Research Article

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On the Microfoundation of Linear Oligopoly Demand

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Abstract: We critically assess the representative consumer with quadratic aggregate utility function which forms the foundation of a well-known class of linear oligopoly demand structures. It is argued that this approach is problematic and redundant. Regarding the latter, we show how the same demand system can be derived directly from a population of heterogeneous buyers for any number of products. Welfare analyses based on aggregate demand is shown to be sensitive to the underlying microfoundation.

Keywords: microfoundations, oligopoly theory, product differentiation, representative consumer models

JEL Codes: B4, L1

1 Introduction

A well-known way of describing the buyers' side of an oligopoly market is through a linear horizontally differentiated demand model. For the case of duopoly, the (direct) demand structure generally takes the following form:

$$x_1(p_1, p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1, p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1,$$

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where price and quantity are positive and respectively given by p_i and x_i, for $i = 1, 2$.[1] It is, moreover, commonly assumed that a_i, b_i, $c > 0$ and $b_i > c$, for $i = 1, 2$, so that a firm's demand depends negatively on its own price, positively on the rival's price and own effects dominate cross effects. This demand system can be roughly interpreted as follows. If, say, firm 1 raises its price slightly, then *ceteris paribus* some of its customers walk away and either go home or visit firm 2 instead. Likewise, lowering price attracts additional buyers, some of whom switch from the competing firm.

The traditional foundation for this demand specification does not come from a group of heterogeneous buyers, however, but from a representative consumer who on behalf of an unspecified buyer population maximizes a quadratic aggregate welfare function. The most popular variations of this type are due to Bowley (1924) and Shubik and Levitan (1980).[2]

In the field of macroeconomics, such a representative agent approach has been heavily criticized by many. One reason for this is that transforming individual preferences into representative aggregate preferences often proves problematic. It is, for instance, quite possible that the representative agent prefers A to B, whereas each and every represented buyer prefers B to A.[3] For this and other reasons, many macroeconomists are reluctant to take this approach and some even went as far as to effectively compose a requiem for the representative consumer.[4] This is in stark contrast to the fields of microeconomics and industrial organization, where the use of such a fictitious agent is widely accepted. What makes this particularly surprising is that a rationale for this approach is commonly missing.[5]

In this paper, we pursue two main goals. The first is to provide a critical assessment of the representative consumer as a foundation for the above linear demand structure. Specifically, we argue that it is both *inaccurate* and *inadequate*. It is inaccurate as the representative agent's aggregate utility function has no

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1 See, for instance, Dixit (1979) and Singh and Vives (1984). Häckner (2000) offers an n-firm variant. Although popular in oligopoly theory, it is noteworthy that this demand system is not necessarily specific to this type of market structure. It may, for example, also describe the demand side of a multi-product monopoly. The welfare analysis that we offer in Section 4 does not apply to monopoly, however, which is why our focus is on oligopoly.

2 See Martin (2002) for a discussion of both these models. For an historical overview of the development of this type of representative consumer model, see the recent work by Choné and Linnemer (2020).

3 This Pareto inconsistency has been clearly established by Jerison (1984). See also Dow and da Costa Werlang (1988).

4 See, for example, Kirman (1992).

5 As a telling example, both Bowley (1924) and Shubik and Levitan (1980) introduce this approach as an illustration and do not provide an explanation or justification for the specification of their representative consumer's utility function.

(clear) connection with the objectives of those represented. Indeed, the existing literature typically remains silent on the characteristics and traits of those represented.[6] It is inadequate because a justification for both the utility specification and the solution approach is missing. Taken together, this leads us to conclude that the popular linear oligopoly demand structure lacks a proper foundation.

