Research Article

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A Model of Access in the Absence of Markets

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Abstract: Economists have long known that properly designed markets allocate resources efficiently. However, in many circumstances markets are unfeasible. In this paper, we construct a general model of access which allows us to value different assignments when resources are allocated in the absence of markets. We demonstrate that marginal value schedules are far less useful in allocating access when property rights are unattainable. The criteria for optimal allocation combine information on both the marginal value schedules and the assignments determining the probabilities of access to the resource. Our approach allows us to rank rationing policies in a wide range of real-world, second-best settings.

Keywords: access, allocation, rationing

JEL Classsification: D23, D45, H41, H44, Q20, Q22

1 Introduction

A fundamental problem in economics is the allocation of resources. Economists have long known that properly designed markets allocate resources efficiently. However, in many circumstances, markets are not feasible, leading economists to design a host of market-like mechanisms. Non-market settings abound, including political offices, grant allocations, spectrum, access to higher-education institutions, intra-company resources, and so on. Mechanisms for allocation in these settings include auctions, lotteries, contests, queues, tradable permits and various forms of rationing (for example, see Myerson 1981; Milgrom and Weber 1982; Sah 1987; Wijkander 1988; Dasgupta and Maskin 2000; Ergin 2002; Che, Gale, and Kim 2013; Ausubel 2006; Platt 2009). Some mechanisms allocate

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the resources efficiently. For private values, the Vickrey auction (for one good) or the Vickrey-Clarke-Groves auction (for multiple goods) is efficient. On the other hand, mechanisms such as lotteries or gueues are not efficient.

In many circumstances, not only are markets absent, but even market-like alternatives such as auctions or tradable permits are simply not viable. Some examples where this may be the case include access to municipal parks, public libraries, street parking, and a variety of natural resources such as surface and ground water, atmospheric resources like air, and biological resources including fish stocks. Access to these resources may be constrained legally, politically, by custom or social norm, or physically. Yet even within the constraints that bind allocation mechanisms for these resources, there are management and policy differences. A public charter school may admit students with a pure lottery or using a part lottery part queuing system. Our goal is to construct models that facilitate valuation of resources under those institutional settings that do not yield efficient outcomes, as in the example of the school just mentioned.

In a second best setting, in which resources are allocated inefficiently, assessing the gains from changes in rules and regulations requires knowing the probabilities of access induced by those rules and regulations. This issue was first addressed by Seneca (1970) and Mumy and Hanke (1975), who studied the value of providing a local public good when access was rationed by arrival. They showed that equally likely access to the public good significantly reduced its expected benefits. The literature following Mumy and Hanke (1975) focused primarily on congestion (Harrington 1988) and allocation by lotteries and by hybrid mechanisms whereby a portion is allocated by price and the remainder through a non-price mechanism (Boyce 1994; Taylor, Tsui, Zhu 2003; Scrogin 2009; Evans, Vossler, and Flores 2009), although Che, Gale, and Kim (2013) have recently studied the assignment of initial ownership of a good when individuals differ in their wealth. They show that market mechanisms favor those able to pay, and may be less efficient than non-market assignments when reselling is allowed. While motivated by the insights in Mumy and Hanke (1975), we find it useful to develop a more general representation of conditions of access in terms of agents' likelihood of obtaining units or services from the resource. Unlike recent work on mechanism design under incomplete information (see, for example, Yoon 2011; Condorelli 2012, 2013), focused primarily on constructing optimal allocation mechanisms when agents invest in costly signals and the designer trades off allocative and expenditure inefficiency, we take the many possible access scenarios -de facto mechanisms for sorting users' valuations- as primitives. We then develop a typology of assignments that will enable us to rank them not only when access is dictated by a thoughtful design but also when it is determined by real-world political and similar constraints. The efficient version of access occurs when it is provided in decreasing order of marginal value, so that the agents with the highest willingness-to-pay gain access first. A simple alternative occurs when a priori all agents have equal probabilities of access, regardless of their marginal values (what we term "uniform random access").

We develop an approach that has efficient access, uniform random access and a variety of other non-price sorting patterns as special cases. This framework will allow us to rank rationing and allocation policies in a second-best world where potential users have heterogeneous values and there is a motivation for rationing access. Throughout we assume that agents must gain access to the resource to receive value. This rules out any non-use values and means that agents that do not have access get no value. We apply our approach to settings (i) in which individuals' valuations are independent of total consumption (non-rival), and (ii) to common-pool resources where external costs associated with intra-season depletion and congestion do not alter the ranking of valuations across individuals (e.g. congestion costs are equal across agents or proportional to the their valuations).¹

Our results have implications for the management of quasi-public goods: knowledge of marginal value functions is insufficient to determine the aggregate value of the good. This conclusion holds across a variety of quasi-public goods. As an example, consider a commercial fishery, where access to fish stocks during a season may be unrelated to marginal values, but the length of the season is subject to policy and may depend on cumulative harvest.

We illustrate how, in the absence of markets for access rights, knowledge of marginal values is insufficient to determine the welfare effects of access. Consider the case of a local agency evaluating the construction of a public facility (e.g., a library). The available budget for constructing the facility is \$7 per year, which provides total capacity for the new facility of 2 units. There are three potential users of this facility, with valuations $v_1 = \$2$, $v_2 = \$4$, and $v_3 = \$6$. Access by these different individuals will depend on traffic, which is unfortunately difficult to predict. If users 1 and 2 arrive systematically earlier than individual 3, the expected welfare associated with the use of the new facility is \$6, which is less than the annual costs and the agency should drop the project. If, on the other hand, little information is available on the odds of any of these individuals arriving before the other two, the agency may assume equal probability of arrival. In these circumstances, the expected welfare associated

¹ In particular, the approach is directly applicable to the case of common pool resources where intra-season depletion is negligible, as in most recreational and commercial fisheries. Indeed, for the majority of fish stocks, managers set effort restrictions and removal targets for each entire season with the objective to ensure the sustainability of the stocks across seasons (rather than controlling the in-season evolution of the stock).

with the use of the facility is \$8; since this amount is higher than the \$7 in annual costs, the project should be undertaken. Many alternative scenarios are possible. Hence, preferences tell us something, but in the absence of a pricing mechanism and the sorting of valuations it induces, they need to be combined with explicit information on the probabilities of access. A corollary is that when access is equally likely across individuals with different valuations, efficient access can be determined with knowledge of the mean values of access.

