Contributions

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Managerial Collusive Behavior under Asymmetric Incentive Schemes

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Abstract: We analyze the effects of asymmetry in incentive contracts on the possibility of collusion between managers. When their compensation is based on the relative performance evaluation contracts, managers can achieve better outcomes by colluding. Using the concept of balanced temptation introduced by Friedman (1971), we find that asymmetry in incentives increases the likelihood of collusion. The result contradicts the general wisdom that asymmetries make collision harder to maintain.

Keywords: asymmetry, collusion, balanced temptation

1 Introduction

It is well established that relative performance evaluation (RPE) makes static interaction more competitive, by inducing managers to behave more aggressively in the output market (see e.g. Miller and Pazgal (2002)). Based on this argument, RPE has been advanced as an important mechanism to hinder or prevent collusion (see e.g. Joh (1999) and Lundgren (1996)). In this paper, we challenge this view and show that the effect of RPE on collusion is, in general, ambiguous and depends on both the intensity and the heterogeneity of RPE in manager pay contracts.

RPE usually means that a manager's pay is increasing in his own firm's performance and decreasing in the performance of other firms in the industry. Recent empirical papers show that firms often use RPE in setting manager pay contracts. For example, Angelis and Grinstein (2011) and Angelis and Grinstein (2013) find that 34% of their sample firms (494 firms that belonged to the

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Standard and Poor's 500 index as of December 2007) are RPE users, i.e. tie manager pay to RPE. On average, RPE users tie 49% of the value of performancebased pay to RPE. Among RPE users, there are large variations in the use of RPE: the standard deviation of weights is 24% and the range of RPE weights is 90% with a minimum of 10% and a maximum of 100%.

The evidence thus supports the idea that, in general, RPE weights are not symmetric. This finding is also consistent with standard models of strategic delegation where the deviation from absolute profit maximization and its level (the incentive weight put on relative performance in manager pay contracts) are endogenized (see e.g. Salas Fumas (1992), Aggarwal and Samwick (1999), Miller and Pazgal (2002) and Vroom (2006)). In these two-stage games, when managers compete à la Cournot in the second stage, there is a continuum of subgame perfect Nash equilibria (because the reaction functions in the reduced form first stage game are superimposed curves) in which, at least one profit-maximizing firm places a negative weight on the profits of the rival firm in the manager's objective function.

Comparing the effects of symmetric versus asymmetric RPE weights, and more generally, appraising the impact of changes in the distribution of RPE weights on managers' ability to collude, is not an easy task. The main difficulty is not technical, but lies in the lack of an obvious focal point on which managers are likely to tacitly coordinate.

We consider a repeated game played by two managers with given, not necessarily identical, RPE pay contracts. We make three key assumptions. First, we assume that managers follow standard grim trigger strategies and collude on Pareto-optimal collusive outcomes. Second, we assume Cournot competition if collusion breaks down. Last but not least, we assume that among the set of sustainable Pareto-optimal subgame perfect Nash equilibria, managers choose the most collusive one. As we will see, this turns out to select, for a given distribution of RPE weights, the market sharing rule which maximizes the sustainability of the monopoly outcome. With this rule, market shares (in the monopoly output) are adjusted between managers until the critical discount factor is the same for all. In that sense, this rule satisfies the property of balanced temptation of Friedman (1971).

This approach is very powerful because it allows us to characterize, for any distribution of RPE weights, the minimum critical discount factor for which (monopoly or Pareto-optimal) collusion can be sustained, and to analyze the variations of this threshold in response to changes in the distribution of RPE weights in manager contracts.

¹ See, Angelis and Grinstein (2013) Table 1, p. 33.

This paper adds the following contributions to the existing literature. First, even though asymmetric RPE contracts have been demonstrated to be the rule and not the exception by several empirical studies, this evidence has not yet received attention in the theoretical literature studying the relationship between RPE and collusion. Some recent papers have analyzed the impact of managerial incentive schemes different from profit maximization on collusion sustainability (see e.g. Spagnolo [2000, 2005], Lambertini and Trombetta [2002] and de Lamirande, Guigou, and Lovat [2013]), but all focus on symmetric incentive schemes only. The closest paper to ours is Matsumura and Matsushima (2012). These authors show that an increase in the degree of competition (i.e. an increase in the intensity of RPE weights) is anti-collusive (i.e. increases the minimum critical discount factor for which monopoly collusion can be sustained). Our main result is to show that the effect of an increase in the intensity (or the sum) of RPE weights can be more than 'compensated' by an increase in the heterogeneity of (or the difference between) RPE weights, such that an increase in RPE weights can support collusion.