We then proceed with our second goal, which is to argue that quadratic representative consumer models are effectively redundant. This is done by showing that the same demand structure can be easily derived directly from a population of heterogeneous consumers making discrete choices. Specifically, we use a variation of the spatial spokes models as described in Chen and Riordan (2007) and Amir et al. (2016). We first present this alternative foundation for duopoly and then extend the framework to allow for any number of firms. The latter is of particular interest because Jaffe and Weyl (2010) establish that the linear oligopoly demand structure is inconsistent with discrete consumer choice when the number of products exceeds two. The critical difference with our approach is that we assume each firm to have a brand loyal, captive customer base.[7] That property appears sufficient for a discrete choice microfoundation of the linear demand differentiation model independent of the number of products involved.

A main advantage of this approach is that it is explicitly based on simple buyer behavior at the micro-level and therefore has a natural interpretation. In addition, however, it also has potentially important welfare implications. To illustrate this, we show that a welfare analysis based on the aggregate linear demand system may lead to wrong conclusions if one ignores the corresponding microeconomic foundation. Specifically, consumer welfare may decrease in the cross-price effect within our setting, while at the same time making the representative consumer better off. Hence, our population of heterogeneous buyers is not properly represented by this fictitious quadratic utility maximizer.

A couple of recent papers have raised some red flags regarding the use of a representative agent with a quadratic utility function. Kopel, Ressi, and Lambertini (2017), for instance, find that seemingly similar quadratic aggregate utility functions may give rise to fundamentally different demand systems. In turn, this might lead to radically different policy implications. In a similar vein, Theilen (2012) illustrates how the consequences of changes in product differentiation can be very

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6 An exception here would be when the population of represented buyers is assumed to all possess a similar type of utility function. Yet, in that particular case it is not clear what exactly would be the added value of a representative agent.

7 More details on strategic interaction among firms with captive consumers can be found in, for example, Armstrong and Vickers (1993). Here, brand-loyalty is taken as given. The possibility of consumer loyalty being partly endogenous is studied in Soeiro and Pinto (2019).

sensitive to the precise linear demand specification. Amir, Erickson, and Jin (2017) contains a thorough study of several characteristics of the quadratic utility specification. Among other things, this paper shows that strict concavity of the utility function is a necessary condition for the corresponding demand system to be well-defined. As to the alternative microfoundation, Martin (2002, p. 53) points out how the linear demand system can be derived from a vertical product differentiation model. Finally, our work is also related to Anderson, De Palma, and Thisse (1989, 1992) who show how other representative consumer models, such as ones with a constant elasticity of substitution (CES) utility function, can be given a discrete choice foundation.

The remainder of this paper is organized as follows. The next section discusses the quadratic representative consumer utility function and highlights several problematic features of this specification. Section 3 presents the alternative microfoundation for linear oligopoly demand. Section 4 offers some welfare implications. Section 5 concludes.

2 Quadratic Representative Consumer Models

In this section, we express some concerns about the above mentioned class of quadratic representative consumer models. Specifically, we raise several issues regarding the shape of the objective function, the derivation of the corresponding demand functions and the relation between the products involved.

2.1 Issue 1: The Objective

Both the Bowley (1924) and the Shubik and Levitan (1980) demand specifications are derived from a representative consumer *gross* utility function that takes the following general form:

$$U(x_1, x_2) = \alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2 - \gamma \cdot (x_2 - x_1)^2,$$

with $\alpha, \beta, \gamma > 0$.[8] Notice that the way in which we present this objective function, it effectively consists of three distinct parts. Starting with the third, $\gamma \cdot (x_2 - x_1)^2$, this part captures the complementarity between products x_1 and x_2. Consistent with classic consumer theory, utility is *ceteris paribus* higher with a more balanced consumption plan. In fact, the representative agent is induced to buy both

8 We use this general form to facilitate the discussion. For the precise details of both utility functions, see Bowley (1924, p. 56) and Shubik and Levitan 1980, p. 69).

products in equal amounts ($x_1 = x_2$) so that the third-term disutility is minimized and *de facto* disappears.