2 A Model of Access

In this section we build a general model of access. The purpose of this model is to value scarce resources that are rationed in the absence of markets, that is, are accessed under different patterns of non-price sorting of marginal values. We start by assuming that there is a collection of agents seeking access to a commonpool resource or a public facility with finite capacity. We assume there is a mass m of agents, each demanding a single unit, and characterized by a willingnessto-pay of v. The valuation (or type) v is distributed independently over $[0,\hat{v}]$ according to the cumulative distribution F(v) and density f(v). Capacity is costly to provide and insufficient to satisfy the entire demand, q < m, making the resource scarce and creating the need for the manager to ration access.² As indicated earlier, valuations ν may be independent of total consumption q, or represent agents' marginal willingness-to-pay net of the external costs associated with total consumption. All that is required for the model to accommodate this later case is that external costs preserve the ranking of agents' willingness-to-pay (e.g. costs are identical across users, or proportional to users' valuations).³ We assume that the regulator's objective is utilitarian efficiency.4

In the next sections we show that total welfare derived from consumption of the q units is contingent upon how the rules that govern access to the resource sort individuals with different willingness-to-pay.

² We assume that the manager lacks the authority to charge users for access, that the willingness-to-pay of the lowest valuation types is insufficient to cover the costs of expanding capacity, or that capacity is otherwise exogenously fixed (e.g. national park, annual harvest quota set by the biologists, etc.).

³ In the former case we write the marginal willingness-to-pay net of external costs as u(v, q) =v-c(q), where $c'(q) \ge 0$, in the later case as $u(v, q) = v(1-\zeta(q))$, where $\zeta'(q) \ge 0$ and $\zeta(q) \in [0, 1]$.

⁴ While we acknowledge that the regulator may adopt alternative social welfare functions in order to advance objectives such as fairness and social inclusion, we focus on utilitarian welfare for being the most common and where our approach can inform policy.

2.1 When Exclusion is Feasible

When exclusion is feasible, efficient utilization of the resource maximizes total willingness-to-pay by granting access to the individuals with the highest valuations. Thus, the marginal valuation z defining efficient access is given by 1-F(z) =q/m. Only agents with willingness-to-pay z and above are granted access. Total willingness-to-pay corresponding to the efficient use is

$$W_e(z) = m \int_z^{\hat{v}} v f(v) d(v) = q \psi(z)$$
 [1]

where

$$\psi(z) = \frac{\int_{z}^{\hat{v}} v f(v) d(v)}{1 - F(z)}$$
 [2]

is the expectation of an individual's valuation conditional on exceeding z, that is, conditional on gaining access (Che, Gale, and Kim 2013).

Note that efficient access, denoted by A_e , describes who receives the resource, but does not specify how this assignment is to be achieved. The efficient outcome in eq. [1] could be attained by allowing the transferability of access rights. In a competitive market, the equilibrium price for the access right would be $p_e = z$, and the subset of individuals with the highest willingness-topay would end up with the access rights, regardless of the initial allocation. Hence, allocative efficiency follows. This decentralized mechanism relies on the assignment of clearly defined user rights and does not require the manager to have information on individuals' types.

In the following sections, we pursue the idea of access to resources when exclusion is not viable, to help understand the interaction between access, sorting and valuation.

2.2 When Exclusion is Infeasible

Let the different sets of controls that the manager could use to ration the resource (e.g. lotteries for charter schools, queuing for driver's licenses, gear restrictions and seasonal closures in commercial fishing) represent distinct access scenarios. Let Ω be the set of agents. Then each access scenario A_i induces an assignment, $a_i:\Omega\times[0,q]\to[0,1]$, which determines the probability of access for each valuation in Ω . Under access scenario A_i , an assignment consists of a family of distributions of the form

$$\rho_i(\eta, y) = \int_0^{\eta} a_i(v, y) dv$$
 [3]

where $a_i(v,y)$ is the probability of access to the y^{th} available unit for valuation v, and $\rho_i(\eta,y)$ the cumulative probability of access to the y^{th} available unit for valuations η or lower. Let's look again at our example with only three agents with willingness-to-pay v_1 = \$2, v_2 = \$4, and v_3 = \$6 and two available units of the public resource. The efficient assignment –scenario e– would allocate the two units to the highest valuation individuals; thus, the first unit would go to the individual with willingness-to-pay of \$6, $a_e(v=6,y=1)=1$ and $a_e(v=4,1)=a_e(v=2,1)=0$, and the second unit to the agent with willingness-to-pay of 4, $a_e(v=4,2)=1$ and $a_e(v=2,2)=a_e(v=6,2)=0$. However, it could also happen that under a given access scenario k all valuations have equal probability to access the resource. We would then have $a_k(v=2,1)=a_k(v=4,1)=a_k(v=6,1)=a_k(v=2,2)=a_k(v=4,2)=a_k(v=6,2)=1/3$. In this latter case, it would hold for the first unit that $\rho_k(v=2,1)=1/3$, $\rho_k(v=4,1)=2/3$, $\rho_k(v=6,1)=1$ and similarly for the second unit $\rho_k(v=2,2)=1/3$, $\rho_k(v=4,2)=2/3$, $\rho_k(v=6,2)=1$. Many cases in between are possible.

For any access scenario A_i , it holds that

$$\rho_i(y) = \int_0^{\hat{v}} a_i(v, y) dv = 1$$
 [4]

since by assumption capacity is scarce and therefore each available unit is consumed. From eq. [4] it follows that:

$$\int_{0}^{q} \int_{0}^{\hat{v}} a_{i}(v, y) dv dy = \int_{0}^{q} \rho_{i}(y) dy = q$$
 [5]

where q denotes the available quantity or the resource capacity.