Second, in the theoretical literature, asymmetry is generally presented as a factor that hinders collusion (see Ivaldi et al. [2003], Motta [2004] or Feuerstein [2005], for a recent survey on factors that affect collusion). However, this result does not hold unambiguously. Regarding the case of cost asymmetry, Rothschild (1999) and Ciarreta and Gutiérrez-Hita (2011) find that cost asymmetry (i.e. the difference between the high-cost firm and the low-cost firm) does not support collusion. This result is obtained assuming that firms follow grim trigger strategies and maximize joint profits. But it is sufficient to abandon this last assumption and consider another selection criterion to obtain different results. Collie (2004), for example, applies Friedman's balanced temptation rule and finds that cost asymmetry supports collusion. da Silva and Pinho (2012) propose a profitsharing rule that maximizes the sustainability of cartel agreements and show that if a cartel applies this rule, then cost asymmetry may not hinder collusion. In our paper as well, the impact of RPE asymmetry on collusion is ambiguous. However, we do not obtain this result by using different models of collusive equilibrium. RPE manager pay contracts, like cross-shareholding links among firms (see Malueg [1992], and Gilo, Moshe, and Spiegel [2006]), have in fact two conflicting effects on collusion. On the one hand, RPE does not support collusion by increasing a manager's incentive to deviate, because RPE not only increases the manager's own firm's profits but also decreases other firms' profits. On the other hand, RPE makes collusion easier to sustain by decreasing all managers' incentives to deviate, because RPE strengthens market competition and induces, in case of defection, a reversion to a less profitable Cournot Nash equilibrium. Our results show that the net effect of these two opposing forces depends critically on both the heterogeneity and the intensity of RPE weights in manager contracts.

The paper is organized as follows. Section 2 presents the model. Section 3 examines the sustainability of collusion. Section 4 considers the impact of asymmetry on collusion sustainability. Section 5 concludes.

2 Model

There are two quantity-setting firms on a market for a homogeneous product. Both firms have a common marginal cost of production, normalized to 0. Market demand is given by the linear inverse demand function P(Q) = 1 - Q, where $Q = q_1 + q_2$ is the aggregate output (P = 0 if Q > 1).

2.1 One-Shot Game

Each firm is run by a manager who receives a compensation based on absolute and relative profits (RP). Following Jansen, van Lier, and van Witteloostuijn (2009), manager i receives a compensation proportional to:

$$W_i = \pi_i - \theta_i \pi_i \tag{1}$$

where $\pi_i = (1 - Q)q_i$ is the profit of firm *i* and θ_i is the weight put on the profit of (the rival) firm j. We assume that $\theta_i \in [0,1]$. A high θ_i makes manager i more aggressive. On the other hand, if θ_i is low, then manager i behaves more like a pure profit maximizer. Let $\theta = (\theta_1, \theta_2)$ denote the distribution of managers' incentives in the economy. We assume that θ is revealed and becomes common knowledge before managers interact on the output market.

It is important to note that θ is not endogenized at any stage of our model. Our focus is to investigate if asymmetry constitutes a factor supporting collusion.

2.2 Repeated Game

The repeated game consists of an infinite repetition of the game described above. We assume that the game is stationary in each period. In particular, the pair of incentive weights (or the distribution managers' types) is the same in each period. Information is complete, time is discrete and δ is the common discount factor.

Managers maximize the discounted sum of their payoffs given by $\sum_{t=0}^{\infty} \delta^t W_i^t$ where $W_i^t = \pi_i^t - \theta_i^t \pi_i^t$ represents manager *i*'s payoff at time *t*.

For simplicity and tractability, we restrict our attention to the case where stationary collusive agreements are enforced by grim trigger strategies. Intuitively, each manager chooses her specified collusive output in each period, as long as both managers continue do to so. If, however, one of the managers defects, the punishment is triggered; both managers revert to the static Cournot Nash outputs forever.

Let q_i^M and q_i^N denote the outputs of manager i in the collusive agreement (M) and the Cournot Nash equilibrium (N) respectively.

