Research Article

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Optimal Penalties for Repeat Offenders – The Role of Offence History

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Abstract: Within an infinite and a corresponding finite game framework we analyse intertemporal punishment for repeat offenders. The legal authority is assumed to maximize social welfare by minimizing the sum of harm from crimes and cost of punishment. We show that the time horizon considerably affects the structure of the optimal penalty scheme. In the finite game framework decreasing as well as escalating penalty schemes may be optimal. For the more appropriate infinite game framework we show three main results: First, any penalty scheme can be replaced by a (weakly) escalating penalty scheme that leads to the same criminal activity and the same social penalization cost. Second, the optimal penalty scheme is of the escalating type. Third, the socially optimal level of crime under escalating penalties may be higher than the level which would be optimal under uniform penalties.

Keywords: crime prevention, repeat offenders, infinite time horizon
JEL classification: K14, C73

1 Introduction

This paper is in the economics of crime literature tradition building upon the fundamental work of Becker (1968). Here, the decision on whether to commit a crime is modelled as a rational choice of the potential offender. It is rational in that it is based on utility maximization balancing expected benefits from crime against expected costs. There is an extensive literature on the benefits and costs

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of crime starting with the seminal papers by Ehrlich (1973, 1975) highlighting the importance of the expected size of punishment.[1]

Of course, criminals impose costs on society, particularly the damage done to their victims but also indirect costs like higher uncertainty in economic and social interactions. Consequentially, society allocates scarce resources to crime prevention. Consistent with the understanding of the potential criminals' decisions sketched above, the decision of society on how much to spend on crime prevention is also interpreted to be a rational one balancing benefits of crime reduction and its costs.[2]

Obviously, the decisions of potential criminals and the decisions of the designers of criminal law are highly interdependent. Not only do potential criminals acknowledge the size and structure of penalties as an important determinant of their decisions for (against) crime. Also, the lawmakers should take the cost-benefit-analysis of potential criminals into account in the making of criminal law. Any assessment of the deterrence effect of a certain form and size of punishment (and thereby of the benefits of this punishment) must be based on a hypothesis about how potential criminals might adjust their decisions to this specific punishment.

To date, most of the contributions to the literature tradition referred to above use static economic models. A price to be paid for the analytical simplicity thereby achieved is that certain intertemporal aspects of crime and punishment cannot be satisfactorily analysed.[3] One of these aforementioned aspects is the intertemporal profile of punishment for repeat offenders.[4]

A central question in this context is: "We observe that repeat offenders are punished more severely than first time offenders. What (if any) is the economic rationale for this escalating penalties structure?"

Papers that explore the escalating (or synonymously: increasing) penalties question address two issues: First, how does a potential criminal react to alternative intertemporal punishment structures for first time and repeat offenders? Second, what is the optimal intertemporal structure of penalties (given the criminal's cost benefit analysis explored in the first step)? The optimality concept used for the choice of the intertemporal penalties structure varies within the "escalating penalties literature". Most authors assume that the lawmaker strives

1 Other important determinants are unemployment rates, income levels and disparities as well as sociological and demographic factors. See Fajnzylber, Lederman, and Loayza (2002) and the literature given therein.

2 Recent papers following this approach are Persson and Siven (2006), Wilhite and Allen (2008).

3 See Persson and Siven (2006, 226).

4 Other examples are punishment by incarceration, opportunities to accumulate human capital inside or outside of prison, and many other issues including the rate of time preference.

to design the penalty structure which maximizes social welfare in that the social cost of crime is minimized.[5] Typically the social cost of crime consists of the damage suffered by the victims and the cost of crime prevention. Some authors differentiate the cost of crime reduction distinguishing between the cost of apprehending offenders and the cost of punishment.[6] Others differentiate the benefits of crime reduction subtracting the utility losses suffered by the criminals from the gains of the potential victims which are spared.[7]

Most of the contributions to the escalating penalties literature use a finite game framework and confine the analysis to two periods and two sanction levels, one for first time and one for repeat offenders.[8] The advantage of this framework is analytical simplicity. The drawback is well-known in the game theoretic literature: The approach suffers from all kinds of "finite horizon paradoxes" (Rubinstein 1998, 165), the discussion of which began with the seminal contributions of Luce and Raiffa (1957) and Selten (1978), and is ongoing.

Apart from this methodological problem the results produced by the established escalating penalties literature are puzzling, in that a decreasing penalty scheme (one that punishes repeat offenders less severely than first time offenders) is optimal at least for certain parameter values in many of these finite models.[9] However, this kind of penalty scheme is nowhere to be observed in reality, and seems implausible.

5 See Chu, Hu, and Huang (2000), Dana (2001), Friehe (2009), Kaplow (1992), Polinsky and Shavell (1998), Rubinstein (1979). Opposed to that a few papers do not assume social welfare maximization as the goal of the lawmaker but cost effectiveness. There, the law is supposed to be designed to achieve a predetermined crime rate at minimum cost. Some authors take this predetermined rate to be zero. (See Emons (2003), (2007), Endres and Rundshagen (2012), Hylton (2005), Miceli and Bucci (2005), Polinsky and Shavell (2000)). Of course, cost effectiveness is a necessary condition for social welfare maximization. The dichotomy between approaches taking the lawmaker to strive for social welfare maximization and for cost minimization, respectively, is also common in the economics of other areas of the law than criminal law. See, e. g. Endres (2011) for an application to the economics of environmental law.

6 Occasionally, the punishment of mistakenly convicted innocent people is included as an element into the cost function for crime reduction. See, e. g., Chu, Hu, and Huang (2000), Rubinstein (1979).

7 Polinsky and Rubinfeld (1991) distinguish between "acceptable" and "illicit" gains. As suggested by these terms the former are counted in the social welfare function, the latter are not.

8 For two period models see, e. g., Chu, Hu, and Huang (2000), Emons (2003), (2007), Friehe (2009), Miceli and Bucci (2005), Mungan (2010), Mungan (2014), Polinsky and Rubinfeld (1991), Polinsky and Shavell (1998), Rubinstein (1980). Unlike the aforementioned papers Burnovski and Safra (1994) and Motchenkova (2014) use multi-period models with fines depending on the number of past convictions. However, just like the two period models these models are finite.

9 Examples for two-period models in which optimal penalty schemes may be of the decreasing as well as of the escalating type are Emons (2007), Friehe (2009), Miceli (2013) and Polinsky and

To solve the "puzzle of escalating penalties" (Dana 2001), we analyse the impact of the time horizon on the structure of the optimal penalty schemes. In the main parts of the paper (Sections 2–5) we extend the model of the "criminal career" of an individual from the two-period case presented in the literature to an infinite game framework in order to formalize the unknown endpoint of lifetime more appropriately. Specifically, we assume that a decision maker who is alive in period t expects still to be alive in the next period with a positive probability $q < 1$.[10,11]

In Section 6 we consider a corresponding traditional two-period framework and show that in this finite model the optimal penalty scheme might be of the escalating as well as of the decreasing type as it is also the case in Emons (2007), Friehe (2009), Miceli (2013) and Polinsky and Rubinfeld (1991). This highlights the fact that the time horizon may have a crucial impact on the optimal penalty structure.[12]

The infinite game framework is similar to the one of Endres and Rundshagen (2012). However, opposed to the paper at hand, in Endres and Rundshagen (2012) only two sanction levels are considered, i. e., the sanction for repeat offenders is not allowed to depend on the number of previous records, which is an important feature of the paper at hand. This allows us to rule out the optimality of hybrid penalty schemes, which combine decreasing and increasing steps of penalization. Moreover, Endres and Rundshagen (2012) do not compare their outcome with a corresponding finite game, which is a central part of the paper at hand.

In both frameworks of this paper, the infinite and the finite one, the game is modelled in three stages. In the first two stages the legal system is chosen. First a regulator, who strives to minimize the sum of harm from crimes and cost of

Rubinfeld (1991). In Emons (2003) the optimal penalty for repeat offenders equals zero, that is, in this model decreasing sanctions are always optimal. Also Burnovski and Safra (1994), Dana (2001) and Motchenkova (2014) argue in favour of the optimality of decreasing penalty schemes under different assumptions. In Motchenkova (2014) declining penalties are optimal under the assumptions that offenders are wealth constrained and the government is resource constrained. Burnovski and Safra (1994) assume that offenders make the decision on the number of crimes ex ante, whereas Dana (2001) considers escalating probabilities of detection for repeat offenders. In Polinsky and Shavell (1998) offenders in period 1 and repeat offenders in period 2 receive the maximal sanction. First time offenders in period 2 may receive a lower sanction. Rubinstein (1980) and Chu, Hu, and Huang (2000) rule out decreasing penalties by assumption.

10 The use of an infinite game is not to be confused with the assumption that the decision maker will live on forever. To the contrary, in our infinite game framework each decision maker passes away some day, since the probability of an infinite life is given by $\lim_{t\to\infty} q^t = 0$ for each $q < 1$.

11 A similar approach is taken by Baker and Westelius (2013) who construct an econometric model of the deterrence hypothesis which takes into account that decisions to commit crimes have long term consequences.

12 Of course, this does not imply that in each of the cited models a transition from a finite to an infinite game framework would lead to a substantial change of the optimal penalty structure.

punishment chooses a target criminality level and in the second stage he chooses the corresponding optimal penalty scheme. In the third stage which comprises the infinite game described above or a traditional two-period game, respectively, potential offenders decide in each period of their life whether they commit a crime.

For the infinite game framework we derive three main results: First, any decreasing or hybrid penalty scheme can be replaced by a (weakly) escalating penalty scheme that leads to the same criminal activity and the same social penalization cost. Second, for any (exogenously) given aspired criminality level (chosen in stage one), the cost minimizing penalty scheme (equilibrium of stage two) is monotonically increasing in the number of previous police records but not strictly increasing. Third, although crime prevention is cheaper under optimal escalating penalty schemes than under uniform penalty schemes, the socially optimal level of crime may be higher than the level which would be optimal under the restriction of uniform penalties.