We now introduce the function $y(\cdot)$, which we need in order to explicitly identify the probability that each valuation has to access the resource. For any assignment a_i we define the weighting function y(v,y) according to $y(v,y) = a_i(v,y)/f(v)$. Thus, y(v,y) is nonnegative and may be smaller, equal or larger than 1. A particularly interesting case arises when assignment is independent of the unit of the resource being accessed. If $a_i(v,y) = f(v)$, then y(v,y) = 1 and we call the scenario *uniform random access*. In this form of assignment, each user has an equal probability of access, regardless of valuation, and the expected value of each accessed unit of the resource is simply $\bar{v} = \int_0^{\bar{v}} v f(v) dv$. For all the other access scenarios, and for each available unit, the function $y(v,y) \neq 1$ redistributes probability mass across valuations v, thus resulting in the modified probability distribution a(v,y) = y(v,y)f(v). It is this modified distribution that defines the probability that each valuation has to access the resource. For example, rationing by waiting (queuing) will generally result in different probabilities of access than in a lottery. From eq. [4] it must hold that

 $\int_0^{\hat{v}} y(v,y) f(v) dv = 1$, that is, the mean weight must equal one for each available unit ν .

Given access scenario A_i and the induced assignment a_i defined by expression [3], the expected value of the y^{th} unit drawn is

$$EV(y) = \int_0^{\hat{v}} v a_i(v, y) dv = \int_0^{\hat{v}} v \gamma_i(v, y) dF(v)$$
 [6]

Thus, for a given distribution of willingness-to-pay F(v), the expected marginal value function EV(y) in eq. [6] completely characterizes the value of the accessed resource units. Embedded in EV(y) is the sorting of agents with different valuations across resource units, that is, how likely is each valuation ν to access the first unit of the resource, how likely to access the second unit, and so on. In turn, EV(y) and its curvature properties are fully determined by assignment a_i

$$\frac{dEV(y)}{dy} = \int_{0}^{\hat{v}} v \frac{\partial a_{i}(v, y)}{\partial v} dv = \int_{0}^{\hat{v}} v \frac{\partial \gamma_{i}(v, y)}{\partial v} dF(v)$$
 [7]

Figure 1 shows the usual marginal willingness-to-pay curve (i.e. the demand curve for the resource), denoted V(y), and the associated expected marginal value function $EV_i(y)$ for a possible assignment $a_i = \gamma_i(v, y) f(v)$, where $\gamma_i(v, y) =$ $[1+\theta_i(y)(v-\bar{v})]$ with $\theta_i(y) \ge \frac{-1}{\bar{v}-\bar{v}}$ for all y and $\int_0^m \theta_i(y) dy = 0$. Note that $\theta_i(y)$ determines how the reweighing of probabilities across valuations changes for the different resource units, $\frac{\partial y_i(v,y)}{\partial y} = \theta_i'(y)(v-\bar{v})$. In Figure 1, $\theta_i(y)$ is increasing in the interval [0, p] and achieves its maximum at y = p. The function V(y) is downwards sloping as it represents the efficient allocation in which resource units are accessed in rank order or marginal values. There are few a priori arguments that help us discern the shape of EV(y). The circumstances that lead to different curvature properties are not so easily intuited when we allow the public sector to dictate access. The functions γ twist the marginal value schedules to meet the feasibility constraints or policy goals. Thus, in some cases we might expect the resulting expected value functions to be non-smooth and their slopes to switch signs, as other forces that determine access to the resource may be lumpy and uncorrelated with values. For example, within emergency room resource allocation, assignments are a mix of triage, allocation by waiting and willingness to pay. In this case there may be a good bit of lumpiness in EV(y), as in Figure 1.

Next, note that in Figure 1 the efficient allocation achieves the maximum value associated with the consumption of any number of units. Indeed, $\int_0^q EV(y)dy < \int_0^q V(y)dy$ for all q < m. Only in the absence of scarcity, when rationing is unnecessary and the entire demand m can be satisfied, is the welfare associated with both scenarios equal. Graphically, this implies that the area

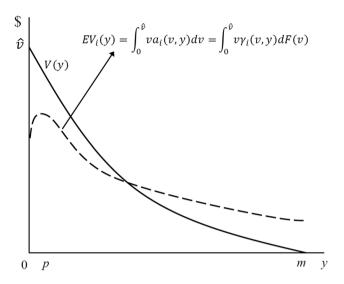


Figure 1: The expected marginal value function corresponding to assignment a_i

under the curve V(y) is always larger than that under EV(y), unless q = m in which case both become equal regardless of the sorting on valuations.

Next we introduce the function Γ , which will allow us to rank different access scenarios and, in turn, rank the rationing policies that induce them.

Definition 1 Let $\Gamma_i(\eta)$ denote the fraction of the q available units (or capacity) assigned by access scenario A_i to individuals with valuations equal or less than η . $\Gamma_i(\eta)$ is defined by

$$\Gamma_i(\eta) = \int_0^\eta d\Gamma_i(v) = \frac{1}{q} \int_0^\eta \int_0^q a_i(v,y) dy dv = \frac{1}{q} \int_0^q \rho_i(\eta,y) dy$$

Figure 2 illustrates graphically how $\Gamma(\cdot)$ is determined for the example of the access scenario defined by $\gamma_i(v,y) = [1 + \theta_i(y)(v - \bar{v})]^{.5}$

We say that access scenario A_i value-dominates A_j if $\int_0^{\hat{v}} \Gamma_i(v) dv < \int_0^{\hat{v}} \Gamma_j(v) dv$. In words, A_i grants access to greater quantities of the resource to high-valuation individuals than A_j does. For example, envision an scenario in which in-state

⁵ It is easy to show that in this case $\Gamma_i(\eta) = F(\eta) \left(1 + \frac{1}{q} \int_0^q \theta_i(y) (E[v|v \le \eta] - \bar{v}) dy\right)$. Thus, $\Gamma_i(\eta) = F(\eta)$ for all η under no rationing, since $\int_0^m \theta_i(y) dy = 0$. On the other hand, from the fact that $\theta_i'(y) \ge 0$ in [0,p] and $\theta_i'(y) < 0$ thereafter, it follows that $\Gamma_i(\eta) < F(\eta)$ for all $\eta < \hat{v}$ under rationing, q < m. Contrast this result with the case of a random access scenario in which $\theta_j(y) = 0$ for all y, where $\Gamma_j(\eta) = F(\eta)$ for all η and $q \in [0,m]$, that is, regardless of the level of rationing.

