Supplement to "Political Support and Civil Disobedience: A Social Interaction Approach"

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Overview

This supplement consists of 6 appendices. Appendix A1 provides the details of population shares of different matches. Appendices A2 and A3 show derivation of the period-to-period dynamics of government support and compliance, correspondingly. Appendix A4 provides proof of Proposition 1. Appendix A5 contains tables referred to in the main text but not there. Appendix A6 presents derivation of population dynamics for the case of segregation.

A1 - population shares

match	share
$R^O R^O$	$(\gamma o_t^r)^2$
$P^O P^O$	$((1-\gamma)\, o_t^p)^2$
$R^O P^O$	$2\gamma\,(1-\gamma)\, o_t^r o_t^p$
$R_C^O R^S$	$2pq_t\gamma^2 o_t^r\,(1-o_t^r)$
$R_C^O P^S$	$2pq_t\gamma\,(1-\gamma)\, o_t^r\,(1-o_t^p)$
$R_N^O R^S$	$2\,(1-p)\, q_t\gamma^2 o_t^r\,(1-o_t^r)$
$R_N^O P^S$	$2\,(1-p)\, q_t\gamma\,(1-\gamma)\, o_t^r\,(1-o_t^p)$
$R_H^O R^S$	$2\gamma^2\,(1-q_t)\, o_t^r\,(1-o_t^r)$
$R_H^O P^S$	$2\gamma\,(1-\gamma)\,(1-q_t)\, o_t^r\,(1-o_t^p)$
$P^O R^S$	$2\gamma\,(1-\gamma)\, o_t^p\,(1-o_t^r)$
$P^O P^S$	$2\,(1-\gamma)^2\, o_t^p\,(1-o_t^p)$
$R^S R^S$	$(\gamma\,(1-o_t^r))^2$
$P^S P^S$	$((1-\gamma)\,(1-o_t^p))^2$
$R^S P^S$	$2\gamma\,(1-\gamma)\,(1-o_t^r)\,(1-o_t^p)$

Table A1: Population shares of matches in support decision.

A2 - derivation of government support/opposition dynamics

Collecting the terms from appendix 1 and multiplying them with corresponding probability for a rich individual to switch ($\Pr(O|S) = \Pr(OO|\cdot,\cdot)$) to or to stay in opposition ($\Pr(O|O) = 1 - \Pr(SS|\cdot,\cdot)$), we get

$$\begin{aligned}
\gamma o_{t+1}^r \;=\;& (\gamma o_t^r)^2 + \gamma\,(1-\gamma)\, o_t^r o_t^p + \\
& pq_t\gamma^2 o_t^r\,(1-o_t^r)\,(1+\alpha\beta) + pq_t\gamma\,(1-\gamma)\, o_t^r\,(1-o_t^p) + \\
& (1-p)\, q_t\gamma^2 o_t^r\,(1-o_t^r)\,(1-(1-\alpha)\,\beta+\alpha\beta) + \\
& (1-p)\, q_t\gamma\,(1-\gamma)\, o_t^r\,(1-o_t^p)\,(1-(1-\alpha)\,\beta) + \\
& \gamma^2\,(1-q_t)\, o_t^r\,(1-o_t^r)\,(1-(1-\alpha)\,\beta+\alpha\beta) + \\
& \gamma\,(1-\gamma)\,(1-q_t)\, o_t^r\,(1-o_t^p)\,(1-(1-\alpha)\,\beta) + \gamma\,(1-\gamma)\, o_t^p\,(1-o_t^r)\,\alpha\beta + \\
& (\gamma\,(1-o_t^r))^2 * 0 + ((1-\gamma)\,(1-o_t^p))^2 * 0 + \gamma\,(1-\gamma)\,(1-o_t^r)\,(1-o_t^p) * 0.
\end{aligned} \tag{1}$$

Collecting terms again and dividing by $\gamma \neq 0$, we arrive at (3) in the main text.

Similarly, for the poor individuals and $\gamma \neq 1$ we first arrive at

$$\begin{aligned} o^p_{t+1} &= (1-\gamma) o^{p2}_t + \gamma o^r_t o^p_t + \\ & pq_t \gamma o^r_t (1-o^p_t)(1-\alpha)\beta + (1-p) q_t \gamma o^r_t (1-o^p_t)(1-\alpha)\beta + \\ & \gamma (1-q_t) o^r_t (1-o^p_t)(1-\alpha)\beta + \gamma o^p_t (1-o^r_t)(1-\alpha\beta) \\ & + (1-\gamma) o^p_t (1-o^p_t)(1-\alpha\beta + (1-\alpha)\beta), \end{aligned}$$

that then simplifies to (4) in the main text.

