DE GRUYTER BE J. Theor. Econ. 2016; 16(1): 119–142

Research Article

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Assortative Outsourcing with Exit

DOI 10.1515/bejte-2014-0063
Published online July 18, 2015

Abstract: A general framework is presented that incorporates dynamics and heterogeneity among both upstream suppliers and downstream producers to mimic the exit strategy of Hirschman (1970) in building vertical relations. An assortative matching develops between producers and suppliers based on their level of efficiency, which leads to an increase in the aggregate industrial productivity but also makes the distribution of firms more dispersed. Further experiments suggest that the nature of outsourcing relations is impacted in certain ways by business cycles and technological advancements.

Keywords: outsourcing, productivity, search friction, exit strategy, business cycle
JEL Classification: D23, D83, L23, L24

1 Introduction

Firms are increasingly faced with the make–or–buy choice, that is, they have to decide whether to produce an intermediate input in-house or simply procure the input through arm's-length trade. The current body of theories dealing with this issue mainly relies on the characteristics of the downstream producer to explain which outsourcing decisions should be implemented; see Antras and Helpman (2004), Grossman and Helpman (2004), and Assche and Schwartz (2009). Much less attention is given to the characteristics of the supplying party in sustaining such relations. This paper pursues a model of two-layered heterogeneity in both upstream suppliers and downstream producers, hereafter called suppliers and producers for brevity, to provide some insight into the outsourcing relations that survive the test of time.

The model embarks on an environment where production is two-stage, an input production stage and a core process that turns the input into the consumption good. The production of input can be trusted to an independent supplier if internalization is inefficient. The efficiency of suppliers is unobserved

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prior to a match, however, after a match is formed both parties reveal their productivities and reach a contract that sets the price and quantity of the input to be supplied. At the same time, future search is made possible, so that both producers and suppliers endogenize their outside options when evaluating a potential contract. In case the present value of contract is inferior to the search option for at least one party, the match is broken and search resumes.

Two certain behaviors arise from such setting: first, in the same line as Shimer and Smith (2000) the matching between producers and suppliers is assortative such that more efficient suppliers, on average, form vertical relations with more efficient producers in the long run and vice versa. Nonetheless, unlike in Shimer and Smith (2000), the matching set in this modeling tends to be non-convex owing to the unboundedness of the productivity space. Second, the thick-market externality of Shimer and Smith (2001) becomes evident as numerous producers that are very efficient in their core processes but relatively inefficient in the intermediate processes feel reluctant to enter the search market in fear of having to search for an extensive period of time, since searching embodies opportunity costs of deviating efforts from production.

The initial findings reaffirm the classical notion that outsourcing firms are low in productivity but challenge the perception that every low-productivity firm outsources. The results also break from the existing literature by indicating that outsourcing can affect both the first and second moments of productivity distribution in an industry. Outsourcing improves productivity, hence, the industry as a whole becomes more productive. Producers also get more dispersed in their productivities, not least because productivity gains are distributed non-uniformly across firms: producers with efficient core processes benefit much more from outsourcing than those with inefficient ones. This non-uniformity in gains makes the outsourcing producers drift apart, so that the distribution of productivity as a whole becomes more dispersed.

The outcome of matches in this model mimics the exit strategy of Hirschman (1970) in the face of an unattractive match. Producers that are efficient in their core processes do not remain matched to inefficient suppliers, mainly because the demanded input price is too high. Suppliers also drop producers with relatively inefficient core processes and go on search again, since their option value is more promising than status quo. This strategy can be summarized in each of producers and suppliers forming a range of acceptable productivities they would be contented with, and the decision to keep or to break a match depends on the productivity of the matched partner falling into that interval.

Suppliers become even more selective when there is a secular reduction in consumer spending as may happen during economic downturns. Under such circumstance, fewer suppliers enter the market, and those that enter toughen up their matching criteria in the hope of finding more profitable matches in order to recoup their perceived losses from the weak demand. With deterioration in the outsourcing market, fewer producers opt for outsourcing. A technological advancement that culminates into an economy-wide boost to productivity has the reverse effect and makes producers much more selective. The prospects of profit making leads to an influx of suppliers, and producers reject matches more easily and seek out more productive suppliers. With the outsourcing market in favor of producers, there will be a larger mass of producers choosing outsourcing over integration.

The model bears resemblance to a few other works in the literature in terms of mechanics and the addressed problem. The aforementioned threshold productivities are the analogous counterparts of the reservation wage in the search and matching models of labor market (see Rogerson, Shimer, and Wright 2005, for a survey). The fundamental difference is that in the case of producer–supplier both sides form a reservation, hence, each party tends to shun matches on two sides and keep the ones in the middle. In contrast, in the labor models worker is the only side with a reservation value. Outsourcing has been implemented in the context of incomplete contracts by Grossman and Helpman (2002) and later by Antras and Helpman (2004) with heterogeneous firms. However, in Antras and Helpman (2004) heterogeneity is one-layered and producers transfer their productivity to their suppliers at no gain or loss, hence, there is no discussion of the types of matches that are to be formed. Van Mieghem (1999) offers another perspective on the outsourcing decision where contracts are incomplete and both producers and suppliers face uncertain outside demand. He is able to show that more uncertainty in demand is conducive to a larger proportion of inputs being outsourced. Alvarez and Stenbacka (2007) use a different real options model and reach the same conclusion. This paper, instead, focuses on another source of uncertainty in the outsourcing decision, which comes from the inability of producers and suppliers to observe each other's productivities ex ante. Where a larger uncertainty in demand pushes for more outsourcing, a larger uncertainty in the quality of matches can have the reverse effect. Real option models of outsourcing, such as those of Van Mieghem (1999) and Alvarez and Stenbacka (2007), have also traditionally addressed only homogeneous firms and suppliers.

The rest of the paper proceeds as such: Next section sets up the theoretical model and articulates the type of matches that are formed in equilibrium. Section 3 applies the theory to find how the downstream industry is affected by outsourcing and follows with a few experiments when the economic conditions change. Section 4 concludes the paper.

2 Theoretical Setup

There is one sector with a continuum of firms. The mass of firms in this sector is normalized to one, and each firm produces a distinct variety $j \in [0, 1]$ of a consumption good and competes monopolistically. Since the space of varieties is continuous, the probability of two firms producing the same variety is zero; hence, j is indexing both the variety and its corresponding firm.