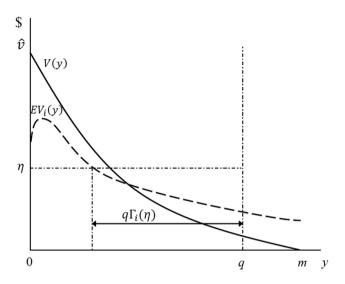


Figure 2: Graphical representation of $\Gamma(\cdot)$

anglers represent the lowest quartile of users' valuations, but access half of the total allowable catch due to permit surcharges and additional restrictions on out-of-state fishermen. Then, this status quo is value dominated by an alternative scenario in which all anglers are charged the same license fee and are subject to identical catch restrictions, regardless of place of residence. Note that while value dominance implies higher expected value, it is a weaker condition than second-order stochastic dominance (SSD), which requires $\int_0^\eta \Gamma_i(v) dv \leq \int_0^\eta \Gamma_j(v) dv \text{ for all } \eta, \text{ and unlike SSD it provides a complete ordering of access scenarios. SSD is a sufficient condition for value dominance, but it is not necessary for ranking different scenarios. <math>\mathcal{A}_i$ and \mathcal{A}_j are value-equivalent if $\int_0^{\hat{\nu}} \Gamma_i(v) dv = \int_0^{\hat{\nu}} \Gamma_j(v) dv. \text{ Finally, the uniform random assignment, characterized by } \gamma_i(\eta, y) = 1 \text{ for all } \eta, y, \text{ awards equal access probability to all users, regardless of their valuations (i.e. zero-price lottery). Writing the welfare function associated with capacity <math>q$ and access scenario \mathcal{A}_i as

$$W_i(q) = \int_0^q EV_i(y)dy = \int_0^{\hat{v}} \int_0^q va_i(v,y)dydv = q \int_0^{\hat{v}} vd\Gamma_i(v)$$
 [8]

we have the following result

Proposition 1 *If access scenario* A_i *value-dominates* A_i , *then* $W_i(q) > W_i(q)$.

Proof See Appendix

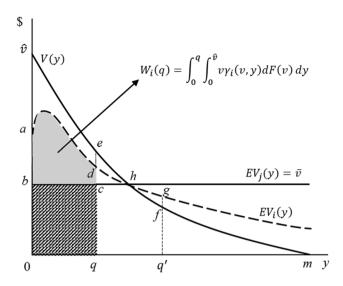


Figure 3: Welfare associated with quota q for different access scenarios.

Figure 3 illustrates how a given number of available units (or capacity) qgenerates different levels of welfare depending on the sorting induced by the access scenario. Under the efficient allocation A_e the total expected value associated with q is given by the area $(0qe\hat{v})$, while under access scenario itotal value is given by the area (0qda), and under uniform random access by the area (0qcb), with $(0qe\hat{v}) > (0qda) > (0qcb)$. Naturally the efficient allocation, given by the area under V(y), yields the highest value, because the individuals with the highest marginal values access first. In terms of our earlier definition, by construction $\Gamma_e(\eta)$ value dominates any other access scenarios, and according to the result above any access scenario A_i that is value-dominated by A_e will result in lower welfare than W_e in eq. [1]. Notice that this is the case for any degree of rationing q < m. Take for example q'. We know that when entire demand is satisfied, welfare associated with use of the resource equals $m\bar{v}$ regardless of the sorting of valuations, that is, regardless of the order in which the available units are accessed. Graphically, this means that in Figure 3 the areas under V(y) and $EV_i(y)$ and up to m are identical. For this to hold, however, it must be the case that area $(\hat{v}ah) > area (hfg)$, which in turns implies that $area(0q'f\hat{v}) > area(0q'ga)$.

We now look at a concrete example to illustrate how proposition 1 can be readily used to establish the conditions under which one mechanism is superior or equivalent to another. We focus on the case of hybrid assignment rules previously studied in the literature. As noted in Evans, Vossler, and Flores (2009), several U.S. states have recently used hybrid mechanisms to allocate big game permits, a setting in which there is often a small number of permits available relative to total demand. In one such hybrid mechanism, a few permits are auctioned off with the remaining permits allocated via lottery. An alternative hybrid mechanism rations permits using a lottery with one-part or two-part entry tariffs (Scrogin 2005). A question of interest for policy makers is the relationship between the fraction σ of available permits to auctioned off in the first mechanism and the entry tariff τ in the second mechanism that makes both hybrid mechanisms equivalent from a welfare standpoint. To explore this matter and according to the proposition, we require the mechanisms to be value-equivalent, $\int_0^{\hat{v}} \Gamma_i(v) dv = \int_0^{\hat{v}} \Gamma_j(v) dv.$

By the efficiency of the discriminatory and uniform auction formats (for single-unit demands and independent private values), we know that the σq permits that are auctioned off in the first mechanism end up in the hands of the highest valuation bidders. The lowest valuation $v^*(\sigma)$ to acquire a permit through the auction is defined by $[1 - F(v^*)] = \frac{\sigma q}{m}$. Thus, for these permits $\Gamma_i(\eta) = 0$ for $\eta < v^*$ and $\Gamma_i(\eta) = \frac{\sigma[F(\eta) - F(v^*)]}{1 - F(v^*)}$ for $v^* < \eta \le \hat{v}$. The remaining $(1 - \sigma)q$ units are subsequently allocated in a lottery in which only valuations $v \in [0, v^*)$ participate. Therefore $\Gamma_i(\eta) = \frac{(1-\sigma)F(\eta)}{F(\nu^*)}$ for $\eta < \nu^*$ and $\Gamma_i(\eta) = 1$ otherwise. Turning now to the second hybrid mechanism, note that the tariff τ discourages any valuation $v < \tau$ from entering the lottery. Since those that enter face the same probability of obtaining a permit, it is immediate that $\Gamma_j(\eta) = \frac{F(\eta) - F(\tau)}{1 - F(\tau)}$ for $\eta \ge \tau$, and $\Gamma_j(\eta) = 0$ otherwise. Using the definition of value equivalence, it is straightforward to show that the two hybrid mechanisms result in identical welfare if $E[v|v \ge \tau] = (1 - \sigma)E[v|v < v^*] + \sigma E[v|v \ge v^*],$ condition that implicitly defines $\tau = \tau(\sigma)$. In words, the tariff should exclude enough low willingness-to-pay individuals for the average valuation in the lottery to equal the mean valuation in the first mechanism, defined as a weighted average of the mean willingnessto-pay in the auction and the ensuing lottery. Furthermore, from the implicit function theorem, it follows that the tariff is increasing in the fraction of available permits that is auctioned off under the first mechanism, $\tau'(\sigma) > 0$. In particular, as $\sigma \to 1$, τ becomes the market clearing price.

