**Online Appendix for**

"The Extent to which Contingent Convertible Leasing Protects Bank Deposits: A Barrier Option Approach"

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## I. Derivation of Equation (3)

At each point in time, the value of the asset's service stream, or the instantaneous rental rate $s_t$ follows a diffusion process:

$$ds_t = \alpha_s \, s_t dt + \sigma_s \, s_t dz_\alpha \quad (\mathbf{I}.1)$$

Let us denote the present value of the use of the asset for $T$ years by $Y(s,T)$. It can be expressed as follows:

$$Y(s,T) = E\left(\int_0^T e^{-rt} s_t \; dt\right) \quad (\mathbf{I}.2)$$

Using the properties of lognormal variables, we can find an explicit expression for $Y(s,T)$. First, we know:

$$s_t = sexp\left(\left(\alpha_s - \frac{{\sigma_s}^2}{2}\right)t + \sigma_s \, z_\alpha\right) \quad (\mathbf{I}.3)$$

Then, $Y(s,T)$ can be written:

$$Y(s,T) = E\left(\int_0^T e^{-rt} sexp\left(\left(\alpha_s - \frac{{\sigma_s}^2}{2}\right)t + \sigma_s \, z_\alpha\right) \; dt\right) \quad (\mathbf{I}.4)$$

By Girsanov's theorem, we can change the probability measure to a new measure $Q^\theta$.

Let $\theta = -\sigma_s$ , then $\mathcal{L}_t^\theta = exp\left(\sigma_s \, z_\alpha - \frac{{\sigma_s}^2}{2} t\right)$ is the Radon Nikodym derivative.

Therefore:

$$Y(s,T) = E\left(\int_0^T e^{-rt} s e^{\alpha_s t} \mathcal{L}_t^\theta \; dt\right) = \int_0^T E^{Q^\theta}\left(s e^{-(r-\alpha_s)t}\right) \; dt \quad (\mathbf{I}.5)$$

Solving the previous equation, we finally have:

$$Y(s,T) = \frac{s}{r-\alpha_s}\left[1 - e^{-(r-\alpha_s)T}\right] \quad (\mathbf{I}.6)$$

## II. Derivation of Equation (4)

The equilibrium rental rate, denoted $R(T)$, will then be the payment stream whose annuity value is equal to $Y(s,T)$. Thus, $R(T)$ can be expressed as:

$$R(T) = \frac{r}{1-e^{-rT}} Y(s,T) \quad (\mathbf{II}.1)$$

Inserting equation (I. 6) into the above equation, we have the explicit expression:

$$R(T) = \frac{r}{1-e^{-rT}} \frac{s}{r-\alpha_s}\left[1 - e^{-(r-\alpha_s)T}\right] \quad (\mathbf{II}.2)$$

## III. Derivations of Equations (6), (7), (8) and (12)

Following the assumptions of Black and Scholes (1973), the risk-free interest rate is constant over time and equal to $r$ and $\alpha_v = r - \frac{{\sigma_v}^2}{2}$. The value of bank assets, denoted $v_t$, is well described under the risk-neutral probability by the following stochastic differential equation:

$$dv_t = \alpha_v v_t dt + \sigma_v v_t dz_v \quad (\mathbf{III}.1)$$

All options expire at time $T$, have a strike price of $R_T$ and a barrier level of $x_k$.

- Down-and-in call option: If $v_t$ reaches the $x_k$ barrier, the option becomes a vanilla European call option with a strike price $R_T$ and expiration date $T$. If $v_t$ does not reach the $x_k$ barrier, the option expires worthless. The value of the option with a barrier that is strictly greater than the strike price is :

$$CB^{din} = v_t N(d_1) - R_T e^{-rT} N(d_2) + v_t \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}+1} N(d_5) - R_T e^{-rT} \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}-1} N(d_6) - v_t N(d_3) + R_T e^{-rT} N(d_4) \quad (\mathbf{III}.2)$$

where $N\ (.)$ represents the standard normal cumulative probability distribution function.

The value of a down-and-in call option with a barrier below the strike price is:

$$CB^{din} = v_t \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}+1} N(d_7) - R_T e^{-rT} \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}-1} N(d_8) \quad (\mathbf{III}.3)$$

- Down-and-out call option: If $v_t$ does not reach the barrier $x_k$, the option becomes a European vanilla call option with a strike price $R_T$ and a maturity $T$. If $v_t$ reaches the $x_k$ barrier, the option expires worthless. The value of the option with a barrier strictly above the strike price is :

$$CB^{dout} = v_t N(d_3) - R_T e^{-rT} N(d_4) - v_t \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}+1} N(d_5) + R_T e^{-rT} \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}-1} N(d_6) \quad (\mathbf{III}.4)$$