Moreover we show that there is a fundamental difference between the equilibrium strategies in the finite and infinite game framework. In the infinite game a decreasing sanction scheme never leads to higher welfare than a uniform penalty scheme. The driving force behind this result is the fundamental divergence between the equilibrium strategy of a potential offender under a decreasing sanction scheme in a game with ex ante known number of periods, and this strategy in an infinite game. In the finite game under a decreasing penalty scheme the decision of the potential offender in the second (or a later) period, depends on whether the offender has been convicted for previously committed crimes. However, in the infinite game a potential offender either commits the crime in each period or in no period under a decreasing penalty scheme as well as under a uniform penalty scheme. In both cases he commits the crime if and only if his benefit from committing the crime exceeds his average expected penalty. Thus, for any decreasing penalty scheme which induces the same criminality level as a given uniform penalty, the average expected penalty coincides with the uniform penalty. Hence, the critical benefit that separates offenders from non-offenders is identical under both penalty schemes, too. This equality also implies identical penalization costs. To prove this conjecture we proceed as follows: for a given penalty scheme we relate the decision of a potential offender to commit a crime to a sequence of critical benefits $\{b_n\}_{n=0,1,\dots}$. For criminal record level n an outsider will observe that the offender commits the crime if and only if his benefit is larger than b_n. We then show that each penalization scheme that leads to the same sequence of critical benefits (as it is the case, e. g. for a decreasing and corresponding uniform penalty scheme) implies identical penalization cost. Moreover, for the infinite game we show that each set of penalty schemes that lead to the same sequence of critical benefits contains an escalating penalty scheme (including the uniform penalty scheme as border case). Hence the task of finding the overall optimal penalty scheme is reduced to the task of finding the optimal penalty scheme within the set of escalating ones (including the uniform penalty schemes as border case). We show that the structure of the unique optimal penalty scheme is of the following type. There exists at most one criminal record level n_0 for which the penalty level is non-degenerate $(0 < s_{n_0} < s_{max})$. All the other records result in either zero penalty ($s_n = 0$ for $n < n_0$), or the maximum penalty ($s_n = s_{max}$ for $n > n_0$).

The intuition behind this result may be explained as follows. Consider a uniform or escalating penalty scheme s which induces the maximum tolerable criminality level, as chosen by society. Now consider a change from this scheme to a scheme of escalating penalties $\hat{s}$ which just induces the same criminality level but differs from s by a lower penalty for offenders with a low criminal record and a higher penalty for offenders with a high criminal record. The change in the penalties must be such that the increased criminal activity of offenders with low criminal record is exactly offset by the decrease in the criminal activity of offenders with high criminal record. However, even though the level of crime is unaffected by the envisaged change in the penalty structure, the aggregate cost of punishment goes down: the cost of punishment for the additional criminal activity of offenders with low criminal record is lower than the saved cost of punishment for the reduced criminal activity of offenders with high criminal record. This is so since the (new) penalty for offenders with low criminal record is lower than the (old) penalty for offenders with high criminal record and thus the aggregate penalization effort is reduced.[13,14]

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13 Of course there are additional (secondary) cost effects for those individuals which commit crimes as offenders with low and/or high criminal record under both penalty schemes. These effects are described in section 4.

14 The structure of the optimal escalating penalty scheme with maximum penalty for offenders with high criminal history and zero sanctions for first time offenders resembles optimal penalty schemes in the marginal deterrence literature. (See, e. g., Shavell 1992; Wilde 1992). Shavell 1992, for instance, shows that if enforcement probability cannot be differentiated between harmful and less harmful criminal acts, the sanction for harmful crimes should be maximal, whereas the expected sanction for low harm crimes should be relatively low (i. e., lower than harm). This is so in order to induce criminals to commit the low harm crime instead of the more harmful ones, even though the low sanction makes the low harm crime attractive for some additional offenders. There are two parallels to our model framework. i) In both settings the differentiation of penalties leads to a shift of criminal behavior: from long criminal history offenders to offenders with short or no criminal history in the escalating penalties model and from harmful to less harmful acts in the marginal deterrence literature. ii) Social cost decrease due to this shift: in our model, the increase of penalization cost for low criminal record offenders is overcompensated by the reduction of penalization cost for high criminal record offenders, whereas in Shavell (1992) additional harm from less severe crimes is overcompensated by the reduction of more harmful crimes.

In Section 6 we demonstrate that in the finite game framework, as well as in the infinite one, cost might be reduced by using escalating instead of uniform penalties. However, due to the final round effect, different from the infinite scenario, they also might be reduced by using decreasing penalties instead of uniform ones. Here it depends on the parameter values whether increasing of decreasing penalties are globally optimal in this setting.

We proceed as follows: In Section 2 we present the basic model. In Section 3 we derive the equilibrium strategies of the potential offenders, that is, we first solve the infinite game of stage 3. In Sections 4 and 5 we derive the equilibrium strategies of the legal authority. In Section 6 we investigate the consequences of a change from our infinite game framework to a corresponding two-period framework. We conclude in Section 7.

2 The Basic Model – Assumptions

Suppose that some individuals, the "potential offenders", receive non-negative, constant gross benefits, b, from crime.[15] We assume that in each period a set of potential offenders with benefits distributed according to a continuous distribution function $F: B \to [0,1]$, $B \subset \mathbb{R}_0^+$, is born.[16] Additionally we assume that the probability density is bounded away from zero and that the hazard rate[17] defined by $h(b) = f(b)/(1 - F(b))$ is increasing.[18] The probability that an individual which

15 The assumption that the benefit b remains constant over time and thus neither depends on the age of the offender nor on the criminal history is conventional in the literature. See, e. g., Polinsky and Shavell (1998), Chu, Hu, and Huang (2000) and Miceli and Bucci (2005). We stick to this assumption in order to make our model better comparable to the aforementioned ones. Note that the assumption of time dependent benefits might change the structure of the optimal penalty schemes. However, this is mainly due to the effect that also in a static model (without repeat offenders) the optimal penalty level may depend on the benefit level. The lower the benefit from crime the lower is the penalty which is necessary to deter the potential offender from criminal behaviour. In our dynamic context this implies that if the benefit from crime decreases with the offender's age, lower penalties might be sufficient to completely deter older offenders or criminal offenders with long criminal history, respectively.

16 Note that $F(\mathfrak{b})$ denotes the probability that the benefit of a potential offender does not exceed $\mathfrak{b}$, since the number of potential offenders born in each period is normalized to one.

17 The hazard rate is mostly known from survival statistics. There the hazard rate is defined as the event rate of the event "death" at time t conditional on "not deceased before time t". In our context the hazard rate expresses the conditional probability of benefit level b conditional on benefit level "equal or larger than b".

18 For instance, the uniform distribution function $F: [0, a] \to [0, 1]$, $F(b) = b/a$ with $a > 0$ fulfills the requirements.

is alive in period t still lives in period $t+1$ is given by $q \in (0, 1)$.[19] Hence in each period the total number of potential offenders is given by $1/(1-q)$.[20] Moreover, each individual is assumed to be able to commit at most one crime per period in which it is alive.[21]

We formalize the decision process as a three stage game.[22] In the first two stages the regulator chooses the maximum tolerable ("aspired") number of crimes c per period (stage 1) and a penalty scheme (stage 2) $\{s_n\}_{n=0,1,...}$ with $0 \le s_n \le s_{max}$ $\forall n \ge 0$, where the index n denotes the number of previous convictions and s_{max} the maximal possible sanction.[23] We illustrate our point using three types of penalty schemes: uniform penalty schemes defined by $s_n = \bar{s}$ $\forall n \ge 0$, decreasing penalty schemes, defined by $s_{n+1} \le s_n$ $\forall n \ge 0$ and increasing penalty schemes, defined by $s_{n+1} \ge s_n$ $\forall n \ge 0$.[24] The Propositions however are also valid for hybrid penalty schemes for which there exist criminal histories n, $m \ge 0$, such that $s_{m+1} < s_m$ and $s_{n+1} > s_n$ hold. In the third stage potential offenders decide whether they commit a crime.

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19 Note that q can also be interpreted as $q = s\delta$ with survival probability s and discount factor δ. The assumption $q < 1$ is the convenient formalization of lifetime with uncertain endpoint. The corresponding argument is that infinitely repeated games efficiently mimic the situation of decision makers who are uncertain about when the game will end. See, e. g., Binmore (2007, 325), Mailath and Samuelson (2006, 106/7). Mathematically, the assumption is necessary to receive finite and hence comparable values in the following analysis. E.g., if an agent lived forever, which is equivalent to the assumption $q = 1$, without discounting the net present value of committing beneficial crimes in any period would be infinite.

20 Note that the size of the population is constant over time. In particular we do not formalize a starting point of the game with a first generation or an initial change of the penalty scheme. Following e. g. Rubinstein (1979) we only consider the stationary equilibrium of the game.

21 For terminological simplicity we equate the time spread for potential criminal behaviour of an individual with his lifetime beginning with his birth and ending with his death. Of course the birth may be interpreted as reaching the minimum age for committing a crime and the death may be interpreted as the date where an individual loses the physical ability to commit a crime. Moreover, we ignore the possibility that being convicted in period t may reduce the leeway to commit a crime in the next period. See Shavell (2004, 531–535), Abrams (2012) and Barbarino and Mastrobuoni (2014) on the "incapacitation" effect of imprisonment. A somewhat sceptical view on this effect is in Cooter and Ulen (2012, 502–504).

22 Note that the first two stages of the game occur simultaneously. However, dividing the simultaneous decision into two substages simplifies the analysis of the game.

23 Assume that life imprisonment is the maximum punishment available to society. Then it is plausible that the expected value of this penalty is finite since the expected lifetime is also finite. The maximum penalty s_{max} society is willing to apply *for most specific crimes* will be lower than lifetime imprisonment. Cooter and Ulen (2012, 476–477) mention the following reasons: First, "powerful deterrence on less serious crimes often precludes using them on more serious crimes". Second, "harsh penalties may violate the moral and constitutional rights of criminals".

24 Note that the last two types contain uniform penalty schemes as border case.

Let $p \in (0, 1)$ denote the (exogenously given) probability of apprehending offenders which is assumed to be independent of the type of offender (defined by his previous number of crimes and his benefit).[25] We assume that $F(ps_{max}) < 1$ holds, i. e., even if the regulator chooses the maximum available penalty for each offender there are some potential offenders who may not be deterred from committing a crime.

In this paper, we follow the lines of neoclassical welfare economics. There, the regulator is stylized to maximize social welfare. In our model the effect of crime on social welfare is two-fold. First, crime inflicts harm to the victim and second, punishing criminals is costly to society.[26] Thereby, the strive to maximize social welfare translates into a strive to minimize the sum of harm from crimes and cost of punishment. This implies that the optimal crime control policy is defined by using "the minimum amount of punishment to achieve any given level of crime control" (Kleiman 2009, 3).[27]

Harm from crimes is given by $H(c)$ with $H'(c) > 0\ \forall c \in (0, c_{max})$, where $c_n(s)$ denotes the number of crimes *per period* committed by the group of potential offenders with n police records given the penalty scheme s, $c = \sum_{n=0}^{\infty} c_n(s)$ denotes the aggregate number of crimes committed per period and c_{max} denotes the aggregate number of crimes committed if there are no sanctions at all. Penalization costs are given by $P(p \sum_{n=0}^{\infty} c_n(s) s_n)$ with P denoting a strictly increasing function of the sum of imposed sanctions.[28] Thus, the objective function of the regulator may be written as

25 In section VII we briefly consider an extension of our basic model allowing for endogenous p. Other papers investigating endogenous p are Chu, Hu, and Huang (2000), Miceli and Bucci (2005) and Persson and Siven (2006).

