we analyze the effect of contingent convertible leasing contract design on risk incentives. We find that for any level of leverage and asset volatility, a financial institution with contingent convertible leasing has a lower probability of default than one without. The effectiveness of contingent convertible leasing as a stabilization tool is therefore more pronounced for a bank's asset risk levels. We then analyze the impact on risk incentives of including contingent convertible leasing in the capital structure. In addition, incentives that are sensitive to asset risk can reduce management focus on maximizing the overall value of bank assets.

2. Literature Review

One area of considerable interest in finance concerns the potential leases. Our study contributes to the contingent convertible leasing contracts of the financial literature. This research contributes to the problem linked to the credit risk that banks incur to deal with situations of financial distress by strengthening their financial base and thus indirectly improving the quality of deposit protection while respecting the prudential rules of banks.

Leasing is the process by which an owner sells the use of an asset for a specified period to a lessee who promises to make payments over the term of the lease. This is the standard assumption for lease valuation according to Miller and Upton (1976), McConnell and Schallheim (1983), Schallheim and McConnell (1985) and Grenadier (1995, 1996). In the older literature, Lewis and Schallheim (1992) and Grenadier (1996) offered traditional rental rate models and recognize the importance of tenant default and hence tenant credit risk. Lease *et al.* (1990) documented high realized returns on leases, although realized returns were lower than expected returns. In addition, they found that realized salvage values tended to greatly exceed the actual salvage values on which the lease was based. Grenadier (1995) was the first to apply a continuous-time model and competitive market rationale to derive the term structure of lease rates. In Grenadier's model, lease rates are endogenously determined, and driven by the trade-off decision between construction cost and developer's profit. Hence, the lease market equilibrium is determined by a firm's decision given a competitive market assumption. However, the credit risk is not considered. Grenadier (1996) incorporated default risk into lease rate determination also by using a competitive market rationale. In this article, the default process is modeled on the first passage time that the lessee firm's asset value hits a given bankruptcy level, which is the so called "structural model" in the broader credit risk literature. In Grenadier's (1996) analysis, the lease rate credit spread, which is defined as the difference between the lease rate and the risk free rate, is influenced by the lease duration, default recovery rate, correlation between leased assets and the lessee firm' assets, etc. However, only the exogenous default level is considered.

From this standpoint, the lessee firm's capital structure decision is not relevant in determining lease rates. In addition, the lessee firm's likelihood to default is not endogenously related to how much it pays for its lease service. Lewis and Schallheim (1992) demonstrated that leasing can induce a firm to take on more debt than it otherwise would.

Our article extends this framework to incorporate the risks of lessee default in the lease rate duration structure and its endogenous effect on the capital structures of lessees. Our article is also linked to work on modeling by default (Duffie and Singleton, 1999; Zhou, 2001; Duffie *et al.* , 2009), and counterparty risk modeling (Jarrow and Yu, 2001). Indeed, Jarrow and Yildirim (2002) derived a simple analytical formula for credit default swaps that incorporates the correlation of credit and market risks.

Currently, prior literature investigates the associations between operating leases and credit risk, in addition to equity risk. Prior studies indicate that operating leases constructively capitalized with footnote information are associated with cost of debt (Bratten *et al*. (2013), Kraft (2015)). For example, Bratten *et al*. (2013) showed that operating lease were risk-relevant in explaining the debt yield gap and documented that the associations between finance lease obligations versus operating lease obligations and cost of debt are not substantially different. However, when operating lease information is less reliable, they indicated that the associations between recognized finance leases versus disclosed operating leases and cost of debt were materially different.

A recent study by Devos and Li (2016) argueed that operating leases were embedded in real option hedging properties, and they found that risk-taking incentives were negatively related to the intensity of firms' operating leases. Triki and Abid (2022) studied the contingent convertible lease as a risk management instrument between the lessor and lessee. They developed three asset lease models to manage the lessee's credit risk using a contingent claim approach. They found that using convertible leases in the firm's capital structure can reduce asset substitution and insolvency inefficiencies for a conversion rate close to 1.