We finish this section by characterizing a class of well-behaved assignments, monotone assignments. This definition will later allow us to derive optimality conditions for allocation of access under these assignments, assignments that Mumy and Hanke (1975) depicted only in graphical form.

Definition 2 (Monotone Assignments) Assignment a_i is said to be:

(i) monotone decreasing if access to the y'th unit second-order stochastically dominates (SSD) access to the y"th unit, $a_i(\cdot, y') \ge_{SSD} a_i(\cdot, y'')$, for all y' < y'', or

$$\int_{0}^{\eta} \rho_{i}(v, y') dv \le \int_{0}^{\eta} \rho_{i}(v, y'') dv \quad \text{for all} \quad \eta, \ y' < y''$$

(ii) monotone increasing if access to the y'th unit is second-order stochastically dominated by access to the y"th unit, $a_i(\cdot, y'') \ge_{SSD} a_i(\cdot, y')$, for all y' < y'', or

$$\int_0^{\eta} \rho_i(v, y'') dv \le \int_0^{\eta} \rho_i(v, y') dv \quad \text{for all} \quad \eta, \ y' < y''$$

In words, *monotone decreasing assignments* grant higher willingness-to-pay units a larger probability of being satisfied before lower willingness-to-pay units. *Monotone increasing assignments*, on the other hand, grant lower willingness-to-pay units a larger probability of being satisfied first than higher willingness-to-pay units (see appendix for proof). The following lemma ranks monotone increasing and decreasing assignments

Lemma 1 For a given distribution of valuations f(v), monotone decreasing assignments always value-dominate monotone increasing assignments.

Proof See Appendix

Figure 4 depicts a marginal value function and the associated expected marginal value functions for access scenarios that induce different monotone assignments: EV_1 , the expected value function corresponding to access scenario 1, is downward-sloping as this scenario induces a monotone decreasing assignment and $a_1(\cdot, y') \ge_{SSD} a_1(\cdot, y'')$ for all y' < y'' implies $\partial EV_1(y)/\partial y \le 0$; similarly EV_2 , the expected value function corresponding to access scenario 2, is upward-sloping as this access scenario induces a monotone increasing assignment, a_2 . Monotone decreasing assignments are easy to envision. With a bit of price discrimination we can induce an assignment similar to EV_1 in Figure 4. While the intuitive nature of EV_2 is obvious, it's less clear how this would arise. One possible case occurs when the arrival rates of individuals with different valuations changes over time, but favors low valuations early on. Lemma 1 allows us to rank different monotone assignments without the need to compute the expected welfare associated with each of them individually. In Figure 4, for example, we know immediately that access scenario A_1 value dominates A_2 -i.e. generates higher welfare- for any level of rationing.

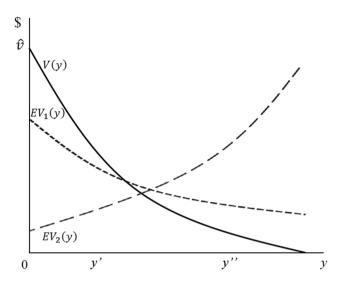


Figure 4: Expected marginal value function for different monotone assignments.

3 Access Across Multiple Sectors

To analyze the implications of different rules of access on optimal access allocation, we now introduce additional sectors whose agents seek to exploit the public resource. Our objective is to demonstrate that our framework can be extended to the case of competing sectors, typically the situation in real world settings such as fisheries and water resources. Let there be n sectors, with mass m_j of agents in sector j, each characterized by willingness-to-pay of v_j , distributed independently over $[0, \hat{v}_j]$ according to the distribution $F_j(v)$ and density $f_j(v)$. We assume that identification of agents with a given sector is based on observables, and thus we rule out problems of asymmetric information. Total capacity, or total available resource units is $X < \sum_j m_j$. The manager's objective is to allocate access to X among the different sectors to maximize welfare.

3.1 The Efficient Allocation

Let $v_j^e > 0$ denote the critical valuation in sector j such that (i) $1 - F(v_j^e) = q_j/m_j$ for a given quota q_j to sector j. Denote z the value that simultaneously satisfies condition (i) for all sectors. Eliminating q_1, \ldots, q_n it follows that z solves $\sum_j m_j [1 - F_j(z)] = X$. Valuation z is the marginal valuation in each sector that

defines the efficient allocation (i.e. as prescribed by the equi-marginal principle). In this efficient allocation only individuals with valuations z or higher gain access. To see that this is necessarily the case, note that if this condition did not hold, that is, if $v_i^e \neq v_k^e$, then it would be possible to reallocate quota between these two sectors and increase total welfare, making the initial allocation a suboptimal solution. In the efficient allocation, sector *i* receives $q_i^e = m_i[1 - F(z)]$ in access rights. Thus, total welfare corresponding to the efficient allocation is

$$W(z) = \sum_{j=1}^{n} m_{j} \left(\int_{z}^{\hat{v}_{j}} v f_{j}(v) d(v) \right) = \sum_{j=1}^{n} q_{j}^{e} \psi_{j}(z)$$
 [9]

where $\sum_{i} q_{i}^{e} = X$ and, as before,

$$\psi_j(z) = \frac{\int_{z}^{\hat{V}_j} v f_j(v) d(v)}{1 - F_i(z)}$$
 [10]

is the expectation of an individual's valuation in sectors *j* conditional on receiving quota.

