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To Bequeath, or Not to Bequeath? On Labour Income Risk and Top Wealth Concentration

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Abstract: Recent theoretical advances suggest that capital income risk, rather than earnings uncertainty, is the key determinant of fat-tailed behavior of stationary wealth distributions. I provide novel insights into this issue by studying an incomplete market model with general time and state separable preferences, where parental altruism and unobservable idiosyncratic shocks engender non-linear bequest rules. I analytically pin down conditions on the preference structure and other model's primitives under which optimal bequest behavior hinders inter-generational wealth transmission for any degree of capital income risk, causing the dynamics of wealth to converge to a unique (stationary) distribution with thin tails. These results imply, in particular, that (i) the stochastic properties of labour income risk (as shaped by, e.g. fiscal policies) may play a role in defining the structure of the upper tail of the limiting distribution of wealth, and that (ii) matching empirically documented fat tails with choice theoretic frameworks of wealth dynamics requires joint restrictions on preferences and calibrated earnings processes to be met.

Keywords: wealth distributions; incomplete markets; earnings uncertainty; capital income risk; labour income taxation

JEL Classification: D31; H20

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1 Introduction

Empirical wealth distributions in advanced economies are almost invariably characterized by an asymmetrically long upper tail, where wealth shares are extremely large and exhibit power-law decay – see e.g. Klaas et al. (2007) and Benhabib and Bisin (2018). Paralleling a rapidly expanding literature on wealth measurement and the ensuing estimation of the size distribution of wealth (e.g. Vermeulen 2018), a number of theoretical studies on the evolution of wealth have aimed at identifying economic mechanisms, grounded in fundamental utilitarian principles, able to produce positive skew and thick tails as robust predictions.

Within this context, capital income risk has been put forward as a key determinant of top wealth concentration in micro-founded, life-cycle models of consumption and saving behavior, e.g. Benhabib, Bisin, and Zhu (2011, 2015), Benhabib, Bisin, and Luo (2019), and Zhu (2019). According to such a view, households' exposure to shocks to returns on capital, against which full insurance is not achievable due to market incompleteness, underpins intergenerational mobility flows (via direct wealth transfers) as well as accumulation patterns over the higher rungs of the wealth ladder, eventually producing an upper tail of the long-run distribution that resembles a Pareto law.

Using a micro-founded OLG model of saving behavior with CRRA utility and warm-glow preferences for altruism (*joy-of-giving*), Benhabib, Bisin, and Zhu (2011) demonstrate that the tail index of the asymptotic equilibrium state to which the wealth accumulation process converges is invariant with respect to features of the probability law governing labour income risk. Since the thickness of the upper tail is inversely related to the within-tail Gini coefficient, such a result suggests that taxing labour earnings produces no effects on top wealth inequality. An analogous result is established in Zhu (2019), who generalizes to altruistic bequest motives the thick-tail property of wealth accumulation processes in the presence of isoelastic preferences and idiosyncratic investment shocks (capital income risk).[1]

I contribute to the this body of theoretical work by studying the properties of the upper tail of the stationary distribution of wealth in a simple OLG model with incomplete markets and altruistic agents. Using general functional utility specifications that conform with time and state separability requirements, I formally derive conditions on the preference representation and other primitives (e.g. the risk-return structure of financial investment) under which optimal bequest behavior of

1 In the presence of preferences for altruism in *joy-of-giving* form, agents derive direct utility from bequeathing wealth to their children (Andreoni 1990); an *altruistic* (or paternalistic) bequest motive rather entails the enjoyment of a child's economic status through the lens of her parents' preferences (Becker and Tomes 1979).

paternalistic agents entails no intergenerational transfers for *any* degree of capital income risk, causing the dynamics of wealth to converge to a unique (ergodic) distribution with thin tails. I then study how such conditions vary with the structural parameters of the economy, and in particular with the main stochastic properties (e.g. size and dispersion) of labour income risk. Two immediate implications of my analysis are that (i) labour income taxation may play a role in defining the structure of the upper tail of the limiting distribution of wealth, and that (ii) the emergence of a fat-tailed stationary wealth distribution in micro-founded models of intergenerational wealth transmission with separable preferences requires that the statistical models of earnings shocks be consistent with the identified restrictions on preferences for altruism and risk attitudes.

The economic intuition behind these results is straightforward. In a world of uninsurable risk, consumption and bequest policies depend non-linearly on parental wealth, which is heterogeneous across lineages experiencing different histories of income shocks. Since children's future human wealth cannot serve as a valid collateral for parents' private borrowing, low-wealth agents exhibit weak incentives towards engaging in risky bequest choices against the perceived benefits accruing to their offspring; in turn, this can trap within-lineage wealth transitions in low-wealth states where capital income risk, however strong, cannot trigger accumulation of non-human wealth beyond some finite level. A natural corollary of this result is that, under some circumstances, the wealth dynamics are necessarily confined in the bounded support of the distribution of labour income in the long run, preventing them from experiencing patterns of wealth growth that would eventually lead to a fat upper tail of the stationary distribution. As far as micro-founded models of bequest choices and wealth inequality are concerned, this observation calls for caution in interpreting the policy implications of comparative statics results about the introduction/modification of fiscal policies that affect the risk-return trade-offs faced by individual agents in a world of incomplete markets.

The remainder of the note is organized as follows. Upon locating my analysis into the many strands of literature it purports to contribute to (Section 2), I introduce and discuss the model economy in Section 3, and then turn to deriving the key analytical results in Section 4. Section 5 investigates the comparative statics properties of the model's solution with respect to variation in salient features of the labour income distribution; in turn, this will allow me to draw some interesting observations about the effects of labour income taxation on the tail behavior of the stationary distribution of wealth. A few concluding remarks are finally provided in Section 6. To facilitate the reading, all the proofs are relegated to the Appendix.

2 Related Literature

Since the seminal work of Becker and Tomes (1979, 1986), the analysis of the determinants of the intergenerational transmission of inequality has gained central stage in the economic research agenda (e.g. Bossmann, Kleiber, and Wälde 2007; Cagetti and De Nardi 2009; Castaneda et al. 2013).

Building on extensive evidence about excess returns to private over public equity investment and their heterogeneous dispersion across households in the US (e.g. Moskowitz and Vissing-Jørgensen 2002), Benhabib, Bisin, and Zhu (2011, 2015) formalize the following intuition: unlike labour earnings shocks, that accrue additively into wealth, uninsurable shocks to the rates of return on wealth enter multiplicatively the accumulation process and pile up over time, leading to extremely large upswings in total wealth. In a world of incomplete markets, the exposure to capital income risk fuels mobility across social classes and enhances opportunities to experience large wealth growth towards the top end of the long-run distribution.