26 We do not count the benefits of potential offenders from committing a crime as elements of the social welfare function.

27 It is worth noting that the traditional welfare economic approach is not the only framework within which the issue of an optimal penalty structure may be analysed. A completely different route would be taken by using a political economy approach. There, the regulator would not be stylized to strive for the common good but for the satisfaction of his own needs. In this point of view it is important that collected fines are often added to the regulator's budget. A budget-maximizing regulator would then strive to maximize the net revenue from these fines instead of minimizing punishment, as the socially benevolent regulator would do.

28 Note that if we would instead assume that penalization is costless, in our model the maximum penalty, irrespective of criminal history would be optimal. Our assumption that the penalty cost is increasing in the level of imposed sanctions, is in accordance with Miceli and Bucci (2005), Miceli (2013), Mungan (2014), Polinsky and Shavell (2007) and in the context of marginal deterrence by Wilde (1992) (see footnote 14). For imprisonment which is the obvious application of our model it is obvious that penalization cost increases in the penalty level (see, e. g., Becker 1968). However also in case of financial penalties (which might at first glance be

$$\min_{s} \ K = H\left(\sum\nolimits_{n=0}^{\infty} c_n(s)\right) + P\left(p \sum\nolimits_{n=0}^{\infty} c_n(s)\, s_n\right). \qquad [1]$$

In the following three sections we analyse the game backwards and begin with the third stage. That is, we first solve the infinite game (where the potential offender plays versus nature, which determines the offender's lifetime). Replacing the third stage by its equilibrium outcome reduces the infinite game to a finite one.

3 Third Stage: Equilibrium Number of Crimes – Individual Choice and Aggregation

3.1 The Choice of a Potential Offender

Consider the equilibrium behaviour of a potential offender with benefit b and n previous convictions in t_0. A potential offender either commits the crime immediately or he does not commit a crime in any period $t \geq t_0$. The reason is that it is never worthwhile to delay an offence to a subsequent period $t_1 > t_0$. If the potential offender would delay committing the offence he would receive the payoff 0 in each period t with $t_0 \leq t < t_1$ and in period t_1 (provided that he is still alive) he would face the same decision problem as in period t_0. Hence, the lifetime returns of the potential offender with n previous records may be expressed by the following recursive equation:

$$V(n) = \max\{0, b - ps_n + q(pV(n+1) + (1-p)V(n))\}. \qquad [2]$$

The potential offender either decides against committing the crime and receives $V(n) = 0$, or he commits the crime. In this case his lifetime returns are composed of three parts. First, there is the benefit from committing the crime, b, second, the offender has to suffer from the expected penalty ps_n and third, his criminal

judged as transfers which do not affect social welfare) social penalty cost arise. Miceli (2013) justifies this assumption as follows: "either there could be administrative costs of collecting fines, or because society simply has an aversion to imposing criminal sanctions of any sort beyond what is deemed "appropriate" (or proportional) to the offense in question, or is minimally necessary to deter the offense" (Miceli 2013, 590–591). Moreover, social losses might result from the stigma effect, the size of which is positively correlated with the penalty, see Dominguez Alvarez and Loureiro (2012) for an empirical assessment. Social losses might also be caused by the conviction of innocent individuals. These losses increase with the size of the penalty. A further argument is that the probability that the convicted parties dispute the court's decision increases with the level of fines. (It may be an interesting subject for empirical law and economics to test this hypothesis based on plausibility.)

history status changes from n to $n+1$ with probability p, which becomes relevant if the potential offender is still alive in the next period, which occurs with probability q.

The potential offender commits the crime if and only if $V(n)>0$ holds. For any sanction scheme $\{s_n\}_{n=0,1,\ldots}$ and any criminal history n lifetime returns $V(n)$ are positive, if the benefit b which the potential offender receives from committing a crime (and which is constant over time) is sufficiently high. In the following we denote the threshold benefit level for an offender with n previous sanctions by $\bar{b}_n$. That is an offender with criminal record n would commit the crime if and only if $b>\bar{b}_n$ holds.[29] However only if also $b>\bar{b}_m$ for all $m<n$ holds, the potential offender may receive n convictions (because otherwise he stops criminal activity earlier). Hence the criminal activity of potential offenders may be represented by a sequence of critical benefit levels $\{b_n\}_{n=0,1,\ldots}$ with $b_n=\max\{\bar{b}_m, 0\le m\le n\}$. An offender with benefit level b becomes a repeat offender with criminal history n (provided that he is alive long enough), if and only if $b>b_n$ holds.

In the following we determine the sequences of critical benefit levels for uniform, decreasing and escalating penalty schemes.

– Under a *uniform penalty scheme* we have $V(n+1)=V(n)$ with $V(n)>0 \Leftrightarrow b>p\bar{s}$. That is, a potential offender commits the crime, if and only if the gross benefit of committing the crime exceeds the expected penalty. Hence the critical benefits are given by $b_n=\bar{b}_n=p\bar{s}$.
– Under an *escalating penalty scheme* we have $0\le V(n+1)\le V(n)$. Hence a potential offender commits a crime as long as his benefit exceeds his expected penalty, i. e., the critical benefits are given by $b_n=\bar{b}_n=ps_n$.
– Under a *decreasing penalty scheme* a potential offender either commits a crime in each or in no period (as in the case of uniform penalties). If there is a criminal record $m\ge n$ such that $V(n)>0$ and the strict inequality $s_{m+1}<s_m$ holds, we have $V(n+1)>V(n)$, from which $\bar{b}_n<ps_n$ follows. Thus, a potential offender is willing to receive an expected loss in the current period if this loss is outweighed by the positive effect of receiving the more favourable status as a repeat offender with higher criminal record for the subsequent periods. Moreover we have $\bar{b}_{n+1}\le\bar{b}_n$ from which follows that $b_n=\max\{\bar{b}_m, 0\le m\le n\}=\bar{b}_0$ holds. To determine $\bar{b}_0$, we consider a potential offender who is (without loss of generality) assumed to be born in period 0 and commits a crime in each period. If the potential offender has got exactly k convictions with penalties $s_0,\ldots,s_{k-1}$ in the t periods 0,…,t–1, he receives

29 We assume that in the indifference case the potential offender does not commit the crime.

penalty s_k in period t provided that he is convicted (again) (which occurs with probability p). The probability for k previous convictions is given by the binomial distribution $B(k|t) = \binom{t}{k}(1-p)^{t-k}p^k$ with the binomial coefficients $\binom{t}{k} = \frac{t!}{k!\,(t-k)!}$. Thus his expected penalty in period t (provided that he is still alive) is given by $p\sum_{k=0}^{t} B(k|t)\cdot s_k$. Hence a potential offender commits a crime in each period if and only if $\sum_{t=0}^{\infty} bq^t > \sum_{t=0}^{\infty} q^t p \sum_{k=0}^{t} B(k|t)\, s_k$ holds, which is equivalent to $b > ps_{\varnothing}$ with $s_{\varnothing} := (1-q)\sum_{t=0}^{\infty} q^t \sum_{k=0}^{t} B(k|t)\, s_k$. Hence the critical benefits are given by $b_n = \bar{b}_0 = ps_{\varnothing}$. Note that the general structure of the optimal strategies under a uniform and decreasing penalty scheme is identical: The potential offender commits a crime in each period if and only if the benefit exceeds the average expected penalty. Otherwise he does not commit a crime.

With respect to the sequence of critical benefits $\{b_n\}_{n=0,1,\ldots}$[30] the following general statements apply:

Proposition 1: Sequence of critical benefit levels

(i) For each type of penalty scheme including the hybrid ones the sequence of critical benefits is weakly increasing, $b_{n+1} \geq b_n \;\forall n \geq 0$.

(ii) Different punishment schemes may give the same sequence of critical benefits $\{b_n\}_{n=0,1,\ldots}$

Proof: See Appendix.

Proposition 1.ii implies that a potential offender behaves identically under two punishment schemes which lead to the same sequence of critical benefits. He stopps commiting crimes when he receives criminal history n for which $b \leq b_n$ holds.

3.2 Aggregation

Building on the previous analysis of the equilibrium behaviour of a single potential offender the equilibrium number of crimes per period may be determined.

30 In the following we omit the lower addition "n = 0,1,…" for notational convenience.

Proposition 2: Equilibrium criminality levels
For a penalty scheme $\{s_n\}$ *with corresponding sequence of critical benefits* $\{b_n\}$ *the aggregate and criminal record specific equilibrium criminality levels are given by* $c = \sum_{n=0}^{\infty}\left((1-F(b_n))\sum_{t=n}^{\infty} q^t B(n|t)\right)$ *and* $c_n = (1-F(b_n))\sum_{t=n}^{\infty} q^t B(n|t)$.

Proof: See Appendix.

The criminal activity of a potential offender for any criminal history only depends on the relation of his individual benefit b to the critical benefit levels. Thereby, the aggregate level of criminality also depends on the sequence of critical benefit levels only. Hence, two punishment schemes with the same sequence of critical benefit levels do also lead to the same criminality level.