The value of a down-and-out call option whose barrier is strictly below the strike price is as follows:

$$CB^{dout} = v_t N(d_1) - R_T e^{-rT} N(d_2) - v_t \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}+1} N(d_7) + R_T e^{-rT} \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}-1} N(d_8) \quad (\mathbf{III}.5)$$

- Down-and-out numerical call option: If $v_t$ does not reach the barrier $x_k$, the option pays one currency unit at expiration $T$ if the value of the assets at expiration is greater than the strike price $R_T$. If $v_t$ reaches the barrier $x_k$, the option expires worthless. The value of the option with a barrier strictly above the strike price is:

$$DB^{dout} = R_T e^{-rT} \left( N(d_4) - \left(\frac{x_k}{v_t}\right)^{\frac{2r}{{\sigma_v}^2}-1} N(d_6) \right) \quad (\mathbf{III}.6)$$

The value of a down-and-out digital call option with a barrier strictly below the strike price is:

$$DB^{dout} = R_T e^{-rT}\left(N(d_2) - \left(\frac{x_k}{v_t}\right)^{\frac{2r}{\sigma_v^2}-1} N(d_8)\right) \quad (\textbf{III}.7)$$

- Down-and-in digital call option (with touch payout): If $v_t$ reaches the barrier $x_k$, the option pays one unit of currency at touch. If $v_t$ does not reach $x_k$, the option expires worthless. The value of the option is:

$$DB^{din} = \left(\frac{x_k}{v_t}\right)^{\frac{2r}{\sigma_v^2}} N(d_5) + \frac{x_k}{v_t} N(d_9) \quad (\textbf{III}.8)$$

where

$d_1 = \frac{ln\left(\frac{v_t}{R_T}\right)+(r+\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$ , $d_2 = \frac{ln\left(\frac{v_t}{R_T}\right)+(r-\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$, $d_3 = \frac{ln\left(\frac{v_t}{x_k}\right)+(r+\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$, $d_4 = \frac{ln\left(\frac{v_t}{x_k}\right)+(r-\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$ ,
$d_5 = \frac{ln\left(\frac{x_k}{v_t}\right)+(r+\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$, $d_6 = \frac{ln\left(\frac{x_k}{v_t}\right)+(r-\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$, $d_7 = \frac{ln\left(\frac{x_k^2}{v_t R_T}\right)+(r+\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$, $d_8 = \frac{ln\left(\frac{x_k^2}{v_t R_T}\right)+(r-\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$,
$d_9 = \frac{ln\left(\frac{x_k}{v_t}\right)-(r+\frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}$.

## IV. Derivations of Equations (9), (10), (11) and (13)

Default occurs at any time before maturity if the asset value of the business touches the barrier, or at maturity if the asset value of the business falls below the face value of the debt.

However, instead of only admitting the possibility of default at maturity, Black and Cox (1976) postulated that default occurs the first time the value of the firm's asset falls below of a certain barrier.

Bankruptcy is defined as follows:

– Either the value of the assets touches a certain barrier at any time before maturity.

$$P_r(T_D'' < T) \quad (\textbf{IV}.\mathbf{1})$$

– Conditional on always above a predefined barrier, at maturity the value of the assets is above this barrier but below the nominal value of the debt.

$$P_r(max(x_k, x_k') < v_T < (R_T + C_s), T_D'' \geq T) \quad (\textbf{IV}.2)$$

We assume that $v_t$ is a Brownian motion with $\alpha_v t$ drift and variance $\sigma_v{}^2 t$.

$f(y)$ is the probability density of $v_t$ such that

$$f(y) = \frac{1}{\sigma_v\sqrt{2\pi t}} exp(-(y-\alpha_v t)^2/2\sigma_v{}^2 t) \quad (\textbf{IV}.3)$$

and $g(y, x)$ is the joint probability density with $x = ln\left(\frac{max(x_k, x_k')}{v_t}\right)$

$$g(y,x) = exp(2x\alpha_v/\sigma_v{}^2)\frac{1}{\sigma_v\sqrt{2\pi t}} exp(-(y-2x-\alpha_v t)^2/2\sigma_v{}^2 t) \quad (\textbf{IV}.4)$$

The probability of breaking the barrier before expiry is given by:

$$P_r\left(T_D^{''} < T\right) = P_r(min_{0<t<T} \quad v_t \le x) = P_r(v_T \le x) + P_r(min_{0<t<T} \quad v_t \le x, v_T > x)$$
$$= \int_{-\infty}^{x} f(y) \; dy + \int_{x}^{+\infty} g(y,x) \; dy$$
$$= \int_{-\infty}^{x} \frac{1}{\sigma_v\sqrt{2\pi t}} exp(-(y-\alpha_v t)^2/2{\sigma_v}^2 t) \; dy$$
$$+ \int_{x}^{+\infty} exp(2x\alpha_v/{\sigma_v}^2) \frac{1}{\sigma_v\sqrt{2\pi t}} exp(-(y-2x-\alpha_v t)^2/2{\sigma_v}^2 t) \; dy$$
$$= N\left(\frac{ln\left(\frac{max(x_k, x_k^{'})}{v_t}\right) - \alpha_v T}{\sigma_v\sqrt{T}}\right)$$
$$+ \left(\frac{max(x_k, x_k^{'})}{v_t}\right)^{(2\alpha_v/{\sigma_v}^2)} N\left(\frac{ln\left(\frac{max(x_k, x_k^{'})}{v_t}\right) + \alpha_v T}{\sigma_v\sqrt{T}}\right) \quad (\mathbf{IV}.5)$$

