Nicolas Pilia*, Christian Ritter, Danila Potyagaylo, Walther H. W. Schulze, Olaf Dössel and Gustavo Lenis

Determination of the excitation origin in the ventricles from the ECG using support vector regression

Abstract: A common treatment of focal ventricular tachycardia is the catheter ablation of triggering sites. They have to be found manually by the physician during an intervention in a catheter lab. Thus, a method for determining the position of the focus automatically is desired. The inverse problem of electrocardiography addresses this problem by reconstructing the source of the ectopic beats using the surface ECG. This problem is ill-posed and therefore needs specific methods for solving it. We propose a machine learning approach for localisation of the ectopic foci in the heart to assist cardiologists with their therapy planning. We simulated 600 120-lead ECGs with different known excitation origins in the heart using a cellular automaton followed by a forward calculation. Features from the ECGs were used as input for a support vector regression (SVR). We assumed a functional relation between features from the ECG and the excitation origin. To benchmark SVR, we also used the well-known Tikhonov 0th order regularisation to reconstruct the transmembrane potentials in the heart and detect the location of the ectopic foci. Parameters for SVR and regularisation were chosen using a grid search minimising the error between estimated and true excitation origin. Compared to the Tikhonov regularisation method, SVR achieved a smaller deviation between estimated and real excitation origin evaluated with 6-fold cross validation. Future work could investigate on the behaviour on data from simulations with other torso and electrophysiological models, the influence of other methods for feature extraction and finally the evaluation with clinical data.

Keywords: Support vector regression, inverse problem, ECG, ECG Imaging, machine learning

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1 Introduction

Ventricular tachycardia caused by ventricular ectopic beats (VEB) is a common heart disease. In some cases, catheter ablation is a suitable treatment. During this, physicians try to spot an ectopic focus in the ventricles, ablate it and hence, terminate the triggering source of the VEB. Nevertheless, especially spotting the ectopic focus is very time consuming as physicians have to find the origin of the pathological excitation manually. By solving the inverse problem of electrocardiography, it is tried to support the physician in the reconstruction of the location of the triggering source using a multi-lead ECG, a body surface potential map (BSPM). With this information, physicians can navigate directly to the point of interest without the need of determining the location of the ectopic focus manually.

The underlying problem can be formulated as a forward problem with a linear equation $\underline{A}\underline{x} = \underline{b}$, where $\underline{b} \in \mathbb{R}^{m\times 1}$ is the BSPM, $\underline{A} \in \mathbb{R}^{m\times n}$ is the lead field matrix projecting the transmembrane voltages (TMV) $\underline{x} \in \mathbb{R}^{n\times 1}$ on the body surface [1]. However, in this case, a solution to the inverse problem, which is a highly under-determined problem as there are many more unknown sources in the heart than electrodes on the body surface, is required. To overcome the problem of ill-posedness, Tikhonov regularisation can be used to solve the problem [2] [3]. In this work, 600 simulations of the heart and the corresponding BSPM were performed. With these data, the inverse problem was solved applying two different methods: first, Tikhonov regularisation based on regularisation as a standard method and second, a regression method based on machine learning.

2 Methods

2.1 Simulated data

In order to test the methods in a controlled environment, 600 simulations with different excitation origins were performed. A cellular automaton was used to simulate the de- and repolarisation waves on a heart mesh with $n = 3340$ nodes (according

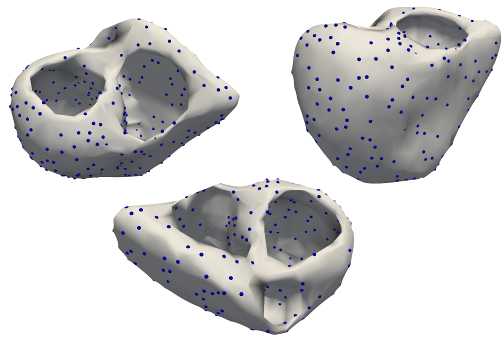

Figure 1: Heart surface from different perspectives. The blue dots show 600 different origins of the simulated VEB.

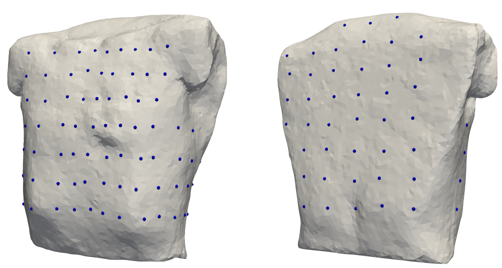

Figure 2: Torso geometry used for the forward calculation. The electrodes measuring the BSPM are visualised as blue dots.

to [4]). The spatial resolution of the mesh was 4 mm, the starting node of the excitation for each VEB was chosen randomly. Figure 1 shows the origin positions on the heart geometries as blue dots. With a torso model including separate conductivities for lungs, lever and other intestines, a 120-lead BSPM was extracted by a forward calculation. Figure 2 visualises the position of the $m = 120$ electrodes on the torso surface.