We denote by $W_i^M = W_i(q_i^M, q_i^M)$ manager i's payoff in the stationary collusive agreement. Let $W_i^D = \max_{q_i} \hat{W}_i(q_i, q_i^M)$ denote the one-shot payoff obtained by manager i when she optimally deviates from the collusive agreement, while manager j sticks to M. Finally, let $W_i^N = W_i(q_i^N, q_i^N)$ denote manager i's perperiod payoff during the punishment phase. Given the common discount factor δ , the collusive agreement is a subgame-perfect Nash equilibrium (SPNE) if and only if the following incentive (no-deviation) constraints are satisfied:

$$\delta \ge \underline{\delta}_i = \frac{W_i^D - W_i^M}{W_i^D - W_i^N}, \quad i = 1, 2, \tag{2}$$

where $\underline{\delta}_i$ is the minimum level of the discount factor at which manager i can support the collusive agreement.² Consequently, collusion is sustainable if and only if $\delta \geq \min[\delta_1, \delta_2]$.

3 Sustainability of Collusion

In most articles, cartel members are assumed to maximize the joint payoff. However, as pointed out by Lambertini and Trombetta (2002) in a framework similar to ours, if managers have asymmetric incentives and try to collude on the maximization of joint payoff, then the solution would be at a corner with one firm producing all the output given the market demand. In our setting, if $\theta_1 > \theta_2$, the joint payoff equals $(1 - \theta_2)\pi_1 + (1 - \theta_1)\pi_2$ and is maximized for $q_1 = 1/4$ and $q_2 = 0$. In words, only the manager with the higher θ is active and her firm produces the optimal (monopoly) output while the other manager sets her production to zero.

² Note that although managers have a common intertemporal discount factor, differences in manager types impose different minimum thresholds.

However, this approach is open to criticisms when firms are asymmetric. As noted by Bain (1948), collusion based on joint profit maximization can arise only when some side payments are involved. If not, the firm producing no output would be worse off under collusion than if collusion does not take place.³ Therefore, such collusion is not an option when managers have asymmetric incentives and side payments are not allowed.

To study the sustainability of collusion, we consider the case where managers are only interested in their own payoffs. Since the objective of this paper is to investigate whether incentive asymmetry supports collusion, we are not interested by the combination of production levels q_1, q_2 that managers would choose under collusion. We investigate conditions that guarantee the existence of a combination of production levels (q_1, q_2) that make collusion sustainable for both managers given shared discount factor δ . This approach, called the balanced temptation, was introduced by Friedman (1971). It consists in finding the combination of (q_1, q_2) that minimizes $\max[\underline{\delta}_1,\underline{\delta}_2]$.

3.1 Collusive Stage

To simplify the demonstration, we define a (collusive) outcome by the pair (Q, s), where Q is the collusive output and s is the associated market share (or output quota) of, say, manager 1. This notation is equivalent to (q_1, q_2) since $q_1 = sQ$ and $q_2 = (1 - s)Q$. Using this notation, it results that managers' payoffs can be written as $W_1 = (s - \theta_1 + s\theta_1)(Q - Q^2)$ and $W_2 = (1 - s - s\theta_2)(Q - Q^2)$. Since $Q \in [0,1]$, manager 1's and manager 2's payoffs are non-negative if $s \in \left[\frac{\theta_1}{1+\theta_1}, \frac{1}{1+\theta_2}\right]$.

Proposition 1 Q = 1/2 maximizes W_1 and W_2 simultaneously for any given value of $s \in \left(\frac{\theta_1}{1+\theta_1}, \frac{1}{1+\theta_2}\right)$.

Note that Q = 1/2 is the aggregate monopoly output. In words, if negative payoffs are ruled out, then the total output which maximizes both managers' payoffs simultaneously is unique and coincides with the monopoly output.

From Proposition 1, we argue that the monopoly output is a "natural" focal point on which managers are very likely to tacitly coordinate. It means that, for

³ In our example, without a transfer from manager 1 to manager 2, the latter would have no incentive to collude in the first place.

any value of *s* belonging to $\left(\frac{\theta_1}{1+\theta_1}, \frac{1}{1+\theta_2}\right)$, managers would agree to produce a total of O = 1/2.

Consequently, the monopoly (collusive) outputs are given by $q_1^M = s/2$ and $q_2^M = (1-s)/2$ and the monopoly (collusive) payoffs by:

$$W_1^M = \frac{s - \theta_1 + s\theta_1}{4}$$
 and $W_2^M = \frac{1 - s - s\theta_2}{4}$. [3]

It must be noted that the monopoly (collusive) output and payoff of each manager depend on the size of s: the larger s is, the higher q_1^M and W_1^M (the lower q_2^M and W_2^M), and vice versa.