4 Second Stage: Lawmaker's Choice – The Optimal Time Profile of Sanctions

Having determined the equilibrium criminality levels, we now consider the optimization task (1) of the regulator. As already mentioned in Section 2 this task is divided into two steps. In the second stage the regulator has already chosen the aspired number of crimes $\bar{c}$ per period. His remaining task is to choose a sanction scheme which achieves $\bar{c}$ with minimal penalization cost. Since the penalization costs are a strictly increasing function of total sanctions ($P(p\sum_{n=0}^{\infty} c_n(s)s_n)$ with $P' > 0$), without loss of generality we may assume that P equals the identity function. Hence in the remaining parts of Section 4 we consider the equivalent optimization task

$$\min_{s} P(c(s), s) := p\sum\nolimits_{n=0}^{\infty} c_n(s)s_n \quad \text{s.t.} \quad c = \sum\nolimits_{n=0}^{\infty} c_n(s) \le \bar{c}. \qquad [3]$$

To assure that a reduction of crime (or equivalently an increase of a sanction) raises punishment cost, we need the following assumption:

Assumption 1: $s_{max} \le \tilde{s}$ with $\tilde{s}$ implicitly defined by $p\tilde{s}\,h(p\tilde{s}) = 1$.[31, 32]

31 E.g., in case of a uniform penalty scheme from $\bar{c} = (1-F(p\bar{s}))/(1-q)$ follows that the cost of punishment is given by $P(\bar{s}) = p\bar{s}(1-F(p\bar{s}))/(1-q)$. Hence we have $dP/d\bar{s} = (1-F(p\bar{s}) - p\bar{s}\,f(p\bar{s}))p/(1-q) > 0 \Leftrightarrow p\bar{s} < (1-F(p\bar{s}))/f(p\bar{s}) = 1/h(p\bar{s}) \Leftrightarrow p\bar{s}h(p\bar{s}) < 1$. Since the hazard rate is assumed to be increasing, there is a unique penalty level $\tilde{s}$ which satisfies $p\tilde{s}\,h(p\tilde{s}) = 1$.
32 For the uniform distribution function $F: [0, a] \to [0, 1]$, $F(b) = b/a$ we have

$$p\tilde{s}h(p\tilde{s}) = 1 \Leftrightarrow \frac{p\tilde{s}(1/a)}{1-F(p\tilde{s})} = 1 \Leftrightarrow \frac{p\tilde{s}/a}{1-p\tilde{s}/a} = 1 \Leftrightarrow \tilde{s} = \frac{a}{2p}.$$

If the regulator chooses the minimal (maximal) sanction $\bar{s}=0$ ($\bar{s}=s_{max}$) the corresponding criminality level is given by $c_{max}=1/(1-q)$ ($c_{min}=(1-F(ps_{max}))/(1-q)$).

In the remaining part of Section 4 we assume that for the criminality target $\bar{c}\in(c_{min},c_{max})$ holds in order to assure that there exist different penalty schemes that enforce $\bar{c}$ and thus the solution of eq. [3] is non-trivial.[33]

In Section 3 we have seen that a given criminality target $\bar{c}$ can be induced by different penalty schemes, which either may lead to the same or to different sequences of critical benefits $\{b_n\}$. Proposition 3 shows that the penalization cost under a given penalty scheme is determined by the sequence of critical benefits.

Proposition 3:
Each penalty scheme $\{s_n\}$ that induces the same sequence of critical benefits $\{b_n\}$ (and hence the same criminality level $\bar{c}\in(c_{min},c_{max})$) leads to the same cost of punishment.

Proof: See Appendix.

The intuition behind Proposition 3 can be explained by the following example. Consider two penalty schemes which lead to the same sequence of critical benefits $\{b_n\}=(1,1,1,5,5,5,5,7,7,8,12,\ldots)$. Under both penalty schemes a potential offender with benefit $b=1$ is indifferent between "never committing the crime" and incurring the cost of "committing the crime until he is caught 3 times". From this indifference condition, it follows that under both penalty scheme the penalization cost of punishing *any criminal* for the first 3 times are identical. Analogously it can be argued that the aggregate penalization cost for punishing offenders for the fourth to seventh time are identical and so on.

Proposition 3 can be interpreted as follows: If two penalty schemes induce the same crime decisions their implementation costs the same to the society. This directly implies the following statement:

Corollary 1:
Any decreasing penalty scheme s *with* $c(s)=\bar{c}$ *leads to the same cost of punishment as the corresponding uniform penalty scheme.*[34]

Proposition 4 shows that any penalty scheme can be replaced by an equivalent escalating penalty scheme (in terms of critical benefits and social cost).

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33 In case of $\bar{c}=c_{min}(\bar{c}=c_{max})$ the solution would be given by $s_n=s_{max}\ \forall n$ ($s_n=0\ \forall n$). In case of $\bar{c}<c_{min}$ or $\bar{c}>c_{max}$ there would be no solution.

34 Using Assumption 1 this implies that also under a decreasing penalty scheme lowering the criminality level $\bar{c}$ raises enforcement costs.

Proposition 4:
Each sequence of critical benefits $\{b_n\}$ *(that corresponds with a criminality level* $\bar{c} \in (c_{min}, c_{max})$*) can be implemented by the (weakly) increasing penalty scheme* $\{s_n\}$ *with* $s_n = b_n/p$.

The assertion directly follows from the fact that under an escalating penalty scheme the critical benefits are given by $b_n = ps_n$ (see Section 3).

Using Proposition 4, the task of finding the globally optimal penalty scheme can be reduced to the task of determining the optimal scheme within the class of escalating penalty schemes. From Proposition 2 directly follows that the corresponding aggregate and type specific criminality levels are given by $c = \sum_{n=0}^{\infty} \left((1-F(ps_n)) \sum_{t=n}^{\infty} q^t B(n|t)\right)$ and $c_n = (1-F(ps_n)) \sum_{t=n}^{\infty} q^t B(n|t)$. That is, in case of escalating penalty schemes, c_n is a function of s_n only.

It turns out (see Proposition 5) that the optimal penalty scheme has the following structure: Offenders with a high criminal record receive the harshest punishment and offenders with a low criminal record are not punished at all. The number of offences which is "necessary" to receive a positive punishment depends on the criminality target. The maximal sanction is imposed the earlier, the lower the aspired criminality level is. In case of a strict criminality target the punishment for the first detected offence is already positive. (Only) the first positive sanction may differ from s_{max} to assure that the criminality target is exactly fulfilled. In particular it is not optimal to choose a strictly increasing penalty scheme with multiple (i. e. more than three) sanction levels.

Proposition 5: Escalating penalty schemes

(a) *Within the class of escalating penalty schemes (including uniform penalty schemes as border case) the cost minimizing one is of the following type:*

$$\exists n_0 \geq 0 : s_n = 0 \ \forall n < n_0, \ 0 < s_{n_0} \leq s_{max}, \ s_n = s_{max} \forall n > n_0.$$

(b) *The penalty scheme* s^I *defined in a) is the unique cost minimizing penalty scheme.*

(c) *For any given criminality level* $c \in (c_{min}, c_{max})$ *the penalization costs are strictly higher when a uniform penalty* s^U *is used instead of the optimal escalating penalty scheme* s^I.

Proof: See Appendix.

The proceeding of the proof of Proposition 5.a is as follows: We assume that in the initial situation we have an escalating penalty scheme s with $c(s) = \bar{c}$ which is not of the type described in Proposition 5.a. We define the minimum number of previous convictions a potential offender must have under s to receive a positive punishment by n_0, i. e. $n_0 := \min\{n|s_n > 0\}$. Since s is not of the type given in Proposition 5.a, there must be a number of previous records $\tilde{n} > n_0$ for which the corresponding sanction $s_{\tilde{n}}$ is not maximal. We show that in this case it is possible to increase one or more sanction(s) for offenders with high criminal record and instead decrease the number of offenders with rather low criminal record in such a way that the following three conditions are fulfilled: i) The new penalty scheme is of the escalating penalties type, too, ii) the criminality level remains constant and iii) the shift of sanctions decreases penalization cost.

To clarify the driving forces that generate smallest penalization cost for the escalating penalty scheme given by Proposition 5 we conclude this section with an example, which compares the penalization cost of two escalating penalty schemes.

Example 1:
Let the distribution function of benefits be given by $F:[0,\ 10] \to [0,1]$, $F(b) = b/10$, the survival probability by $q = 9/10$, the probability of apprehending offenders by $p = 1/2$ and the maximum available penalty by $s_{max} = 10$. Assume that the initial escalating penalty scheme is given by $s = (1,\ 2,\ 10,\ 10, \ldots)$. This penalty scheme is of the escalating penalty type, but not optimal, since it is possible to decrease s_0 and instead increase s_1 in such a way that the criminality level remains constant. More formally we consider a second escalating penalty scheme $\hat{s} = (0, \hat{s}_1, 10, 10, \ldots)$, which fulfills $c(\hat{s}) = c(s)$.[35] For s the sequence of critical benefits is given by $\{b_n\} = \{1/2,\ 1,\ 5,\ 5, \ldots\}$ and for $\hat{s}$ it is given by $\{\hat{b}_n\} = \{0,\ \hat{s}_1/2,\ 5,\ 5, \ldots\}$. Hence we have two potential offender types that change their criminality behaviour if the penalty scheme s is replaced by $\hat{s}$. Under s potential offenders with benefit $b < 1/2$ do not commit crimes at all, whereas under $\hat{s}$ they commit crimes until their first conviction. Potential offenders with $b \in [1,\ \hat{s}_1/2)$ do only under s proceed with committing crimes after their first conviction (until they receive their second one). Hence we have

35 Note that our example mainly proceeds along the proof of Proposition 4.a. However, it does not consider a marginal change of the penalty scheme but a discrete change from a suboptimal to the optimal one.

$$
\begin{aligned}
c(\hat{s})-c(s) &= c_0(\hat{s})-c_0(s)+c_1(\hat{s})-c_1(s)\\
&= ((1-0)-(1-F(1/2)))\cdot\sum\nolimits_{t=0}^{\infty} q^t B(0|t)\\
&\quad+\left((1-F(\hat{s}_1/2))-(1-F(1))\right)\cdot\sum\nolimits_{t=1}^{\infty} q^t B(1|t)\\
&= (F(1/2))\cdot\sum\nolimits_{t=0}^{\infty} q^t B(0|t)-\left(F(\hat{s}_1/2)-F(1)\right)\cdot\sum\nolimits_{t=1}^{\infty} q^t B(1|t)\\
&= \frac{1}{20}\cdot\sum\nolimits_{t=0}^{\infty} q^t B(0|t)-\left(\frac{\hat{s}_1}{20}-\frac{1}{10}\right)\cdot\sum\nolimits_{t=1}^{\infty} q^t B(1|t)\\
&= \frac{1}{20}\cdot 1.81-\left(\frac{\hat{s}_1}{20}-\frac{1}{10}\right)\cdot 1.487603306.
\end{aligned}
$$

Solving $c(\hat{s})-c(s)=0$ for $\hat{s}_1$ reveals $\hat{s}_1=3,\bar{2}$, i. e. $\hat{s}=(0,3.\bar{2},10,10,\ldots)$.

Since the criminality level and hence also harm from crime is identical under both penalty schemes, we only have to show that penalization cost under $\hat{s}$ is lower than under s.