3.2 Allocation when Exclusion is Infeasible

We now derive the optimality condition for the allocation of capacity among multiple sectors for any access scenarios, when exclusion of low willingness-topay agents is infeasible. The optimal allocation maximizes total welfare, which is given by

$$W(q_1, \ldots, q_n) = \sum_{j=1}^n \left(\int_0^{q_j} EV_j(y) dy \right)$$

$$= \sum_{j=1}^n \left(\int_0^{q_j} \int_0^{\hat{v}_j} v a_j(v, y) dv dy \right)$$

$$= \sum_{j=1}^n \left(\int_0^{q_j} \int_0^{\hat{v}_j} v \gamma_j(v, y) dF_j(v) dy \right)$$
[11]

where $q_1 + ... + q_n = X$.

Proposition 2 For any collection A_1, \ldots, A_n of access scenarios, the (nx1) vector q^* of capacity shares allocated to different sectors maximizes total welfare if the following conditions hold

$$\int_{0}^{\hat{v}_{i}} v \gamma_{i}(v, q_{i}^{\star}) dF_{i}(v) = \int_{0}^{\hat{v}_{j}} v \gamma_{j}(v, q_{j}^{\star}) dF_{j}(v), \qquad \forall i, j$$
 [SC-1]

$$\int_{0}^{\hat{v}_{i}} v \frac{\partial \gamma_{i}(v, y)}{\partial y} \bigg|_{y = q_{i}^{*}} dF_{i}(v) < 0 \qquad \forall i$$
 [SC-2]

Proof See Appendix

According to this proposition, total capacity (or quota) should be allocated until the expected value of the last unit is equal across sectors (condition SC-1), with the expected value of that last unit in each sector given by the corresponding probability distribution of access to that unit across valuations.⁶ Condition SC-2 is necessary to guarantee a local maximum. This proposition characterizes optimality over a wide range of access settings as it does not impose any restrictions on the shape of the expected value functions. Uniqueness is not guaranteed unless monotonicity assumptions are made on the functions $EV_i(\gamma)$ (see lemma below); consequently, the search for the optimal allocation may entail comparing total welfare for the various critical points. Finally, note that conditions SC-1 and SC-2 characterize an interior solution; to rule out potential corner solutions in which some sectors receive zero quota, it is sufficient for the first unit of quota allocated to any of the sectors to be more valuable than the same unit optimally allocated instead across the remaining sectors: $\lim_{\varepsilon \to 0} EV_i(\varepsilon) > EV_k(q_k^{\star\star})$, where $q_k^{\star\star}$ is defined by the conditions $EV_k(q_k^{\star\star}) = EV_z(q_z^{\star\star}) \ \forall k, z \neq i \text{ and } \sum_{j \neq i} EV_j(q_j^{\star\star}) = X - \varepsilon$.

The following lemma characterizes the optimal allocation for the special case of monotone assignments. For both decreasing and increasing assignments uniqueness is guaranteed as monotonicity implies, respectively, global concavity and convexity.

Lemma 2 (i) Let the collection A_1, \ldots, A_n of access scenarios induce monotone decreasing assignments: vector \mathbf{q}^* maximizes total welfare if and only if

$$\int_{0}^{\hat{v}_{i}} v \gamma_{i}(v, q_{i}^{\star}) dF_{i}(v) = \int_{0}^{\hat{v}_{j}} v \gamma_{j}(v, q_{j}^{\star}) dF_{j}(v), \quad \forall i, j$$

⁶ For external costs associated with congestion or in-season depletion, SC-1 above, $EV_i(q_i^*) = EV_j(q_j^*)$ for all i, j, needs to be slightly modified. In this case $EV_i(q, y) = \int_0^{v} (v - c(q)) \gamma(v, y) f(v) dv$ (or $v(1 - \zeta(q))$) instead of v - c(q) if external costs are proportional to agents' valuations), Accordingly, the condition becomes $EV_i(q_i^*,y)|_{y=q_i^*} + \int_0^{q_i^*} \partial EV_i(q,y)/\partial q|_{q=q_i^*} dy = EV_j(q_j^*,y)|_{y=q_j^*} + \int_0^{q_j^*} \partial EV_j(q,y)/\partial q|_{q=q_j^*} dy$. The second term in each side of the equality is simply the effect on each sector's welfare of a marginal increase in its allocated quota, through the effect of total consumption (i.e. sector's quota) on external costs. Condition SC-2, $EV_i'(q_i^*) < 0$ for all *i*, remains necessary for a local maximum.

(ii) Let Λ_1 denote the set of w_i , where $w_i = \int_0^{m_i} EV_i(y) dy$ for $i = 1, \ldots, n$, and define $\Lambda_{k+1} = \Lambda_k \setminus \{ max\{\Lambda_k\} \}$ and $\Omega_k = \Lambda_k \setminus \Lambda_{k+1}$. Let the collection A_1, \ldots, A_n of access scenarios induce monotone increasing assignments: vector \boldsymbol{q}^* maximizes total welfare if and only if

$$q_i^{\star} = \begin{cases} m_i & \text{if } \mathbf{w}_i \in \bigcup_{k=1}^z \Omega_k \text{ with z defined by } \sum_{j=1}^z m_j = X \\ 0 & \text{Otherwise} \end{cases}$$

Proof See Appendix

According to the lemma, the optimal allocation under monotone decreasing assignments is defined by the vector of quotas that equates marginal expected benefits across all sectors (condition (i)).⁷ Figure 5 illustrates the case for the allocation between two sectors. The optimum is found at the point of intersection of the expected value functions. On the other hand, under monotone increasing assignments, that is, when high valuations are expected to access last, condition (ii) states that the optimal allocation is a corner solution that satisfies the entire demand of the

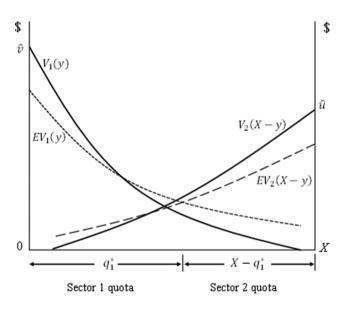


Figure 5: Optimal allocation for two sectors under monotone decreasing assignments.

⁷ In the case of external costs due to congestion or in-season depletion, condition i) is modified as indicated in note 6.