3.2 Deviation Stage

When deviation occurs, the deviating manager i considers manager j's output q_i^M as given, and optimally selects the output that maximizes her own payoff. Let q_i^D denote the optimal deviation output for manager i. Thus, q_i^D is obtained maximizing $W_i = (1 - q_i - q_i^M)(q_i - \theta_i q_i^M)$. Calculations $q_1^D = (1 + \theta_1 + s - s\theta_1)/4$ and $q_2^D = (2 - s + s\theta_2)/4$ and the resulting deviation payoffs are:

$$W_1^D = \frac{(s - \theta_1 + s\theta_1 + 1)^2}{16}$$
 and $W_2^D = \frac{(s + s\theta_2 - 2)^2}{16}$. [4]

3.3 Competitive Stage

In the competitive stage, managers independently and simultaneously choose their firm outputs q_i that maximize their own payoffs $W_i = (1 - q_i - q_i)(q_i - \theta_i q_i)$. Solving $\partial W_i(q_i,q_j)/\partial q_i=0$ for q_i , one obtains manager i's best response output function: $R_i(q_i) = (1 - (1 - \theta_i)q_i)/2$. The Cournot Nash equilibrium is the pair of outputs (q_1^N, q_2^N) such that $q_i^N = R_i(q_i^N)$ holds for both managers, $i \neq j = 1, 2$. It is easy to check that $q_i^N = (1 + \theta_i)/(3 + \theta_1 + \theta_2 - \theta_1\theta_2)$, i = 1, 2. Substituting these output levels into (3) yields, for a pair of incentives (θ_1, θ_2) , the Nash equilibrium payoff of manager i:

$$W_i^N = \frac{(1 - \theta_1 \theta_2)^2}{(3 + \theta_1 + \theta_2 - \theta_1 \theta_2)^2} \quad i = 1, 2.$$
 [5]

In the static Cournot Nash equilibrium, the manager with the higher incentive toward relative profits sets a larger quantity but does not earn a larger payoff than the manager with the lower incentive: if, say, $\theta_1 > \theta_2$, then $q_1^N > q_2^N$ but $W_1^N = W_2^N$.

3.4 Critical Discount Factors

Before looking at the critical discount rates, it is important to identify some restrictions on the market share. To guarantee the existence of a discount rate that sustains collusion, compensations under collusion must be higher that those under the competitive equilibrium. In other words, we must find conditions on s such that $W_i^M \geq W_i^N$.

Lemma 1 For any market share s such that

$$s \in \left(\frac{\theta_1^2 + \theta_2^2 \theta_1^2 - 2\theta_2 \theta_1^2 - 2\theta_2 \theta_1 + 5\theta_1 + \theta_2^2 \theta_1 + 4}{(3 + \theta_1 + \theta_2 - \theta_1 \theta_2)^2}, \frac{\theta_2 - 2\theta_2 \theta_1 - 3\theta_2 \theta_1^2 + 5 + 6\theta_1 + \theta_1^2}{(3 + \theta_1 + \theta_2 - \theta_1 \theta_2)^2}\right)$$
 [6]

there exists a discount rate $\overline{\delta} \in (0,1)$ such that for any $\delta \in [\overline{\delta},1]$, collusion is sustainable.

Lemma 1 is an application of the folk theorem. It is easy to show that the interval of Expression 6 is not empty for any value of θ_1, θ_2 . Consequently, for any pair (θ_1, θ_2) , collusion can be sustainable if managers are patient enough.

Now, we find the minimum discount rate that supports collusion. By appropriate substitution of eqs [3]-[5] into eq. [2], we obtain critical discount factors for both managers as a function of the market sharing rule s:

$$\underline{\delta_1}(s) = \frac{(1+\theta_1)\phi^2(1-s)^2}{(2+2\theta_2-(1-s)\phi)(\psi-(1-s)(3+4\theta_1+\theta_2+\theta_1^2-\theta_2\theta_1^2))}$$

$$\underline{\delta_2(s)} = \frac{(1+\theta_2)\phi^2 s^2}{(2+2\theta_1 - s\phi)(\psi - s(3+\theta_1 + 4\theta_2 + \theta_2^2 - \theta_1\theta_2^2))},$$
 [7]

where $\phi \equiv (3 + \theta_1 + \theta_2 - \theta_1 \theta_2)$ and $\psi \equiv 2\phi + 4(1 - \theta_1 \theta_2)$.