The representative consumer has a Constant Elasticity of Substitution (CES) utility function over these varieties, where the elasticity of substitution between any two varieties is equal to $1/(1-\alpha),\ \alpha \in (0, 1)$. It is well known that this type of utility function gives rise to the following demand function for each variety (Grossman and Helpman 2002):

$$p_j = Ay_j^{-(1-\alpha)}, \tag{1}$$

in which p_j is the price of variety j, y_j is the demand for output,

$$A = \left(E \Big/ \int_0^1 p_j^{\frac{\alpha}{1-\alpha}} dj\right)^{1-\alpha}, \tag{2}$$

and E is the aggregate consumer expenditure. In this context, A is an aggregate index of the economy that is exogenously driven by consumer expenditures, but also depends on the collective action of all firms in setting their prices. Individual firms, on the other hand, are atomistic in the continuous space and take A as given when making decisions.

2.1 Integrated Firm

The production of a variety is in two stages, a firm has to produce an intermediate input first (intermediate process) and then adapt the input into the distinct variety (core process). Only the intermediate process can be outsourced. Both the intermediate and core processes are constant returns using labor as the only input. Producers are differentiated in their efficiency in both the intermediate and core processes. Specifically, producer j needs $1/\phi_j$ units of labor to produce one unit of input and $1/\lambda_j$ unit of labor to transform the input into one unit of final product, where ϕ_j and λ_j are jointly and randomly drawn from a known cumulative distribution $F(\lambda, \phi)$ with support $\lambda, \phi \geq 0$ and observed after entry. The marginal densities associated with $F(\cdot, \cdot)$ are assumed to have falling upper tails in accordance with the empirical observations for distribution of firms (Axtell 2001).

The pair (λ_j, ϕ_j) defines the overall production efficiency of firm j. Alternatively, total labor required by producer j to produce one unit of output can be conveniently described in terms of a *total labor productivity*, which is the harmonic mean of ϕ_j and λ_j, or

$$\chi_j = \left(\frac{1}{\lambda_j} + \frac{1}{\phi_j}\right)^{-1}. \qquad [3]$$

In the remainder, index j is dropped where not causing confusion.

For the moment, focus on the one-period behavior of a producer in steady state. The wage rate is fixed in steady state and, without loss of generality, normalized to one. I am also abstracting from fixed costs and time variations in productivities to keep the model tractable and to help single out the key results.[1] Dynamics and entry costs are discussed later in Section 2.3.

Producers plan their production by maximizing the profit function $\pi_V = py - y/\chi$ subject to eq. [1]; subscript V refers to vertical integration. The optimal profit perceived by a producer with productivity χ results from plugging the solution to the first-order condition with respect to y back into the profit function, yielding

$$\pi_V(\chi) = (1-\alpha)A^{\frac{1}{1-\alpha}}(\alpha\chi)^{\frac{\alpha}{1-\alpha}}, \qquad [4]$$

All the derivations can be found in Appendix. Note that, in a classic way, more profit is accrued to more efficient firms (those with higher χ's). In view of this feature, I make the following assumption to ensure that the expected profit of an integrated firm stays finite.

Assumption 1 *The upper tail of the productivity distribution $F(\cdot,\cdot)$ goes to zero at such a rate that*

$$\pi_V(\chi)\frac{d^2F(\lambda,\phi)}{d\lambda d\phi} \in o\left(\lambda^{-1}\phi^{-1}\right).$$

2.2 Outsourcing Firm

A producer can also decide to procure the required input from an independent supplier after having observed own internal productivity. Upon finding a supplier, both sides observe each other's productivities and reach an agreement on

1 Including nonzero fixed costs merely introduces a cutoff productivity that limits the range of "operational" productivities from below. However, the theoretical implications concerning outsourcing behavior would not be affected.

the price and quantity of the supplied input. Given their option values from continuing search, each party then decides whether to accept the contract or exit the match. For production to go ahead, both parties must be willing to accept the contract. Once the agreement is mutual, the producer forfeits its ability to the production of input and relies solely on the supplier for its input delivery. If either side opts for exiting the match, both parties earn zero profit in that period and resume search in the next period. For the moment, focus on the one-period operation of producers and suppliers.

Suppliers are heterogeneous in their productivities, ϕ_o. ϕ_o is drawn independently from a cumulative distribution $H(\phi_o)$ with support $\phi_o \geq 0$ and $E[\phi_o] > E[\phi]$; the supplying industry is, on average, more sophisticated in the production of input, though this assumption is not crucial to the results. Akin to Assumption 1, I am assuming that the upper tail of the density function associated with $H(\cdot)$ is also falling at a fast enough rate. The productivity of an individual supplier is, nonetheless, unobserved by producers prior to a match. This uncertainty is an additional risk factor producers should take into account when making a make–or–buy decision.

In return, outsourcing producers experience an improvement to their core processes as a proportional increase in λ by a factor of $\mu\,(>1)$. The boost symbolizes the producer focusing on its core competencies, such as developing unique features that make the product more attractive, and accounts for improvement in efficiency as a result of the producer reinvesting some of the extra revenue to innovate and improve its current technology (Breunig and Bakhtiari 2013).

To formulate the problem, let p_o be the price of the supplied input and $x_o = y$ be the quantity. Incorporating these variables, the profit functions for producer and supplier become

$$P: \ \pi_P(\lambda, \phi_o) = py - y/(\mu\lambda) - p_o y, \qquad [5]$$

$$S: \ \pi_S(\lambda, \phi_o) = p_o y - y/\phi_o, \qquad [6]$$

where P and S stand for producer and supplier, respectively. The solution to this problem is in two stages: first, a producer taking price p_o as given decides the size of production subject to eq. [1], then, given the perceived size of production, the supplier decides what price would maximize its profit. The solution to the first stage is:

$$x_o = y = \left(\frac{\alpha A}{\frac{1}{\mu\lambda} + p_o} \right)^{\frac{1}{1-\alpha}}. \qquad [7]$$

Plugging eq. [7] into the supplier's profit function and maximizing with respect to p_o yield the price of input as

$$p_o(\lambda, \phi_o) = \frac{1}{\alpha}\left(\frac{1-\alpha}{\mu\lambda} + \frac{1}{\phi_o}\right). \quad [8]$$

The price of input has a similar structure to that of the price of final good in a model of monopolistic competition. It is related to the inverse of productivity inflated by a constant markup; the markup is driven by the inverse of α. The main difference is the presence of an appropriation effect $1-\alpha$, reflecting the inability of suppliers in capturing the full match surplus.

Using this solution, the perceived profit for each party becomes

$$\pi_P(\lambda, \phi_o) = (1-\alpha)A^{\frac{1}{1-\alpha}}(\alpha^2\chi_o)^{\frac{\alpha}{1-\alpha}}, \quad [9]$$

$$\pi_S(\lambda, \phi_o) = (1-\alpha)(\alpha^{1+\alpha}A\chi_o^{\alpha})^{\frac{1}{1-\alpha}}, \quad [10]$$

in which χ_o is the total labor productivity of an outsourcing pair and is defined as

$$\chi_o = \left(\frac{1}{\mu\lambda} + \frac{1}{\phi_o}\right)^{-1}. \quad [11]$$

Again, in a classic sense, profit functions for both the producer and the supplier are monotonically increasing in the total labor productivity of the match, χ_o.