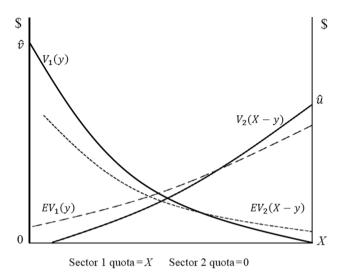


Figure 6: Optimal allocation for two sectors under monotone increasing assignments.

sectors where access generates the largest welfare and excludes all other sectors. Figure 6 depicts this case for the allocation of access between two sectors. Total welfare in sector 1, when it receives the entire quota X, exceeds that in sector 2 when its entire demand is satisfied. Accordingly, the entire demand of sector 1 is satisfied, and sector 2 receives no quota. If, for example, Figure 6 depicted access to campgrounds along the Inca trail and sector 2 represented independent backpackers, then optimality would call for commercial, professionally guided trips only. The rationale for the result is straightforward. Backpackers would otherwise displace some high-value guided tourism by getting to campsites first.⁸

4 Conclusion

We have constructed a general model of access which has allowed us to value different assignments when resources are allocated in the absence of markets. These "assignments" appear in a variety of forms. They include lotteries, queues, priority lists, and restrictions based on personal characteristics uncorrelated with the privately held values. Our approach can be used to establish the design conditions

⁸ Indeed, since 2002 the permits required for access to the Inca Trial are sold exclusively through licensed tour operators. Independent, unguided visitors are no longer allowed. For details see http://www.mincetur.gob.pe/TURISMO/proyectos/regl_uso_turistico.htm.

under which one mechanism is superior to another. Because the assignment of transferable property rights tends to achieve the highest returns and most efficient use of a resource, economists have since long recommended its adoption. Those cases that resist tend do so for good reason -the high costs of imposing property rights, undesirable outcomes that emerge as a consequence, entrenched political interests, and the desire to allocate access for merit reasons can all contribute to the allocation of resources without property rights. We demonstrate that marginal value schedules are far less useful in allocating access when property rights are unfeasible. In these settings, the criterion for optimal allocation –a generalized version of the marginal principle-combines information on both the marginal value schedules and the assignments determining the probabilities of access to the resource. Our results highlight the need for practitioners handling scarce resources to think explicitly on how their rule making sorts heterogeneous users. The design of empirical strategies and survey instruments for assessing users' probabilities of access under alternative scenarios should rank high in the agenda of managers limited in their ability to use price signals. Our framework has allowed us to rank rationing policies (and the access scenarios they induce) in a wide range of real-world, second-best settings. Alternatively, the approach could help inform managers' optimal supply of (costly) capacity under various access scenarios. Information on who is likely to benefit from access is essential for the optimal provision of public resources.

Appendix

Proof of Proposition 1

We need to demonstrate that $W_i(q) > W_i(q)$ if A_i value-dominates A_i . Write

$$W_{i}(q) > W_{j}(q)$$

$$q \int_{0}^{\hat{v}} v d\Gamma_{i}(v) > q \int_{0}^{\hat{v}} v d\Gamma_{j}(v)$$

$$\Rightarrow q \int_{0}^{\hat{v}} v d(\Gamma_{i}(v) - \Gamma_{j}(v)) > 0$$
[12]

Integrating by parts yields

$$\int_0^{\hat{v}} v d(\Gamma_i(v) - \Gamma_j(v)) = \left[\Gamma_i(v) - \Gamma_j(v)\right] v \Big|_0^{\hat{v}} - \int_0^{\hat{v}} \left[\Gamma_i(v) - \Gamma_j(v)\right] dv$$

The first term on the right-hand side is zero because $\Gamma_i(0) = \Gamma_i(0) = 0$ and $\Gamma_i(\hat{v}) = \Gamma_i(\hat{v}) = 1$. We can therefore rewrite eq. [12] as

$$q\int_{0}^{\hat{v}} \left[\Gamma_{j}(v) - \Gamma_{i}(v)\right] dv > 0$$

which holds due to value dominance of A_i over A_i .

Proof: Monotone Assignments and Probability of Access

Monotone decreasing assignments grant higher willingness-to-pay units a larger probability of being satisfied before lower willingness-to-pay units. Monotone increasing assignments, on the other hand, grant lower willingness-to-pay units a larger probability of being satisfied first than higher willingness-to-pay units. To see that this is indeed the case, rewrite $\int \rho_i(v, y') dv$ as

$$\int_0^{\eta} \left[\int_0^{v} \gamma_i(t, y') f(t) dt \right] dv = \int_0^{\eta} \left[\int_0^{v} \gamma_i(t, y') \frac{f(t)}{F(v)} dt \right] F(v) dv$$

and note that the term in parenthesis on the right-hand-side is the conditional expectation of the weighting function γ_i when willingness-to-pay are truncated at ν . Thus, we can rewrite the condition for monotone decreasing assignments as

$$\int_0^{\eta} \left[\bar{\gamma}_i(t, y'|t \le v) - \bar{\gamma}_i(t, y''|t \le v) \right] F(v) dv \le 0$$

since this condition must hold for all y' < y'' and $\eta \in [0, \hat{v}]$, in particular for any sufficiently small η , it necessarily follows that $\bar{\gamma}_i(t, y'|t \le v) \le \bar{\gamma}_i(t, y''|t \le v)$ for low willingness-to-pay. Thus, the monotone decreasing assignment a_i is shifting a larger probability mass, from lower to higher valuations, for the early accessed units y' than for the later accessed units y''. A similar argument shows that monotone increasing assignments do the opposite.