On the other hand, our article proposes the introduction of contingent capital in the leasing sector. In this sense, our article contributes to research on bank failure. We offer a contingent convertible credit lease contract which represents a commitment between a lessor (Leasing) and a lessee (bank) and is automatically converted into equity on the basis of a predetermined conversion rate in the event of bank default. The advantage of our model is to examine a capital structure with a contingent convertible credit lease contract. Our model also makes it possible in capturing the credit risk in order to indirectly protect bank deposits.

3. A Model for Pricing Bank Capital with CoCo-Leasing and Default Probability

We define a bank capital structure that includes a contingent convertible Leasing (CoCo-Leasing) contract, deposits and equity as follows:

$$V = CL + D + E \tag{1}$$

In this article, we will present a continuous time valuation to determine the pricing of bank securities, in particular contingent convertible leases. In this model, the economic benefits of the service flow are realized by the user of the asset, while the asset owner retains the right to sell the service flow to potential lessees. At each point in time, the value of the asset's service flow, or the instantaneous rental rate $s(t)$ follows a diffusion process:

$$ds_t = \alpha_s s_t\, dt + \sigma_s s_t dz_\alpha \tag{2}$$

where α_s denotes the expected instantaneous conditional percentage change of s per unit of time, σ_s is the expected instantaneous conditional standard deviation per unit of time and dz_α is a standard Wiener process. The sign of the asset service cash flow growth rate is not restrictive which means that the asset service cash flow appreciates or depreciates over time. We assume risk neutrality.

According to Grenadier (1996) and in the case where the company does not default and all promised rental payments are refunded, the value of the use of the asset for the year T, $Y(s,T)$ is expressed by:

$$Y(s,T) = E\left(\int_0^T e^{-rt} s_t\, dt\right) = \frac{s}{r-\alpha_s}\left[1-e^{-(r-\alpha_s)T}\right] \tag{3}$$ [1]

The risk-free breakeven rental, $R(T)$, is equal to the payment flow including the rental value $Y(s,T)$, and $R(T)$ satisfies the following equilibrium equality:

$$R(T) = \frac{r}{1-e^{-rT}} Y(s,T) = \frac{r}{1-e^{-rT}} \frac{s}{r-\alpha_s}\left[1-e^{-(r-\alpha_s)T}\right] \tag{4}$$

We assume that the value of bank assets v_t follows a diffusion process in the risk-

[1] For the derivations, please refer to the Appendix of this paper on the Journal's website.

neutral probability measure:

$$dv_t = \alpha_v v_t dt + \sigma_v v_t dz_v \tag{5}$$

where α_v denotes the drift of the process in the risk-neutral measure, σ_v is the volatility and z_v is a standard Brownian motion.

To model the default event, we follow Black and Cox (1976).

When the bank encounters financial difficulties, the value of its assets declines. Specifically, the event of default occurs and the bank cannot reimburse its payments, the triggering of the conversion occurs and the holder of the CoCo-leasing becomes the owner in the bank. In this case, the value of the bank rises and the conversion prevents the default. However, if the bank's default persists, bank deposits may be damaged and the bank can only recover a fraction of its deposits.

In this case, the promised lease payments are not repaid. Thus, default in leasing is contractually defined and may consist of a delay in payment (Grenadier, 1996). Whereas in our case, default occurs when the first lease payment is not repaid. Thus, we assume that default occurs when the value of the assets banking falls below a predefined threshold. In effect, the lease default occurs when the bank (lessee) becomes insolvent.

As a result, the default of payment occurs at the moment T_D where $T_D = inf\left\{0 < t < T \text{ , } v_t \le x_k\right\}$ and if $T_D = \infty$ implies that there is no default and the payment is refunded.

If v_t reaches a default barrier x_k, such as $x_k = R_T\left(1-\psi\right)$, $(0 \le \psi \le 1)$, then the first lease payment is not paid and the default occurs. In this event, the conversion will occur, the CoCo-Leasing holder receives a share β $\left(0 \le \beta \le 1\right)$ of the equity and the previous shareholders receive the remaining $\left(1-\beta\right)$. β indicates the number of shares into which each CoCo-Leasing will be converted, and it is the ratio of the conversion amount to the conversion price.

This assumption follows Black and Cox (1976) and captures the fact that the regulator has limited ability to seize the bank at the moment it becomes insolvent either because of imperfect information due to discrete audit frequency (Duffie and Lando, 2001), or simply choosing a policy where banks are not immediately seized. Assuming $\psi = 0$ represents a perfect ability of the regulator to seize the bank immediately when insolvency is reached and results in an inability of shareholders to transfer wealth from depositors by changing the volatility of the asset.

