

COMPUTATION OF MUSCLE FIBER CONTRACTION DYNAMICS

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Abstract: This paper contains the modeling of muscle fiber contraction dynamics as a result of electrical stimulation. The novelty of the model is the possibility of running simultaneous analyses of the fiber's action potential propagation and its biomechanical response. Simulations for different muscle fiber types, stimulus rates and contraction types can be run. The model correctly predicts the occurrence of smooth tetanic contractions due to the expected fusion frequencies and the behavior of the fiber's elastic elements. It can be used as a base for developing higher dimensional models of motor units or a complete muscle that is stimulated by electrodes.

Keywords: Myodynamics, COMSOL, Excitation process, Myocybernetics

Introduction

Various factors influence the contraction behavior of skeletal muscles. Despite biomechanical aspects, such as force-length or force-velocity dependencies, the type of the exciting stimulus is of great importance. In order to analyze the dynamic response of a single muscle fiber to direct electrical stimulation a model is developed in COMSOL Multiphysics (COMSOL, Inc., CA 94301, USA) that computes the fiber's excitation and subsequent contraction process. These processes are computed and coupled by using two models: the Wallinga et al. muscle fiber model [1] and Hatze's myocybernetic control models [2, 3, 4].

The link between these models lies within the release of Ca^{2+} ions from the sarcoplasmic reticulum due to a depolarization of the muscle fiber's membrane voltage. Owing to the dependency of the Ca^{2+} concentration on the stimulus rate [6], the model simulates the effects of various stimulus rates. Furthermore, the different behavior of fast and slow-twitch fibers is analyzed and an agonist-antagonist model based on Mehnen [5] is used in order to compute concentric contractions.

Methods

COMSOL's cPDE module was used for the complete computation process. All required differential equations were implemented by properly setting the coefficients of the cPDE module's general equation (1). The fiber model was defined as a 1D interval from point A to point B with a length of 5 cm.

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + \alpha u = f$$

$$\nabla = \frac{\partial u}{\partial x} \quad (1)$$

In (1) u is called dependent variable, f is called source term and e_a , d_a , c , α , γ and β are the coefficients that have to be set.

The Wallinga et al. model [1] was mainly used for the computation of the fiber's membrane voltage V (i.e. action potential), which triggers the contraction process. A rectangular function served as the stimulus (50 μA over 1 ms) and was set as source for the cPDE module.

(2) contains the implementation of the dependent variable V into the cPDE module (1) [1].

$$u = V$$

$$f = \frac{-I_{ion} - V_T}{C_m}$$

$$d_a = 1, \quad c = \frac{r}{2 \cdot \rho}, \quad [a, e_a, \alpha, \beta, \gamma] = 0 \quad (2)$$

In (2) I_{ion} denotes the total ionic current, C_m the fiber's membrane capacity, V_T the T-System membrane voltage, r the fiber's radius and ρ the fiber's intracellular specific resistance [1].

The Wallinga et al. model and the myocybernetic control models were coupled during the computation of the Ca^{2+} concentration γ . Two dependent variables were added to the cPDE module: γ (3) and $\beta(t)$ (4), a function of the membrane voltage V (2).

(3) contains the implementation γ into the cPDE module (1) [2, 3, 4].

$$u = \gamma$$

$$f = c_3 \cdot V_T \cdot \beta_t$$

$$a = c_2, \quad e_a = 1, \quad d_a = c_1, \quad [c, \alpha, \beta, \gamma] = 0 \quad (3)$$

(4) contains the implementation of $\beta(t)$ into the cPDE module (1) [2, 3, 4].

$$u = \beta_t$$

$$f = c_6 \cdot V$$

$$a = c_5, \quad e_a = 1, \quad d_a = c_4, \quad [c, \alpha, \beta, \gamma] = 0 \quad (4)$$

$c_1 - c_6$ are constants [2, 3, 4].

Three main influences are factorized to result in the force f^{CE} generated by the fiber (5): the velocity dependency $g(d\eta/dt)$, the active state q and the filamentary overlap $k(\xi)$ minus an internal force f_0 . All variables were computed via analytical solutions based on [5].

$$f^{CE} = g\left(\frac{d\eta}{dt}\right) \cdot k(\xi) \cdot q(\gamma, \xi) - f_0 \quad (5)$$

ξ denotes the relative length of the fiber's contractile element with respects to its optimum length, under which a normalized force of 1 occurs.

In order to compute concentric contractions, a damping component was added to the force F^{PE} of the fiber's parallel elastic element PE (6) [5, 14].

$$F^{PE} = f_1(\zeta) + f_2(\zeta, \dot{\zeta}) = c'_1 \cdot (e^{c'_2 \zeta} - 1) + \beta_d \cdot \frac{dl}{dt} \quad (6)$$

In (6) $f_1(\zeta)$ is the PE's force-elongation relation, c'_1 and c'_2 denote constants [4], ζ is the normalized elongation, l the fiber's absolute length and $f_2(\zeta, \dot{\zeta})$ the PE's damping component with β_d based on [5] and [14].

Results

Fig. 1 contains the single twitch (ST) response of a fast-twitch muscle fiber to a 50 μ A stimulus over 1 ms. Fig. 2 shows the force output due to various stimulus rates.