The first two parts, $\alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2$, capture utility coming from total rather than relative consumption and express the extent to which the goods are considered substitutes. These components indicate that the consumer derives utility from consuming more products, but only up to a certain amount. That is, utility increases at low levels of total consumption through the first term, but at higher levels of total consumption the second term starts to dominate the first. This implies that there is a point at which the consumer is satiated. Notice that this is true also when the representative agent does not face a budget constraint and holds even when the goods would be offered, in the broadest sense, for free. Contrary to the third term favoring balance in consumption, this therefore is at odds with traditional consumer theory. Indeed, the fact that the objective function has a unique maximum makes that the common assumption of nonsatiation is violated.[9]

2.2 Issue 2: The Solution

To solve the representative consumer problem, one naturally needs to take account of the cost of consumption. In the following, we point out that the linear oligopoly demand structure will only result from the representative agent's maximization problem under fairly specific, and arguably strong, assumptions.

Towards that end, let F be a set of vectors $(x, m) \in \mathbb{R}^n \times \mathbb{R}$ and consider the utility function $V: F \mapsto \mathbb{R}$. For $(x, m) \in F$, therefore, utility $V(x, m)$ is obtained from consuming an amount of $x \in \mathbb{R}^n$ goods as well as from the unspent money m.[10] Moreover, let the vector of prices be given by $p \in \mathbb{R}^n$. If the available income is I, then the representative consumer faces the following general maximization problem:

$$\begin{aligned} &\max_{x \in F} V(x, m) \\ &\text{s.t.}: \quad m = I - p \cdot x, \\ &\qquad\quad\; m \geq 0, \end{aligned}$$

where $p \cdot x$ is total expenditure.

9 It further violates the free disposal assumption. For a detailed discussion of classic consumer theory, see Chapter 1 of Jehle and Reny (2001). It is worth noting that this conclusion also holds for the net utility function, i.e., the gross utility function minus expenditures (see, e.g., Shubik and Levitan 1980, p. 69)). In that case, the representative agent would prefer to have less of x_1 and x_2 at given prices.

10 Alternatively, m can be interpreted as an Hicksian composite commodity with a price normalized to 1.

It can be easily verified, however, that the linear oligopoly demand system is the solution to:

$$\max_{x \in F} U(x) + m$$
$$\text{s.t.}: \qquad m = I - p \cdot x.$$

Thus, in light of the general maximization problem, $V(x, m) = U(x) + m$ and the constraint $m \geq 0$ is either ignored or income is (implicitly) assumed large enough. Observe that this specification effectively treats expenditures as a disutility, which is linearly subtracted from the gross utility function. At first sight this may seem natural and innocuous, but it does imply two strong assumptions:

[A1] The representative consumer's utility function is quasi-linear in money;

[A2] The representative consumer can borrow unlimited amounts of money for free.

The first condition means the absence of a *wealth effect* as utility is linearly increasing in money. Irrespective of whether utility is high or low, the marginal gain of an extra dollar is the same.[11] The second states that the budget restriction has no bite and is effectively non-existent. Given the quadratic nature of the objective function, this holds trivially when the representative consumer has a sufficiently large income.[12] Note, however, that this approach makes it irrelevant how much budget is available. Indeed, even when the representative agent loses all his income ($I = 0$) or would have severe debts ($I < 0$), demand for both products stays the same.

2.3 Issue 3: The Products

Following the textbook approach and thus assuming A1 and A2, the representative consumer picks x_1 and x_2 to maximize:

$$V(x_1, x_2) = \alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2 - \gamma \cdot (x_2 - x_1)^2 + I - p_1 \cdot x_1 - p_2 \cdot x_2.$$

11 Interestingly, LaFrance (1985) finds that the linear demand system requires the underlying utility function to be quadratic when income effects are absent. Also, income effects may well be present when assuming a different class of utility functions, such as the ones with a constant elasticity of substitution (CES). See, for example, Spence (1976) and Dixit and Stiglitz (1977). It is further noteworthy that A1 is violated in the original derivation provided by Bowley (1924, p. 56) and that the resulting demand functions are, in fact, nonlinear in prices.

12 As Amir, Erickson, and Jin (2017) point out, there indeed is an interior solution in which income effects are absent when the representative consumer's budget is sufficiently high.