We can think of the size of as being related to the ability and willingness of the regulator to closely monitor and enforce bank solvency. In fact, the depletion of the Federal Deposit Insurance Corporation (FDIC) Deposit Insurance Fund (DIF) during the financial crisis is likely a consequence of the difficulty faced by the regulator in shutting down banks exactly at the moment when asset value reaches the value of liabilities. See also Prescott (2011), who pointed out that, despite the policy of ‘prompt corrective action’ by the FDIC, during the crisis losses to the deposit insurance fund were substantial.

A new default time for deposits is defined as : $T_D' = inf\{0 < t < T\ , v_t \leq x_k'\}$, with x_k' is the a default barrier for the deposits which is expressed as follows: $x_k' = C_s(1-\psi)$, with $0 \leq \psi \leq 1$, and C_s is coupon deposit. When v_t reaches the a default barrier x_k' , the default event occurs and the bank becomes insolvent and can only recover a fraction $(1-w)$ of $v_{T_D'}$ where $(1-w)$ represents the recovery rate in the event of default.

First of all, to determine the value of CoCo-Leasing, debt direct and the equity, Table 1 below defines the following possible scenarios:

(1) No default event: in this case, the value of the bank’s assets does not touch the conversion threshold until the debt matures and the CoCo-Leasing holder is fully paid.

(2) Financial distress level triggering lease conversion preventing from default: as the bank falls into financial distress, conversion will take place, the value of the bank’s assets has reached the lower conversion threshold and, as a result, the value of the bank’s assets rises and the CoCo-Leasing holder receives a predetermined ratio of the bank’s assets in exchange for its debt while the former shareholders receive the residual assets.

(3) Default only on deposits: in this case, the bank capital structure is assumed to include only deposits and equity.

(4) Financial distress level triggering lease conversion insufficient to preventing from default: if the conversion occurs but the value of the bank’s assets is still declining, then the default occurs on the bank’s deposits and the bank becomes insolvent.

Table 1. Payoffs to Claimholders for Capital Structures with CoCo-Leasing

Liability Type	Scenarios			
	(1)	(2)	(3)	(4)
CoCo-Leasing	R_T	βE_{T_D}	0	βE_{T_D}
Deposits	C_s	C_s	$(1-w)v_{T_D'}$	$(1-w)v_{T_D'}$
Equity	$v_T - R_T - C_s$	$v_{T_D} - \beta E_{T_D} - C_s$	$wv_{T_D'}$	$wv_{T_D'} - \beta E_{T_D}$

where v_{T_D} is the value of asset at the time of default for CoCo-Leasing and deposits.

To evaluate a CoCo-Leasing contract, we use a combination of differential barrier options (form-closed solution). The peculiarity of this type of option is that their exercise can be activated or deactivated because the underlying reaches (or not) a predefined threshold (the barrier).

The value of the deposits is derived from future cash flows that can be received in these two events: if the default does not occur during the life period of the option then the yield to maturity equal to C_s and consequently this yield can be replicated by C_s units of a long position in a "down and out" barrier option rated $DB^{dout}\left(x_k^{'}\right)$ and its yield at maturity is $DB^{dout}\left(x_k^{'}\right)=C_s 1_{\{T_D{'}>T\}}$; if the default occurs, then the yield at maturity can be replicated by $(1-w)$ units in a "down and in" barrier option rated $DB^{din}\left(x_k^{'}\right)$ and it equal to $DB^{din}\left(x_k^{'}\right)=v_{T_D{'}} 1_{\{T_D{'}<T\}}$.

Hence the value of deposits can be expressed as follows:

$$D = DB^{dout}\left(x_k^{'}\right)+(1-w)DB^{din}\left(x_k^{'}\right) = E^Q\left[e^{-rt}\left(C_s 1_{\{T_D{'}>T\}} + (1-w)v_{T_D{'}} 1_{\{T_D{'}\leq T\}}\right)\right] \tag{6}$$

with E^Q as the expectation in the neutral risk measure Q.