A3 - derivation of compliance dynamics

The derivation of compliance dynamics is a bit more involved, as there is no rematching between the support and compliance decision, so the latter takes place given a match described by the support decision. Thus, every term in the support equation of the rich (1) must be multiplied by the corresponding probability to comply that in turn depends on the history of compliance and auditing. In general then, dividing table values by $\gamma o^r_t \neq 0$, we have

$$\begin{aligned} q_{t+1} &= \gamma o^r_{t+1} \left((1-q) pq \left(Q\left(R^O_H R^O_C\right) + Q\left(R^O_C R^O_H\right)\right) + (1-q)(1-p) q \left(Q\left(R^O_H R^O_N\right) + Q\left(R^O_N R^O_H\right)\right)\right) + \\ & \gamma o^r_{t+1} \left((1-q)^2 Q\left(R^O_H R^O_H\right) + p^2 q^2 Q\left(R^O_C R^O_C\right) + (1-p)^2 q^2_t Q\left(R^O_N R^O_N\right)\right) + \\ & \gamma o^r_{t+1} p (1-p) q^2 \left(Q\left(R^O_N R^O_C\right) + Q\left(R^O_C R^O_N\right)\right) + \\ & (1-\gamma) o^p_{t+1} \left((1-q) Q\left(R^O_H P^O\right) + pqQ\left(R^O_C P^O\right) + (1-p) qQ\left(R^O_N P^O\right)\right) + \\ & pq_t \gamma \left(1-o^r_{t+1}\right) \left(\alpha\beta \left(Q\left(R^O_C R^O_-\right) + Q\left(R^O_- R^O_C\right)\right) + (1-\alpha\beta) Q\left(R^O_C R^S\right)\right) + \\ & pq_t (1-\gamma) \left(1-o^p_{t+1}\right) \left((1-\alpha)\beta Q\left(R^O_C P^O\right) + (1-(1-\alpha)\beta) Q\left(R^O_C P^S\right)\right) + \\ & (1-p) q_t \gamma \left(1-o^r_{t+1}\right) (1-\alpha\beta + \beta(2\alpha-1)) Q\left(R^O_N R^S\right) + \\ & (1-p) q_t (1-\gamma) \left(1-o^p_{t+1}\right) \left((1-\alpha)\beta Q\left(R^O_N P^O\right) + (1-(1-\alpha)\beta) Q\left(R^O_N P^S\right)\right) + \\ & \gamma (1-q_t) \left(1-o^r_{t+1}\right) \left(\alpha\beta \left(Q\left(R^O_H R^O_-\right) + Q\left(R^O_- R^O_H\right)\right) + (1-\beta) Q\left(R^O_H R^S\right)\right) + \\ & (1-\gamma)(1-q_t) \left(1-o^p_{t+1}\right) \left((1-\alpha)(1-\beta) Q\left(R^O_H P^O\right) + \alpha Q\left(R^O_H P^S\right)\right) + \\ & (1-\gamma) \frac{o^p_{t+1}}{o^r_{t+1}} \left(1-o^r_{t+1}\right) \alpha\beta Q\left(R^O_- P^O\right). \end{aligned}$$

Using the compliance probabilities $Q\left(\right)$, we arrive then at

$$\begin{aligned}
q_{t+1} &= \gamma o^{r}_{t+1}\left(\left(1-q_t\right)^2\eta+\left(1-p\right)^2 q_t^2+\left(1-q_t\right)\left(1-p\right)q_t\left(1+\eta\right)+p\left(1-p\right)q_t^2\left(1-\eta\right)\right)\\
&\left(1-\gamma\right)o^{p}_{t+1}\left(\left(1-q_t\right)\eta+\left(1-p\right)q_t\right)+\left(1-p\right)q_t\gamma\left(1-o^{r}_{t+1}\right)\left(\beta\left(1+\eta\right)\alpha+1-\beta\right)+\\
&\left(1-p\right)q_t\left(1-\gamma\right)\left(1-o^{p}_{t+1}\right)\left(\left(1-\alpha\right)\beta+1-2\left(1-\alpha\right)\beta\right)+\\
&\gamma\left(1-q_t\right)\left(1-o^{r}_{t+1}\right)\left(2\alpha\beta\eta+\left(1-\beta\right)\eta\right)+\\
&\left(1-\gamma\right)\left(1-q_t\right)\left(1-o^{p}_{t+1}\right)\eta\left(\left(1-\alpha\right)\beta+1-2\beta\left(1-\alpha\right)\right)+\left(1-\gamma\right)\frac{o^{p}_{t+1}}{o^{r}_{t+1}}\left(1-o^{r}_{t+1}\right)\alpha\beta\eta,
\end{aligned}$$