Goods are substitutes in *utility* when for every $\eta_1 > 0$ there is an $\eta_2 > 0$ such that:

$$U\left(x_1 - \eta_1, x_2 + \eta_2\right) = U\left(x_1, x_2\right).$$

That is, every decrease in utility resulting from a reduction in the consumption of good 1 can be compensated by an increase in the consumption of good 2.[13] If we consider x_2 an implicit function $X_2\left(x_1\right)$ along a level curve of V, then $X_2' < 0$. Implicit differentiation yields:

$$\alpha \cdot \left(1 + X_2'\right) - 2\beta \cdot \left(x_1 + X_2\right) \cdot \left(1 + X_2'\right) - 2\gamma \cdot \left(x_1 - X_2\right) \cdot \left(1 - X_2'\right) = 0.$$

Rearranging gives:

$$X_2'\left(x_1\right) = \frac{2\gamma \cdot \left(x_1 - x_2\right) + 2\beta \cdot \left(x_1 + x_2\right) - \alpha}{2\gamma \cdot \left(x_1 - x_2\right) - 2\beta \cdot \left(x_1 + x_2\right) + \alpha}.$$

Thus, $X_2' < 0$ requires

$$2\gamma \cdot \left(x_1 - x_2\right) + 2\beta \cdot \left(x_1 + x_2\right) - \alpha < 0 \Rightarrow \left(\beta + \gamma\right) \cdot x_1 + \left(\beta - \gamma\right) \cdot x_2 < \frac{\alpha}{2}$$

and

$$2\gamma \cdot \left(x_1 - x_2\right) - 2\beta \cdot \left(x_1 + x_2\right) + \alpha > 0 \Rightarrow \left(\beta + \gamma\right) \cdot x_2 + \left(\beta - \gamma\right) \cdot x_1 < \frac{\alpha}{2}$$

or the reverse.

Note that the above inequalities do not hold for all consumption bundles (x_1, x_2). For instance, $x_1 = 0$, $x_2 = 1$ and $\beta - \gamma < \frac{\alpha}{2} < \beta + \gamma$ violates the inequalities. In this model, therefore, utility can remain constant when the representative agent receives more or less of both goods.[14] One therefore may need a rather peculiar quadratic aggregate utility function to obtain a relatively straightforward demand system. This is arguably problematic since the representative consumer's utility function is ultimately intended to serve as a microfoundation for linear oligopoly demand.

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13 It is noteworthy that there is an alternative interpretation of substitution in *utility* in the literature that dates back as far as Edgeworth (1881). In that case, two goods, x and y, are considered substitutes in utility when $\partial^2 U/\partial x \partial y < 0$ and $\partial^2 U/\partial y \partial x < 0$. It can be shown that when both goods are substitutes in utility according to this definition, then they are indeed also substitutes in price and *vice versa*. As shown in Amir, Erickson, and Jin (2017), however, this need no longer be the case for three or more goods.

14 Indeed, it can be easily verified that the level curves are ellipses. To illustrate, let $\alpha = 4$ and $\beta = \gamma$ be (approximately) equal to 1 so that $U = 2 - (x_1 - 1)^2 - (x_2 - 1)^2$. At $U = 1$, the indifference curve is therefore a circle with center $(x_1, x_2) = (1, 1)$ and radius 1.

3 A Microeconomic Foundation for Linear Oligopoly Demand

In the preceding section, we have highlighted some problematic features of quadratic representative consumer models. We now proceed by presenting an alternative microeconomic foundation for the linear oligopoly demand system. Below, we study the two-good case first before showing the derivation for any number of products.

3.1 Duopoly

Consider the linear duopoly demand system as described above:

$$x_1(p_1, p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1, p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1.$$

In the following, we will show how this demand structure can be derived directly from a population of heterogeneous consumers.