Proposition 2 $\underline{\delta}_1(s)$ is strictly decreasing in s and $\underline{\delta}_2(s)$ strictly increasing in s.

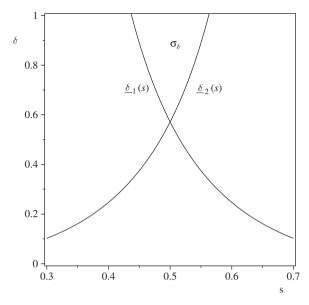


Figure 1: Minimum discount factor for collusion when $\theta_1 = \theta_2 = 1/3$.

Figure 1 depicts the critical discount factors of the two managers as a function of s for the particular case where the pair of incentive weights $(\theta_1, \theta_2) = (\frac{1}{3}, \frac{1}{3})$.

Using $\underline{\delta}_1(s)$ and $\underline{\delta}_2(s)$, we can study the conditions that make collusion sustainable. By definition, if for a given market share s, δ is greater than $\underline{\delta}_1(s)$ and $\underline{\delta}_2(s)$, then collusion is sustainable. Looking at Figure 1, the area σ_{δ} represents the set of all combinations of s and δ such that $\delta \geq \max[\underline{\delta}_1(s),\underline{\delta}_2(s)]$. Explicitly, all pairs (s,δ) belonging to the set σ_{δ} make collusion sustainable.

Since we study the sustainability of collusion and not the collusive equilibrium itself, our focus is now on finding the condition that guarantees the existence of market shares that sustain collusion. Let $s_1(\delta)$ be the minimum market share that sustains collusion for manager 1. Similarly, let $s_2(\delta)$ be the maximum market share that sustains collusion for manager 2. Consequently, collusion is sustainable for any market share s belonging to the interval $I_s(\delta) = [s_1(\delta), s_2(\delta)]$. Figure 2 illustrates a case where, if the discount factor is high enough, the set of all market shares that support collusion as a subgame perfect Nash equilibrium outcome is non-empty.

The following proposition states the minimum condition on the discount factor for collusion to be sustainable.

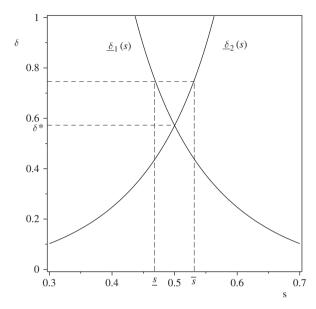


Figure 2: Non-empty collusive market share set.

Proposition 3 Let

$$\delta^* = \frac{(3 + \theta_1 + \theta_2 - \theta_1 \theta_2)^2}{17 + 10\theta_1 + 10\theta_2 - 12\theta_1 \theta_2 - 6\theta_1 \theta_2^2 - 6\theta_1^2 \theta_2 + \theta_1^2 + \theta_2^2 + \theta_1^2 \theta_2^2}.$$
 [8]

For any pair (θ_1, θ_2) , $\delta^* < 1$ and for any $\delta \in [\delta^*, 1]$, $I(\delta)$ is non-empty and for $\delta < \delta^*$, $I(\delta) = \emptyset$.

If Proposition 3 provides the minimum discount factor sustaining collusion, the following proposition states the market share attached to it.

Corollary 1 When $s=s^*\equiv \frac{1+\theta_1}{2+\theta_1+\theta_2}$, critical discount factors are equal, i.e. $\underline{\delta}_1(s^*)=\underline{\delta}_2(s^*)=\delta^*$.

In Figure 2, we identify by δ^* the minimum discount rate sustaining collusion. The discount factor δ^* corresponds to the balanced temptation equilibrium discount factor proposed by Friedman (1971). This criterion refers to the idea that managers split the market in such a way that both have the same incentive to deviate (same discount factor threshold). For the remaining of the paper, we use δ^* to analyze the effect of incentive asymmetry on collusion: an increase in incentive asymmetry supports collusion if δ^* increases.

4 Collusion under Incentive Asymmetry

Incentive asymmetry is defined as the difference between θ_1 and θ_2 . Before studying incentive asymmetry, let us first investigate the impact of an increase of θ_i on the sustainability of collusion.