And the second event,

$$P_r(max(x_k, x_k^{'}) < v_T < (R_T + C_s), T_D^{''} \ge T)$$
$$= P_r(max(x_k, x_k^{'}) < v_T < (R_T + C_s), \; min_{0<t<T} \quad v_t > x)$$
$$= P_r\left(x < v_T < ln\left(\frac{R_T + C_s}{v_t}\right)\right)$$
$$- P_r\left(x < v_T < ln\left(\frac{R_T + C_s}{v_t}\right), \; min_{0<t<T} \quad v_t \le x\right)$$
$$= \int_{x}^{ln\left(\frac{R_T+C_s}{V_t}\right)} f(y) \; dy - \int_{x}^{ln\left(\frac{R_T+C_s}{V_t}\right)} g(y,x) \; dy$$
$$= \int_{x}^{+\infty} f(y) \; dy - \int_{ln\left(\frac{R_T+C_s}{V_t}\right)}^{+\infty} f(y) \; dy$$
$$- \int_{x}^{+\infty} g(y,x) \; dy + \int_{ln\left(\frac{R_T+C_s}{V_t}\right)}^{+\infty} g(y,x) \; dy$$
$$= N\left(\frac{ln\left(\frac{v_t}{max(x_k, x_k^{'})}\right) + \alpha_v T}{\sigma_v\sqrt{T}}\right) - N\left(\frac{ln\left(\frac{v_t}{R_T + C_s}\right) + \alpha_v T}{\sigma_v\sqrt{T}}\right)$$
$$- \left(\frac{max(x_k, x_k^{'})}{v_t}\right)^{(2\alpha_v/{\sigma_v}^2)} \left( N\left(\frac{ln\left(\frac{max(x_k, x_k^{'})}{v_t}\right) + \alpha_v T}{\sigma_v\sqrt{T}}\right)\right.$$
$$\left. - N\left(\frac{ln\left(\frac{(max(x_k, x_k^{'}))^2}{v_t(R_T + C_s)}\right) + \alpha_v T}{\sigma_v\sqrt{T}}\right)\right) (\mathbf{IV}.6)$$

Then, the probability of default for a capital structure with deposits and CoCo-Leasing is equal to the sum of the two previous events:

$$PD = P_r(T_D{}'' < T) + P_r(max(x_k, x_k{}') < v_T < (R_T + C_s), T_D'' \geq T)$$

$$= 1 - N\left(\frac{ln\left(\frac{v_t}{R_T + C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right)$$

$$+ \left(\frac{max(x_k, x_k{}')}{v_t}\right)^{(2\alpha_v/\sigma_v{}^2)} N\left(\frac{ln\left(\frac{(max(x_k, x_k{}'))^2}{v_t(R_T + C_s)}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right) \quad (\mathbf{IV}.7)$$

For a capital structure including only deposits, the probability of default is as follows:

$$PDD = P_r(T_D{}' < T) + P_r(x_k{}' < v_T < C_s, T_D' \geq T)$$

$$= N\left(\frac{ln\left(\frac{x_k{}'}{v_t}\right) - \alpha_v T}{\sigma_v \sqrt{T}}\right) + \left(\frac{x_k{}'}{v_t}\right)^{(2\alpha_v/\sigma_v{}^2)} N\left(\frac{ln\left(\frac{x_k{}'}{v_t}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right)$$

$$+ N\left(\frac{ln\left(\frac{v_t}{x_k{}'}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right) - N\left(\frac{ln\left(\frac{v_t}{C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right)$$

$$- \left(\frac{x_k{}'}{v_t}\right)^{(2\alpha_v/\sigma_v{}^2)} \left(N\left(\frac{ln\left(\frac{x_k{}'}{v_t}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right) - N\left(\frac{ln\left(\frac{(x_k{}')^2}{v_t C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right)\right)$$

$$= 1 - N\left(\frac{ln\left(\frac{v_t}{C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right) + \left(\frac{x_k{}'}{v_t}\right)^{(2\alpha_v/\sigma_v{}^2)} N\left(\frac{ln\left(\frac{x_k{}'^2}{v_t C_s}\right) + \alpha_v T}{\sigma_v \sqrt{T}}\right) \quad (\mathbf{IV}.8)$$