Consider a price-setting duopoly where both firms are located on the boundary of an interval [0, 2]. In particular, and without loss of generality, firm 1 and firm 2 are respectively situated at 0 and 2. There are three types of consumers, each with uniform population density λ_i on [0, 2], $i = 1, 2, 3$. The total number of type i buyers is thus given by $2 \cdot \lambda_i$. Type 1 customers are assumed to obtain positive gross utility when buying from firm 1, $s > 0$, and no utility when buying from firm 2. By contrast, Type 2 customers attach no value to the products of firm 1 and derive positive gross utility from buying at firm 2, $v > 0$. Finally, Type 3 customers value both equally and have a willingness to pay of 4 for each.[15] Consumers either buy one unit of the product or do not buy and are characterized by their location. In the spirit of spatial IO settings, there are costs associated with distance between buyer and seller and these are assumed to be linearly increasing.

Let us now specify the utility function of a Type 1 consumer located at $z \in [0, 2]$. This customer has basically three options: (1) buy from firm 1 (value $s - z - p_1$), (2) buy from firm 2 (value $z - 2 - p_2$) or (3) buy nothing (value 0). Notice that the third

15 As an illustrative interpretation, one may view both firms as competing ice cream vendors where firm 1 sells strawberry flavor and firm 2 sells vanilla ice. Type 1 buyers are then those customers who only like strawberry ice, for example, whereas Type 2 buyers exclusively prefer vanilla. Type 3 customers consider both and let their buying decision depend on the prices set.

choice dominates the second, because $z-2-p_2 \le 0$ for positive prices. Thus, the utility function of a Type 1 buyer located at $z \in [0,2]$ effectively is

$$u_1(p_1,p_2,z) = \max\{s-z-p_1, 0\}.$$

The Type 1 customer who is indifferent between option (1) and option (3) is located at $z = s - p_1$.

The utility function of a Type 2 customer located at $z \in [0,2]$ can be determined in a similar fashion and is given by

$$u_2(p_1,p_2,z) = \max\{v-2+z-p_2, 0\}.$$

The Type 2 customer who is indifferent between buying and not buying is thus located at $z = p_2 - v + 2$. Finally, the utility function of a Type 3 customer at $z \in [0,2]$ is

$$u_3(p_1,p_2,z) = \max\{4-z-p_1, 2+z-p_2, 0\}.$$

Under the assumption that prices are sufficiently low, the indifferent Type 3 buyer is located at $z = 1 + \frac{1}{2}(p_2 - p_1)$.[16]

On the basis of these utility specifications, we can now derive the corresponding demand functions. For a given combination of prices (p_1, p_2) in the relevant range, demand for firm 1 is given by the sum of consuming Type 1 buyers and the part of Type 3 buyers preferring the product of firm 1.[17]

$$\begin{aligned} x_1(p_1,p_2) &= \lambda_1 \cdot (s-p_1) + \lambda_3 \cdot \left(1 + \frac{1}{2}(p_2-p_1)\right) \\ &= s \cdot \lambda_1 + \lambda_3 - \left(\lambda_1 + \frac{1}{2}\lambda_3\right) \cdot p_1 + \frac{1}{2}\lambda_3 \cdot p_2. \end{aligned}$$

Demand for the products of firm 2 can be derived in a similar way.

$$x_2(p_1,p_2) = v \cdot \lambda_2 + \lambda_3 - \left(\lambda_2 + \frac{1}{2}\lambda_3\right) \cdot p_2 + \frac{1}{2}\lambda_3 \cdot p_1.$$

The above approach therefore allows one to derive any linear duopoly demand system of the form:

$$x_1(p_1,p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1,p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1,$$

where $\lambda_1 = b_1 - c$, $\lambda_2 = b_2 - c$, $\lambda_3 = 2c$, $s = \dfrac{a_1 - 2c}{b_1 - c}$ and $v = \dfrac{a_2 - 2c}{b_2 - c}$.

16 A sufficient condition to ensure that all Type 3 consumers buy a product is $p_1, p_2 \le 2$.

17 Indifferent Type 1 and Type 2 customers are located in the interval [0, 2] when $s - 2 \le p_1 \le s$ and $v - 2 \le p_2 \le v$. Type 3 customers prefer to buy a product when $p_1 + p_2 \le 6$.