Lemma 2 For any values of $\theta_1, \theta_2 \in [0, 1]$,

$$\frac{\partial \delta^*}{\partial \theta_i} = \frac{4(1+\theta_i)(1+\theta_j)^3(3+\theta_1+\theta_2-\theta_1\theta_2)}{(17+10\theta_1+10\theta_2-12\theta_1\theta_2-6\theta_1\theta_2^2-6\theta_1^2\theta_2+\theta_1^2+\theta_2^2+\theta_1^2\theta_2^2)^2} > 0$$
 [9]

This result can be explained using the following intuitive interpretation. When RP contracts are used, manager 1's payoff increases when the profit of firm 2 decreases. Because a firm's defection from collusion lowers the rival firm's profit and gets more profit, manager 1 has an incentive to defect. This destabilizes collusion. To curb such an incentive, future payoffs to manager 1 from collusion must be raised. That is accomplished by increasing the output quota of manager 1 in the collusive output. The adjustment of output quotas continues until the critical discount factor is the same for the two managers and both have the same incentive to defect from the cartel. However, this transfer is not enough to completely compensate the increase in manager i's aggressiveness. Furthermore, after the transfer of production from firm *j* to firm *i*, manager *j* has less incentive to collude, making collusion more difficult to sustain thereafter (δ^* increases).

Lemma 2 itself cannot provide a clear and complete picture of the effect of an increase in asymmetry. Indeed, an increase in θ_i ceteris paribus does not support collusion both when $\theta_i > \theta_i$ (asymmetry increases) and when $\theta_i < \theta_i$ (asymmetry decreases). Consequently, the study of the first derivative cannot provide a clear understanding of the effect of asymmetry on collusion.

4.1 Iso-stability Curves

To illustrate the effect of incentive asymmetry on collusion, we develop the concept of an iso-stability curve. An iso-stability curve is the set of all pairs (θ_1, θ_2) for which managers have the same discount rate threshold while the market share is given by s^* . Explicitly, an iso-stability curve $IS(\delta)$ is the set of all pairs (θ_1, θ_2) such that $\delta^*(\theta_1, \theta_2) = \delta$.

⁴ The equation $\delta^*(\theta_1, \theta_2) = \delta$ defines the surface in $(\theta_1, \theta_2, \delta)$ -space along which the cartel stability remains constant and equal for both managers. A cross-section of the surface in (θ_1, θ_2) -space, at a given value of δ , yields an iso-stability curve.

To graph iso-stability curves, we isolate θ_2 in the equation $\delta^*(\theta_1, \theta_2) = \delta$ and we obtain θ_2 as a function of θ_1 and δ .

$$\theta_{2}(\theta_{1}, \delta) = \frac{6\delta\theta_{1} - 2\theta_{1} + 3\delta\theta_{1}^{2} + 3 - \theta_{1}^{2} - 5\delta - 2\sqrt{\delta(-1 + 2\delta)(1 + \theta_{1})^{4}}}{\delta\theta_{1}^{2} - 6\delta\theta_{1} + \delta - 1 + 2\theta_{1} - \theta_{1}^{2}}$$
[10]

While eq. [10] seems difficult to analyze, it is easy to show that, for any $\theta_1 \in [0,1]$ and $\delta \in [9/17,1]$,⁵

- IS curves are decreasing and convex;
- IS curves are symmetric with respect to the line $\theta_2 = \theta_1$;
- when $\theta_2(\theta_1, \delta) = \theta_1$, the slope of the IS curve is -1.

Figure 3 illustrates some iso-stability curves for different values of δ .

We can see that δ^* increases when (θ_1, θ_2) moves closer to (1,1). This result is consistent with Lemma 2: an increase in θ_i does not support collusion.

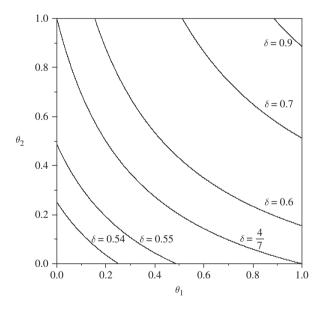


Figure 3: Iso-stability curves.

⁵ The case $\delta^* = 9/17$ arises when $\theta_1 = \theta_2 = 0$ which is equivalent to the traditional Cournot game with symmetric firms. Since δ^* is increasing in θ , δ^* takes values only between 9/17 and 1.

4.2 Incentive Asymmetry

The next step consists in the separation of the effect of an increase in the θ_i , i=1,2, from an increase in incentive asymmetry. Indeed, when (θ_1,θ_2) changes, the difference between θ_1 and θ_2 changes but the level of θ_i changes as well. As we have seen above, an increase in either θ_i increases δ^* . However, the effect of asymmetry (difference between θ_1 and θ_2) is, as yet, unclear.