that further simplifies to (5) in the main text.

A4 - proof of Proposition 1

We start with part (i): Our steady state system is described by the three quadratic equations. The solution to the rich support (equation (3) in the main text) can then be generally written as

$$\begin{aligned}
o^r &= \frac{A+G+C\pm\sqrt{\left(A+G+C\right)^2-4AC}}{2A},\\
A &:= \gamma\left(1-2\alpha-pq\left(1-\alpha\right)\right),\\
C &:= \left(1-\gamma\right)o^p\alpha,\\
G &:= \left(1-\gamma\right)\left(1-o^p\right)\left(1-\alpha\right)\left(1-pq\right).
\end{aligned}$$

for $\beta\neq 0$.

Analogously, the solution to the poor support (equation (4) in the main text) can be written as

$$\begin{aligned}
o^p &= \frac{a+h+c\pm\sqrt{\left(a+h+c\right)^2-4ac}}{2a},\\
a &:= \left(1-\gamma\right)\left(2\alpha-1\right),\\
c &:= \gamma o^r\left(1-\alpha\right),\\
h &:= \gamma\left(1-o^r\right)\alpha.
\end{aligned}$$

Clearly, only one of the solutions to each of the two equations describes a necessary condition for a stable steady state (it is the smaller root if $A\left(a\right)>0$, the larger root, if $A\left(a\right)<0$, the only root otherwise). It can be shown that both relevant solutions belong to the unit interval (the proof is trivial, but lengthy, so we do not include it here).

Moreover, each of the relevant solutions may be represented as a function, $o^r(o^p)$ and $o^p(o^r)$ correspondingly. An intersection of these functions describes a stable steady state for given q^{ss} and parameters, if after a small deviation in o^p or o^r the system converges back to this intersection. Depending on α, there are following possibilities:

1) $\alpha < \frac{1-pq}{2-pq}$ $(A > 0, a < 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} - \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} + \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

2) $\alpha > \frac{1}{2}$ $(A < 0, a > 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} + \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} - \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

3) $\frac{1-pq}{2-pq} < \alpha < \frac{1}{2}(A < 0, a < 0)$

$$\begin{aligned} o^r &= \frac{1}{2} + \frac{G+C}{2A} + \sqrt{\left(\frac{1}{2} + \frac{G+C}{2A}\right)^2 - \frac{C}{A}}, \\ o^p &= \frac{1}{2} + \frac{h+c}{2a} + \sqrt{\left(\frac{1}{2} + \frac{h+c}{2a}\right)^2 - \frac{c}{a}}; \end{aligned}$$

And the special cases are

$$\begin{aligned} o^r &= \frac{C}{G+C}, \alpha = \frac{1-pq}{2-pq}; \\ o^p &= \frac{c}{c+h}, \alpha = \frac{1}{2}. \end{aligned}$$

Inspecting these expressions closely, one can establish that the intersection defining the stable steady state is unique. To show this, we first check the limiting expressions for each function. For the rich opposition we have with $\alpha < \frac{1-pq}{2-pq}$ $(A > 0)$:

$$
\begin{aligned}
\lim_{o^p\to 0} o^r &= \frac{1}{2}+\frac{G}{2A}-\sqrt{\left(\frac{1}{2}+\frac{G}{2A}\right)^2}=0;\\
\lim_{o^p\to 1} o^r &= \frac{1}{2}+\frac{C}{2A}-\left|\frac{1}{2}-\frac{C}{2A}\right|=\frac{C}{A}, C<A,\\
&= 1, C>A.
\end{aligned}
$$