2.2 Tikhonov regularisation method

A standard method for solving the inverse problem of electrocardiography is Tikhonov regularisation. With Tikhonov regularisation, the sought TMV distribution $\underline{x}$ in the ventricles is formulated as a minimisation problem [3]:

$$\underline{x} = \arg\min_{\underline{x}}(||\underline{b} - \underline{A}\underline{x}||_2^2 + \lambda^2 ||\underline{L}\underline{x}||_2^2) \tag{1}$$

λ^2 is the squared regularisation parameter and balances the squared norm of the regularisation term $||\underline{L}\underline{x}||_2^2$ and the squared norm of the residual $||\underline{b} - \underline{A}\underline{x}||_2^2$. $\underline{L} \in \mathbb{R}^{n\times n}$ was chosen to the identity matrix, yielding 0th order Tikhonov regularisation. The lead field matrix $\underline{A}$ was known from simulation. Through the significant influence of the regularisation parameter λ on the inverse solution of the TMV, we applied a simple grid search for values of the regularisation parameter in the range of 10^{-8} to 10^{-2}. The λ yielding the minimal distance between estimation and truth was selected for calculating the TMV.

After the determination of the TMV courses of the single heart nodes, we needed to detect the location of the VEB origin analysing these signals. This was performed by evaluating the signals 0.01 s after the start of the depolarisation wave that is known from the simulation. We set a signal dependent threshold by calculating the 5%-quantile of all TMV and took the center of mass (according to [5]) of all signals exceeding this threshold and took the heart node closest to the calculated center of mass as the reconstructed VEB origin. In contrast to many methods proposed in literature, the estimation of the VEB origin was done for each coordinate separately to be comparable to the second method. Hence, the estimation of each coordinate could use its individual optimal regularisation parameter.

2.3 Support vector regression

The problem of reconstructing the origin of a VEB was considered as a regression problem. We assumed, that there exists a functional connection between the features of the BSPM and the origin of the VEB and tried to find a function describing exactly this connection. This can be done by a common regression technique from the field of machine learning called support vector regression (SVR), or support vector machine regression [6]. It is tried to approximate the function $f(\underline{x})$ by a linear combination of the non-linear functions Φ:

$$f(\underline{x}, \underline{w}) = \langle \underline{w}, \Phi(\underline{x}) \rangle \tag{2}$$

$\langle\cdot\rangle$ is the dot product. The regression can be formulated as a convex optimization problem with its associated dual problem formulation as the maximisation of:

$$\begin{aligned}\tilde{L}(a, \hat{a}) = &-\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}(a_n - \hat{a}_n)(a_m - \hat{a}_m)k(x_n, x_m)\\ &- \epsilon\sum_{n=1}^{N}(a_n + \hat{a}_n) + \sum_{n=1}^{N}(a_n + \hat{a}_n)t_n\end{aligned} \tag{3}$$

subject to: $\sum_{i=1}^{l}(a - \hat{a}) = 0$ and $a, \hat{a} \in [0, C]$.

N is the number of data points. The ϵ-SVR contains $k(x, \hat{x})$ as the kernel function defined as $k(x, \hat{x}) := \langle \Phi(\underline{x}), \Phi(\underline{x}^T) \rangle$. A possible choice for $k(x, \hat{x})$ is the gaussian function: $k(x, \hat{x}) = \exp(-\frac{||x-\hat{x}||^2}{2\sigma^2})$ [6]. For an appropriate performance of the SVR, the regularisation parameter C, the loss function parameter ϵ and the kernel parameter σ have to be chosen in advance.

2.3.1 Reconstruction of excitation origins

With the SVR method, we can try to estimate the cartesian coordinates of the excitation origin of a VEB. As the formulation of the SVR only allows one output, three different SVRs were trained, each estimating one coordinate in the 3D space. A principal component analysis of the matrix containing the BSPM of all corresponding VEB origins was determined. The first 10 principal component scores were used as input values for the SVR. The transformation was calculated with the training data and applied equally to the test dataset. 6-fold cross validation was used as evaluation method. Thus, the SVR was trained using 500 of the 600 simulated data (approximately 83.3%) with known excitation origins. The performance of this method was tested with the remaining 100 samples. The tuning parameters C, ϵ and σ were chosen in an iterative process. Therefore, parameters were varied in the interval $[10^{-5}, 10^{5}]$ similar to the grid search applied to determine the regularisation parameter in the Tikhonov regularisation method. The parameter combination yielding the lowest error was chosen for evaluation. For each coordinate there was an individual combination of C, ϵ and σ.

3 Results

3.1 Tikhonov regularisation method

Table 1 gives an overview on the results obtained with the standard method for solving the inverse problem. If we define the global estimation error as the euclidean distance $d_{global} = \sqrt{d_x^2 + d_y^2 + d_z^2}$ we achieve a global error of $d_{global,Tikh} = 10.03$ mm. This has to be seen alongside the heart mesh resolution of 4 mm. Figure 3 shows the spatial distribution of the errors on the heart mesh.