It is also noteworthy that the outcome of this outsourcing arrangement bears some similarities to the results of Grossman and Helpman (2002), despite the difference in modeling. In particular, comparing profit functions [9] and [10] reveals that

$$\pi_S(\phi_o; \lambda) = \alpha\pi_P(\lambda; \phi_o). \quad [12]$$

In other words, the division of match surplus is the same as the outcome of a Nash bargaining in which the bargaining powers of producer and supplier are $\frac{1}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, with a smaller share going to the supplier. This result emulates the assumption in Grossman and Helpman (2002) in which an ex ante and irreversible investment by suppliers left them in a weaker bargaining position and led to a holdup situation in which suppliers under-invested in the mutual relationship. Similarly, profit function [9] exhibits a holdup situation, where $\pi_P(\lambda; \phi_o)$ is $(1+\alpha)\alpha^{\alpha/(1-\alpha)}$ fraction of the profit the producer would have earned had it owned the supplier.[2] Cooper, Haltiwanger, and Willis (2007) estimate α to be around 0.65 amongst the US establishments. Using this value, the producer

2 Note that

$$\pi_P(\lambda; \phi_o) = \frac{\pi_P(\lambda; \phi_o) + \pi_S(\phi_o; \lambda)}{1+\alpha} = (1+\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{\pi_V(\mu\lambda, \phi_o)}{1+\alpha}\right).$$

receives a 60% share, whereas the supplier is entitled to a 40% share of the profits. The share of profit lost to holdup is about 26%.

2.3 Dynamics

The operation of integrated and outsourcing producers specified above repeats in every period, and future values are discounted for both producers and suppliers by a factor of $\delta \in (0, 1)$. In every period, firms fail at an exogenous rate of $\xi \in (0, 1)$. If they are in an outsourcing relationship, then both the producer and supplier exit. In what follows, $1 - \xi$ typically acts as an additional discount factor alongside δ. Therefore, I define and use $\hat{\delta} = (1 - \xi)\delta$ for brevity.

Also, in the interest of simplicity and tractability, the mass of potential suppliers is assumed much larger than the mass of searching producers. The assumption has empirical justification since the US Census Bureau's counts of businesses in 2009 shows that, for instance, for every motor vehicle manufacturer (NAICS 3361) in the US there has been on average 5.7 body and trailer manufacturers (NAICS 3362) and 14.8 parts manufacturers (NAICS 3363).[3] These counts still exclude foreign body and parts suppliers available to American auto manufacturers. As a result, in each period a searching producer can find a match with probability one while a potential supplier finds a match with probability $\rho \in (0, 1)$ that, for the moment, firms take as given.[4] In this context, probability ρ is the same as market tightness in the search and matching models of labor.

Finally, I am considering a long-run situation where the dynamics of the industry has settled on a steady state path. As a result, ρ is time invariant. As stated earlier, the mass of producers is normalized to one, but suppliers enter freely into the market paying an entry cost of $c_e > 0$ to cover their start-up costs, such as setting up the physical plant and commencing their search.

3 The number of businesses in 2009 are reported as 355 motor vehicle manufacturers, 2,007 body and trailer manufacturers, and 5,270 parts manufacturers; see Census Bureau's County Business Patterns.

4 Let n_p and n_s be the numbers for producers and suppliers, then using the matching function $m(n_p, n_s) = n_p(1 - e^{-n_s/n_p})$ from Petrongolo and Pissarides (2001) one finds that the matching probabilities for producers, ρ_p, and suppliers, ρ, are

$$\rho_p = m(n_p, n_s)/n_p = 1 - e^{-1/\theta}, \quad \rho = m(n_p, n_s)/n_s = \theta\rho_p,$$

where $\theta = n_p/n_s$ is the market tightness. The application of $\theta \ll 1$ leads to the approximations $\rho_p \simeq 1$ and $\rho \simeq \theta$. For instance, using the ratio 5.7 of body to auto manufacturers generates $\rho_p = 0.997$ and $\rho = 0.176$. Alternatively, using the ratio 14.8 of parts to auto manufacturers yields $\rho_p \simeq 1$ and $\rho = 0.068$.

Definition 1 *A steady state equilibrium for the outsourcing problem with heterogeneous producers and suppliers is fully described by the match probability* ρ*, the cumulative distribution of searching producers* $G(\lambda)$*, and a set of cutoff productivities* $(\underline{\phi}_o(\lambda), \underline{\lambda}(\phi_o))$ *and the set* $(x_o, p_o, \phi^*(\lambda))$ *that characterize the matching set and the outsourcing decision, respectively.*

In what follows, each of the cutoff productivities, their roles and important features, and also the conditions that give rise to the steady state will be described in details.

2.4 The Matching Set

The possibility of further search leads to non-zero option values and gives rise to each of the producers and suppliers forming a selection criteria of their own. Given the monotonicity of the profit functions, these selection criteria manifest themselves as lower cutoff productivities $\underline{\phi}_o(\lambda)$ and $\underline{\lambda}(\phi_o)$, such that producers exit any match with $\phi_o < \underline{\phi}_o$ and suppliers exit any match with $\lambda < \underline{\lambda}$. As the following lemma shows, the matchings that are accepted by both sides are assortative in nature and have implied upper cutoff productivities as part of the selection criteria.

Lemma 1 *There exists a vector of cutoff productivities* $(\underline{\phi}_o, \bar{\phi}_o, \underline{\lambda}, \bar{\lambda})$ *such that a producer stays in a match conditional on* $\phi_0 \in [\underline{\phi}, \bar{\phi}_0]$ *and a supplier stays in a match conditional on* $\lambda \in [\underline{\lambda}, \bar{\lambda}]$*. Moreover, the matching set is non-empty for any supplier with* $\phi_o > 0$ *and any producer with* $\lambda > 0$*, that is*

$$\bar{\phi}_o(\lambda) > \underline{\phi}_o(\lambda), \quad \lambda > 0,$$
$$\bar{\lambda}(\phi_o) > \underline{\lambda}(\phi_o), \quad \phi_o > 0.$$

Finally, the matching is assortative, or formally put

$$\left(\frac{d\bar{\phi}_o}{d\lambda}, \frac{d\underline{\phi}_o}{d\lambda}, \frac{d\bar{\lambda}}{d\phi_o}, \frac{d\underline{\lambda}}{d\phi_o}\right) > 0.$$

Proof See Appendix.