In a linear setting, this mechanism is an instance of a fairly general law labeled *proportional random growth*, according to which a state variable (here, wealth) evolves over time following a recurrence equation that involves a stochastic multiplicative term (hear, random returns on wealth) and a stochastic additive one (here, labour earnings). Under some mild regularity conditions, the tail of the unique stationary and causal process solving the random recurrence can be fatter than that of a Normal distribution (e.g. Gabaix 2009; Kesten 1973). Benhabib, Bisin, and Zhu (2011) extend the analysis to the case of positive autocorrelations within (and across) random driving forces behind wealth dynamics, to capture e.g. variations in social mobility in the underlying economy, establishing an analogous asymptotic equivalence result. In a similar vein, Benhabib, Bisin, and Zhu (2015) show that capital income risk can support ergodic and fat-tailed behavior of the limiting distribution of wealth in infinite-horizon Bewley economies.[2]

As acknowledged in Zhu (2019), the key assumption that warrants linear bequest policies pertains to both (i) the nature of the bequest motive (joy-of-giving vs. altruism), and (ii) the information set upon which agents formulate optimal consumption and bequest plans. In fact, on the assumption that agents correctly anticipate the rate of return on life-cycle savings/bequeathed wealth, and also perfectly foresee the labour earnings of their children (as assumed in, e.g. Benhabib, Bisin, and Zhu 2011), uncertainty about capital and labour income is immaterial for the

2 Further studies in applied asset pricing theory and dynamic macroeconomics exploiting proportional random growth mechanisms to match salient features of financial and/or business cycle data include, among others, Benhabib and Dave (2014), Dave and Malik (2017), and Dave and Sorge (2020, 2021).

characterization of optimal bequest choices under risk. When this assumption is relaxed, even in the presence of CRRA preferences, further restrictions on the stochastic features of the labour earnings and investment returns processes are to be imposed in order to guarantee that consumption (hence, bequest) policies are asymptotically linear and that the dynamics of wealth can converge to a fat-tailed distribution in the long-run, see Benhabib, Bisin, and Zhu (2015), Zhu (2019), and D'Amato, Di Pietro, and Sorge (2024). I build on all these insights and characterize several conditions under which altruistic agents who face idiosyncratic and unpredictable income shocks may optimally decide to refrain from bequeathing wealth to their offspring, thereby fully neutralizing the capital income risk mechanism.[3]

On the policy front, Benhabib, Bisin, and Zhu (2011) prove that the implementation of (possibly non-linear) taxes on capital income unambiguously reduces capital income risk and hence produces an equalizing effect in the top end of the wealth distribution. Using a simple consumption-saving model with unanticipated income shocks, Di Pietro and Sorge (2018) uncover the ambiguous impact on wealth concentration of alternative capital income tax systems (e.g. proportional versus progressive), insofar as they trigger different behavioral reactions of taxpayers against uninsured risk. In particular, the introduction of a progressive tax on capital income is shown to induce risk-averse households to over-save in order to counterbalance the adverse effects of taxation on inherited wealth; in such a case, the underlying wealth accumulation process is allowed to converge to a stationary distribution displaying higher wealth inequality at the top. Di Pietro, Pietroluongo, and Sorge (2023) derive a number of further stochastic ordering comparisons in the context of linear random recurrences with non-negative coefficients. When applied to models of wealth growth and inequality, their analysis reveals that, irrespective of whether the ergodic stationary distribution exhibits fat-tailed behavior or not, (i) proportional earnings taxation necessarily affects expected concentration of wealth in higher quantiles; and that (ii) under mild regularity assumptions on the distributions of shocks, the conditional right tail variability of wealth monotonically increases with the tax rate on labour earnings, suggesting the existence of a link between top wealth inequality (as measured by the dispersion of wealth holdings in extreme right tail quantiles) and the earnings tax regime. Relative to this last set of articles, I here focus on first- and second-order dominance shifts in the distribution of labour earnings and show how they can end up entangling individual wealth

3 D'Amato, Di Pietro, and Sorge (2024) develop a model of wealth accumulation where uninsurable incomes shocks, indivisibilities in educational investment and borrowing constraints all interact in shaping the properties of the stationary distribution of wealth. One of their main results is that a thick upper tail of the stationary wealth distribution can obtain along with a mass point at the bottom of the its support, where upward mobility is bound to occur via human capital formation within lineages.

transitions in the lower part of the wealth space where optimal bequests are zero, implying thin-tailed behaviour of the ensuing long-run distribution of wealth.

3 A Simple Model of Intergenerational Wealth Transmission

Following Zhu (2019), I consider an economy populated by a measure-one continuum of agents that live for one period. Each agent generates one child at the end of the period, so that the population is stationary over time. At the beginning of each period t, each agent receives wealth inheritances I_t from their parents (via bequests) and idiosyncratic labour earnings $\tilde{y}_t$, and then optimally makes their own consumption $c_t \geq 0$ and bequest choices $b_t \geq 0$ out of their current wealth

$$\omega_t = I_t + \tilde{y}_t = \tilde{R}_t b_{t-1} + \tilde{y}_t \tag{1}$$

where $\tilde{R}_t$ is the gross random return on financial wealth.

I assume that $\tilde{R}_t$ and $\tilde{y}_t$ are absolutely continuous, mutually independent, non-negative random variables, each defined on a bounded support. Formally

Assumption 1. The processes $\{\tilde{y}_t\}$ and $\{\tilde{R}_t\}$ are i.i.d. along generations, and mutually independent. $\tilde{R}_t$ has a smooth probability density $g(\cdot)$ on $[0, \overline{R}]$, with $\overline{R} < \infty$ and $\Pr(\tilde{R}_t > 1) > 0$. $\tilde{y}_t$ has a smooth probability density $f(\cdot)$ on $[\underline{y}, \overline{y}]$, with $0 < \underline{y} < \overline{y} < \infty$.

Remark 1. In keeping with previous studies that emphasize the role of capital income risk in determining power-law decay of top wealth shares in the long run, Assumption 1 maintains that agents can experience sufficiently large rates of return on financial bequests – i.e. $\Pr(\tilde{R}_t > 1) > 0$ – so that the wealth accumulation process can expand with positive probability (Benhabib, Bisin, and Zhu 2011, 2015; Zhu 2019).