3.2 Oligopoly

The above duopoly demand structure has been generalized to any number of firms by Häckner (2000). If the set of firms is N, then demand for each firm $k \in N$ takes the following form:

$$x_k(p_k, p_{-k}) = a_k - b_k \cdot p_k + c \cdot \sum_{j \neq k} p_j,$$

with a_k, b_k, $c > 0$. We will now use a variation on the 'spokes-model' introduced by Chen and Riordan (2007) to show how this demand system can be derived with the approach laid out in the previous subsection.[18]

Consider a price-setting oligopoly where each firm is 'located' at an end node of a star-shaped network. The edge from the center to firm i is denoted by E_i. Let the distance to the center (i.e., the length of E_i) be equal to 1 so that the distance between each pair of firms is 2.

For each firm $i \in N$, there is a Type i consumer with uniform population density λ_i on E_i. The total number of type i buyers is thus given by λ_i. Type i customers are assumed captive in the sense that they obtain positive gross utility $v_i > 0$ when buying from firm i and no utility when buying from any other firm $j \in N$, $j \neq i$.

Next, for each pair of firms (i, j), type ij customers value the goods from both firm i and firm j equally and have a willingness to pay of 4 for each. Type ij customers are distributed with uniform population density λ_{ij} along the path $E_i \cup E_j$ from firm i to firm j. Consumers either buy one unit of the product or do not buy and are characterized by their location. As before, there are costs associated with distance between buyer and seller and these are assumed to be linearly increasing.

Let us now specify the utility function of a Type i consumer located at $z \in E_i$, where $z = 0$ indicates the location of firm i at the endpoint of E_i and $z = 1$ is the location of the center. Such a customer has basically three options: (1) buy from firm i (value $v_i - z - p_i$), (2) buy from another firm j, $j \neq i$, (value $z - 2 - p_j$) or (3) buy nothing (value 0). Notice that the third choice dominates the second since $z - 2 - p_j \leq 0$ whenever p_j is positive. Thus, the utility function of a Type i buyer located at $z \in E_i$ is effectively given by:

$$u_i(p_i, p_{-i}, z) = \max\{v_i - z - p_i, 0\}.$$

The Type i customer who is indifferent between option (1) and option (3) is located at $z = v_i - p_i$.

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18 Chen and Riordan (2007) assume all consumers to be interested in two brands. By contrast, we distinguish between brand loyal buyers who only consider buying from their preferred supplier and non-loyal customers who consider buying from any supplier. Amir et al. (2016) study monopoly pricing within a similar star-shaped setting assuming that one shop is located in the center.

The utility function of a Type *ij* customer located at $z \in E_i \cup E_j$ can be determined in a similar fashion and is given by

$$u_{ij}\left(p_i, p_j, p_{-ij}, z\right) = \max\left\{4 - z - p_i, 2 + z - p_j, 0\right\}.$$

Under the assumption that prices are sufficiently low, the indifferent Type *ij* buyer is located at $z = 1 + \frac{1}{2}\left(p_j - p_i\right)$.[19]

On the basis of these utility specifications, we can now derive the corresponding demand functions. For a given combination of prices $p = (p_1, \dots, p_n)$ in the relevant range, demand for firm *i* is given by the sum of consuming Type *i* buyers and the part of Type *ij* buyers preferring the product of firm *i*.

$$\begin{aligned} x_k\left(p_k, p_{-k}\right) &= \lambda_k \cdot \left(v_k - p_k\right) + \sum_{j \neq k} \lambda_{kj} \cdot \left(1 + \frac{1}{2}\left(p_j - p_k\right)\right) \\ &= v_k \cdot \lambda_k + \sum_{j \neq k} \lambda_{kj} - \left(\lambda_k + \frac{1}{2}\sum_{j \neq k} \lambda_{kj}\right) \cdot p_k + \frac{1}{2}\sum_{j \neq k} \lambda_{kj} \cdot p_j. \end{aligned}$$

The above approach therefore allows one to derive any linear oligopoly demand structure of the form

$$x_k\left(p_k, p_{-k}\right) = a_k - b_k \cdot p_k + c \cdot \sum_{j \neq k} p_j, \forall k \in N,$$

where $\lambda_k = b_k - c$, $\lambda_{kj} = \dfrac{2c}{n-1}$, and $v_k = \dfrac{a_k - 2c}{b_k - c}$.