3.2 Support vector regression

As described above, we allowed different combinations of the parameters C, ϵ and σ to achieve a minimal error for each coordinate separately. The optimal parameter combinations for each coordinate are given in table 2. Additionally, the estimation error of the excitation origin for the respective parameter combination is given. The global error was $d_{global,SVR} = 1.73$ mm. This has to be again seen alongside the heart mesh resolution of 4 mm. Figure 4 shows again the spatial distribution of the errors on the heart mesh.

Table 1: The results obtained with a standard method for reconstruction. Regularisation parameter and estimation errors are given for each coordinate separately.

Parameter/ Estimation error	X coordinate	Y coordinate	Z coordinate
$\log_{10}(\lambda)$	-2.5	-3.5	-3
d in [mm]	6.16	5.90	5.27

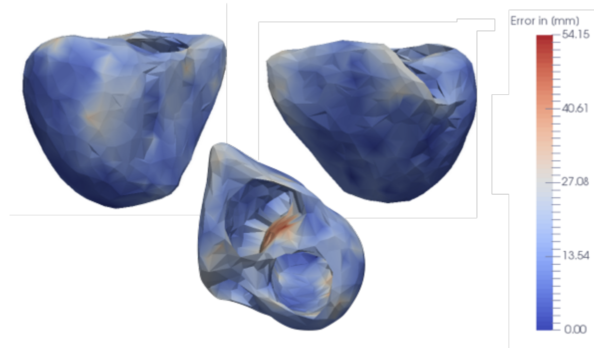

Figure 3: Estimation errors resulting from the Tikhonov regularisation method for the different VEB origins plotted on the heart geometry viewed from different perspectives. Highest deviations are visible in the region of the septum.

Table 2: Estimation errors of VEB origin obtained with SVR. Optimal parameters and estimation errors are given for each coordinate separately.

Parameter/ Estimation error	X coordinate	Y coordinate	Z coordinate
$\log_{10}(\epsilon)$	0.65	-0.74	-10.63
$\log_{10}(C)$	5.02	5.04	4.40
$\log_{10}(\sigma)$	2.18	2.19	1.95
d in [mm]	0.92	1.08	0.99

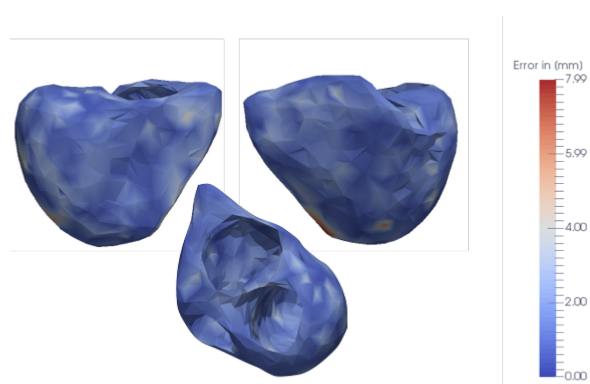

Figure 4: Estimation errors resulting from the SVR method for the different spatial excitation origins plotted on the geometry viewed from different perspectives. Highest deviations are visible in the apex of the heart.

4 Discussion and conclusion

In this study, we used 600 simulations for the reconstruction of the excitation origins using SVR and Tikhonov regularisation for comparison. The suitability of SVR in the context of localising the VEB origins was underlined by a global error of $d_{global,SVR} = 1.73$ mm. The method for comparison using Tikhonov regularisation delivered a higher error of $d_{global,Tikh} = 10.03$ mm although the lead field matrix $\underline{A}$ was known from the simulation. A drawback of the Tikhonov regularisation method is the requirement of an anatomical model of the patient e.g. for calculating the lead field matrix since it is not known a priori in the clinical environment. SVR, however, needs only known data points for training which could be determined without the patient's heart geometry. Nevertheless, this work can only be seen as a proof of concept study. Simulated data lack noise, which is usually part of realistic signals - even after filtering. This could influence the performance of the method. Furthermore, only one patient model and one TMV simulation method was used and therefore the patient dependency of the method was not investigated. These drawbacks, however, do not contradict the general suitability of a reconstruction with SVR in the field of the inverse problem of electrocardiography as shown in this work.

5 Outlook

A few follow-up steps result directly from the discussion. To overcome the point of using simulated data created with just one patient model and one simulation method, two possible follow-up studies should be considered: First, the additional use of data created with another simulation method (e.g. a monodomain simulation) for the evaluation of the SVR method should be named. Second, other patient geometries can be considered for simulation. These could be used to further investigate the independency of the SVR method and the used features from other geometries, as well as to evaluate a possibly needed calibration. For sure, if patient data are available, these should always be used. A further point to be extended is feature extraction. By using other features from the BSPM, both temporal and spatial, we could create a method for feature extraction that is independent from the used signals. Moreover, a multi-output SVR, that estimates all three coordinates at the same time, could be used to incorporate cross relations between the different coordinates. Finally, the comparison of the methods for solving the inverse problem could be extended to more of the standard methods as proposed in literature (e.g. in [7]).

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