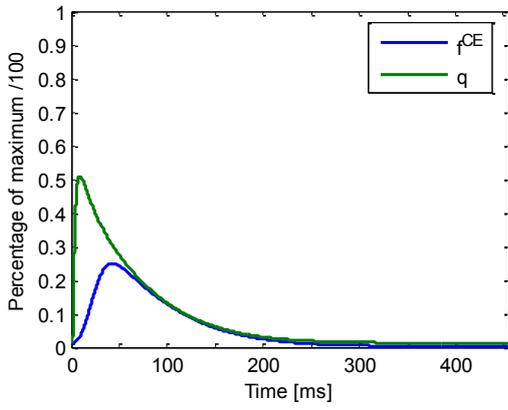


Fig. 1: Single twitch response of a fast-twitch fiber (active state q and normalized force f^{CE})

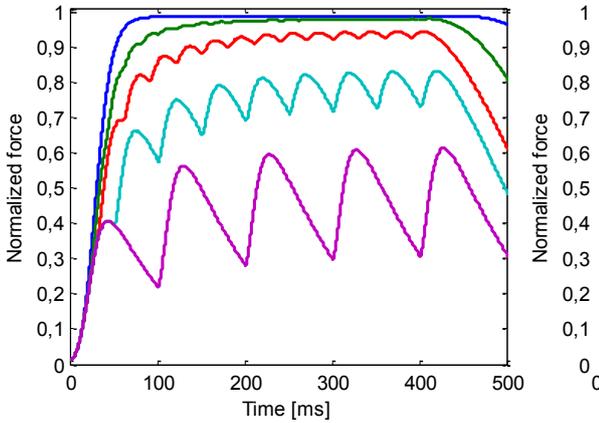


Fig. 2: Effects of various stimulus rates on the normalized force output of a fast-twitch fiber

Tab. 1 contains the most important parameters for evaluating the model.

Tab. 1: Output parameters of the model

	Fast-twitch	Slow-twitch
Max. Ca^{2+} release (ST)	0.31 μmol	0.36 μmol
Contraction time (ST)	43.3 ms	140.5 ms
Max. normalized force (ST)	0.25	0.26
Active state peak time (ST)	10 ms	12 ms
Tetanic contraction	100 Hz	33 Hz

Fig. 3 shows the viscoelastic stress-strain hysteresis computed by using an agonist-antagonist model based on [5] combined with a damped PE (6).

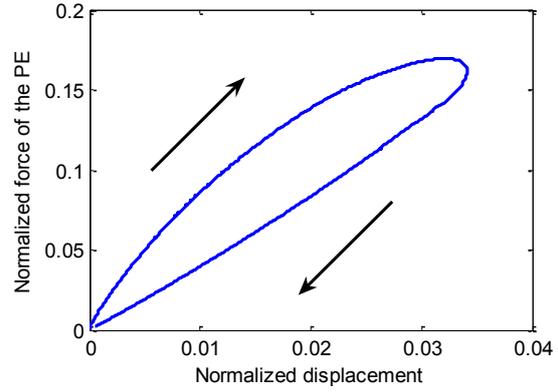


Fig. 3: The fiber's viscoelastic stress-strain hysteresis

Fig. 4 highlights the effects of the damped PE (6) on the fiber's displacement due to a single twitch.

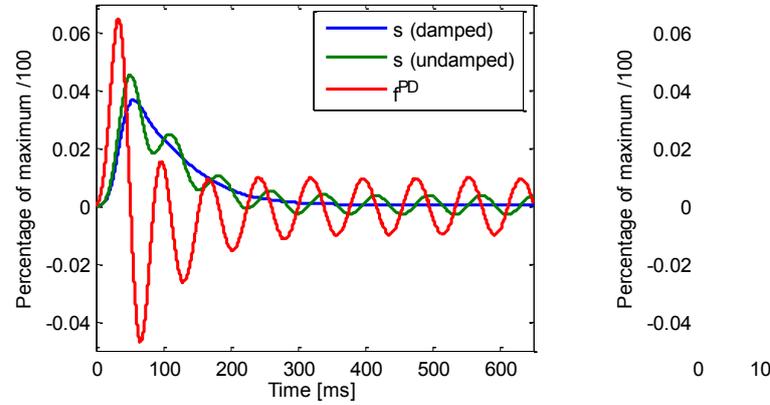


Fig. 4: Effects of the damping force f^{PD} , i.e. $f_2(\zeta, \dot{\zeta})$ from (6), on the fiber's displacement s

Discussion

The model that has been developed provides data that agree well with experimental and computed data from literature. The amount of Ca^{2+} ions released due to a controlled depolarization of the fiber's membrane voltage accords with experimental data [6], specific contraction patterns of slow and fast-twitch fibers agree with descriptions from [7] and [8], the muscle's active state progress with data from [9], stimulation rates due to which smooth tetanic contractions occur accord with [10], [11] and [12], and the behavior of the fiber's elastic elements with [13] and [15]. These comparisons reveal that the model predicts the contraction dynamics of a muscle fiber due to electrical stimulation properly. The model, furthermore, offers a flexible way of computing many processes between the electrical stimulation of a muscle fiber and its dynamic response. Adjustable parameters, such as fiber length, fiber type, stimulus rate and contraction type, allow for expanding the model to higher dimensional ones, such as motor units or a complete muscle that is stimulated by electrodes. This would be of explicit help for simulating the effects of functional electrical stimulation.

Literatur

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Abbreviations

ST	Single Twitch
cPDE	Coefficient Partial Differential Equation
PE	Parallel Elastic Element
CE	Contractile Element