The value of the CoCo-Leasing is derived from future cash flows that can be received in these two events: if the default does not occur during the life period of the option then the yield to maturity equal to R_T and hence this performance can be replicated by R_T long position units in a "down and out" barrier option noted $DB^{dout}\left(x_k\right)$ and its yield to maturity is $DB^{dout}\left(x_k\right)=R_T 1_{\{T_D>T\}}$; in the event of conversion and default, the yield at maturity is replicated by β units of a "down and in" barrier option noted $DB^{din}\left(x_k\right)$ and it equal to $DB^{din}\left(x_k\right) = E_{T_D} 1_{\{T_D<T\}}$.

Then, the value of a CoCo-Leasing is expressed by:

$$CL = DB^{dout}\left(x_k\right)+\beta DB^{din}\left(x_k\right) = E^Q\left[e^{-rt}\left[R_T 1_{\{T_D>T\}} + \beta E_{T_D} 1_{\{T_D<T\}}\right]\right] \tag{7}$$

Finally, the equity value is derived from the future cash flows that can be received in these events: if the default does not occur on the deposits or on the CoCo-Leasing during the life of the option and therefore the performance can be replicated by a "down and out" barrier call option denoted note $CB^{dout}\left(min(x_k, x_k^{'}), R_T + C_s\right)$

with an exercise price $\left(R_T + C_s\right)$ and a barrier $min(x_k, x_k^{'})$ and its yield to maturity is $CB^{dout}\left(min(x_k, x_k^{'}), R_T + C_s\right) = max\left(v_T - \left(R_T + C_s\right), 0\right)1_{\{T_D > T,\ T_D' > T\}}$; in the event of conversion and by preventing the default, the yield to maturity is replicated by a "down and in" barrier call option denoted $CB^{din}\left(x_k, \beta E_{T_D} + C_s\right)$ with an exercise price $\left(\beta E_{T_D} + C_s\right)$ and a barrier x_k and its yield to maturity is $CB^{din}\left(x_k, \beta E_{T_D} + C_s\right) = max\left(v_{T_D} - \left(\beta E_{T_D} + C_s\right), 0\right)1_{\{T_D \le T,\ T_D' > T\}}$. If the conversion is insufficient to prevent default, the yield to maturity is replicated by a down and in barrier call option denoted $CB^{din}\left(min(x_k, x_k^{'}), \beta E_{T_D} + (1-w)v_{T_D'}\right)$ with an exercise price $\left(\beta E_{T_D} + (1-w)v_{T_D'}\right)$ and a barrier $min(x_k, x_k^{'})$ and its yield to maturity is $CB^{din}\left(min(x_k, x_k^{'}), \beta E_{T_D} + (1-w)v_{T_D'}\right) = max\left(v_{T_D'} - \left(\beta E_{T_D} + (1-w)v_{T_D'}\right), 0\right)1_{\{T_D \le T,\ T_D' \le T\}}$.

The value of equity is determined by the same mutually exclusive events. Thus, the value of equity is given by:

$$
\begin{aligned}
E &= CB^{dout}\left(min(x_k, x_k^{'}), R_T + C_s\right) + CB^{din}\left(x_k, \beta v_{T_D} + C_s\right) \\
&\quad + CB^{din}\left(min(x_k, x_k^{'}), \beta E_{T_D} + (1-w)v_{T_D'}\right) \\
&= E^Q\Big[e^{-rt}\big[\ (v_T - R_T - C_s)1_{\{T_D > T, T_D' > T\}} + \left((1-\beta)v_{T_D} - C_s\right)1_{\{T_D \le T,\ T_D' > T\}} \\
&\quad + \left(wv_{T_D'} - \beta E_{T_D}\right)1_{\{T_D \le T, T_D' \le T\}}\ \big]\Big]
\end{aligned}
\tag{8}
$$

In order to quantify the effect of the introduction of a CoCo-Leasing and deposits into the capital structure, we calculate the probability of default with a CoCo-Leasing and deposits and perform an analysis of the risk change incentives.

Policymakers are interested in monitoring the probability of default of a financial institution because of the deleterious effect of default on the real economy. Indeed, an important motivation for the introduction of a CoCo-Leasing is its ability to absorb losses and reduce the probability of default. In effect, this reduction is quantified by comparing the case of CoCo-Leasing with deposits. In addition, the capital structure includes CoCo-Leasing, deposits and equity.