Agents exhibit an altruistic (or paternalistic) bequest motive, i.e. a concern for the total wealth of their child. Preferences are additively separable in consumption and bequests, and satisfy standard regularity conditions (e.g. monotonicity, concavity and smoothness), as stated in the following

Assumption 2. The expected utility of agents is in the form

$$U(c_t, b_t) = u(c_t) + \chi\, \mathbb{E} v(\omega_{t+1}(b_t; \tilde{R}, \tilde{y})) \tag{2}$$

where functions $u\colon [0,\infty) \to \mathbb{R} \cup \{-\infty\}$ and $v\colon (0,\infty) \to \mathbb{R} \cup \{-\infty\}$ are strictly increasing, strictly concave and smooth everywhere on $(0,\infty)$, with $\lim_{c\downarrow 0} u'(c) = \infty$; $\chi > 0$ is the bequest motive intensity and $\mathbb{E}$ is the statistical expectation operator.

Remark 2. The class of (u, v) functions complying with Assumption 2 encompasses, but is apparently not confined to, the routinely employed time/state separable CRRA representation under the expected utility paradigm (e.g. Benhabib, Bisin, and Zhu 2011, 2015; Zhu 2019). Importantly, even when restricting attention to fully parametric families of utility functions, u and v can be both selected to reflect salient features of rational choice under risk (e.g. prudence), but need not share the same set of parameters (e.g. the coefficient of absolute prudence).

Notice that agents' expectations in (2) are conditional on the probability distributions of their offspring's wealth components: bequest strategies must be formulated prior to the realization of the two sources of uncertainty. Since the marginal utility from bequests depends non-linearly on child's wealth, intergenerational altruism entails a precautionary motive for saving, and produces non-linear decision rules and the possibility of corner solutions, as shown next.

For random variables $(\tilde{R}, \tilde{y})$ complying with Assumption 1, the agent's problem thus takes the static form

$$\begin{aligned} &\max_{c_t,\, b_t}\ U(c_t, b_t), \\ &\text{s.t. } c_t + b_t \le \omega_t, \\ &c_t \ge 0,\ \ b_t \ge 0, \end{aligned} \tag{3}$$

where the non-negative bequest constraint $b_t \ge 0$ captures the idea that parents' private borrowing against children's future earnings is not viable.

A solution to the expected utility maximization problem (3) comprises optimal consumption and bequest policies (or decision rules) as time invariant functions of the state variable ω_t, i.e. $c_t^* = c^*(\omega_t)$ and $b_t^* = b^*(\omega_t)$. Idiosyncratic income shocks will generate ex-post wealth heterogeneity by forcing the individual wealth accumulation process

$$\omega_{t+1} = \tilde{R}_{t+1}\, b^*(\omega_t) + \tilde{y}_{t+1} \tag{4}$$

whose limiting properties can be established using techniques borrowed from the theory of Markov chains (e.g. Meyn and Tweedie 2009).

4 The Evolution of the Wealth Distribution

4.1 Optimal Bequest Behavior

To study the role of optimal bequest strategies in shaping fat-tailed behavior of the stationary wealth distribution, I first characterize bequest decision rules as follows

Lemma 1. *Consider problem* (3)*, and let Assumptions* 1 *and* 2 *hold. Then*

(i) *The bequest policy function* b_t^* *exists and is unique; it is non-decreasing and continuous in* ω_t*;*

(ii) *There exists a unique wealth threshold* $\bar{\omega} > 0$ *such that*

$$b_t^* = \begin{cases} \hat{b}(\omega_t) > 0 & \textit{if and only if} \quad \omega_t > \max\left\{\underline{y}, \bar{\omega}\right\} \\ \\ 0 & \textit{otherwise} \end{cases}$$

where $\hat{b}(\omega_t)$ *solves*

$$\hat{b}(\omega_t) = \omega_t - u'^{-1}\left(\chi\, \mathbb{E}\left(\tilde{R}_{t+1}\, v'\left(\tilde{R}_{t+1}\, \hat{b}(\omega_t) + \tilde{y}_{t+1}\right)\right)\right)$$

Proof. See the Appendix. □

The previous Lemma stipulates that optimal bequest policies are uniquely defined, non-linear functions of parental wealth that entail a kink in $\bar{\omega}$, provided $\bar{\omega} > \underline{y}$, so that zero bequests prevail in the lower region of the wealth space.

It is instructive to inspect conditions on the preference representation (u, v) and/or market risk (encoded in the stochastic features of the returns on wealth $\tilde{R}$ and labour earnings $\tilde{y}$) under which bequest incentives are in fact inactive for low-wealth agents, as this property will be key for the features of the process of intergenerational wealth transmission and the enusing tail behavior of the limiting wealth distribution. This is the goal of the following

Lemma 2. *Consider problem* (3)*, and let Assumptions* 1 *and* 2 *hold. Then the threshold* $\bar{\omega}$ *satisfies* $\bar{\omega} > \underline{y}$ *if and only if*[4]

4 Notice that $\mathbb{E}[v'(\tilde{y})] < \infty$ since the stochastic process for labour income is restricted to a compact subset of the positive reals.

$$\chi\mathbb{E}[\tilde{R}] < \frac{u'(\underline{y})}{\mathbb{E}[v'(\tilde{y})]} \tag{5}$$

Proof. See the Appendix. □

Intuitively, Lemma 2 suggests that, for given properties of the labour income process, wealth-constrained agents are more likely to engage in bequest behavior the larger the average rate of return on financial bequests and the stronger their altruistic concerns toward children. Notice that, since v is assumed to be strictly decreasing over its domain (see Assumption 2), a sufficient condition for $\bar{\omega} > \underline{y}$ to obtain in the case the function u coincides with the function v – not necessarily from the CRRA family – is simply that the average rate of return on wealth be sufficiently low, i.e. $\mathbb{E}[\tilde{R}] < 1/\chi$.

Given the relevance of the restriction $\bar{\omega} \geq \bar{y}$ for the limiting properties of the wealth accumulation process (4), I also present the following

Lemma 3. *Consider problem* (3), *and let Assumptions* 1 *and* 2 *hold. Then the threshold* $\bar{\omega}$ *satisfies* $\bar{\omega} \geq \bar{y}$ *if*

$$u'(\bar{y}) \geq \chi\mathbb{E}[\tilde{R}]v'(\underline{y}) \tag{6}$$

Proof. See the Appendix. □

Remark 3. The economic interpretation of Lemma 3 is straightforward: at the margin, agents with zero financial wealth and yet enjoying the highest possible labour earnings will not prefer engaging in bequest behavior even when rationally expecting their children to be at the bottom at the income distribution.