To clarify this point, we divide the change in the pair (θ_1,θ_2) in two components: a change in average and a change in asymmetry. First, let Θ be the average of θ_1 and θ_2 . Second, we define Δ as the Euclidean distance between θ_1 and θ_2 . Without loss of generality, we suppose $\theta_1 \geq \theta_2$ and therefore, $\Delta = \theta_1 - \theta_2$. Using these two notations, we are able to separate the effect of an increase in the level of the θ_i from that of a change in asymmetry.

Figure 4 illustrates the effects of an increase in Θ and Δ on δ^* . A change in Θ when Θ remains constant is represented by a movement along the line of slope 1 while a change in Δ when Θ remains unchanged by a movement along the line of slope -1.

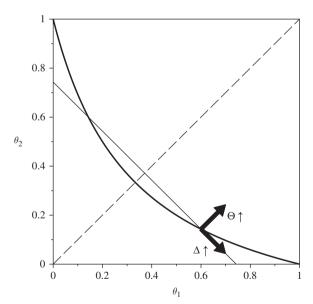


Figure 4: Effects on δ^* of a change in Θ and Δ .

Proposition 4 *The average of* θ_1 , θ_2 *remaining constant, an increase in incentive asymmetry supports collusion* (δ^* *decreases*).

The formal proof of Proposition 4 is in the Appendix. Graphically, the result is clear: an increase in the incentive asymmetry permits to reach a lower iso-stability curve. We already know that δ^* increases when an incentive parameter increases $(\partial \delta_*/\partial \theta_i > 0)$. However, when θ_1 increases and θ_2 decreases, the effect on collusion is mitigated and finally negative. It seems that the effect of the decision to use incentive compensations is relatively more important when the incentive parameter θ is low. In other words, the gain from the reallocation of production from firm 2 to firm one decreases as θ_1 increases. This is due to the relative lower gain under deviation since firm 1 already produces a larger quantity.

Another way to look at this result is to see the impact of a change in asymmetry on the set of market shares supporting collusion.

Corollary 2 If $I(\delta)$ is not empty and incentive asymmetry increases, then $I(\delta)$ remains non-empty.

As stated previously, our focus is on the link between incentive asymmetry and collusion and not the actual equilibrium market shares. From Corollary 2, collusion remains sustainable if it was sustainable before an increase in asymmetry. However, this does not mean that the initial set is a subset of the new set of market shares. In other words, if a market share supported collusion before a change in asymmetry, it does not mean it still supports collusion thereafter. When asymmetry increases while the average remains unchanged, $\underline{\delta_1}$ and $\underline{\delta_2}$ move in the same direction. Consequently, the interval $I(\delta)$ moves as well. Some market shares that belonged to the interval of sustainable market shares do not belong to the interval anymore.

4.3 Collusion Rate of Substitution

If Proposition 4 is useful to isolate the effect of an increase in asymmetry from an increase in the average value of the incentive parameters, it does not cover all possible cases. Indeed, if asymmetry and average increase altogether, Proposition 4 cannot predict the final effect on δ^* . However, there exists another approach that covers more cases. This second approach involves the introduction of a new concept: the collusion rate of substitution.

The collusion rate of substitution (CRS) is given by the negative of the ratio of derivatives of δ^* with respect to θ_1 and θ_2 , i.e.

$$CRS = -\frac{\frac{\partial \delta^*}{\partial \theta_1}}{\frac{\partial \delta^*}{\partial \theta_2}} = -\frac{(1+\theta_2)^2}{(1+\theta_1)^2}$$
[11]

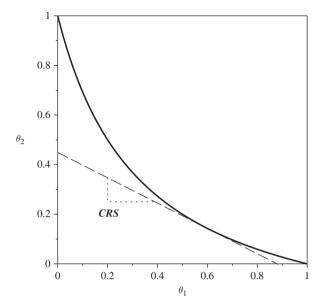


Figure 5: Collusion rate of substitution.

Since θ_1 and θ_2 belong to [0,1], then CRS varies between -1/4 and -4. Technically, CRS is the slope of the iso-stability curve passing through (θ_1, θ_2) (see Figure 5).

To study the effect of a change in θ_1 and θ_2 on δ^* , we compare the variation in θ_1 and θ_2 with CRS at the initial pair (θ_1, θ_2) . Let us still assume that $\theta_1 > \theta_2$ and let $(\Delta\theta_1, \Delta\theta_2)$ be the variation in θ_1 and θ_2 .