Referring to Black and Cox (1976), the default occurs the first time the value of the assets falls below a certain barrier $max\left(x_k, x_k^{'}\right)$. The default event occurs for the first time at $0 < t < T$ at which the value of v_t falls below the level where the default event occurs for the first time at maturity. This is explained by the right of bondholders to exercise a “security clause” which enables them to liquidate the firm if, at any moment, its value falls below the specified threshold. Thus, the default time is given by: $T_D^{"} = inf\left\{0 < t < T \text{ , } v_t \le max\left(x_k, x_k^{'}\right)\right\} = inf\left\{0 < t < T \text{ , } ln\left(\frac{max\left(x_k, x_k^{'}\right)}{v_t}\right) \ge 0\right\}$.

Thus, the default is defined as follows: the value of the assets touches the barrier $max\left(x_k, x_k^{'}\right)$ from above at any time before T or at maturity, the asset value is greater than $max\left(x_k, x_k^{'}\right)$ but lower than $\left(R_T + C_s\right)$.

According to Black and Cox, let $m_t = min_{0<t<T}\, v_t$ the first time that the asset value process crosses the barrier and v_t a Brownian motion with $\alpha_v t$ drift and variance $\sigma_v{}^2 t$.

Suppose $f\left(y\right)$ is the probability density of v_t and $g\left(y,x\right)$ is the joint probability density with $x = ln\left(\frac{max\left(x_k, x_k^{'}\right)}{v_t}\right)$.

Let $N\left(.\right)$ is the normal distribution.

The probability of default before maturity is given by:

$$
\begin{aligned}
P_r\left(T_D" < T\right) &= P_r\left(min_{0<t<T}\, v_t \le x\right) \\
&= N\left(\frac{\ln\left(\frac{max\left(x_k, x_k^{'}\right)}{v_t}\right) - \alpha_v T}{\sigma_v \sqrt{T}}\right) \\
&+ \left(\frac{max\left(x_k, x_k^{'}\right)}{v_t}\right)^{\left(2\alpha_v / \sigma_v{}^2\right)} \\
&N\left(\frac{\ln\left(\frac{max\left(x_k, x_k^{'}\right)}{v_t}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right)
\end{aligned} \tag{9}
$$

Regarding the second event above,

$$
\begin{aligned}
&P_r(max\left(x_k,x_k^{'}\right)<v_T<\left(R_T+C_s\right),T_D^{"}\geq T) \\
&=P_r\left(max\left(x_k,x_k^{'}\right)<v_T<\left(R_T+C_s\right),min_{0<t<T}\,v_t>x\right) \\
&=N\left(\frac{\ln\left(\frac{v_t}{max\left(x_k,x_k^{'}\right)}\right)+\alpha_v T}{\sigma_v\sqrt{T}}\right)-N\left(\frac{\ln\left(\frac{v_t}{R_T+C_s}\right)+\alpha_v T}{\sigma_v\sqrt{T}}\right) \\
&-\left(\frac{max\left(x_k,x_k^{'}\right)}{v_t}\right)^{\left(2\alpha_v/\sigma_v^2\right)}\left(N\left(\frac{\ln\left(\frac{max\left(x_k,x_k^{'}\right)}{v_t}\right)+\alpha_v T}{\sigma_v\sqrt{T}}\right)-N\left(\frac{\ln\left(\frac{\left(max\left(x_k,x_k^{'}\right)\right)^2}{v_t\left(R_T+C_s\right)}\right)+\alpha_v T}{\sigma_v\sqrt{T}}\right)\right)
\end{aligned}
\tag{10}
$$

In summary, following the two probabilities of default at maturity and anticipated, we obtain the following formula of the probability of default for a capital structure with deposits and CoCo-Leasing as follows:

$$
\begin{aligned}
PD &= P_r\left(T_D"<T\right)+P_r(max\left(x_k,x_k^{'}\right)<v_T<\left(R_T+C_s\right),T_D^{"}\geq T) \\
&=1-N\left(d_2^{"}\right)+\left(\frac{max\left(x_k,x_k^{'}\right)}{v_t}\right)^{\left(2\alpha_v/\sigma_v^2\right)}N\left(d_8^{"}\right)
\end{aligned}
\tag{11}
$$

with $d_2^{"}=\frac{\ln\left(\frac{v_t}{R_T+C_s}\right)+\alpha_v T}{\sigma_v\sqrt{T}}$ and $d_8^{"}=\frac{\ln\left(\frac{max\left(x_k,x_k^{'}\right))^2}{v_t\left(R_T+C_s\right)}\right)+\alpha_v T}{\sigma_v\sqrt{T}}$.