4.2 Wealth Dynamics: Convergence and Limiting Support

The nature of the bequest motive, the agents' preference structure and the stochastic properties of the asset return all interact in determining the existence, uniqueness and tail features of long-run wealth distributions in the economy under scrutiny.

Establishing ergodicity is however a non-trivial task in the presence of general preference specifications along with the occurrence of random multiplicative and additive shocks to the wealth accumulation process. In fact, the structure of (4) allows the probability mass contained in the wealth distribution to possibly drift to infinity (preventing convergence to a unique invariant regime); by the same token, not enough mixing across wealth states, as enforced by idiosyncratic income shocks, may prevent social mobility within and across families to take place (implying some form of history dependence) – see Ma, Stachurski, and Toda (2020) for an analysis of these issues within a generalized income fluctuation problem with stochastic discount factors and interest rates.

The actual location of the threshold $\bar{\omega}$ in the wealth space is key to the transmission of non-human wealth across generations: when such a threshold level is sufficiently high, under any possible sequence of (income) shocks the economy converges – in a strong probabilistic sense – to a unique stationary wealth distribution with bounded support. This is established in the following

Proposition 1. *Let Assumptions* 1 *and* 2 *hold, and suppose* $\bar{\omega} \geq \bar{y}$*. Then there exists a unique limiting (stationary) wealth distribution* ω_∞ *with full measure on* $\left[\underline{y}, \bar{y}\right]$.

Proof. See the Appendix. □

In words, when the (uniquely identified) wealth threshold below which optimal bequests are zero is sufficiently large, the random component of (4) embodying the multiplicative shock becomes almost surely null in a finite period of time; as a consequence, for any initial wealth distribution the Markov chain produced by the law (4) is bound to converge to a unique invariant regime described by a finite (probability) measure, and the unique stationary distribution of wealth will inherit the features of the (exogenous) distribution of labour income.[5]

A simple yet crucial implication of the previous result is that, under the stated conditions, the stationary distribution of wealth will exhibit a thin upper tail no matter how strong the capital income risk mechanism is. Formally:

Proposition 2. *Let Assumptions* 1 *and* 2 *hold, and suppose* $\bar{\omega} \geq \bar{y}$*. Then* ω_∞ *is thin-tailed.*

Proof. See the Appendix. □

Remark 4. While Propositions 1 and 2 are derived under an i.i.d. representation for the labour income process, they naturally extend to more general settings where

5 Taking advantage of Lemma 1, one can possibly show that the Markov chain associated with the law (4) converges, in a strong probabilistic sense, to a unique stationary distribution defined on a possibly unbounded support. This would be an interesting result in and of itself, for it would generalize previous theoretical investigations who stick to CRRA preferences which either enforce linear bequest rules (under warm-glow preferences for altruism, as in e.g. Benhabib, Bisin, and Zhu 2011) or rather simplify checking technical conditions adapted from renewal theory that prove jointly sufficient for ergodicity of the limiting distribution of wealth in the presence of capital and labour income risk (as in e.g. Benhabib, Bisin, and Zhu 2015). Given the focus of the present work on the case $\bar{\omega} \geq \bar{y}$, I leave this analysis to future work.

$\tilde{y}_t$, defined on a bounded support, is irreducible, ergodic and admits a density representation.[6] This includes the realistic case where labour earnings exhibit some degree of serial correlation.

Remark 5. Let $\bar{\omega} \geq \bar{y}$. Since the threshold $\bar{\omega}$ does not depend on higher moments of the distribution of idiosyncratic shocks $\tilde{R}_t$, any pure increase in the volatility of returns on wealth that leaves their average unaffected (i.e. a mean preserving spread) has no impact on the convergence properties of the dynamic Equation (4). It follows that the ensuing limiting distribution of wealth will display a thin upper tail for any degree of capital income risk: in the long run, mobility across wealth levels and wealth inequality will ultimately by driven by labour income fluctuations.

Remark 6. Previous studies exploiting homothetic CRRA representations for the (u, v) functions explicitly assume the $\bar{\omega} \geq \bar{y}$ inequality away – see e.g. Assumption 1.i in Benhabib, Bisin, and Zhu (2015) and Assumption $5'''$ in Zhu (2019). As a matter of fact, these peculiar assumptions are imposed to guarantee that the wealth accumulation process resulting from optimal life-cycle (and/or bequest) choices of forward-looking agents is not trapped in the lower part of the wealth space in which intertemporal savings and/or intergenerational financial transfers are null, which is required for positive expansion of wealth toward arbitrarily large levels.

Remark 7. When the bequest motive is in *joy-of-giving* form (e.g. Andreoni 1990), i.e. when altruistic agents derive direct utility from bequeathing wealth to their offspring, positive financial bequests over the entire wealth space obtain in the CRRA case, eluding any of the arguments developed in the present study – see, among others, Benhabib, Bisin, and Zhu (2011) and Benhabib, Bisin, and Luo (2019). Relaxing the CRRA assumption and/or allowing for paternalistic concerns can dramatically change the picture, and thus calls for identifying necessary conditions under which the capital income risk mechanism is able to produce theoretical tails as fat as those documented in the real world. This is exactly where my analysis steps in. The above results suggest that the role of capital income risk on wealth dynamics in microfounded models of bequest behavior are far more involved – and their relationship with measures of wealth inequality more difficult to characterize – relative to the CRRA/joy-of-giving case.

6 See Zhu (2019) and Ma, Stachurski, and Toda (2020) for formal definitions.

5 Labour Income Risk and the Wealth Distribution

Having established a link between the stochastic features of the labour earnings process and the existence of a region of the wealth space where, conditional on the preference structure, optimal bequest strategies of altruistic agents entail zero financial transfers, it is instructive to understand how this link responds to well-defined changes in the fundamental features of the earnings distribution. To discipline this exercise, I restrict attention to first- and second-order dominance shifts in the latter and characterize their effects on the wealth threshold $\bar{\omega}$; I then relate those shifts to the imposition of labour income taxes in the model economy and to the ensuing features of the limiting distribution of wealth.