Proof of Proposition 2

 $W = W(q_1, \ldots, q_{n-1})$, since $q_n = X - \sum_{j=1}^{n-1} q_j$ is the residual quota, and we write the Hessian

$$H = \begin{bmatrix} \frac{\partial^2 W}{\partial q_1^2} & \cdots & \frac{\partial^2 W}{\partial q_1 \partial q_i} & \cdots & \frac{\partial^2 W}{\partial q_1 \partial q_{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 W}{\partial q_i \partial q_1} & \cdots & \frac{\partial^2 W}{\partial q_i^2} & \cdots & \frac{\partial^2 W}{\partial q_i \partial q_{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 W}{\partial q_{n-1} \partial q_1} & \cdots & \frac{\partial^2 W}{\partial q_{n-1} \partial q_i} & \cdots & \frac{\partial^2 W}{\partial q_{n-1}^2} \end{bmatrix}$$

or, equivalently as $H = (h_{ij})_{(n-1)\times(n-1)}$, where

$$h_{ij} = \begin{cases} EV'_n(q_n^{\star}) & \text{if} \qquad i \neq j \\ EV'_i(q_i^{\star}) + EV'_n(q_n^{\star}) & \text{if} \qquad i = j \end{cases}$$
[13]

where ${m q}^{\star}$ is defined by the first order conditions SC-1 and $EV_i^{\prime}(q_i^{\star})$ = $\int_0^{\hat{v}_i} v \frac{\partial y_i(v,y)}{\partial v} \big|_{y=q_i^*} dF_i(v).$

The kth leading principal minor of expression [13] can be determined by Gaussian elimination. First, subtract the kth column from each of the first k-1 columns. Then, for each of the columns $i = 1, \ldots, k-1$, multiply column i by $\left(\frac{EV_n'}{EV'}\right)$ and subtract it from the kth column, to obtain a lower triangular determinant whose diagonal components are

$$h_{ii}^{k} = \begin{cases} EV_{i}' & \text{if} & i < k \\ EV_{k}' + EV_{n}' + \sum_{j=1}^{k-1} \left(\frac{EV_{n}'}{EV_{j}}\right) EV_{k}' & \text{if} & i = k \end{cases}$$
 [14]

and thus the kth leading principal minor is given by

$$|H_k| = \prod_{j=1}^k EV_j' + EV_n' \left(\prod_{j=1}^{k-1} EV_j' + \sum_{j=1}^{k-1} \prod_{\substack{r=1\\r\neq j}}^k EV_r' \right)$$
[15]

where the term in parentheses represents the combinations C(k, k-1) of k-1elements out of the k possible EV_i in the kth order principal submatrix of H. Thus, it is sufficient that $EV_i^{\prime}(q_i^{\star}) < 0$ for all i for the kth leading principal minor to have the same sign as $(-1)^k$, that is, for *H* to be negative definite. Consequently, SC-2 is a sufficient condition for q^* to be a strict local maximum of W^9 .

Proof of Lemma 1

By definition of monotone decreasing assignments (case (i)), $EV_i(y) < 0$ for $y \in [0, m_i]$, and $W(q_1, \ldots, q_n)$ is a strictly concave function. Thus, the first order conditions are necessary and sufficient for a global maximum.

By definition of monotone increasing assignments (case (ii)), $EV_i'(0) > 0$ for $y \in [0, m_i]$, and $W(q_1, \ldots, q_n)$ is a strictly convex function. Thus, the first order

⁹ For the case of external costs, $h_{ii}^k = Z_{ii}$ for $i \neq j$ and $h_{ii}^k = (Z_{ii} + Z_{nn})$ for i = j, where $Z_{ii} = EV_i'(q_i^*) - \int_0^{\hat{v}} c'(q_i^*) \gamma(v, q_i^*) f(v) dv - \int_0^{q_i} \int_0^{\hat{v}} c''(q_i^*) \gamma_i(v, y) f(v) dv dy$, and an equivalent expression for Z_{nn} . Note that since $c' \geq 0$ and $c'' \geq 0$ (or, for external costs proportional to valuation, $\zeta' \geq 0$ and $\zeta'' \ge 0$), it follows that $Z_{ii} < 0$ if $EV'_i(q_i^*) < 0$. Thus, as stated SC-2 remains a necessary condition for a local maximum.

conditions are necessary and sufficient for a global minimum. Consequently, maximization of welfare calls for a corner solution. The optimal allocation is found by satisfying the entire demand of the sectors where access generates the largest welfare and excluding the other agents. \Box

Proof of Lemma 2

Let \mathcal{A}_i be the monotone decreasing assignment and \mathcal{A}_j the monotone increasing assignment. By definition, $EV_i(y) < 0$ and $EV_j(y) > 0$ for $y \in [0, m]$. These conditions combined with the fact that welfare must be the same when the entire demand is satisfied, regardless of the sorting of valuations, $\int_0^m EV_i(y)dy = \int_0^m EV_j(y)dy = m\bar{v}$, implies that expected value functions $EV_i(y)$ and $EV_j(y)$ intersect exactly once in [0, m]. Let y^* be the intersection point and write

$$\int_{0}^{m} EV_{i}(y)dy - \int_{0}^{m} EV_{j}(y)dy = 0$$

$$\int_{0}^{y^{*}} [EV_{i}(y) - EV_{j}(y)]dy + \int_{y^{*}}^{m} [EV_{i}(y) - EV_{j}(y)]dy = 0$$

$$\int_{0}^{y^{*}} [EV_{i}(y) - EV_{j}(y)]dy + \int_{y^{*}}^{q} [EV_{i}(y) - EV_{j}(y)]dy > 0$$

$$W_{i}(q) - W_{j}(q) > 0$$

where the first inequality follows from the fact that q < m and $EV_i(y) < EV_j(y)$ for $y \in [y^*, m]$. We need to demonstrate that, for monotone assignments, $W_i(q) > W_i(q)$ implies A_i value-dominates A_j . Rewrite the last inequality above

$$q \int_{0}^{\hat{v}} v d(\Gamma_{i}(v) - \Gamma_{j}(v)) > 0$$
 [16]

Integrating by parts yields

$$\int_{0}^{\hat{\mathbf{v}}} v d(\Gamma_{i}(\mathbf{v}) - \Gamma_{j}(\mathbf{v})) = \left[\Gamma_{i}(\mathbf{v}) - \Gamma_{j}(\mathbf{v})\right] v \Big|_{0}^{\hat{\mathbf{v}}} - \int_{0}^{\hat{\mathbf{v}}} \left[\Gamma_{i}(\mathbf{v}) - \Gamma_{j}(\mathbf{v})\right] d\mathbf{v}$$

The first term on the right-hand side is zero because $\Gamma_i(0) = \Gamma_j(0) = 0$ and $\Gamma_i(\hat{v}) = \Gamma_i(\hat{v}) = 1$. We can therefore rewrite eq. [16] as

$$q\int_0^{\hat{v}} [\Gamma_j(v) - \Gamma_i(v)] dv > 0$$

which is the definition of value dominance of A_i by A_i .

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