With $\alpha > \frac{1-pq}{2-pq}\,(A<0)$:

$$
\begin{aligned}
\lim_{o^p\to 0} o^r &= \frac{1}{2}+\frac{G}{2A}+\sqrt{\left(\frac{1}{2}+\frac{G}{2A}\right)^2}=1+\frac{G}{A}, 1+\frac{G}{A}>0,\\
&= 0, 1+\frac{G}{A}<0;\\
\lim_{o^p\to 1} o^r &= \frac{1}{2}+\frac{C}{2A}+\sqrt{\left(\frac{1}{2}-\frac{C}{2A}\right)^2}=1.
\end{aligned}
$$

The limiting expressions for the poor if $\alpha > \frac{1}{2}\,(a>0)$

$$
\begin{aligned}
\lim_{o^r\to 0} o^p &= \frac{1}{2}+\frac{h}{2a}-\sqrt{\left(\frac{1}{2}+\frac{h}{2a}\right)^2}=0,\\
\lim_{o^r\to 1} o^p &= \frac{1}{2}+\frac{c}{2a}-\left|\frac{1}{2}-\frac{c}{2a}\right|=\frac{c}{a}, c<a;\\
&= 1, c>a.
\end{aligned}
$$

If $\alpha < \frac{1}{2}\,(a<0)$, we get

$$
\begin{aligned}
\lim_{o^r\to 0} o^p &= \frac{1}{2}+\frac{h}{2a}+\left|\frac{1}{2}+\frac{h}{2a}\right|=1+\frac{h}{a}, 1+\frac{h}{a}>0;\\
&= 0, 1+\frac{h}{a}<0;\\
\lim_{o^r\to 1} o^p &= \frac{1}{2}+\frac{c}{2a}+\sqrt{\left(\frac{1}{2}-\frac{c}{2a}\right)^2}=1.
\end{aligned}
$$

From the limiting expressions we see that each of the functions $o^r\,(o^p)$ and $o^p\,(o^r)$ has a unit interval as its domain; its range is contained in unit interval. We also know that these functions are parts of ellipses describing the whole set of solutions to the corresponding quadratic equations.

By the nature of ellipse, its upper part is concave and the lower is convex. So whenever our solution is the higher root, we know it is concave; when it is the lower root, it is convex. So, with $A\,(a)>0$ we have a smaller root - a convex curve, in the opposite case a concave one. We must also remember that inverse of a convex function is a concave function.

We can see that for $\alpha \in \left[\frac{1-pq}{2-pq}, \frac{1}{2}\right]$ $(A < 0, a < 0)$, as $\lim_{o^r \to 1} o^p = 1$ and $\lim_{o^p \to 1} o^r = 1$, $(1, 1)$ is an intersection. As $\lim_{o^r \to 0} o^p \geq 0$ and $\lim_{o^p \to 0} o^r \geq 0$, $o^r(o^p)$ is concave and inverse of $o^p(o^r)$ is convex, the only other possible intersection is at $o^r = o^p = 0$. But this is an unstable steady state. To see that intuitively, one can plot the corresponding curves and check how the dynamic converges to $(1, 1)$ for arbitrarily small deviation from $(0, 0)$. Therefore, the unique stable steady state in this case is $(1, 1)$.

For $\alpha < \frac{1-pq}{2-pq}$ $(A > 0, a < 0)$, $\lim_{o^r \to 1} o^p = 1$ and $\lim_{o^p \to 1} o^r \leq 1$; $\lim_{o^r \to 0} o^p \geq 0$ and $\lim_{o^p \to 0} o^r = 0$. There are at most 3 intersections; both curves are now convex. The stable steady state is again unique: it is an interior intersection, if it exists, and a corner intersection ($(0, 0)$ or $(1, 1)$) otherwise.

For $\alpha > \frac{1}{2}(A < 0, a > 0)$, $\lim_{o^r \to 1} o^p \leq 1$ and $\lim_{o^p \to 1} o^r = 1$; $\lim_{o^r \to 0} o^p = 0$ and $\lim_{o^p \to 0} o^r \geq 0$. Again, there are at most 3 intersections while both curves are concave. $(0, 0)$ is always an intersection, but it is only a stable steady state, if no other intersection exists. Again, if an interior intersection exists, it describes a unique stable steady state.

To sum up, for any given parameter combination and q^{ss} a stable steady state (o^r, o^p) exists and is unique.