Given the specifics of the modeling, it is useful to seek more details about the option values of each party and the decisions that follow in order to construct a platform for further experimentation. Beginning with the supplier, there is a ρ probability that a match is found in the current period. With the cumulative distribution of producers on the search being $G(\lambda)$, the probability that this new match is acceptable is $G(\bar{\lambda}) - G(\underline{\lambda})$. The supplier's expected profit from accepting the match is

$$E\left[\pi_S(\phi_o;\lambda)|\lambda \in [\underline{\lambda},\bar{\lambda}]\right],$$

per period thereafter, with the expectation taken over the conditional distribution of $G(\lambda)$. But, with probability $1-\rho(G(\bar{\lambda})-G(\underline{\lambda}))$ there are no new matches or the match fails and the game repeats. Denote the amortized per period option value of breaking a match and resuming search by $V_S(\phi_o)$, then

$$V_S(\phi_o)=\frac{\rho(G(\bar{\lambda})-G(\underline{\lambda}))}{1-\hat{\delta}+\hat{\delta}\rho(G(\bar{\lambda})-G(\underline{\lambda}))}E\left[\pi_S(\phi_o;\lambda)|\lambda \in [\bar{\lambda},\underline{\lambda}]\right]. \quad [13]$$

The option value of a supplier directly depends on the supplier's perception of how profitable future matches will be on average. However, the probability of finding those profitable matches is a drag on the option value: if suppliers are very selective, it becomes almost impossible for them to match at all, driving the option value to zero. The equilibrium value is a trade-off between profitability and the feasibility of matches.

If a supplier keeps the match, it earns $\pi_S(\phi_o;\lambda)$ per period, but upon exiting a match it earns zero in the current period and expects to receive $V_S(\phi_o)$ per period afterwards, with the appropriate time discounting applied. The cutoff productivity $\underline{\lambda}(\phi_o)$ is where the profit from accepting the match equals the option value, or

$$\pi_S(\phi_o;\underline{\lambda})=\hat{\delta}V_S(\phi_o). \quad [14]$$

In the case of producers, the option value can be found basically by setting $\rho=1$ in eq. [13] and making the right adjustments to arrive at

$$V_P(\lambda)=\frac{H(\bar{\phi}_o)-H(\underline{\phi}_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))}\; E\left[\pi_P(\lambda;\phi_o)|\phi_o \in [\underline{\phi}_o,\bar{\phi}_o]\right], \quad [15]$$

where $V_P(\lambda)$ is the amortized option value of a λ-type producer. The value of $\underline{\phi}_o(\lambda)$ is the solution to the following equation:

$$\pi_P(\lambda;\underline{\phi}_o)=\hat{\delta}V_P(\lambda). \quad [16]$$

With $\underline{\lambda}$ and $\underline{\phi}_o$ known and given the assortative nature of the matches, the value of $\bar{\lambda}$ and $\bar{\phi}_o$ can be determined from

$$\bar{\lambda}(\phi_o)=\min\left\{\lambda|\phi_o<\underline{\phi}_o(\lambda)\right\}, \quad [17]$$

$$\bar{\phi}_o(\lambda)=\min\left\{\phi_o|\lambda<\underline{\lambda}(\phi_o)\right\}. \quad [18]$$

In other words, $\bar{\lambda}(\phi_o)$ is the least productive producer that marginally rejects a ϕ_o-type supplier and similarly for $\bar{\phi}_o$. These definitions, in turn, imply that

$$\frac{d\bar{\lambda}}{d\underline{\phi}_o} < 0, \quad \text{and} \quad \frac{d\bar{\phi}_o}{d\underline{\lambda}} < 0. \qquad [19]$$

The derivatives above basically reflect the conflict of interest between suppliers and producers, such that higher selectivity on the part of producers disadvantages suppliers and forces them to settle with matches that have lower productivities and vice versa.

Another interesting feature of the cutoff productivities is their limit behavior as described in the following lemma:

Lemma 2 *There exist* $\Phi, \Lambda < \infty$ *such that* $\lim_{\lambda\to\infty} \underline{\phi}_o(\lambda) = \Phi$ *and* $\lim_{\phi_o\to\infty} \underline{\lambda}(\phi_o) = \Lambda$.

Proof See Appendix.

The lemma posits that at some level of productivity producers and suppliers do not get any more selective mainly because the distribution of more productive firms on the other side of the matching market is diminishing. Being over-selective for these firms could mean that they will never be able to form a lasting relationship. The same lemma is also a proof that the matching set is non-convex (Figure 1).

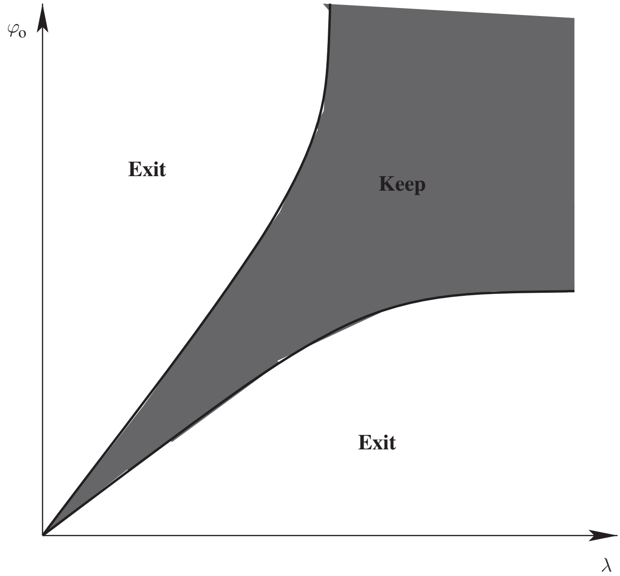

Figure 1: The assortative nature of the match and the decisions to keep or exit a match by the two sides.

2.5 The Decision to Outsource

A producer that decides to outsource expects to receive $V_P(\lambda)$ per period. The same producer can stay fully integrated and earn $\pi_V(\chi)$ per period. A λ-type producer outsources only when

$$\pi_V(\lambda, \phi) < V_P(\lambda). \qquad [20]$$

The right-hand side is constant for any given λ, while the left-hand side is monotonically increasing in ϕ. However, $\pi_V(\lambda, \infty)$ is bounded, and an intersection is not guaranteed. Especially, if μ is much larger than one, then all producers find it optimal to outsource as the benefits overshadow any transaction cost. For an industry to be a mix of both integrated and outsourcing producers, the cost and benefit of outsourcing should be in some balance as described below:

Proposition 1 *If $\alpha\mu \le 1$, then the equilibrium is a mix of vertically integrated and outsourcing producers.*

Proof See Appendix.