5.1 Stochastic Ordering

I first show that a larger probability of relatively high labour earnings – thus, a larger average return on non-financial wealth – discourages risk-averse agents to bear capital income risk by abstaining from leaving positive financial bequests at low wealth levels. Formally

Lemma 4. *Consider two distinct i.i.d. processes* $\{\tilde{y}\}$ *and* $\{\tilde{y}'\}$ *complying with* Assumption 1. *All else equal, if* $\tilde{y}'$ *first-order stochastically dominates* $\tilde{y}$, *then the wealth threshold of* Lemma 1 *under* $\{\tilde{y}'\}$ *is larger than under* $\{\tilde{y}\}$, *for any preference structure satisfying* Assumption 2.

Proof. See the Appendix. □

By contrast, when labour income risk is sufficiently strong, it stimulates bequest behavior at relatively lower wealth levels for agents who exhibit downside risk-aversion (or prudence, after Kimball 1990). Formally

Lemma 5. *Consider two distinct i.i.d. processes* $\{\tilde{y}\}$ *and* $\{\tilde{y}'\}$ *complying with* Assumption 1. *All else equal, if* $\tilde{y}'$ *is a mean preserving spread of* $\tilde{y}$, *then the wealth threshold of* Lemma 1 *under* $\{\tilde{y}'\}$ *is smaller than under* $\{\tilde{y}\}$, *for any preference structure satisfying* Assumption 2 *and such that* $v'''(\cdot) > 0$.

Proof. See the Appendix. □

Remark 8. Upon quantifying (via simulated moment estimation) the relative importance of competing drivers of wealth accumulation and social mobility in the United States, Benhabib, Bisin, and Luo (2019) argue that stochastic earnings do not contribute to filling the upper tail of the stationary distribution of wealth, and yet prove fundamental in supporting social mobility from the bottom end by limiting the emergence of poverty traps. Lemma 5 offers theoretical support to this claim by showing that, in the economy under scrutiny, bequest incentives of prudent agents at low wealth levels are enhanced when earning uncertainty is relatively high.

5.2 Labour Income Taxation and the Structure of the Tail

Quantitative investigations of the relative importance of the stochastic earnings mechanism in shaping the fat-tailed behavior of wealth distributions appear to suggest that such a mechanism is very limited, unless coupled with overly counterfactual assumptions about either the skewness of earnings or the length of the working life of agents – see Benhabib, Bisin, and Luo (2019) for a discussion of this point. A natural implication of this substantial body of work is that fiscal policies affecting labour income risk are hardly important for top wealth concentration.

The analysis conducted so far can be fruitfully exploited to shed further light on the dependence of the stationary distribution of wealth on alternative earnings tax treatments, that embody a number of realistic features such as some form of tax progressivity. Governments routinely exploit more or less sophisticated tax policies (e.g. individual income taxes, corporate income taxes, property taxes) to raise revenue and support redistribution programs (e.g. universal transfers, unemployment insurance, public education). In the US, the Federal income tax system adopts a tax code with brackets under which inflation-indexed marginal tax rates (MRTs) remain constant by bracket and several exemptions apply – in 2023, income tax brackets ranged from 10 percent to 37 percent. In terms of wage taxation, which aggregates personal income taxes, social security contributions and payroll taxes on gross earnings, both tax wedges and net personal average taxes vary greatly across OECD countries, and heavily depend on the household structure (e.g. single workers, one-earner couple, two-earner couples with and without children), see OECD (2024).

In the following, I will study the dependence of the tail properties of the stationary wealth distribution on two simple fiscal policies: proportional earnings taxation without redistribution, and proportional earnings taxation cum a lump-sum transfer. Formally, the model is extended to encompass either of the following

(i) *Proportional taxation without redistribution.* Labour income is taxed at a fixed rate $\tau_y \in (0,1)$, which is common knowledge and time invariant. Tax revenues are assumed not to be redistributed to taxpayers, e.g. they are used by the Government to finance public expenditures that do not affect private decisions. Stochastic after-tax earnings $\tilde{y}_\tau$ thus are $\tilde{y}_\tau = (1-\tau_y)\tilde{y}$.

(ii) *Proportional taxation with redistribution.* Labour income is redistributed through a proportional tax $\tau_y \in (0, 1)$ on income and a lump-sum transfer $T(\tau_y) > 0$ to all the agents in the economy. Both τ_y and $T(\tau_y)$ are common knowledge and time invariant. Letting the Government run a balanced budget in each time period, one has $T(\tau_y) = \tau_y \mathbb{E}[\tilde{y}]$. Notice that a flat tax plus a lump-sum transfer is equivalent to a progressive tax since the effective average tax rate is increasing in labour income. Stochastic after-tax earnings thus are $\tilde{y}_\tau = (1 - \tau_y)\tilde{y} + \tau_y \mathbb{E}[\tilde{y}]$.

The presence of labour income taxation impacts the trade-offs faced by altruistic agents, in particular those near the borrowing constraint, who have a lower propensity to save at the margin, and may therefore respond to the fiscal policy measure by cutting back bequests. The question of interest is whether, and under what conditions, the introduction of a tax on labour income can prevent patterns of intergenerational wealth transmission along which exposure to idiosyncratic shocks to wealth returns operates to bolster extreme concentration in the upper tail of the long-run distribution.

In the CRRA setting, if altruism reflects a joy-of-giving motive, then, provided the degree of capital income risk is sufficiently strong, the upper tail of the stationary wealth distribution (if it exists) will be asymptotically equivalent to a Pareto law. An analogous result holds true in the presence of paternalistic altruism and CRRA preferences, provided both returns on inherited wealth and labour earnings shocks hitting children are perfectly anticipated by their parents at the time bequest choices are made, or when the elasticity of the marginal utility of consumption equals that of the marginal utility of children's wealth – e.g. Zhu (2019). On the policy front, this result implies that fiscal policies targeting labour earnings do not affect the accumulation mechanism that is responsible for the emergence of large and slowly declining top wealth shares. Is this property robust to (i) more general preference structures, and (ii) the non-observability of incomes shocks $(\tilde{R}, \tilde{y})$ on the part of altruistic agents? The following Proposition offers some insights into this issue:

Proposition 3. *Consider problem* (3), *and let Assumptions* 1 *and* 2 *hold. Suppose that the unique stationary distribution* ω_∞ *is fat-tailed in the no tax benchmark* ($\tau_y = 0$). *Then, for either tax scheme, there exist utility specifications* (u, v) *and tax rates* $\tau_y \geq \bar{\tau}$ ($\bar{\tau} < 1$) *under which a thin-tailed stationary wealth distribution emerges.*

Proof. See the Appendix. □

This above result suggests that, in economies where agents exhibit an altruistic (paternalistic) concern for the well-being of their children, labour income taxation may play a role in defining the structure of the upper tail of the limiting distribution of wealth: by reducing the volatility of earnings shocks, it countervails precautionary savings incentives, possibly tipping the economy into the zero bequest region of the wealth space, which, in turns, makes the capital income risk channel ineffective. Remarkably, a higher degree of progressivity of the tax/transfer structure, as typically prevailing across countries in the world, reinforces this mechanism, for it more strongly operates as a partial insurance device against idiosyncratic earnings risk.