Finally, the compliance equation has the following coefficients:

$$
\begin{aligned}
A_0 &= \gamma o^r p^2 \eta, \\
B_0 &= \gamma o^r (Q - p\eta) + QD + \gamma (1 - o^r) (Q (1 - \beta (1 - \alpha)) - \alpha\beta\eta) - 1, \\
C_0 &= \eta \left((1 - o^r) \left(\gamma (2\alpha\beta + 1 - \beta) + (1 - \gamma) \frac{o^p}{o^r} \alpha\beta \right) + D + \gamma o^r \right), \\
Q &:= 1 - p - \eta, \\
D &:= (1 - \gamma) (1 - \beta (1 - \alpha - o^p + o^p \alpha)) .
\end{aligned}
$$

Since $A_0 \geq 0$, only the smaller solution characterizes the stable steady state. Thus, for each parameter combination we have at most one triple (o^r, o^p, q) that defines a stable steady state.

Proof of Proposition 1 (ii): From part (i) we know that each of the 3 equations characterizing the stable steady state always have a solution, but it is not always an interior solution. We have already seen that for $\alpha \in \left[\frac{1-pq}{2-pq}, \frac{1}{2}\right]$ no interior solution is possible, and a unique stable steady state in this case is $(1, 1, q^{ss})$.

Inspecting the limiting expressions again (and plotting corresponding pictures), we can also see that sometimes the interior solution is sure regardless of the slope of our functions:

1) $A > 0, a < 0$ ("small alpha"). In this case, $C < A, 1 + \frac{h}{a} > 0$ give intersection at

the interior. The conditions may be rewritten as

$$\frac{\alpha}{(1-p)\,\delta q+(1-q)\,(1-\alpha)}<\gamma<\frac{1-2\alpha}{1-\alpha}.$$

2) $A<0, a>0$ ("big alpha"). For large government spending, $c<a, 1+\frac{G}{A}>0$ give an interior solution. Rewritten, we have

$$\frac{1-\alpha}{\alpha}\frac{P}{(2-q-P)}<\gamma<\frac{1}{(q\beta\,(1-\delta\,(1-p))\,/\,(1-\beta)+(1-\beta)\,(1-q))\,(1-\alpha)\,/\,(2\alpha-1)+1}.$$

If these conditions are not satisfied, we may still have the interior solution under certain conditions on the slope of the two curves at the corners. We have then to compute the slope:

$A>0$:

$$\begin{aligned}
o^{r\prime} &= \frac{G'+C'}{2A}-\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)^2-\frac{C}{A}\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
o^{r\prime}(0) &= \frac{G'+C'}{2A}-\frac{G'+C'}{2A}+\frac{C'}{2A}\left(\frac{1}{2}+\frac{G}{2A}\right)^{-1}=\frac{C'}{2A}\left(\frac{1}{2}+\frac{G}{2A}\right)^{-1}=\frac{C'}{A+G}>0,\\
o^{r\prime}(1) &= \frac{C-A+G'}{A}\frac{C}{C-A}>0, C<A,\\
o^{r\prime}(1) &= \frac{G'+C}{2A}+\left(\frac{1}{2}-\frac{C}{2A}\right)^{-1}\left(\left(\frac{1}{2}+\frac{C}{2A}\right)\frac{G'+C}{2A}-\frac{C}{2A}\right)=\frac{G'}{A-C}, C>A.
\end{aligned}$$

$A<0$:

$$\begin{aligned}
o^{r\prime} &= \frac{G'+C'}{2A}+\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)^2-\frac{C}{A}\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G+C}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
o^{r\prime}(1) &= \frac{G'+C}{2A}+\left(\frac{1}{2}-\frac{C}{2A}\right)^{-1}\left(\left(\frac{1}{2}+\frac{C}{2A}\right)\frac{G'+C}{2A}-\frac{C}{2A}\right),\\
&= \frac{G'}{A-C}>0;
\end{aligned}$$

$$\begin{aligned}
o^{r\prime}(0) &= \frac{G'+C'}{2A}+\left(\left(\frac{1}{2}+\frac{G}{2A}\right)^2\right)^{-1/2}\left(\left(\frac{1}{2}+\frac{G}{2A}\right)\frac{G'+C'}{2A}-\frac{C'}{2A}\right),\\
&= -\frac{1}{A}\frac{G}{A+G}\left(A+G-C'\right), 1+\frac{G}{A}>0,\\
&= \frac{C'}{A+G}, 1+\frac{G}{A}<0,\\
&= \frac{C'-G}{2A}, 1+\frac{G}{A}=0.
\end{aligned}$$

Note that o^r (o^p) is monotonic.