Corollary: if $CL=0$, then the price of deposits is the same as in the general case but the equity price is affected by the result of the same two mutually exclusive events as $\left\{T_D^{'}>T,T\geq T_D{'}\right\}$ in the case of: no default and default. The value of the equity is given by:

$$E' = CB^{dout}\left(x_k^{'}, C_s\right) + wDB^{din}\left(x_k^{'}\right) = E^{Q}\left[e^{-rt}\left((v_T - C_s)1_{\{T_D{'}>T\}} + wv_{T_D{'}}1_{\{T_D{'}\leq T\}}\right)\right] \tag{12}$$

with $CB^{dout}\left(x_k^{'}, C_s\right)$ and $DB^{din}\left(x_k^{'}\right)$ are a "down and out" barrier call option with an exercise price C_s and a barrier $x_k^{'}$ and a "down and in" barrier option with a barrier $x_k^{'}$, respectively.

For this case, the probability of default can be written as follows:

$$PDD = 1 - N\left(d_2^{'}\right) + \left(\frac{x_k^{'}}{v_t}\right)^{\left(2\alpha_v/\sigma_v^2\right)} N\left(d_8^{'}\right) \tag{13}$$

with $d_2^{'} = \dfrac{\ln\left(\dfrac{v_t}{C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}$ and $d_8^{'} = \dfrac{\ln\left(\dfrac{x_k^{'2}}{v_t C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}$.

Indeed, this default probability for a capital structure including only deposits is the same as a capital structure with CoCo-Leasing and deposits that the default occurs as soon as the value of the assets is less than C_s instead of $R_T + C_s$. So, simply replace $R_T + C_s$ by C_s, T_D'' by $T_D{'}$ and $max\left(x_k, x_k^{'}\right)$ by $x_k^{'}$ in the previous equation of the default probability with a CoCo-Leasing and deposits.

4. Numerical Results

The values of the basic parameters in our work are displayed as follows:

Given that major audits are scheduled once a year, we have chosen a maturity $T=1$ year.

The resulting bank asset value implied by a leverage ratio of 0.907 is equal to $v_t = 108.02$. By referring to the literature, the nominal values of CoCo-Leasing and Deposits are $D=100$ and $CL=3$, respectively. Indeed, the nominal value of CoCo-Leasing represents 3% of deposits.

The conversion ratio is $\beta=0.5$. A constant risk-free interest rate is $r=5\%$. The risk of the asset is equal to $\sigma_v = 9\%$. The drift of the service asset cash flow process and the asset value process are respectively $\alpha_s = 0.2\%$ and $\alpha_v = 4.6\%$ (referred to Black and Scholes (1973) and under the neutral risk measure, the value of α_v must be calibrated under the following equality $\alpha_v = r - \frac{\sigma_v^2}{2}$). It is assumed that the default occurs for the first time as soon as the value of the assets reaches $\varphi = 3\%$.

Based on the equations of the probability of default in the previous section and the basic parameter values, the default probability with deposits and CoCo-Leasing is 0.44 and that with deposits is 0.46.

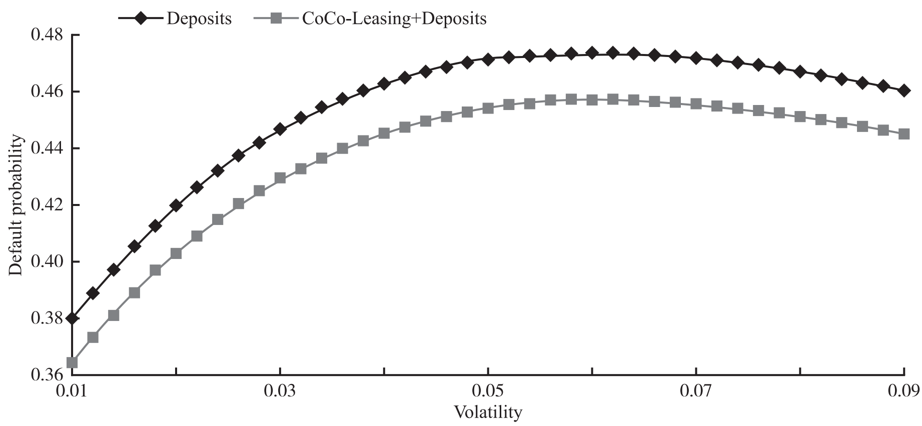

Figure 1. Default Probabilities for a Capital Structure with and without CoCo-Leasing