In a mixed equilibrium, the marginal producer has the intermediate productivity $\phi^*(\lambda)$ that satisfies

$$\pi_V(\lambda, \phi^*) = V_P(\lambda), \qquad [21]$$

such that a λ-type producer will outsource if $\phi < \phi^*(\lambda)$ and stays integrated otherwise. The functional form of $\phi^*(\lambda)$ can be readily found by combining eqs [16] and [21] to get

$$\phi^*(\lambda) = \lambda \left[\frac{\hat{\delta}^{\frac{1-\alpha}{\alpha}}}{\alpha\mu} \left(1 + \frac{\mu\lambda}{\underline{\phi}_o(\lambda)} \right) - 1 \right]^{-1} \qquad [22]$$

In general, the functional form of $\phi^*(\lambda)$ resembles a line emanating from the origin whose slope gradually fades and converges to zero as shown by the following lemma.

Lemma 3 $\lim_{\lambda\to\infty} \phi^*(\lambda) = \Phi^*$, *where* $\Phi^* = \alpha\Phi/(\hat{\delta}^{\frac{1-\alpha}{\alpha}}) < \infty$.

In other words, $\phi^*(\lambda)$ eventually saturates mainly because the distribution of ϕ_o has a falling upper tail. Figure 2 illustrates one such relationship. In line with the existing literature (see Antras and Helpman 2004, for instance), firms with low productivities – those with $\chi \le \Phi^*$ – are the ones that outsource. The argument does not spin the other way: not all low-productivity firms outsource, but only those with inefficient intermediate processes. This last prediction is more accommodating to the existing empirical evidence that finds that outsourcing is more prevalent but not universal among inefficient firms; see Federico (2010) and Pieri and Zaninotto (2011) among others.

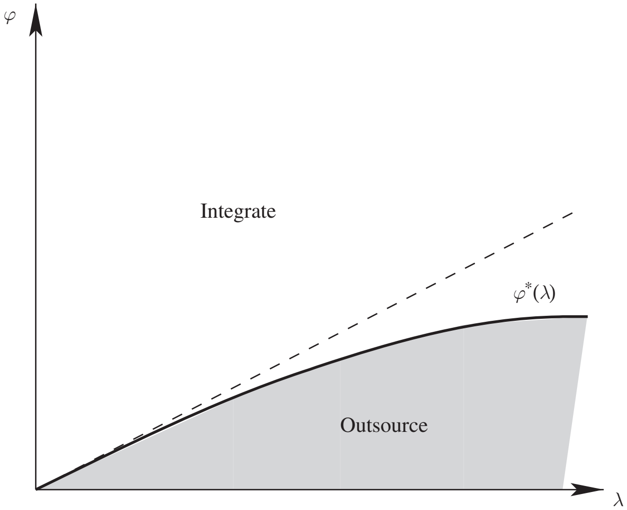

Figure 2: The decision to outsource or to integrate in (λ, ϕ) space. The dashed line shows the hypothetical decision rule in the absence of distributions' thin upper tails.

The general picture in Figure 2, in particular, exhibits two phases of the outsourcing decision. The initial steep rise in $\phi^*(\lambda)$ is mostly driven by the increasing opportunity cost of operating with a low ϕ when the producer is getting more productive in its core process. However, the thick-market externality of Shimer and Smith (2001) kicks in the later stage and the enthusiasm ebbs away and ϕ^* eventually saturates as producers in the upper tail of core productivity have to search much longer to build a lasting outsourcing relation, mainly because the distribution of highly-productive suppliers (which is affecting the term in parentheses above) diminishes.

2.6 Closing the Model

With the mass of producers normalized to one, any adjustment to the equilibrium rate of matching has to be made by the supplying side. Suppliers enter the market freely, and in an equilibrium they expect to earn zero profits. Put formally,

$$\int_0^\infty \frac{V_S(\phi_o)}{1-\hat{\delta}} dH(\phi_o) = c_e. \qquad [23]$$

On the producer's side, exits from the market must be replaced with the same mass of entrants in equilibrium, so that the mass of incumbent producers stays constant over time. With a unit mass of producers, a ξ mass exits and enters every period. The implication of this entry and exit for the search market is that

$$G(\lambda) = \xi \int_{l=0}^{\lambda} \int_{\phi=0}^{\phi^*(l)} d^2F(l, \phi). \quad [24]$$

Solving eqs [23] and [24] along with eq. [21] provides the unknowns ρ, $G(\lambda)$ and $\phi^*(\lambda)$.

Remark 1 *The steady state equilibrium in Definition 1 requires that (i) x_o and p_o satisfy eqs [7] and [8], respectively, (ii) $\underline{\phi}_o(\lambda)$ and $\underline{\lambda}(\phi_o)$ satisfy eqs [14] and [16], respectively, (iii) $\phi^*(\lambda)$ satisfies eq. [21], and (iv) eqs [23] and [24] are satisfied, given the values of α, A, c_e, δ, ξ and the distributions $F(\lambda, \phi)$ and $H(\phi_o)$.*

3 Theoretical Results

Producers that enter the search market do so at the prospects of benefiting from improved productivity due to accessing a more efficient intermediate production. After realizing a match, however, it remains to see whether producers indeed improve their productivity. The decision to accept or reject a match for the producer is a trade off between making profit in the current period and rejecting a match in the hope of more profitable future matches. Producers that vie for fast results – because they heavily discount the future – might settle for a match that leaves them worse off with some positive probability. The problem is compounded by the fact that the profit for an outsourcing producer is driven not only by the production capacities of the supplier–producer pair but also by the hold-up that keeps the pair from achieving full production potentials.

For a proper consideration of these issues in measuring productivity for outsourcing producers, I define the *implied productivity* of an outsourcing producer equal to the productivity level of a hypothetical integrated producer that can generate the same amount of profit. A comparison between eqs [4] and [9] makes it clear that the implied productivity of an outsourcing producer is $\alpha\chi_o$. The productivity gain, γ, specific to the producer from outsourcing can then be expressed as

$$\gamma(\lambda, \phi, \phi_o) = \alpha\chi_o/\chi. \quad [25]$$

By definition, $\gamma = 1$ for integrated producers.

Proposition 2 *The following statements are true for the downstream industry when outsourcing is allowed:*

1. There exists a non-zero mass of outsourcing producers that experience a drop in their productivity as a result of outsourcing, that is, $\gamma < 1$ with non-zero probability. The maximum loss, however, is bounded below since $\gamma > \hat{\delta}^{\frac{1-\alpha}{\alpha}}$.
2. For large enough $\hat{\delta}$, the average gain from outsourcing is strictly increasing with core productivity. Formally, $dE[\gamma|\lambda]/d\lambda > 0$ for the outsourcing producers.
3. For large enough $\hat{\delta}$, the average productivity grows and productivity becomes more dispersed in the downstream industry. Formally, $E[\gamma\chi] > E[\chi]$ and $\sigma_{\gamma\chi} > \sigma_{\chi}$.

Proof See Appendix.