Therefore we consider the change of penalization cost, which is given by

$$\Delta P = p\left(c_0(\hat{s})\cdot\hat{s}_0-c_0(s)\cdot s_0\right)+p(c_1(\hat{s})\cdot\hat{s}_1-c_1(s)\cdot s_1).$$

with

$$c_0(\hat{s})=(1-0)\sum_{t=0}^{\infty} q^t B(0|t)=1.81818,\ c_0(s)=(1-0.05)\sum_{t=0}^{\infty} q^t B(0|t)=1.72727,$$

$$c_1(\hat{s})=(1-3.\bar{2}/20)\sum_{t=1}^{\infty} q^t B(1|t)=1.24793,\ c_1(s)=(1-0.1)\sum_{t=1}^{\infty} q^t B(1|t)=1.33884,$$

from which follows

$$\Delta P=0.5(1.81818\cdot 0-1.72727\cdot 1)+0.5(1.24793\cdot 3.\bar{2}-1.33884\cdot 2)=-0.19191<0.$$

Hence, the increase of penalization cost for offenders with one previous record is overcompensated by the decrease of penalization cost for offenders without criminal record. The main reason behind this assertion is that the cost of punishment for the additional criminal activity of offenders without criminal record and with $b\in[0,\ 0.5)$ is lower (in our example it takes the value zero) than the saved cost of punishment for the reduced criminal activity of offenders with one previous sanction and $b\in[1,\ 1.6\bar{1})$. This is so since the (new) penalty for offenders with low criminal record ($\hat{s}_0=0$) is lower than the (old) penalty for offenders with high criminal record ($s_1=2$) and thus the aggregate penalization effort is reduced. In addition to that we have the following effects. For offenders without criminal record and benefit $b>0.5$ penalization cost is also reduced (in our example they even vanish) due to the reduced sanction for first time offenders. For offenders with benefit $b > 1.6\bar{1}$ there is an additional increasing cost effect for offenders with one previous sanction. However for our equal distribution function this cost effect is just offset by the decreasing cost effect for offenders without previous sanction and with benefit $b > 1.6\bar{1}$.[36] Adding all effects reveals that aggregate penalization cost is decreasing if the system switches from the suboptimal escalating penalty scheme to the optimal one.

5 First Stage: Lawmaker's Choice – The Optimal Level of Crime

In the previous section we have compared penalization cost under different penalty schemes for a given criminality target $c \in (c_{min}, c_{max})$. Knowing the optimal penalty scheme for each criminality level, the regulator may determine the optimal level of crime after inserting the minimized value of punishment cost for each level of crime in the objective function [1]. We denote the optimal level of crime by c^{I^*} in order to indicate that the optimal increasing penalty scheme is used to induce the criminality target. Only in the cases $c^{I^*} \in \{c_{min}, c_{max}\}$ the penalty scheme is of the uniform type as a border case.

Economic intuition might suggest that $c^{I^*} \leq c^{U^*}$, with c^{U^*} denoting the level of crime that would be optimal under the constraint of uniform penalties. After all the costs of reducing crime go down if society switches from the latter to the former penalisation scheme. However, Proposition 6 shows that this plausible presumption does not always hold.

Proposition 6: (Penalty type specific) socially optimal level of crimes
The socially optimal level of crime, c^{I^*}*, may be strictly higher than the criminality target,* c^{U^*}*, which would be optimal under the restriction of uniform penalties.*

Proof: See Example 2 below.

36 $\Delta P(b > p\hat{s}_1) = (1 - F(p\hat{s}_1)) \cdot \left(\left(\sum_{t=1}^{\infty} q^t B(1|t) \right) (\hat{s}_1 - s_1) + \left(\sum_{t=0}^{\infty} q^t B(0|t) \right) (\hat{s}_0 - s_0) \right)$

$$= \frac{a(1 - F(p\hat{s}_1))}{p} \cdot \left(\left(\sum_{t=1}^{\infty} q^t B(1|t) \right) (F(p\hat{s}_1) - F(ps_1)) + \left(\sum_{t=0}^{\infty} q^t B(0|t) \right) (F(p\hat{s}_0) - F(ps_0)) \right)$$

$$= \frac{a(1 - F(p\hat{s}_1))}{p} \cdot 0 = 0.$$

The statement of Proposition 6 is rather counterintuitive. To understand this counterintuition consider a marginal increase of the advised criminality level starting from the criminality level c_{min}, which is induced if the maximal sanction, s_{max}, is applied irrespective of the number of previous convictions. We compare the welfare effects in the following two scenarios: in scenario 1 the criminality level is induced by the corresponding uniform penalty and in scenario 2 the criminality level is induced by the optimal escalating penalty scheme. In both scenarios the welfare effect is composed of two components: the increase of social harm from crimes and the decrease of enforcement cost. Whereas the increase of social harm coincides in both scenarios the decrease of enforcement cost is larger in the escalating penalty scenario than in the uniform scenario. Therefore it is possible to construct an example, as is done below, where in scenario 1 marginal welfare is strictly decreasing in the crime level, and thus c_{min} is the equilibrium criminality level, whereas in scenario 2 marginal welfare is positive in a non-empty interval $(c_{min}, \tilde{c})$ and thus the optimal level of criminality exceeds c_{min}.

Example 2:
We again consider example 1. In the relevant range $c \in [c_{min}, c_{max}] = [5, 10]$ the marginal harm function is assumed to be given by $H'(c) = g(c) + \varepsilon c$ with $g(c) = 2c - 10$ and $\varepsilon > 0$. Moreover, as in Section 4, we assume that P equals the identity function.

First note that $H, H' > 0 \forall c \in (c_{min}, c_{max})$, i. e., the marginal utility of crime prevention is positive and decreasing. Moreover note that $g(c) = -dP^U/dc$ holds.[37] Thus, for the uniform penalty scheme we obtain $\frac{dK^U}{dc} = H' + \frac{dP^U}{dc} = \varepsilon > 0$ from which follows the optimal criminality level $c^{U\star} = c_{min} = 5$, with corresponding penalty scheme $\tilde{s} = (s_{max}, s_{max}, s_{max}, \ldots)$. Hence, to prove Proposition 6 we have to show that there exists an $\varepsilon > 0$ such that within the class of escalating penalty schemes the penalty scheme of border type $\tilde{s} = (s_{max}, s_{max}, s_{max}, \ldots)$ is not optimal. Therefore it suffices to demonstrate that there exists an $\varepsilon > 0$ such that the penalty scheme $s = (0, s_{max}, s_{max}, \ldots)$ (with corresponding higher criminality level) leads to lower total cost than $\tilde{s}$. Since under penalty scheme s the corresponding criminality levels c_n, $n \geq 1$ coincide with those under $\tilde{s}$ and only differ for potential offenders without criminal record, the aggregate criminality level under s can be determined by

37 From $\bar{c} = (1 - F(p\bar{s}))/(1 - q) \Leftrightarrow \bar{s} = F^{-1}(1 - \bar{c}(1 - q))/p$ follows $P^U = p\bar{c}\bar{s} = \bar{c}F^{-1}(1 - \bar{c}(1 - q)) = \bar{c} \cdot 10(1 - \bar{c}/10) = 10\bar{c} - \bar{c}^2$ and hence $dP^U/d\bar{c} = 10 - 2\bar{c}$.

$$c(s) = c_{min} - c_0(\tilde{s}) + c_0(s) = c_{min} + (F(ps_{max}) - F(0)) \cdot \sum_{t=0}^{\infty} q^t B(0|t)$$
$$= 5 + 0.5 \cdot \sum_{t=0}^{\infty} q^t B(0|t) = 5.90909.$$

Thus a change from $\tilde{s}$ to s increases harm from crimes by

$$\Delta H = \int_{c_{min}}^{c(s)} H'(c)dc = \int_{5}^{5.90909} (2c - 10 + \varepsilon c)dc = 0.82646 + 4.95868\varepsilon$$

On the other hand penalization costs under s are lower than under $\tilde{s}$.

Since the penalty and the criminality level coincide under both penalty schemes for potential offenders with at least one criminal record the change of penalization costs can be determined by

$$\Delta P = ps_0 c_0(s) - p \cdot 10 \cdot c_0(\tilde{s}) = 0 - 5 \cdot 0.5 \sum_{t=0}^{\infty} 0.9^t B(0|t) = -4.54545$$

Hence the change of total cost is given by $\Delta K = \Delta H + \Delta P = 4.95868\varepsilon - 3.71901$ which implies that for $\varepsilon < 0.75$ we have $c^{I*} > c^{U*}$.

To clarify the driving forces in our example in a first step we assume $\varepsilon = 0$. In this case we obtain $dK^U/dc = 0$. Hence in case of uniform penalization each enforceable level of criminality and thus also c_{min} is optimal. I.e., if c is increased, the increase of harm is just outweighed by the corresponding decrease of penalization cost $H'(c) = g'(c) = -dP^U/dc$. However under the escalating penalty regime s, penalization cost is lower, from which follows

$$K^{I*}(c(s)) = H(c(s) + P^I(c(s)) < H(c(s) + P^U(c(s)) = K^U(c(s)) = K^U(c_{min}) = K^I(c_{min})$$

i. e., under escalating penalties c_{min} can't be the optimal criminality level.

In the second step, we depart from the assumption $\varepsilon = 0$ and discuss how the additional term $\varepsilon c > 0$ in the marginal harm function affects equilibrium criminality levels. Under uniform penalties $\varepsilon > 0$ assures that the social cost function is strictly increasing in c, and c_{min} is the unique equilibrium. On the other hand if ε is small enough, the advantage of the escalating penalty regime with respect to marginal penalization cost is not overcompensated.

6 Infinite Versus Finite Horizon – The Consequences

In the following we show that if we change the infinite game framework presented above to a two-period and thus finite one, the optimal penalty scheme can be of the escalating as well as of the decreasing type. The result that a decreasing penalty scheme is optimal at least for certain parameter values has been received in many of the finite models presented in the literature. Other papers do not consider decreasing penalty schemes at all and only compare welfare under uniform and escalating penalty schemes.[38]

We restrict our attention to the cost minimizing implementation of a given criminality target. Hence we (only) proceed along the structure of Sections 2–4 of our main model. Obviously, what will be said for any given target also holds for the socially optimal one.