Considering the incentives to increase or decrease asset volatility, asset volatility levels that are significantly above the normal range may attract the attention of the regulator who will try to limit these levels of risk taking. The conversion rate can vary from 0 to 1.

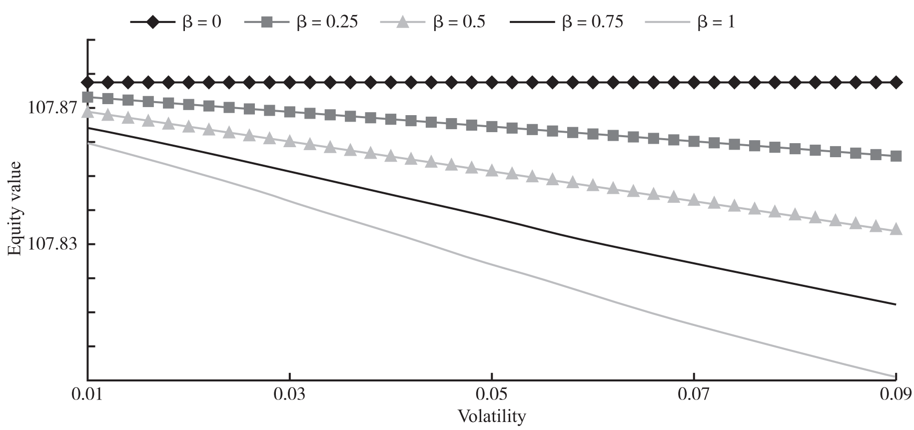

Figure 2. Equity Value vs. Asset Risk for Different Conversion Rates β

Additionally, we clarify the influence of asset value in different scenarios with basic parameter values for different service flow values $\left(s = 7,\ s = 6,\ s = 2,\ s = 3\right)$.

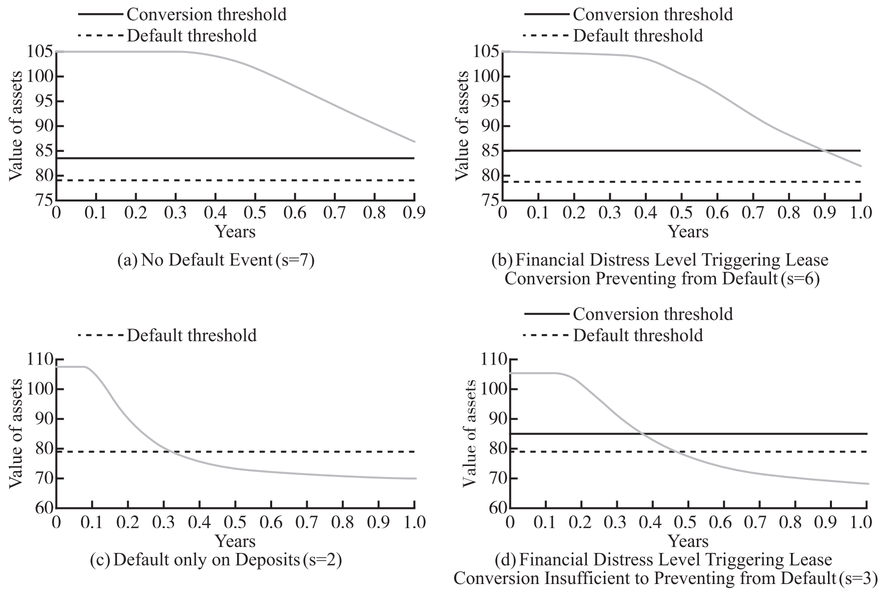

Figure 3. Asset Values for Different Possible Scenarios under Different Service Flow Values (s=7, s=6, s=2, s=3)

From Figure 1, we note that a structure including a CoCo-Leasing and deposits is less risky in default than a structure with only deposits.