Despite some firms settling for matches that are detrimental to their productivities, average gains of higher than one are realized when $\hat{\delta}$ is large enough, that is when producers are patient and think long-term. For a practical estimate of $\hat{\delta}$, note that the annual discount rate is commonly chosen around 0.95 in the literature, while the annual exit rate of establishments in the U.S. is around 10%.[5] When firms are looking for suppliers on a weekly basis, these rates translate to $\delta = 0.999$ and $\xi = 0.002$, respectively. If search is repeated on a monthly basis, one has $\delta = 0.996$ and $\xi = 0.009$.[6] Using $\alpha = 0.65$ from Cooper, Haltiwanger, and Willis (2007), one finds that even the worst matches come within 99.3% of breaking even with integration. In this situation, achieving average gains of larger than one can be quite at hand.

So far, the discussion has been mostly static, looking at the outcome of outsourcing in a long-run equilibrium. But the same framework can also be used to get some insight into the effects of low-frequency movements of an economy on the dynamics of outsourcing. Since the consumer budgeting is assumed exogenous, in my first experimentation I allow for the budget to get tight under adverse economic conditions, so that consumer expenditure falls. With lower expenditure comes lower demand for output, in turn, suppliers and producers are confronted with lower expected profits. Some adaptation on the part of suppliers and producers follows as outlined below.

Proposition 3 *An exogenous reduction in consumer spending increases the chances of finding a potential partner for suppliers, but also forces suppliers to be more selective, whereas producers become less selective. Formally, if $E' < E$, then (prime denotes the new value)*

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5 See Business Dynamics Statistics (BDS) released by the Center for Economic Studies of the US Census Bureau (http://www.census.gov/ces).

6 The weekly rates can be found by setting $\delta^{52} = 0.95$ and $(1 - \xi)^{52} = 0.9$. For monthly rates, the exponent is set to 12 months.

$$\rho' > \rho, \quad (\underline{\lambda}', \bar{\lambda}') > (\underline{\lambda}, \bar{\lambda}), \quad (\underline{\phi}'_o, \bar{\phi}'_o) < (\underline{\phi}_o, \bar{\phi}_o).$$

The mass of outsourcing producers is smaller in the new equilibrium. An increase in consumer spending has the reverse effect.

Proof See Appendix.

In principle, during economic downturns the number of suppliers that enter the search market plummets, effectively raising the chances of suppliers finding a potential partner. At the same time, the prospects of profit making in a relationship are now dim. The higher chance of matching and lower profitability of matches entice suppliers to reject matches more easily and keep searching for more profitable partnerships (Figure 3(a)). This increased selectivity on the part of suppliers discourages producers from entering the search market. Those producers that participate have to lower their standards in order to have a reasonable chance of building a relationship.

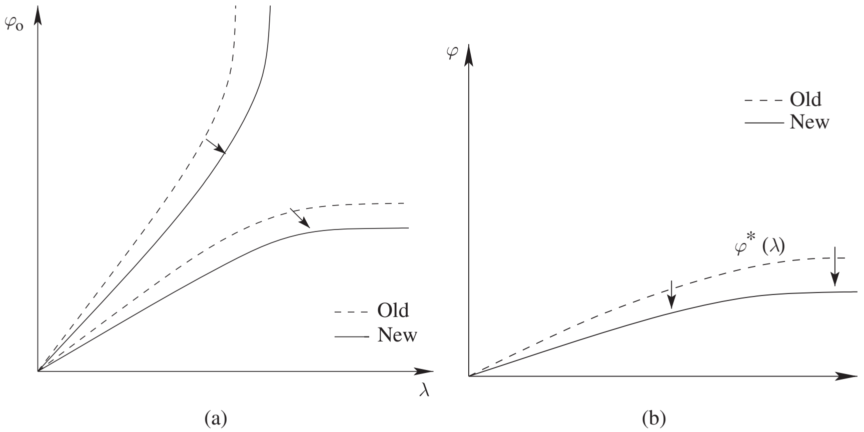

Figure 3: The effect of an exogenous cut in consumer spending on (a) the quality of matches and (b) the cutoff productivity of outsourcing.

In some cases, suppliers are required to make a sunk relation-specific investment $K > 0$ to adapt their production line according to some requirements demanded by the producer. The first implication of such requirement is that the least efficient suppliers are driven out of the search market; their feasible matches can only endure negative lifetime profits. As for the rest, the impact is very similar to that of an economic downturn as long as K is not too large to discourage outsourcing altogether.

Proposition 4 *When forming a vertical relation requires an irreversible relation specific investment, suppliers become more selective and producers become less selective about whom they want to match. Suppliers also have higher chances of finding a match. Formally, with* $K > 0$:

$$\rho' > \rho, \quad (\underline{\lambda}', \bar{\lambda}') > (\underline{\lambda}, \bar{\lambda}), \quad (\underline{\phi}'_o, \bar{\phi}'_o) < (\underline{\phi}_o, \bar{\phi}_o).$$

Proof See Appendix.

Growing productivity is another aspect of modern economies with non-trivial implications on outsourcing. In the next proposition, I consider the case when productivity in the whole economy grows by the same proportion.

Proposition 5 *An economy-wide proportional increase in productivity by a factor* $\tau > 1$ *scales the cutoff productivities of producers by more than* τ *but scales the cutoff productivities of suppliers by less than* τ*. Also, it becomes harder for suppliers to find a match. Put formally, if* $\lambda' = \tau\lambda$, $\phi' = \tau\phi$ *and* $\phi_0 = \tau\phi_0$, *then*

$$(\underline{\phi}'_o, \bar{\phi}'_o) > \tau(\underline{\phi}_o, \bar{\phi}_o), \quad (\underline{\lambda}', \bar{\lambda}') < \tau(\underline{\lambda}, \bar{\lambda}), \quad \rho' < \rho.$$

The mass of outsourcing producers is larger in the new equilibrium.

Proof See Appendix.

Intuitively, an increase in productivity raises the standards of matching on both sides. Nevertheless, the prospect of better profits also encourages entry and makes the market more competitive for suppliers. Producers take note and react by becoming much more selective and rejecting matches more easily. In return, suppliers lower their standards, by not fully implementing the proportional effect, to give themselves a reasonable chance of building a relationship. The matching market is clearly in favor of producers, and accordingly a larger number of them opt for outsourcing. This situation is illustrated in Figure 4.

An alternative to Proposition 5 would be the case where productivity drops across the economy. However, economic downturns are in general marked by a slowdown in the rate of productivity growth but not by a reversal in productivity itself (see Griliches 1980; Hall 2007, for instance). A fall in productivity could happen only under sever economic hardship as could be the case with Greece in the aftermath of its sovereign debt crisis. The outcome of a productivity decline of this kind can be readily obtained by reversing the argument above. Specifically, a decline in productivity will adversely affect the entry of suppliers, but those suppliers that enter have a better chance of matching. In equilibrium, cutoff productivities drop on all sides, but suppliers stay more selective than producers by lowering their standards less than proportionally.