6.1 The Finite Model – Assumptions

We now assume that each individual lives exactly two periods $t \in \{0, 1\}$ and is able to commit at most one crime in each period. Since generations are overlapping, the population size of each period is normalized to two. A sanction scheme is given by $s = (s_0, s_1)$ with $0 \le s_i \le s_{max}$, where $i \in \{0, 1\}$ denotes the number of previous convictions. For a given criminality target $\bar{c}$ the optimization task of the regulator may be written as

$$\min_{s} \; P(c(s), s) := p(c_0(s)s_0 + c_1(s)s_1) \text{ s.t. } c = c_0(s) + c_1(s) \le \bar{c}. \tag{4}$$

6.2 Third Stage: Equilibrium Number of Crimes – Individual Choice and Aggregation

6.2.1 The Choice of a Potential Offender

Since in the finite game model there are only two criminal record levels, the sanction scheme is either of the decreasing, the uniform or the escalating type. Under a uniform or escalating penalty scheme the decision rule of a potential defender in the finite game framework is the same as in the infinite scenario: a potential offender commits the crime as long as his benefit exceeds the expected penalty. In case of decreasing penalties (i. e., $s_1 < s_0$) the choice of a potential offender in period 1 fundamentally differs from the general structure of the infinite game framework. In period 1 the decision depends not only on the benefit but also on whether the offender has been convicted in period 0. A potential offender with

38 See footnote 9 for examples of both types of papers.

(without) criminal record commits the crime if $b > ps_1$ $(b > ps_0)$ holds. In period 0 the decision is essentially the same as in the infinite game: a potential offender commits the crime if and only if $b > p(s_0 - (b - ps_1))$ holds, which is equivalent to $b > ps_\varnothing$ with $s_\varnothing := (s_0 + ps_1)/(1+p)$.

Summarizing, the optimal strategy under a decreasing penalty scheme is given by:

- $b \le ps_\varnothing$: No offences at all.
- $ps_\varnothing < b \le ps_0$: Offence in period 0. Offence in period 1, if and only if the offender has been convicted in period 0.
- $b > ps_0$: Offence in each period.

6.2.2 Aggregation

In Proposition 7, the following notation is used: $c^{U,\,(D\,,I)}$ denotes the aggregate equilibrium criminality level for a given uniform (decreasing, increasing) penalty scheme and $c_n^{U,\,(D\,,I)}$ denotes the criminality level of potential offenders with n previous records, $n \in \{0, 1\}$.

Proposition 7: Equilibrium criminality levels

(a) *Under a uniform penalty scheme the aggregate and criminal record specific equilibrium criminality levels are given by* $c^U = 2(1 - F(p\bar{s}))$ *and* $c_0^U = c_1^U = 1 - F(p\bar{s})$.

(b) *Under a decreasing penalty scheme the aggregate and criminal record specific equilibrium criminality levels are given by* $c^D = (1+p)(1 - F(ps_\varnothing)) + (1-p)(1 - F(ps_0))$, $c_0^D = (1 - F(ps_\varnothing)) + (1-p)(1 - F(ps_0))$ *and* $c_1^D = p(1 - F(ps_\varnothing))$ *with* $s_\varnothing = (s_0 + ps_1)/(1+p)$.

(c) *Under an increasing penalty scheme the aggregate and criminal record specific equilibrium criminality levels are given by* $c^I = (2-p)(1 - F(ps_0)) + p(1 - F(ps_1))$, $c_0^I = (2-p)(1 - F(ps_0))$ *and* $c_1^I = p(1 - F(ps_1))$.

Proofs: See Appendix.

6.3 Second Stage: Lawmaker's Choice – The Optimal Time Profile of Sanctions

Having determined the equilibrium criminality levels, we now consider the optimization task eq. [4] of the regulator.

We proceed as follows. At first, the cost of punishment is derived under the assumption of uniform penalties. This serves as a benchmark against which the cost of punishment under decreasing and increasing penalties are measured. It is shown that the cost of punishment may be reduced by choosing an appropriate escalating penalty scheme instead of a uniform one but also by choosing a decreasing penalty scheme instead of the uniform one.

We consider an example to demonstrate that it depends on the parameter values whether a decreasing or increasing penalty scheme is optimal for a given criminality target.

6.3.1 Uniform Penalties

Lemma 1: Uniform penalty scheme

(a) *The uniform penalty that exactly enforces the criminality level* $\bar{c}$ *equals* $\bar{s} = F^{-1}(1 - \bar{c}/2)/p$.

(b) *The corresponding cost of punishment is* $P^U = p\bar{s}2(1 - F(p\bar{s})) = \bar{c}\,F^{-1}(1 - \bar{c}/2)$.

Proof: See Appendix.

As for the infinite game the derivative $dP^U/d\bar{s} = 2p(1 - F(p\bar{s}) - \bar{s}pf(p\bar{s}))$ becomes negative if and only if $p\bar{s}h(p\bar{s}) > 1$ holds. Hence, to assure that a reduction of crime raises punishment cost, we again make Assumption 1. In the remaining part of Section 6 we assume that $\bar{c} \in (c_{min}, c_{max})$ with $c_{min} = 2(1 - F(ps_{max}))$ and $c_{max} = 2$ holds.

6.3.2 Escalating and Decreasing Penalties

In the following we show that (as in the infinite game framework) starting from $s_0 = s_1 = \bar{s}$ penalization cost may be reduced by increasing s_1 (and correspondingly decreasing s_0). That means penalization cost may be reduced by choosing an escalating penalty scheme.

However, we also show that (in contrast to the infinite game framework) starting from $s_0 = s_1 = \bar{s}$ penalization cost may be reduced by increasing s_0 (and correspondingly decreasing s_1). This means that penalization cost may also be reduced by choosing a decreasing penalty scheme. Proposition 8 discusses the structure of the overall optimal penalty scheme.

Proposition 8

(a) *The uniform penalty scheme leads to the highest penalization cost.*

(b) *From the set of escalating penalty schemes the cost minimizing one is of the border type with* $s_0 = 0$ *or* $s_1 = s_{max}$.

(c) *From the set of decreasing penalty schemes the cost minimizing one is of the border type with* $s_1 = 0$ *or* $s_0 = s_{max}$.

(d) *The penalization cost under the optimal decreasing penalty scheme may be higher than, lower than or equal to the penalization cost under the optimal escalating penalty scheme.*

Proof: See Appendix.

The fundamental divergence between the finite and infinite game framework is given by Proposition 8.c. Starting from a uniform penalty scheme penalization cost can not only be reduced by shifting the penalty from first time to repeat offenders but also by shifting the penalty from repeat to first time offenders in such a way that the crime level remains constant.

The driving forces behind this result are illustrated by the following example, which demonstrates that in the two period framework the (overall) optimal penalty scheme may be of the decreasing as well as of the escalating penalty type.

Example 3:

Consider the uniform distribution function $F: [0, a] \to [0, 1]$, $F(b) = b/a$ and assume $s_{max} = \frac{a}{2p}$.

According to Proposition 7 the equilibrium criminality levels are given by

– $c^U = 2(1 - p\bar{s}/a)$ and $c_0^U = c_1^U = 1 - p\bar{s}/a$ under the uniform penalty scheme $(s_0, s_1) = (\bar{s}, \bar{s})$.

– $c^D = (1+p)(1 - ps_\varnothing/a) + (1-p)(1 - ps_0/a)$, $c_0^D = (1 - ps_\varnothing/a) + (1-p)(1 - ps_0/a)$ and $c_1^D = p(1 - ps_\varnothing/a)$ with $s_\varnothing = (s_0 + ps_1)/(1+p)$ under a decreasing penalty scheme and

– $c^I = (2-p)(1 - ps_0/a) + p(1 - ps_1/a)$, $c_0^I = (2-p)(1 - ps_0/a)$ and $c_1^I = p(1 - ps_1/a)$ under an increasing penalty scheme.

According to Lemma 1, the uniform penalty that exactly enforces the criminality level $\bar{c}$ is given by $\bar{s} = a(1 - \bar{c}/2)/p$. The corresponding cost of punishment is $P^U = p\bar{s}2(1 - p\bar{s}/a)$.

For $a = 10$ and alternative values of p (column 2) and $\bar{c}$ (column 3) Table 1 displays the maximal sanction (column 4), the cost minimizing decreasing (column 5) and escalating (column 6) penalty scheme, as well as the uniform

Table 1: (Type) specific optimal sanction schemes.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	p	$\bar{c}$	s_{max}	s^{D*}	s^{I*}	$\bar{s}$	P^{I*}	P^{D*}	P^{U}	c^{I*}	c^{D*}
(I)	0.5	1.25	10	$(10, 0)$	$(6.\bar{6}, 10)$	7.5	4.583	4.583	4.688	(1,0.25)	(0.92, 0.33)
(II)	0.5	1.5	10	$(6.\bar{6}, 0)$	$(3.\bar{3}, 10)$	5	3.333	3.704	3.75	(1.25,0.25)	(1.11,0.39)
(III)	0.5	1.125	20	$(20, 0)$	(17.143, 20)	17.5	4.911	4.875	4.922	(1,0.125)	(0.98,0.15)

penalty scheme (column 7). Columns 8 to 10 display the corresponding penalization costs and columns 11 and 12 show the record specific criminality levels under the optimal decreasing and escalating penalty scheme.

The example demonstrates that for each parameter constellation the penalization costs take the highest value under the uniform penalty scheme. Moreover the number of first time offenders is lower than under the other types of penalty schemes.[39] The reason is as follows: Under an escalating as well as under a decreasing penalty scheme only a subgroup of the offenders that commit a crime in $t = 0$ do also commit a crime in $t = 1$.[40] Whereas under an increasing penalty scheme the critical benefit level of period 1 increases in case of conviction in period 0, under a decreasing penalty scheme the critical benefit level of period 1 increases in case of non-conviction in period 0. Whether an increasing or decreasing penalty scheme leads to lower penalization cost depends on the relative strength of the following two countervailing effects. i) From $s_0^{I*} < \bar{s} < s_0^{D*}$ follows that under an escalating penalty scheme penalization cost for a crime committed by a member of the larger group of offenders without criminal history may be reduced. This effect works in favor of the escalating penalty scheme being optimal. ii) Since not every offender of $t = 0$ reaches the "criminal record"-status for the next period we have $s_0^{D*} + s_1^{D*} < s_0^{I*} + s_1^{I*}$. That is, aggregate sanctions for offenders that commit a crime in both periods are lower under the decreasing penalty scheme. This effect works in favor of decreasing sanctions being optimal. As Table 1 shows, it depends on the parameter values which effect is stronger. For parameter constellation (I) the two effects just offset each other. Whereas in case (II) the optimal penalty scheme is of the escalating type, in case (III) the decreasing penalty scheme leads to lowest penalization cost.

39 Note that under the uniform penalty scheme we have $c^U = (\bar{c}/2,\ \bar{c}/2)$.

40 In the infinite game this only holds for the escalating penalty scheme.

7 Summary and Outlook

We have analysed positive and normative issues of criminal law in an intertemporal framework. In contrast to the earlier literature our analysis compared an infinite game setting with a finite one. Criminals have been assumed to maximize their net benefits from crime and the lawmaker to minimize social cost. The decision variables have been the number of crimes committed, and the time profile of punishments.