This result contrasts with Jaffe and Weyl (2010) which shows that this type of linear demand structure cannot be derived from a discrete choice setting when the number of goods exceeds 2.[20] In Jaffe and Weyl (2010), the distribution of customers over (realizations of) valuations is given by a smooth probability density function. As a result, the probability of observing a customer who does not value a particular product is zero. The above model does not fit within this framework due to the presence of captive consumers.[21] We thus conclude that linear oligopoly demand can be consistent with a discrete choice foundation *for any number of products* when each firm has a loyal customer base.

19 Similar to the analysis in the preceding subsection, a sufficient condition to ensure that all Type *ij* consumers buy a product is $p_i, p_j \leq 2$.

20 Note that, strictly speaking, this is already the case in the previous subsection in which we considered two goods and an outside option.

21 Indeed, our model induces a probability distribution over customer valuations that does not have a density. Specifically, the probability of a consumer not valuing a particular product is strictly positive. This is consistent with Armstrong and Vickers (2015) who observe that a probability distribution with density cannot generate a linear demand system.

4 Welfare

Since the traditional representative agent approach and our alternative lead to the same demand system, it may be tempting to think that the implications are relatively innocuous. However, and as already pointed out in the introduction, welfare analyses are known to be sensitive to the precise specification of the representative consumer's utility function. We now add to this by showing that results of a welfare analysis based on the aggregate linear demand system depend non-trivially on the underlying microeconomic foundation. Specifically, we illustrate how a stronger cross-price effect can be harmful for customers within our setting, while at the same time be seen as beneficial by the representative consumer.

In the following, we consider the simplest possible case of a symmetric duopoly. For some given prices p_1 and p_2, the representative consumer's surplus is:

$$\begin{aligned}
V &= \alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2 - \gamma \cdot (x_2 - x_1)^2 - p_1 \cdot x_1 - p_2 \cdot x_2 \\
&= \alpha \cdot \left(2a - (b-c) \cdot p_1 - (b-c) \cdot p_2\right) - \beta \cdot \left(2a - (b-c) \cdot p_1 - (b-c) \cdot p_2\right)^2 \\
&\quad -\gamma \cdot \left((b+c) \cdot (p_1 - p_2)\right)^2 - p_1 \cdot x_1 - p_2 \cdot x_2 \\
&= \frac{2a^2}{b-c} - a \cdot p_1 - a \cdot p_2 - \frac{1}{4} \cdot (b-c) \cdot \left(\frac{2a}{b-c} - p_1 - p_2\right)^2 - \frac{1}{4} \cdot (b+c) \cdot (p_1 - p_2)^2 \\
&\quad -ap_1 + bp_1^2 - cp_1p_2 - ap_2 + bp_2^2 - cp_1p_2 \\
&= \frac{a^2}{b-c} - a \cdot p_1 - a \cdot p_2 - c \cdot p_1 \cdot p_2 + \frac{b}{2} \cdot p_1^2 + \frac{b}{2} \cdot p_2^2.
\end{aligned}$$