6 Concluding Remarks

Recent theoretical advances suggest that capital income risk, rather than earnings uncertainty, is the key determinant of fat-tailed behavior of stationary wealth distributions. The present papers offers novel insights into this issue by studying a standard incomplete market model with general time and state separable preferences, where parental altruism and unobservable idiosyncratic shocks engender non-linear bequest rules. A number of joint restrictions on the preference structure and other fundamentals is herein derived, under which optimal bequest behavior entails no financial transfers across generations for any degree of capital income risk, causing the dynamics of wealth to converge to a unique (stationary) distribution with a thin upper tail. These results can be thought of as providing necessary conditions for the emergence of fat-tailed wealth distributions as endogenous outcomes of choice theoretic models of intergenerational wealth transmission, when altruistic agents entertain a paternalistic bequest motive. As such, they can be useful in disciplining simulation-based estimation exercises aimed at evaluating the ability of theoretical models to match the salient long-term features of wealth distributions and wealth inequality.

Appendix

Proof of Lemma 1

(i) By Assumption 2, the objective function is twice-differentiable and strictly concave. Notice that all of the constraints are linear, with the borrowing constraint $b_t \geq 0$ implying $c_t \leq \omega_t$. The following first-order (Karush-Kuhn-Tucker) conditions are thus necessary and sufficient for existence of unique policy functions $\left(c_t^*, b_t^*\right)$ solving the expected maximization problem (3):

$$c_t^* = \min\{\omega_t,\ \phi(\chi\,\mathbb{E}(\tilde{R}_{t+1}\,v'(\tilde{R}_{t+1}\,(\omega_t - c_t^*) + \tilde{y}_{t+1})))\} \tag{7}$$

$$b_t^* = \omega_t - c_t^* = \max\{0, \omega_t - \phi(\chi\,\mathbb{E}(\tilde{R}_{t+1}\,v'(\tilde{R}_{t+1}\,b_t^* + \tilde{y}_{t+1})))\} \tag{8}$$

where, to ease the notational burden, I use $\phi := u'^{-1}$. By smoothness of functions (u, v) it follows that both c_t^* and b_t^* are continuous functions of parental wealth ω_t.

Define now

$$h(b_t, \omega_t) = b_t - \max\left\{0, \omega_t - \phi\left(\chi\,\mathbb{E}\left(\tilde{R}_{t+1}\,v'\left(\tilde{R}_{t+1}\,\hat{b}(\omega_t) + \tilde{y}_{t+1}\right)\right)\right)\right\}$$

so that $h(b_t, \omega_t)$ is increasing in b_t and $h\left(b_t^*, \omega_t\right) = 0$. For any $\check{\omega}_t > \omega_t \geq 0$ one has

$$h(b_t, \check{\omega}_t) \leq 0$$

and thus, since $h\left(b_t^*, \omega_t\right) = 0$, it must be the case that $\check{b}_t$ solving $h(\check{b}_t, \check{\omega}_t) = 0$ satisfies $\check{b}_t \geq b_t^*$. Thus, the bequest policy function is non-decreasing in parental wealth.

(ii) From the first-order condition (8), and the fact that b_t^* is non-decreasing in ω_t, the supremum $\bar{\omega}$ of the set of wealth states $\omega_t \geq 0$ for which $b_t^* = 0$ is defined by the following

$$u'(\bar{\omega}) = \chi\mathbb{E}\left(\tilde{R}_{t+1}v'(\tilde{y}_{t+1})\right) \tag{9}$$

Notice $\bar{\omega}$ is unique given the strict monotonicity of u' and v' – see Assumption 2. Since (from time $t = 1$ onwards) $\underline{y}$ is the lowest possible wealth state – and thus $\omega_{t+1} \geq \underline{y}$ for all $\omega_t > 0$, $t \geq 0$ – then $b_t^* > 0$ if and only if $\omega_t > \max\left\{\underline{y}, \bar{\omega}\right\}$; in this case, $b_t^* = \hat{b}(\omega_t)$ with the latter fulfilling

$$\hat{b}(\omega_t) = \omega_t - \phi\left(\chi\,\mathbb{E}\left(\tilde{R}_{t+1}\,v'\left(\tilde{R}_{t+1}\,\hat{b}(\omega_t) + \tilde{y}_{t+1}\right)\right)\right)$$

Proof of Lemma 2 From the Equation (9) that uniquely defines the threshold $\bar{\omega}$ one has

$$\bar{\omega} > \underline{y} \quad \Longleftrightarrow \quad \chi\mathbb{E}\left(\tilde{R}_{t+1}v'(\tilde{y}_{t+1})\right) < u'(\underline{y}) \tag{10}$$

By virtue of the mutual independence between $\tilde{R}$ and $\tilde{y}$ (Assumption 1), the assertion follows.

Proof of Lemma 3 From the Equation (9) that uniquely defines the threshold $\bar{\omega}$ one has

$$\bar{\omega} \geq \bar{y} \quad \Longleftrightarrow \quad \chi\mathbb{E}\left(\tilde{R}_{t+1}v'(\tilde{y}_{t+1})\right) \leq u'(\bar{y}) \tag{11}$$

which, by the mutual independence between $\tilde{R}$ and $\tilde{y}$ (Assumption 1), and the fact that $\tilde{y} \geq \underline{y}$ almost surely and $v'(\cdot)$ is monotonically decreasing (Assumption 2), delivers the assertion.

Proof of Proposition 1 Let $\bar{\omega} \geq \bar{y}$. I will show that, for any initial wealth distribution $\omega_0 > 0$, there exists a unique limiting distribution w_∞ with an invariant probability measure μ^* to which the process (4) converges. To this end, I will rely on three key properties for the Markov chain $\{\omega_t\}$ generated by (4), which are known to be jointly sufficient for existence and uniqueness of the stationary distribution (see e.g. Meyn and Tweedie 2009): (1) irreducibility (every state in the wealth space is accessible from any other); (2) a-periodicity (accessible states are visited at irregular times), and (3) stability. Accordingly, I will (a) identify the irreducible state space for the Markov chain; (b) verify that the chain evolving on such a state space is ψ-irreducible and strongly a-periodic, which jointly warrant existence and uniqueness of the invariant probability measure; and finally (c) invoke the strong-sense stationarity of the i.i.d. stochastic process for labour income to conclude on the convergence of the chain to the distribution with the uniquely identified measure.