Thus, it can be seen that the much higher default probability with deposits is the reflection of the stabilizing effect that CoCo-Leasing has on the capital structure. Hence, the introduction of the CoCo-Leasing reduces the probability of going bankrupt and improves the solvency of the financial institution.

Figure 2 presents the value of equity versus asset risk for different conversion ratios (β) where the bank capital structure is composed of CoCo-Leasing, deposits and equity. In fact, it presents the effect of the conversion ratio (β) on the risk-taking motivation of the stockholder. All else equal, the value of the equity decreases as the conversion ratio increases. In addition, the stockholder's choice of asset risk strongly depends on the conversion ratio. For a relatively low conversion ratio (β= 0 or 0.25, "equity-friendly"), the value of the stock increases with asset risk. The reverse relationship is present for a relatively high conversion ratio (β = 0.75 or 1, "CoCo-Leasing-friendly"). Importantly, the value of the equity is almost insensitive to asset risk for intermediate levels of the conversion ratio. For example, the value of the equity is close to constant with respect to asset risk when the conversion ratio is equal to 50%. Our findings suggest that a particular choice of conversion ratio may have implications for risk-shifting.

Figure 3 illustrates how, in various scenarios, the asset value falls as the service flow increases. For a range of initial service flow values, it indicates the impact of the bank's asset value over time. A fall in service flow lowers the bank's assets.

Consequently, the bank's maximum value will increase if a contingent convertible leasing contract with a high service flow is added to its capital structure. The service flow value increases with the stability of the bank's asset value.

Figure 3(a) illustrates the scenario when there is no default event $\left(T < T_D < T_D^{'}\right)$. When the service flow $s = 7$, the asset value is higher and remains over the conversion and default thresholds until maturity $\left(T\right)$.

Figure 3(b) shows that the conversion occurs, and the asset value reaches the lower conversion threshold $(v_t \leq x_k)$ with service flow $s = 6$. As a result, the bank's assets value increases while the conversion prevents default $(T_D < T < T_D')$.

The bank capital structure in Figure 3(c) is considered to consist solely of deposits and equity, and the value of the assets approaches the default threshold $(v_t \leq x_k')$ if the service flow $s = 2$ Nevertheless, the value of the bank's assets cannot be increased, and the default directly affects depositors.

With service flow $s = 3$, the value of assets is displayed in Figure 3(d). It indicates that the value of the assets reaches the conversion threshold $(v_t \leq x_k)$. But, even after the conversion, the value of the assets keeps falling, leading to deposit default and the bank's insolvency.

5. Conclusion

In this article, we propose a new model of contingent convertible lease contracts called CoCo-Leasing, a new financial instrument that automatically converts into equity where the coupons for the use of the asset are reimbursed if there is an event of default, the conversion trigger occurs and the lessor receives a certain predefined fraction of the company's equity. A CoCo-Leasing contract between a lessor (leasing) who transfers the enjoyment of the property for a predefined period and a lessee (bank) who undertakes to pay specific rents during the duration of the contract.

The main reason for introducing CoCo-Leasing into the legal system was to improve the ability to absorb losses before the bankruptcy of an institution and the indirect protection of deposits. However, we analyze the effect of including CoCo-Leasing in the capital structure. We provide closed pricing solutions for CoCo-Leasing and other securities by replicating payments through sets of barrier options. We demonstrate that CoCo-Leasing can be effective in stabilizing the financial situation: the introduction of CoCo-Leasing is significantly less likely to default and it can be designed to reduce incentives for risk taking. We find that a capital structure with CoCo-Leasing has a lower probability of default than a bank with a capital structure with only deposits. It is important to note that the conversion rate has a significant impact on the risk incentives, and that for intermediate levels of the conversion rate, the incentives to modify the risk can be virtually eliminated. Indeed, for relatively low conversion rates, shareholders are encouraged to increase the risk of assets, while a high conversion rate implies a desire to reduce risk. The intuition behind this effect is that increasing the risk level of the asset makes conversion more likely.

Therefore, CoCo-Leasing is an effective tool to stabilize the financial system and indirectly protect bank deposits.

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