Since problems for o^r and o^p are 'dual', it is enough to establish that $c(0) = 0, h(1) = 0, c(1) = c', h(0) = -h'$, and we can immediately write

for $a > 0$

$$\begin{aligned} o^{p\prime}(0) &= \frac{c'}{a+h}, \\ o^{p\prime}(1) &= \frac{c-a+h'}{a}\frac{c}{c-a}. \end{aligned}$$

for $a < 0$

$$\begin{aligned} o^{p\prime}(1) &= \frac{h'}{a-c} > 0, \\ o^{p\prime}(0) &= -\frac{a+h-c'}{a}\frac{h}{a+h}. \end{aligned}$$

Note that o^p (o^r) is monotonic as well.

Once we know this, we can consider all the possible combinations of the parameters and formulate necessary and sufficient conditions for the existence of the interior stable steady state:

Necessary and sufficient conditions

1. $A > 0, a < 0:$

a) $C < A, 1 + h/a \leq 0, o^{r\prime}(0) = \frac{C'}{A+G} > o^{p\prime-1}(0) = -\frac{a+h}{a+h-c'}\frac{a}{h};$

b) $C > A, 1 + h/a > 0, o^{r\prime}(1) = \frac{C-A+G'}{A}\frac{C}{C-A} > o^{p\prime-1}(1) = \frac{a-c}{h'};$

c) $C < A, 1 + h/a > 0;$

d) $C > A, 1 + h/a < 0, o^{r\prime}(0) > o^{p\prime-1}(0), o^{r\prime}(1) > o^{p\prime-1}(1).$

2. $A < 0, a > 0:$

a) $1 + G/A > 0, c \geq a, o^{r\prime}(1) = \frac{G'}{A-C} > o^{p\prime-1}(1) = \frac{a}{c-a+h'}\frac{c-a}{c};$

b) $1 + G/A \leq 0, c < a, o^{r\prime}(0) = -\frac{1}{A}\frac{G}{A+G}(A+G-C') > o^{p\prime-1}(0) = \frac{a+h}{c'};$

c) $c < a, 1 + \frac{G}{A} > 0;$

d)$c < a, 1 + \frac{G}{A} > 0, o^{r\prime}(1) > o^{p\prime-1}(1), o^{r\prime}(0) > o^{p\prime-1}(0).$

Working out these conditions, we can get the following formulation:

1. $\alpha < \frac{P}{1+P}$, where $P := 1 - pq^{ss}$, *and*

a) $\alpha < \min\left\{\frac{\gamma P}{1+\gamma P}, \frac{1-\gamma}{2-\gamma}\right\}$

or b) $\frac{1-\gamma}{2-\gamma} < \alpha < \frac{\gamma P}{1+\gamma P}$

or c) $\frac{\gamma P}{1+\gamma P} < \alpha < \frac{1-\gamma}{2-\gamma}$ together with the following condition:

$$\frac{(1-\gamma)(1-\alpha)P}{\alpha - P\gamma + P\alpha\gamma} > \frac{\alpha\gamma - 2\alpha + 1}{\gamma\alpha}. \tag{2}$$

or d) $\alpha > \max\left\{\frac{\gamma P}{1+\gamma P}, \frac{1-\gamma}{2-\gamma}\right\}$ together with condition (2) and the following condition:

$$\frac{\gamma\alpha^2}{(1-\alpha)P-\gamma\alpha} > -(1-\gamma)(2\alpha-1)-\gamma\alpha.$$

or 2. $\alpha > \frac{1}{2}$ *and*

a) $\frac{P}{P+\gamma} < \alpha < \frac{1}{2-\gamma}$

or b) $\alpha > \max\left\{\frac{1}{2-\gamma}, \frac{P}{P+\gamma}\right\}$

or c) $\alpha < \min\left\{\frac{1}{2-\gamma}, \frac{P}{P+\gamma}\right\}$ together with the following conditions:

$$\begin{aligned}\frac{-(1-\gamma)(1-\alpha)P}{P\gamma-\alpha-P\alpha\gamma} &> \frac{2\alpha-\alpha\gamma-1}{-\gamma\alpha},\\ \frac{(1-\gamma)\alpha}{P-P\alpha-\alpha\gamma} &> \frac{2\alpha+\gamma-\alpha\gamma-1}{\gamma(1-\alpha)}.\end{aligned}$$

To sum up, we see that the interior stable steady state may exist only if $\alpha < \frac{P}{1+P} \leq \frac{1}{2}$ or if $\alpha > \frac{1}{2}$, and that is exactly the statement of the Proposition 1(ii) .