(a) Notice that, at each time period t and for any agent i, by virtue of Assumption 1

$$\omega_t > \underline{y} \quad \Leftrightarrow \quad \omega_t > \min\{\mathbb{S}_R\} b_t^* + \min\{\mathbb{S}_y\} \tag{12}$$

where $\mathbb{S}_R$ (respectively $\mathbb{S}_y$) is the smallest closed subset of the Borel σ-algebra in $\mathbb{R}$ such that the probability measure associated with the random variable $\tilde{R}$ (respectively $\tilde{y}$) satisfies $P_R(\mathbb{S}_R) = 1$ (respectively $P_y(\mathbb{S}_y) = 1$). This in turn implies that, for any $\omega_0 \geq \underline{y}$, there is a positive probability that there exists a finite time $n \geq 0$ at which $\omega_n < \bar{y} < \bar{\omega}$, and thus

$$\Pr\left(\omega_{n+1} > \bar{y} \mid \omega_n\right) = 0, \quad \forall \omega_n \leq \bar{\omega} \tag{13}$$

by Lemma 1. The set $\underline{B} = [\underline{y}, \bar{y}]$ is therefore accessible from any other wealth state $\omega_t > \bar{y}$; since $b_t^*(\omega_t) = 0$ for all $\omega_t \in \underline{B}$, then the set $\underline{B}$ is absorbing and full: the process (4) reduces to a Markov chain $\{\hat{\omega}_t\}$ evolving on the compact state space $[\underline{y}, \bar{y}]$.

(b) Since $\hat{\omega}_t = \tilde{y}_t$ for $t \geq n$, the chain $\hat{\omega}_t$ is ψ-irreducible and strongly a-periodic since $\{\tilde{y}_t\}$ is an i.i.d. sequence as per Assumption 1.

(c) Since the chain $\{\hat{\omega}_t\}$ evolves on a bounded set, it cannot drift to infinity.

Thus, a unique limiting (stationary) distribution of wealth ω_∞ exists with support $\underline{B}$ and invariant measure μ^* as induced by $\tilde{y}$, i.e. $\mu^*(\mathcal{B}) = \int_{\mathcal{B}} f(y) \mathrm{d}\lambda(y)$ for all the Borel sets $\mathcal{B}$ (λ is the Lebesgue measure).

Proof of Proposition 2 Since $|\omega_\infty| \leq \bar{y}$ almost surely, then

$$\mathbb{E}\left(|\omega_\infty^n|\right) = \mathbb{E}\left(|\omega_\infty|^n\right) \leq \bar{y}^n < \infty, \quad n \in \mathbb{N} \tag{14}$$

i.e. all of the moments of the limiting distribution of wealth exist, implying thin tails for ω_∞ on its support.

Proof of Lemma 4 Let G' (respectively G) denote the CDF of the random variable $\tilde{y}'$ (respectively $\tilde{y}$) on the support $[\underline{y}, \bar{y}]$. Recall that $\tilde{y}'$ first-order stochastically dominates $\tilde{y}$ – written $\tilde{y}' \geq_1 \tilde{y}$ – if and only if $G'(x) \leq G(x)$ for all $x \in [\underline{y}, \bar{y}]$.

Since $v'(\cdot)$ strictly decreases on its domain, by virtue of Theorem 1.A.3, part (a) in Shaked and Shanthikumar (2007) it holds $v'(\tilde{y}') \leq_1 v'(\tilde{y})$, which in turn implies (by the definition of first-order dominance) $\mathbb{E}[v'(\tilde{y}')] \leq \mathbb{E}[v'(\tilde{y})]$.

Since the wealth thresholds under the two processes for labour income are uniquely defined as follows (see Equation (9))

$$u'(\bar{\omega}) = \chi \mathbb{E}\left(\tilde{R}\right)\mathbb{E}\left(v'(\tilde{y})\right), \tag{15}$$

$$u'(\bar{\omega}') = \chi \mathbb{E}\left(\tilde{R}\right)\mathbb{E}\left(v'(\tilde{y}')\right) \tag{16}$$

and since $u'(\cdot)$ is strictly decreasing on its domain by Assumption 2, the assertion follows.

Proof of Lemma 5 Let G' (respectively G) denote the CDF of the random variable $\tilde{y}'$ (respectively $\tilde{y}$) on the support $[\underline{y}, \bar{y}]$. Recall that $\tilde{y}'$ is a mean-preserving spread of $\tilde{y}$ – written $\tilde{y}' \geq_{MPS} \tilde{y}$ – if and only if $\tilde{y}' =^d \tilde{y} + \epsilon$, $\mathbb{E}\left(\epsilon \mid \tilde{y}\right) = 0$ where $=^d$ denotes equality in distribution. Recall also that $\tilde{y}' \geq_{MPS} \tilde{y}$ if and only if, when $\mathbb{E}\left(\tilde{y}'\right) = \mathbb{E}\left(\tilde{y}\right)$, it holds $\int_\kappa (x)\mathrm{d}G' \geq \int_U \kappa(x)\mathrm{d}G$ for all x, where $U = [\underline{y}, \bar{y}]$ is the smallest closed set including the union of the supports of the two stochastic processes, and $\kappa(\cdot)$ is any concave function defined on U – see Rothschild and Stiglitz (1970); or, equivalently, if and only if, when $\mathbb{E}\left(\tilde{y}'\right) = \mathbb{E}\left(\tilde{y}\right)$, $\tilde{y}$ is smaller than $\tilde{y}'$ in the convex order, since $-v(\cdot)$ is a globally (strictly) convex function – see Shaked and Shanthikumar (2007).

Assume now $v'''(\cdot) > 0$ (or *prudence*, after Kimball 1990), so that the function $v'(\cdot)$ is globally (strictly) convex on its domain. By the definition of the convex order, it follows that $\mathbb{E}[v'(\tilde{y}')] \geq \mathbb{E}[v'(\tilde{y})]$ must hold. Since the wealth thresholds under the two processes for labour income are uniquely defined as follows (see Equation (9))

$$u'(\bar{\omega}) = \chi \mathbb{E}\left(\tilde{R}\right)\mathbb{E}\left(v'(\tilde{y})\right), \tag{17}$$

$$u'(\bar{\omega}') = \chi \mathbb{E}\left(\tilde{R}\right)\mathbb{E}\left(v'(\tilde{y}')\right) \tag{18}$$

and since $u'(\cdot)$ is strictly decreasing on its domain by Assumption 2, the assertion follows.