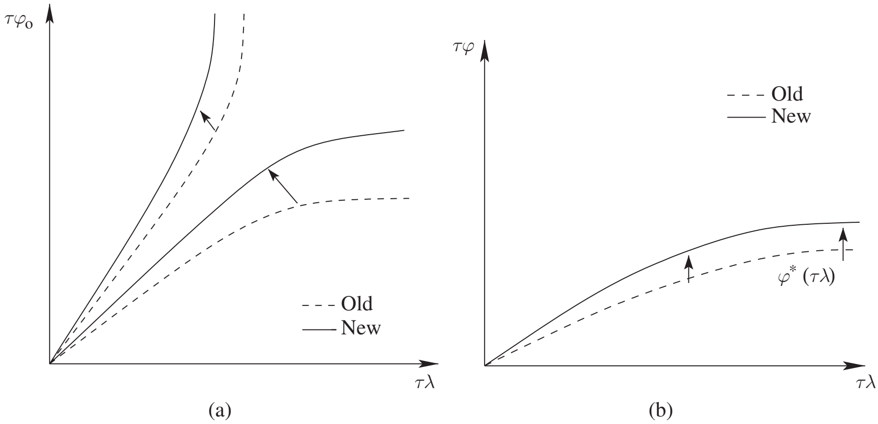

Figure 4: The effect of an economy-wide proportional growth in productivities on (a) the quality of matches and (b) the cutoff productivity of outsourcing.

4 Conclusion

The hallmark of classical models used to explain outsourcing among heterogeneous firms, such as that of Antras and Helpman (2004), has been that the producer is able to fully transfer its productivity to its subordinates, whether at home or abroad. This paper moves past the simplification in an extension that introduces a two-layered heterogeneity in which upstream and downstream firms each exercise the exit strategy of Hirschman (1970). The model is meant to mimic a more realistic and managerial approach to forming vertical relations. The analysis goes beyond replicating the classical findings and explores new grounds that are off-limits to models with one-layered heterogeneity. One central point of the paper is that the possibility of exiting a match and further search leads to an approximate sorting of producers and suppliers in an outsourcing relation. This sorting, in turn, leads to observable changes in the productivity distribution of an industry following a rash of outsourcing. The model is also suited for conducting various thought experiments to better understand the nature of outsourcing relations and their dynamics. This paper offers two examples in which outsourcing decisions are shown to be impacted by business cycles as well as technological changes to the economy. The door is still open to other experiments that might be interesting to researchers in specific fields of economics.

There are caveats too. Throughout the model, it is assumed that producers can outsource to only one supplier, while in reality producers can potentially use many. However, Asanumi (1989) observes that Japanese car manufacturers that initially start with many suppliers, eventually drop several and focus on a narrow set of suppliers that are the closest matches to the firm's operation.

The main challenge that remains at the end is to test these predictions using micro-level data on firms and their suppliers. Currently, such level of details has been absent in the available panels of firms. With the rapid advances in the production and dissemination of firm-level data, however, there is hope that the empirical validation of the ideas presented in this paper will not be put off for too long.

Technical Appendix

Deriving eqs [4], [9] and [10]: Substituting from eq. [1] simplifies the profit function into $\pi_V(\chi) = Ay^{\alpha} - y/\chi$. Taking derivatives with respect to y gives $y_V(\chi) = (\alpha A\chi)^{\frac{1}{1-\alpha}}$. Plugging $y_V(\chi)$ into the profit function gives the required result. Profit functions [9] and [10] can be derived in a similar way. ■

Proof of Lemma 1: When types are confined within the unit square, Shimer and Smith (2000) prove that the supermodularity of the payoff function, the log of its first derivative and the log of its cross second derivative are necessary conditions for an assortative matching that is characterized by a non-empty and convex matching set. That finding can be generalized by a transformation such as $T(\lambda) = \frac{\lambda}{1+\lambda}$ or $T(\lambda) = 1 - e^{-\lambda}$ (likewise for ϕ_o) that maps the infinite space of (λ, ϕ_o) into the unit square. Moreover, the transformations are continuous and monotonic, hence, they preserve signs of derivatives. Consequently, the supermodularity conditions can be equally tested using $\pi_P(.;.)$ or $\pi_S(.;.)$. For instance, using $\pi_P(.;.)$ one has

$$\begin{aligned} \frac{\partial^2 \pi_P(\lambda;\phi_o)}{\partial\lambda\partial\phi_o} &= \frac{(\alpha^{1+\alpha}A)^{\frac{1}{1-\alpha}}}{\mu(1-\alpha)} \frac{\chi_o^{\frac{2-\alpha}{1-\alpha}}}{\lambda^2\phi_o^2} > 0, \\ \frac{\partial}{\partial\lambda\partial\phi_o}\left(\log\frac{\partial\pi_P(\lambda;\phi_o)}{\partial\lambda}\right) &= \frac{1}{1-\alpha}\frac{\mu}{(\mu\lambda+\phi_o)^2} > 0, \\ \frac{\partial}{\partial\lambda\partial\phi_o}\left(\log\frac{\partial^2\pi_P(\lambda;\phi_o)}{\partial\lambda\partial\phi_o}\right) &= \frac{2-\alpha}{1-\alpha}\frac{\mu}{(\mu\lambda+\phi_o)^2} > 0. \end{aligned}$$

Therefore, the matching set is non-empty and assortative. The cutoff productivities are defined by the borders of the matching set, and the rest of the results follow from the non-emptiness and assortativeness of the set. Note, however, that the inverse of $T(.)$ does not preserve convexity. ■

Deriving eqs [13] and [15]: Let

$$P_1 = \rho(G(\bar{\lambda}) - G(\underline{\lambda})), \quad \Pi = E\left[\pi_S(\lambda,\phi_o)|\lambda \in [\underline{\lambda},\bar{\lambda}]\right].$$

Note that the probability of an unsuccessful match is $1-P_1$. Then

$$\frac{V_S(\phi_o)}{1-\hat{\delta}} = P_1\frac{\Pi}{1-\hat{\delta}} + \hat{\delta}(1-P_1)\left(P_1\frac{\hat{\delta}\Pi}{1-\hat{\delta}} + \hat{\delta}(1-P_1)\left(P_1\frac{\hat{\delta}\Pi}{1-\hat{\delta}} + \ldots\right)\right)$$
$$= \frac{P_1}{1-\hat{\delta}(1-P_1)}\frac{\Pi}{1-\hat{\delta}}.$$

Replacing P_1 and Π generates eq. [13]. The derivation of eq. [15] is done similarly. ■