Let us now derive consumer surplus under the same conditions within our alternative setting. Using symmetry, it holds that $\lambda_1 = \lambda_2 = b - c$, $\lambda_3 = 2c$, and $s = v = \frac{a - 2c}{b - c}$. The consumer surplus is then given by:

$$\begin{aligned}
W &= \frac{1}{2} \cdot \lambda_1 \cdot (s - p_1)^2 + \frac{1}{2} \cdot \lambda_2 \cdot (v - p_2)^2 \\
&\quad + \lambda_3 \cdot \left[6 - p_1 - p_2 + \frac{1}{2} + \frac{1}{8} \cdot (p_2 - p_1)^2 + \frac{1}{2} + \frac{1}{8} \cdot (p_2 - p_1)^2\right] \\
&= \frac{b-c}{2} \cdot \left(\frac{a-2c}{b-c} - p_1\right)^2 + \frac{b-c}{2} \cdot \left(\frac{a-2c}{b-c} - p_2\right)^2 \\
&\quad + 2c \cdot \left[7 - p_1 - p_2 + \frac{1}{4} \cdot (p_2 - p_1)^2\right] \\
&= \frac{(a-2c)^2}{b-c} + 14c - a \cdot p_1 - a \cdot p_2 - c \cdot p_1 \cdot p_2 + \frac{b}{2} \cdot p_1^2 + \frac{b}{2} \cdot p_2^2.
\end{aligned}$$

Comparing V and W, we see that the consumer surplus generally differs in absolute terms. Note, however, that the price effects are precisely the same in both cases so that ordinal welfare implications of price changes would lead to similar conclusions.

Yet, the same does not hold for changes in demand system parameters that may result from innovations, for instance. To see this, note that

$$\frac{\partial V}{\partial c} = \frac{a^2}{(b-c)^2} - p_1 \cdot p_2,$$

whereas

$$\frac{\partial W}{\partial c} = \frac{(a-2c)\cdot(a+2c-4b)}{(b-c)^2} + 14 - p_1 \cdot p_2.$$

It can be easily verified that the signs may not be the same.[22] In particular, an increase in the cross-price effect can be beneficial for the representative consumer (i.e., $\frac{\partial V}{\partial c} > 0$), while being harmful for buyers within our framework (i.e., $\frac{\partial W}{\partial c} < 0$). Conducting a welfare analysis at the aggregate demand level can therefore lead to wrong conclusions when one does not take account of the corresponding microeconomic foundation.

5 Concluding Remarks

The use of representative agents in economic theory dates back at least as far as the late 1800s when Marshall's manuscript *Principles of Economics* saw the light of day.[23] Marshall introduced the notion of a 'representative firm', but also considered employing this approach in other areas of economics.[24] In fact, he is claimed to have said:[25]

> "I think the notion of 'representative firm' is capable of extension to labour; and I have had some idea of introducing that into my discussion of standard rates of wages. But I don't feel sure I shall: and *I almost think I can say what I want to more simply in another way*…"

22 For example, for given prices and b and c sufficiently close, $2c > a > 2c - 4b$ gives $\frac{\partial V}{\partial c} > 0$ and $\frac{\partial W}{\partial c} < 0$.

23 The first edition of this work was published in 1890. A flavor of the representative agent approach can also be found in Edgeworth (1925), an English translation of an Italian version from 1897.

24 A detailed discussion is provided by Hartley (1996).

25 See Pigou (1956, p. 437). Italics is ours.

In this paper, we have shown this hunch might hold true for a well-known class of quadratic representative consumer models. Indeed, one can quite simply derive the corresponding linear oligopoly demand structure directly from a population of heterogeneous buyers. This renders the use of a fictitious agent in this case effectively redundant. Moreover, the resulting microeconomic foundation can be easily extended to other demand specifications.

It is, however, not only for the sake of simplicity that one should discard this representative buyer model. In line with other recent work discussed above, we have pointed out some problematic traits of this approach. In particular, we have argued that it is inaccurate as the representative agent's aggregate utility function has no clear connection with the represented buyers' objectives and that it is inadequate as it requires an unsatisfactory solution approach to obtain the linear oligopoly demand system.

Together, this should raise strong doubts about welfare analyses based on this type of representative consumer models. Indeed, we have illustrated how welfare implications of changes in the aggregate demand system depend non-trivially on its microeconomic foundation. This naturally warrants critical assessment of other settings with a similar approach. We leave this issue for future research.

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