A5 - miscellaneous figures

Figure A1. The effect of varying social spending, $\gamma = 0.5$.

Figure A2. The effect of varying auding probability, $\alpha = 0.6$

Figure A3. Optimal α as a function of γ and θ.

Figure A4. The effect of varying social spending, $\varepsilon = 2$.

A6 - segregation

population shares

For the ease of notation, denote

$$\begin{aligned}\Sigma \quad : & = \gamma\left(1-\varepsilon_H \gamma\right)+(1-\gamma)\left(1-\varepsilon_L(1-\gamma)\right) \\ & = \ 2\gamma\left(1-\varepsilon\gamma\right).\end{aligned}$$

The population shares of matches are presented in Table A2.

match	share with segregation
$R^O R^O$	$\varepsilon_H (\gamma o_t^r)^2$
$P^O P^O$	$\varepsilon_L ((1-\gamma) o_t^p)^2$
$R^O P^O$	$\Sigma o_t^r o_t^p$
$R_C^O R^S$	$2pq_t \varepsilon_H \gamma^2 o_t^r (1-o_t^r)$
$R_C^O P^S$	$pq_t \Sigma o_t^r (1-o_t^p)$
$R_N^O R^S$	$2(1-p) q_t \varepsilon_H \gamma^2 o_t^r (1-o_t^r)$
$R_N^O P^S$	$(1-p) q_t \Sigma o_t^r (1-o_t^p)$
$R_H^O R^S$	$2\varepsilon_H \gamma^2 (1-q_t) o_t^r (1-o_t^r)$
$R_H^O P^S$	$\Sigma (1-q_t) o_t^r (1-o_t^p)$
$P^O R^S$	$\Sigma o_t^p (1-o_t^r)$
$P^O P^S$	$2\varepsilon_L (1-\gamma)^2 o_t^p (1-o_t^p)$
$R^S R^S$	$\varepsilon_H (\gamma (1-o_t^r))^2$
$P^S P^S$	$\varepsilon_L ((1-\gamma)(1-o_t^p))^2$
$R^S P^S$	$\Sigma (1-o_t^r)(1-o_t^p)$

Table A2: Population shares of matches in support decision with segregation.

support dynamics

Collecting the terms from Table A2 and simplifying, we arrive at the following equation that describes the dynamics of government support for the rich,

$$\begin{aligned} o_{t+1}^r &= \varepsilon\gamma (o_t^r)^2 + (1-\varepsilon\gamma) o_t^r o_t^p \\ &+ ((1+\alpha\beta) pq_t + (1-pq_t)(1-\beta+2\alpha\beta)) \varepsilon\gamma o_t^r (1-o_t^r) \\ &+ (pq_t + (1-pq_t)(1-\beta+\alpha\beta))(1-\varepsilon\gamma) o_t^r (1-o_t^p) \\ &+ (1-\varepsilon\gamma) o_t^p (1-o_t^r) \alpha\beta, \end{aligned}$$

and the following equation that describes these dynamics for the poor:

$$\begin{aligned} (1-\gamma) o_{t+1}^p &= \varepsilon_L ((1-\gamma) o_t^p)^2 + \gamma (1-\varepsilon\gamma) o_t^r o_t^p + \\ & \gamma (1-\varepsilon\gamma) o_t^r (1-o_t^p)(1-\alpha)\beta + \gamma (1-\varepsilon\gamma) o_t^p (1-o_t^r)(1-\alpha\beta) + \\ & + \varepsilon_L (1-\gamma)^2 o_t^p (1-o_t^p)(1-\alpha\beta + (1-\alpha)\beta). \end{aligned}$$

compliance dynamics

Analogously to Appendix A3, we derive compliance dynamics with segregation. Firstly, we get