For the infinite game framework it has been shown that for any given criminality level penalization costs under decreasing and uniform penalty schemes are identical. However penalization cost may be reduced by choosing an escalating penalty scheme. We have shown that the optimal penalty scheme shifts the punishment as far into the future as possible. We further have shown that the optimal criminality level under escalating penalties may exceed the optimal criminality level under uniform penalties. Moreover we have shown that in the corresponding finite game framework the optimal penalty scheme may be of the escalating as well as of the decreasing type.

Our basic infinite game approach may be generalized in several directions. One is to endogenize the apprehension probability. Following Polinsky and Shavell (1998) this can be done by adding an additional term $A(p)$ (with $A' > 0$) which captures the costs of apprehending offenders. If we additionally assume that neither the choice of $p = 1$ nor the choice of $p = 0$ is socially optimal,[41] it can easily be proven by contradiction that also in this framework the optimal escalating penalty scheme still dominates the uniform and decreasing ones. Once the optimal level of p is determined, finding the optimal penalty levels boils down to the problem that is solved in Sections 4 and 5 of this paper.

Examples for future extensions allow for age-dependent benefits, risk averse individuals or more general distribution functions of offenders' benefits. Moreover, all kinds of "criminal hysteresis" (Fajnzylber, Lederman, and Loayza 2002, 1328) are particularly suggestive to be integrated into our intertemporal model. For example the criminal history of a certain offender might affect his cost of carrying out a particular criminal activity, the probability of apprehension and other determinants of the decision for crime.

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41 Sufficient conditions are given by $A(p) \to \infty$ for $p \to 1$ and $H(c) \to \infty$ for $c \to 1/(1-q)$.

Appendix

Proof of Proposition 1

(i) $b_{n+1} = \max\{\bar{b}_m, 0 \le m \le n+1\} \ge \max\{\bar{b}_m, 0 \le m \le n\} = b_n$.

(ii) E.g., for any given decreasing penalty scheme $\{s_n\}$ there is a corresponding uniform penalty scheme given by $s_n = \bar{s}$ with $\bar{s} = s_\varnothing = (1-q)\sum_{t=0}^{\infty} q^t \sum_{k=0}^{t} B(k|t)\, s_k$. (q.e.d.)

Proof of Proposition 2

The criminal record specific criminality levels c_n of potential offenders with n previous records can be derived as follows: We consider an offender who is born in t_0, commits a crime in each period $t_0, \ldots, t_0 + t - 1$ since his benefit exceeds b_n and is still alive in period $t_0 + t$. The probability that this offender has n previous convictions at the beginning of period $t_0 + t$ is given by $B(n|t)$ (see Section 3.1). Thus the criminality level of potential offenders with n previous convictions is given by $c_n = (1 - F(b_n))\sum_{t=n}^{\infty} q^t B(n|t)$. Finally note, that $c = \sum_{n=0}^{\infty} c_n$ holds. (q.e.d.)

Proof of Proposition 3

Let $1 \le m \le \infty$ denote the number of different elements in the sequence $\{b_n\}$.[42] Further define the sequence of critical indices as follows: $\{j_k\}_{k=1\ldots m}$ with $j_1 := \max_n\{b_n = b_0\}$ and $j_{k+1} := \max_n\{b_n = b_{j_k+1}\}$.[43] Further in case of $m < \infty$ define $b_{j_{m+1}} = \infty$. Potential offenders with benefit $b \le b_{j_1}$ do not commit a crime at all. Potential offenders with benefit $b_{j_k} < b \le b_{j_{k+1}}$ commit crimes until they have $j_k + 1$ previous records. The penalization costs are unambigously determined by the critical benefits as follows: A potential offender with benefit b_{j_1} is indifferent between "never committing the crime" and incurring the cost of "committing the crime until he is caught $j_1 + 1$ times". From this indifference condition, we can deduce the penalization cost of punishing any criminal for the first $j_1 + 1$ times as $\sum_{\ell=0}^{j_1}\sum_{t=\ell}^{\infty} q^t p B(\ell|t)\, b_{j_1}$. Analogously a potential offender with benefit $b_{j_{k+1}}$ will commit the crime until he is caught $j_k + 1$ times. Once he gets to the criminal record level of $j_k + 1$ he is indifferent between "stopping committing crimes from now on" and "continuing committing crimes" until he is caught $j_{k+1} - j_k$ more

42 In case of a uniform or decreasing penalty scheme we have $m = 1$ and in case of a strictly escalating penalty scheme we have $m = \infty$.

43 E.g., in case of $\{b_n\} = (1, 1, 1, 5, 5, 5, 5, 7, 7, 8, 12, \ldots, 12, \ldots)$ we have $m = 5$ and $\{j_k\} = (2, 6, 8, 9, \infty)$.

times and gets to the record level $j_{k+1}+1$. From this second indifference condition follows that the penalization costs of punishing a criminal for the $j_{k+1}-j_k$ more times must equal $\sum_{\ell=j_k+1}^{j_{k+1}} \sum_{t=\ell}^{\infty} q^t pB(\ell|t)\, b_{j_{k+1}}$. In particular, they are identical for any penalty scheme that induces the same sequence of critical benefits. This proves the assertion. (q.e.d.)

Proof of Proposition 5

(a) We assume that in the initial situation we have an escalating penalty scheme s with $c(s)=\bar{c}$ and $n_0=\min\{n|s_n>0\}$. We show that the corresponding penalization cost may be decreased by marginal changes of the sanction levels if s is not of the type determined by Proposition 5.a. If s is not of this type it belongs to one of the following classes of escalating penalty schemes:

$$\text{Type a}: \exists n_1 > n_0 : s_{n_1} < s_{n_1+1}.$$

$$\text{Type b}: s_n = \tilde{s} < s_{max}\ \forall n > n_0.$$

(i) We show for penalty schemes of *type a* that penalization cost may be reduced by a reduction of s_{n_0} by $\varepsilon>0$ and a corresponding increase of s_{n_1} in such a way that the crime level remains constant: $c_{n_0}(s_{n_0}-\varepsilon)+c_{n_1}(s_{n_1}+\delta)=c_{n_0}(s_{n_0})+c_{n_1}(s_{n_1})$.

The term ε must be small enough so that $s_0-\varepsilon\geq 0$ and $s_1+\delta\leq s_{max}$ holds.
Total differentiation of the equality above reveals that $\frac{d\delta}{d\varepsilon}=\frac{c'_{n_0}(s_{n_0}-\varepsilon)}{c'_{n_1}(s_{n_1}+\delta)}$.
Now we consider the change in the objective function.
Differentiation of the penalization costs with respect to ε and δ yields

$$\begin{aligned} dP = {} & p\Big(-c_{n_0}(s_{n_0}-\varepsilon)-(s_{n_0}-\varepsilon)c'_{n_0}(s_{n_0}-\varepsilon)\Big)d\varepsilon \\ & + p\Big(c_{n_1}(s_{n_1}+\delta)+(s_{n_1}+\delta)c'_{n_1}(s_{n_1}+\delta)\Big)d\delta. \end{aligned}$$

Using the value of $d\delta/d\varepsilon$ this equals

$$\begin{aligned} & p\left(-c_{n_0}(s_{n_0}-\varepsilon)-(s_{n_0}-\varepsilon)c'_{n_0}(s_{n_0}-\varepsilon)+c_{n_1}(s_{n_1}+\delta)\frac{c'_{n_0}(s_{n_0}-\varepsilon)}{c'_{n_1}(s_{n_1}+\delta)}+(s_{n_1}+\delta)c'_{n_0}(s_{n_0}-\varepsilon)\right)d\varepsilon \\ & = -pc'_{n_0}(s_{n_0}-\varepsilon)\left(\frac{c_{n_0}(s_{n_0}-\varepsilon)}{c'_{n_0}(s_{n_0}-\varepsilon)}+(s_{n_0}-\varepsilon)-\frac{c_{n_1}(s_{n_1}+\delta)}{c'_{n_1}(s_{n_1}+\delta)}-(s_{n_1}+\delta)\right)d\varepsilon. \end{aligned}$$

According to Proposition 2 we have $c_n=(1-F(ps_n))\sum_{t=n}^{\infty}q^tB(n|t)$ and $c'_n=-pf(ps_n)\sum_{t=n}^{\infty}q^tB(n|t)$.

Hence we obtain

$$dP = -pc'_{n_0}(s_{n_0}-\varepsilon)\left(-\frac{1}{p}\cdot\frac{1-F(p(s_{n_0}-\varepsilon))}{f(p(s_{n_0}-\varepsilon))}+\frac{1}{p}\cdot\frac{1-F(p(s_{n_1}+\delta))}{f(p(s_{n_1}+\delta))}+(s_{n_0}-\varepsilon)-(s_{n_1}+\delta)\right)d\varepsilon$$

$$= \underbrace{-c'_{n_0}(s_{n_0}-\varepsilon)}_{>0}\left(\underbrace{\frac{1}{h(p(s_{n_1}+\delta))}-\frac{1}{h(p(s_{n_0}-\varepsilon))}}_{<0}+\underbrace{p((s_{n_0}-\varepsilon)-(s_{n_1}+\delta))}_{<0}\right)d\varepsilon<0,$$

since the hazard rate has been assumed to be increasing.

(ii) The general structure of *type b* penalty schemes is given by $s=(0,\dots,0,s_{n_0},\tilde{s},\tilde{s},\dots)$ with $0<s_{n_0}\le\tilde{s}$. We show that penalization cost may be reduced by a reduction of s_{n_0} by $\varepsilon>0$ and a corresponding increase of $s_n=\tilde{s}\ \forall n>n_0$ in such a way that the crime level remains constant:

$$c_{n_0}(s_{n_0}-\varepsilon)+\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}+\delta)=c_{n_0}(s_{n_0})+\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}).$$

Total differentiation reveals that $\frac{d\delta}{d\varepsilon}=\frac{c'_{n_0}(s_{n_0}-\varepsilon)}{\sum_{n=n_0+1}^{\infty}c'_n(\tilde{s}+\delta)}$.