Proof of Proposition 3 The proof is constructive. I consider the continuously infinite set of CRRA utility functions with different elasticities (e.g. Benhabib, Bisin, and Luo 2019):

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad v(\omega_{t+1}) = \frac{\omega_{t+1}^{1-\mu}}{1-\mu} \tag{19}$$

with $0 < \mu < \sigma$, $\mu \neq 1$, $\sigma \neq 1$. Notice that the utility functions (u, v) in (19) fulfill Assumption 2. I shall emphasize that the CRRA assumption does not impose any knife-edge condition on the model under scrutiny: as shown below, the set of parameter values satisfying the conditions under which the stated result holds true has a non-empty interior in the space of all admissible preference (elasticity) parameters.

The expected utility maximization problem faced by the agents is now in the form

$$\begin{aligned} &\max_{c_t,\, b_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{\omega_{t+1}^{1-\mu}}{1-\mu}, \\ &\text{s.t. } c_t + b_t \leq \tilde{R}_t b_{t-1} + \tilde{y}_{\tau,t}, \\ &\qquad c_t \geq 0,\ b_t \geq 0, \end{aligned} \tag{20}$$

where $\tilde{y}_{\tau,t}$ denote stochastic after-tax earnings, given the tax regime in place (proportional taxation with or without redistribution).

By assumption, in the no tax benchmark, the stationary wealth distribution ω_∞ is fat-tailed; by virtue of Proposition 2, it must be the case that $\bar{\omega} < \bar{y}$, which is equivalent to the following

$$\chi \mathbb{E}[\tilde{R}] \mathbb{E}\left[\tilde{y}^{-\mu}\right] > \bar{y}^{-\sigma} \tag{21}$$

Consider first the case of a proportional tax without redistribution, i.e. $\tilde{y}_\tau = (1-\tau_y)\tilde{y}$ with $\tau_y \in (0,1)$. Using Lemma 1, there exists a unique threshold $\bar{\omega}^\tau > 0$ defined as

$$\bar{\omega}^\tau = \left(\chi \mathbb{E}[\tilde{R}]\right)^{-\frac{1}{\sigma}} \left(\mathbb{E}\left[(1-\tau_y)\tilde{y}\right]^{-\mu}\right)^{-\frac{1}{\sigma}} \tag{22}$$

such that $b_t^* = 0$ if and only if $\omega_t \leq \bar{\omega}^\tau$. A thin-tailed stationary wealth distribution will emerge – again by virtue of Proposition 2 – provided $\bar{\omega}^\tau \geq (1-\tau_y)\bar{y}$.[7]

From (22), a sufficient condition for $\bar{\omega}^\tau \geq (1-\tau_y)\bar{y}$ to occur is

$$\chi \mathbb{E}[\tilde{R}] \left[(1-\tau_y)\underline{y}\right]^{-\mu} \leq \left[(1-\tau_y)\bar{y}\right]^{-\mu} \tag{23}$$

Thus, the problem of showing emergence of a thin-tailed wealth distribution under labour income taxation can be framed as follows: are there any tax rates $\tau_y \in (0,1)$ such that both (21) and (23) are satisfied? Clearly, the answer depends on (i) the intensity of the bequest motive χ, (ii) the magnitude of the average return on wealth $\mathbb{E}[\tilde{R}]$ and (iii) the range of variation of labour income $\tilde{y}$. Fix any $\chi > 0$ and $\mathbb{E}[\tilde{R}]$ consistent with Assumption 1. Since $\mu < \sigma$, the sufficient condition in (23) holds for all tax rates $\tau_y \geq \bar{\tau}$, where

7 Notice that $\bar{\omega}^\tau = (1-\tau_y)^{\frac{\mu}{\sigma}} \bar{\omega}$; hence $\mu < \sigma$, as assumed, is necessary for having $\bar{\omega} < \bar{y}$ and $\bar{\omega}^\tau \geq (1-\tau_y)\bar{y}$ both hold.

$$\bar{\tau} := 1 - \left(\frac{y^{\mu}}{\bar{\bar{y}}^{\sigma}} \frac{1}{\chi \mathbb{E}[\tilde{R}]} \right)^{\frac{1}{\sigma-\mu}} < 1 \tag{24}$$

for any pair $(\underline{y}, \bar{y})$ for which (21) is satisfied. This proves the assertion.

Consider now the case of proportional taxation with redistribution via the (budget balancing) lump-sum transfer $T(\tau_y) = \tau_y \mathbb{E}[\tilde{y}]$. Notice that $\mathbb{E}[\tilde{y}^{\tau}] = \mathbb{E}[\tilde{y}]$, and that the functions

$$\kappa(x) = x \in [0, \infty) \mapsto x, \qquad \kappa_{\tau}(x) = x \in [0, \infty) \mapsto (1 - \tau_y)x + \tau_y \mathbb{E}[\tilde{y}]$$

are non-negative and increasing on their domain (with $\kappa(x) > 0$ and $\kappa_{\tau}(x) > 0$ for $x > 0$), and such that $\kappa(x)/\kappa_{\tau}(x)$ is increasing in $x > 0$. By Theorem 3.A.26 of Shaked and Shanthikumar (2007) the after-tax income process $\tilde{y}_{\tau}$ dominates the before-tax income process $\tilde{y}$ according to the Lorenz order, i.e. the Lorenz curve corresponding to $\tilde{y}$ is larger than the Lorenz curve corresponding to $\tilde{y}_{\tau}$. By the equality of the means of the two income processes, Theorem 1.5.13 part (c) in Müller and Stoyan (2002) and Theorem 3.A.1 part (b) in Shaked and Shanthikumar (2007) jointly imply $\tilde{y} \geq_{MPS} \tilde{y}_{\tau}$. As the chosen CRRA specification entails $\upsilon''' > 0$, Lemma 5 then implies $\bar{\omega}^{\tau} \geq \bar{\omega}$. Together with the fact that $\bar{y} > (1 - \tau_y)\bar{y} + \tau_y \mathbb{E}[\tilde{y}]$, it follows that any tax rate τ_y equal to or larger than the threshold identified in (24) will, a *fortiori*, be such that $\bar{\omega}^{\tau} \geq \bar{y}$, delivering the assertion.

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