$$
\begin{aligned}
\gamma q_{t+1} o_{t+1}^{r} = {} & \varepsilon_H \left(\gamma o_t^r\right)^2 \left((1-q)\,pq\left(Q\left(R_H^O R_C^O\right)+Q\left(R_C^O R_H^O\right)\right)+\right. \\
& (1-q)(1-p)\,q\left(Q\left(R_H^O R_N^O\right)+Q\left(R_N^O R_H^O\right)\right)+(1-q)^2 Q\left(R_H^O R_H^O\right)+ \\
& p^2 q^2 Q\left(R_C^O R_C^O\right)+(1-p)^2 q_t^2 Q\left(R_N^O R_N^O\right)+p(1-p)\,q^2\left(Q\left(R_N^O R_C^O\right)+Q\left(R_C^O R_N^O\right)\right))+ \\
& \Sigma o_t^r o_t^p/2\left((1-q)\,Q\left(R_H^O P^O\right)+pqQ\left(R_C^O P^O\right)+(1-p)\,qQ\left(R_N^O P^O\right)\right)+ \\
& pq_t\varepsilon_H\gamma^2 o_t^r\left(1-o_t^r\right)\left(\alpha\beta\left(Q\left(R_C^O R_-^O\right)+Q\left(R_-^O R_C^O\right)\right)+(1-\alpha\beta)\,Q\left(R_C^O R^S\right)\right)+ \\
& pq_t\Sigma o_t^r\left(1-o_t^p\right)\left((1-\alpha)\,\beta Q\left(R_C^O P^O\right)+(1-(1-\alpha)\,\beta)\,Q\left(R_C^O P^S\right)\right)/2+ \\
& (1-p)\,q_t\varepsilon_H\gamma^2 o_t^r\left(1-o_t^r\right)\left((1-\beta)\,Q\left(R_N^O R^S\right)+\alpha\beta Q\left(R_N^O R_-^O\right)+\alpha\beta Q\left(R_-^O R_N^O\right)\right)+ \\
& (1-p)\,q_t\Sigma o_t^r\left(1-o_t^p\right)/2\left((1-\alpha)\,\beta Q\left(R_N^O P^O\right)+(1-2(1-\alpha)\,\beta)\,Q\left(R_N^O P^S\right)\right)+ \\
& \varepsilon_H\gamma^2\left(1-q_t\right)o_t^r\left(1-o_t^r\right)\left(\alpha\beta\left(Q\left(R_H^O R_-^O\right)+Q\left(R_-^O R_H^O\right)\right)+(1-\beta)\,Q\left(R_H^O R^S\right)\right)+ \\
& \left(1-q_t\right)\Sigma o_t^r\left(1-o_t^p\right)/2\left((1-\alpha)\,\beta Q\left(R_H^O P^O\right)+(1-2(1-\alpha)\,\beta)\,Q\left(R_H^O P^S\right)\right)+ \\
& \Sigma o_t^p\left(1-o_t^r\right)\alpha\beta/2Q\left(R_-^O P^O\right).
\end{aligned}
$$

Using the compliance probabilities $Q\,()$, we then arrive at the following expression:

$$
\begin{aligned}
q_{t+1} o_{t+1}^{r} = {} & \varepsilon\gamma\left(o_t^r\right)^2\left((1+\eta)(1-p)(1-q)\,q+(1-q)^2\eta+(1-p)^2 q_t^2+p(1-p)\,q^2(1-\eta)\right)+ \\
& (1-\varepsilon\gamma)\,o_t^r o_t^p\left((1-q)\,\eta+(1-p)\,q\right)+ \\
& \left((1-p)\,q_t(1-\beta+\alpha\beta+\alpha\beta\eta)+\left(1-q_t\right)(2\alpha\beta+1-\beta)\,\eta\right)\varepsilon\gamma o_t^r\left(1-o_t^r\right)+ \\
& \left((1-p)\,q_t+\eta\left(1-q_t\right)\right)(1-\varepsilon\gamma)\,o_t^r\left(1-o_t^p\right)\left((1-\alpha)\,\beta+(1-2(1-\alpha)\,\beta)\right)+ \\
& (1-\varepsilon\gamma)\,o_t^p\left(1-o_t^r\right)\alpha\beta\eta.
\end{aligned}
$$