Differentiation of the penalization costs $P=p(s_{n_0}-\varepsilon)c_{n_0}(s_{n_0}-\varepsilon)+p(\tilde{s}+\delta)\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}+\delta)$ with respect to ε and δ yields

$$dP=p\Bigg(\left(-c_{n_0}(s_{n_0}-\varepsilon)-(s_{n_0}-\varepsilon)c'_{n_0}(s_{n_0}-\varepsilon)\right)d\varepsilon$$

$$+\left(\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}+\delta)+(\tilde{s}+\delta)\sum_{n=n_0+1}^{\infty}c'_n(\tilde{s}+\delta)\right)d\delta\Bigg)$$

$$=p\Bigg(-c_{n_0}(s_{n_0}-\varepsilon)-(s_{n_0}-\varepsilon)c'_{n_0}(s_{n_0}-\varepsilon)$$

$$+\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}+\delta)\frac{c'_{n_0}(s_{n_0}-\varepsilon)}{\sum_{n=n_0+1}^{\infty}c'_n(\tilde{s}+\delta)}+(\tilde{s}+\delta)c'_{n_0}(s_{n_0}-\varepsilon)\Bigg)d\varepsilon$$

$$=-pc'_{n_0}(s_{n_0}-\varepsilon)\left(\frac{c_{n_0}(s_{n_0}-\varepsilon)}{c'_{n_0}(s_{n_0}-\varepsilon)}+(s_{n_0}-\varepsilon)-\frac{\sum_{n=n_0+1}^{\infty}c_n(\tilde{s}+\delta)}{\sum_{n=n_0+1}^{\infty}c'_n(\tilde{s}+\delta)}-(\tilde{s}+\delta)\right)d\varepsilon$$

$$= -pc'_{n_0}(s_{n_0}-\varepsilon)\left(\frac{1-F(p(s_{n_0}-\varepsilon))}{-pf(p(s_{n_0}-\varepsilon))}+(s_{n_0}-\varepsilon)-\frac{\sum\limits_{n=n_0+1}^{\infty}\left[[1-F(p(\tilde{s}+\delta))]\sum\limits_{t=n}^{\infty}q^tB(n|t)\right]}{\sum\limits_{n=n_0+1}^{\infty}\left[-pf(p(\tilde{s}+\delta))\sum\limits_{t=n}^{\infty}q^tB(n|t)\right]}-(\tilde{s}+\delta)\right)d\varepsilon$$

$$= -c'_{n_0}(s_{n_0}-\varepsilon)\left(\frac{[1-F(p(\tilde{s}+\delta))]\sum\limits_{n=n_0+1}^{\infty}\left[\sum\limits_{t=n}^{\infty}q^tB(n|t)\right]}{f(p(\tilde{s}+\delta))\sum\limits_{n=n_0+1}^{\infty}\left[\sum\limits_{t=n}^{\infty}q^tB(n|t)\right]}-\frac{1-F(p(s_{n_0}-\varepsilon))}{f(p(s_{n_0}-\varepsilon))}+p((s_{n_0}-\varepsilon)-(\tilde{s}+\delta))\right)d\varepsilon$$

$$= -c'_{n_0}(s_{n_0}-\varepsilon)\left(\frac{1-F(p(\tilde{s}+\delta))}{f(p(\tilde{s}+\delta))}-\frac{1-F(p(s_{n_0}-\varepsilon))}{f(p(s_{n_0}-\varepsilon))}+p((s_{n_0}-\varepsilon)-(\tilde{s}+\delta))\right)d\varepsilon$$

$$= \underbrace{-c'_{n_0}(s_{n_0}-\varepsilon)}_{>0}\left(\underbrace{\frac{1}{h(p(\tilde{s}+\delta))}-\frac{1}{h(p(s_{n_0}-\varepsilon))}}_{<0}+\underbrace{p((s_{n_0}-\varepsilon)-(\tilde{s}+\delta))}_{<0}\right)d\varepsilon<0$$

Hence a marginal decrease of s_{n_0} and a corresponding increase of $s_n\ \forall n>n_0$ decreases penalization cost.

(b) The sequence of critical benefits of the penalty scheme s^I is given by $\{b_n\}$ with $b_n=0\ \forall n<n_0$, $b_{n_0}=ps_0$ and $b_n=s_{max}\ \forall n>n_0$. Since this sequence contains only uniform regions of the border type $b_n=0$ or $b_n=s_{max}$, these regions cannot be induced by decreasing regions of the penalty scheme.

(c) Follows directly from Proposition 5.b. (q.e.d.)

Proof of Proposition 7

We prove exemplarily Proposition 7.b. The number of offenders without criminal record (c_0^D) is composed of $(1-F(ps_\varnothing))$ young offenders and $(1-p)(1-F(ps_0))$ old offenders, which have not been convicted in period 0. The number of offenders with criminal record (c_1^D) follows from the fact that each potential offender with benefit $b>ps_\varnothing$ commits a crime in period 0, and also in period 1, provided that he received the status of a repeat offender. Finally c^D follows from $c^D=c_0^D+c_1^D$. (q.e.d.)

Proof of Lemma 1:

Solving $c^U=\bar{c}$ for $\bar{s}$ and inserting the results in $P^U=p\bar{c}\bar{s}$ proves the assertions. (q.e.d.)

Proof of Proposition 8:

(a) Follows from the proof of Proposition 8. b and c.

(b) We assume that in the initial situation we have a uniform or increasing penalty scheme $s = (s_0, s_1)$ with $c(s) = \bar{c}$ and with $0 < s_0 \le \bar{s} \le s_1 < s_{max}$. We show that the corresponding penalization cost may be reduced by a reduction of s_0 by $\varepsilon > 0$ and a corresponding increase of s_1 in such a way that the crime level remains constant. $c_0(s_0 - \varepsilon) + c_1(s_1 + \delta) = c_0(s_0) + c_1(s_1)$. The term ε must be small enough so that $s_0 - \varepsilon \ge 0$ and $s_1 + \delta \le s_{max}$ holds.

Total differentiation of the equality above reveals that $\frac{d\delta}{d\varepsilon} = \frac{c_0'(s_0 - \varepsilon)}{c_1'(s_1 + \delta)}$. Now we consider the change of penalization cost.
Differentiation of the penalization cost with respect to ε and δ yields

$$dP = p\big(-c_0(s_0 - \varepsilon) - (s_0 - \varepsilon)c_0'(s_0 - \varepsilon)\big)d\varepsilon + p\big(c_1(s_1 + \delta) + (s_1 + \delta)c_1'(s_1 + \delta)\big)d\delta.$$

Using the value of $d\delta/d\varepsilon$ this equals

$$p\left(-c_0(s_0 - \varepsilon) - (s_0 - \varepsilon)c_0'(s_0 - \varepsilon) + c_1(s_1 + \delta)\frac{c_0'(s_0 - \varepsilon)}{c_1'(s_1 + \delta)} + (s_1 + \delta)c_0'(s_0 - \varepsilon)\right)d\varepsilon$$

$$= -pc_0'(s_0 - \varepsilon)\left(\frac{c_0(s_0 - \varepsilon)}{c_0'(s_0 - \varepsilon)} + (s_0 - \varepsilon) - \frac{c_1(s_1 + \delta)}{c_1'(s_1 + \delta)} - (s_1 + \delta)\right)d\varepsilon.$$

According to Proposition 7.c we have $c_0 = (2 - p)(1 - F(p(s_0 - \varepsilon)))$, $c_1 = p(1 - F(p(s_1 + \delta)))$ and $c_0' = -(2 - p)pf(p(s_0 - \varepsilon))$, $c_1' = -p^2 f(p(s_1 + \delta))$.
Hence we obtain

$$dP = -pc_0'(s_0 - \varepsilon)\left(-\frac{1 - F(p(s_0 - \varepsilon))}{pf(p(s_0 - \varepsilon))} + \frac{1 - F(p(s_1 + \delta))}{pf(p(s_1 + \delta))} + (s_0 - \varepsilon) - (s_1 + \delta)\right)d\varepsilon$$

$$= \underbrace{-c_0'(s_0 - \varepsilon)}_{>0}\left(\underbrace{\frac{1}{h(p(s_1 + \delta))} - \frac{1}{h(p(s_0 - \varepsilon))}}_{<0} + \underbrace{p((s_0 - \varepsilon) - (s_1 + \delta))}_{<0}\right)d\varepsilon < 0$$

since the hazard rate has been assumed to be increasing.

(c) Now we assume that in the initial situation we have a uniform or decreasing penalty scheme $s = (s_0, s_1)$ with $c(s) = \bar{c}$ and with $0 < s_1 \le \bar{s} \le s_0 < s_{max}$.

We show that the corresponding penalization cost may be reduced by a marginal increase of s_0 by $\varepsilon > 0$ and a corresponding reduction of s_1 in such a way that the crime level remains constant. $c_0(s_0 + \varepsilon) + c_1(s_1 - \delta) = c_0(s_0) + c_1(s_1)$. The term ε must be small enough so that $s_0 + \varepsilon \le s_{max}$ and $s_1 - \delta \ge 0$ holds. Total differentiation of the equality above reveals that $\frac{d\delta}{d\varepsilon} = \frac{c_0'(s_0 + \varepsilon)}{c_1'(s_1 - \delta)}$.

Now we consider the change of penalization cost.

Differentiation of the penalization cost with respect to ε and δ yields

$$dP = p\big(c_0(s_0+\varepsilon) + (s_0+\varepsilon)c_0'(s_0+\varepsilon)\big)d\varepsilon + p\big(-c_1(s_1-\delta) - (s_1-\delta)c_1'(s_1-\delta)\big)d\delta.$$

Using the value of $d\delta/d\varepsilon$ this equals

$$p\left(c_0(s_0+\varepsilon) + (s_0+\varepsilon)c_0'(s_0+\varepsilon) - c_1(s_1-\delta)\frac{c_0'(s_0+\varepsilon)}{c_1'(s_1-\delta)} - (s_1-\delta)c_0'(s_0+\varepsilon)\right)d\varepsilon$$

$$= pc_0'(s_0+\varepsilon)\left(\frac{c_0(s_0+\varepsilon)}{c_0'(s_0+\varepsilon)} + (s_0+\varepsilon) - \frac{c_1(s_1-\delta)}{c_1'(s_1-\delta)} - (s_1-\delta)\right)d\varepsilon$$

According to Proposition 7.c we obtain

$$dP = pc_0'(s_0+\varepsilon)\left(\frac{(1-F(p(s_1-\delta)))}{pf(p(s_1-\delta))} - \frac{(1-F(p(s_0+\varepsilon)))}{pf(p(s_0+\varepsilon))} + (s_0+\varepsilon) - (s_1-\delta)\right)d\varepsilon$$

$$= \underbrace{c_0'(s_0+\varepsilon)}_{<0}\left(\underbrace{\frac{1}{h(p(s_1-\delta))} - \frac{1}{h(p(s_0+\varepsilon))}}_{>0} + \underbrace{p((s_0+\varepsilon)-(s_1-\delta))}_{>0}\right)d\varepsilon < 0.$$

(d) See Example 3. (q.e.d).

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