Proposition 5 δ^* decreases when incentive parameters move from (θ_1, θ_2) to $(\theta_1 + \Delta\theta_1, \theta_2 + \Delta\theta_2)$ if

- $\Delta\theta_1, \Delta\theta_2 < 0$;
- or $\Delta\theta_1 < 0$, $\Delta\theta_2 > 0$ and

$$\mathit{CRS}(\theta_1, \theta_2) = -\frac{(1+\theta_2)^2}{(1+\theta_1)^2} \le \frac{\Delta \theta_1}{\Delta \theta_2}$$

- or $\Delta\theta_1 > 0$, $\Delta\theta_2 < 0$ and

$$CRS(\theta_1, \theta_2) = -\frac{(1+\theta_2)^2}{(1+\theta_1)^2} \ge \frac{\Delta\theta_1}{\Delta\theta_2}$$

Figure 6 illustrates the three possibilities presented in Proposition 5.

First, point A represents the case when θ_1 and θ_2 decrease. As discussed above, a decrease in θ_1 and θ_2 leads to a decrease of δ^* . The second case

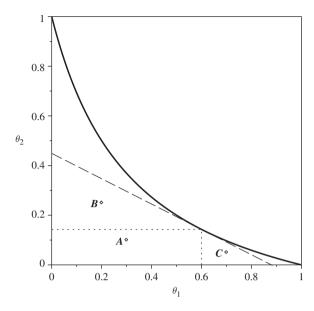


Figure 6: Collusion rate of substitution.

 $(\Delta\theta_1 < 0, \Delta\theta_2 > p0)$ is illustrated by point B. In this case, the decrease in θ_1 must be large enough to compensate the effect of an increase in θ_2 in order to support collusion. The third case (point C) is analogous to the second case.

5 Discussion and Conclusion

There are two major findings in this paper. First, using an incentive compensation based on relative performance, a focal point exists for the total quantity when managers minimize the probability of deviation. This result is completely independent of the demand function and the constant marginal cost. The existence of a focal quantity depends on the type of incentive compensation used. As an example, when incentive compensation is based on profits and sales (as in Lambertini and Trombetta [2002]), no focal quantity exists when managers' incentives are asymmetric.

The main result of this paper constitutes the second major finding. We demonstrate that an increase in incentive asymmetry has an ambiguous effect on the sustainability of collusion. In facts, by isolating changes in incentive asymmetry from changes in incentive level, asymmetry supports collusion. While this conclusion is in opposition to the conventional wisdom that

asymmetries do not constitute a factor supporting collusion, it seems to reinforce the conclusions of some recent papers (Collie [2004] and Miklós-Thal [2011]). However, this result holds only when we keep the level on incentives constant (constant average for θ). When both move in the same direction (both increase or decrease), the effect on collusion is ambiguous and depends on the relative changes.

Appendix

Proof of Proposition 4. First, we substitute θ_1 and θ_2 by $\Theta = \frac{\theta_1 + \theta_2}{2}$ and $\Delta = \theta_1 - \theta_2$ into δ^* . We obtain

$$\delta^* = \frac{\left(12 + 8\Theta - 4\Theta^2 + \Delta^2\right)^2}{16\Theta^4 - 8\Theta^2\Delta^2 - 192\Theta^3 + \Delta^4 + 48\Delta^2\Theta - 160\Theta^2 + 56\Delta^2 + 320\Theta + 272}$$

Now, we can take the derivative of δ^* with respect to Δ .

$$\frac{\partial \delta^*}{\partial \Delta} = \frac{64 \big(12 + 8\Theta - 4\Theta^2 + \Delta^2\big) \Delta \left(-4\Theta^3 + \Delta^2\Theta - 12\Theta^2 + \Delta^2 - 12\Theta - 4\right)}{\big(16\Theta^4 - 8\Theta^2\Delta^2 - 192\Theta^3 + \Delta^4 + 48\Delta^2\Theta - 160\Theta^2 + 56\Delta^2 + 320\Theta + 272\big)^2}$$

The first bracket is strictly positive since $\Theta \in [0,1]$. Δ is positive as well. The last bracket is strictly negative. Consequently, $\frac{\partial \delta^*}{\partial \Lambda}$ is negative for any value of Θ, Δ and strictly negative when $\Delta > 0$.

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