Proof of Lemma 2: Let $\underline{\phi}_o(\lambda) \to \infty$ as $\lambda \to \infty$. An immediate implication is that $\bar{\phi}_o(\lambda) \to \infty$ in the limit. Expanding eq. [15] gives:

$$\begin{aligned} V_P(\lambda) &= \frac{H(\bar{\phi}_o)-H(\underline{\phi}_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))} \int_{\underline{\phi}_o}^{\bar{\phi}_o} \pi_P(\lambda;\phi_o)\frac{dH(\phi_o)}{H(\bar{\phi}_o)-H(\underline{\phi}_o)} \\ &= \frac{\int_{\underline{\phi}_o}^{\bar{\phi}_o} \pi_P(\lambda;\phi_o)dH(\phi_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))}. \end{aligned}$$

With both $\underline{\phi}_o$ and $\bar{\phi}_o$ going to infinity in the limit, the right-hand side above goes to zero because the tail of $dH(\phi_o)$ is falling at such a rate to keep the overall expected profit finite. Therefore, $V_P(\lambda) \to 0$ in the limit. But, from eq. [16], it has to be that *lim* $\pi_P(\lambda;\underline{\phi}_o) \to 0$ as $\lambda \to \infty$, which is a contradiction. Subsequently, it must be that $\underline{\phi}_o(\lambda)$ converges to some $\Phi < \infty$ in the limit. The proof for a limit on $\underline{\lambda}$ is done similarly but using $V_S(\phi_o)$. ■

Proof of Proposition 1: Writing an algebraically simplified version of eq. [20] shows that outsourcing producers are those with

$$\chi < \alpha\left(\frac{H(\bar{\phi}_o)-H(\underline{\phi}_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))}\right)^{\frac{1-\alpha}{\alpha}} E\left[\chi_o^{\alpha/(1-\alpha)}|\phi_o\in[\underline{\phi}_o,\bar{\phi}_o]\right]^{\frac{1-\alpha}{\alpha}}. \qquad [26]$$

When $\phi \to 0$, the inequality is certainly satisfied and the firm outsources. In case $\phi \to \infty$, the inequality simplifies to

$$\begin{aligned} \lambda &< \alpha\left(\frac{H(\bar{\phi}_o)-H(\underline{\phi}_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))}\right)^{\frac{1-\alpha}{\alpha}} \quad E\left[\chi_o^{\alpha/(1-\alpha)}|\phi_o\in[\underline{\phi}_o,\bar{\phi}_o]\right]^{\frac{1-\alpha}{\alpha}} \\ \Rightarrow \quad 1 &< (\alpha\mu)\left(\frac{H(\bar{\phi}_o)-H(\underline{\phi}_o)}{1-\hat{\delta}+\hat{\delta}(H(\bar{\phi}_o)-H(\underline{\phi}_o))}\right)^{\frac{1-\alpha}{\alpha}} \quad E\left[\left(\frac{\phi_o}{\mu\lambda+\phi_o}\right)^{\alpha/(1-\alpha)}|\phi_o\in[\underline{\phi}_o,\bar{\phi}_o]\right]^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

The term in parentheses on the right-hand side is bounded between zero and one (note that its inverse is larger than or equal to one). The expected value is also always less than one for $\lambda > 0$. If $\alpha\mu \leq 1$, then the inequality is guaranteed to get violated and these firms will integrate. It follows that, when $\alpha\mu \leq 1$, at every level of λ both integrated and outsourcing producers are present. ■

Proof of Proposition 2: The worst case scenario for a λ-type producer is to match with a supplier whose productivity is $\underline{\phi}_o(\lambda)$. Subsequently,

$$\begin{aligned}
\gamma^{\frac{\alpha}{1-\alpha}} &= \frac{\pi_P(\lambda;\phi_o)}{\pi_V(\lambda,\phi)} \geq \frac{\pi_P(\lambda;\underline{\phi}_o)}{\pi_V(\lambda,\phi)} \\
&= \frac{\hat{\delta}V_P(\lambda)}{\pi_V(\lambda,\phi)} && \text{using eq. [16]} \\
&> \hat{\delta} && \text{using eq. [20].}
\end{aligned}$$

Outsourcing producers with $\gamma \in (\hat{\delta}^{(1-\alpha)/\alpha}, 1)$ experience a drop in their productivity. Despite this, the conditional expected gains can be larger than one. When $\delta = 1$, it is immediate from the inequality above that $E[\gamma|\lambda,\phi] > 1$. By continuity, for large enough $\hat{\delta}$ the same conditions will be satisfied. At the same time, $\gamma = 1$ for integrated firms. Now, to show that gains are increasing with core productivity, I first take the derivative of γ with respect to λ to get

$$\begin{aligned}
\frac{d\gamma}{d\lambda} = \frac{d}{d\lambda}\left(\frac{\alpha\chi_o}{\chi}\right) &= \alpha\mu\frac{\phi_o}{\phi}\frac{\phi_o - \mu\phi}{(\mu\lambda+\phi_o)^2} \\
&= \alpha\mu\frac{\phi_o^2}{(\mu\lambda+\phi_o)^2}\left(\frac{1}{\phi}-\frac{\mu}{\phi_o}\right) \\
&= \alpha\mu\frac{\chi_o^2}{\mu^2\lambda^2}\left(\frac{1}{\chi}-\frac{1}{\lambda}-\frac{\mu}{\chi_o}+\frac{1}{\lambda}\right) \\
&= \frac{\chi_o}{\mu\lambda^2}(\gamma-\alpha\mu) = \frac{\chi}{\alpha\mu\lambda^2}\gamma(\gamma-\alpha\mu).
\end{aligned}$$

In expectation:

$$\begin{aligned}
\frac{dE[\gamma|\lambda]}{d\lambda} &= E\left[\frac{d\gamma}{d\lambda}\middle|\lambda\right] = E\left[\frac{\chi}{\alpha\mu\lambda^2}\gamma(\gamma-\alpha\mu)\middle|\lambda\right] \\
&= E_\phi\left[\frac{\chi}{\alpha\mu\lambda^2}\left(E_{\phi_o}[\gamma^2|\lambda,\phi]-\alpha\mu E_{\phi_o}[\gamma|\lambda,\phi]\right)\right] && \text{(Nested expectations)} \\
&\geq E_\phi\left[\frac{\chi}{\alpha\mu\lambda^2}\left(E_{\phi_o}[\gamma|\lambda,\phi]^2-\alpha\mu E_{\phi_o}[\gamma|\lambda,\phi]\right)\right] && \text{(Jensen's inequality).}
\end{aligned}$$

Given that $E[\gamma|\lambda,\phi] > 1$ for large enough $\hat{\delta}$ and $\alpha\mu \leq 1$ (Proposition 1), one concludes that $dE[\gamma|\lambda]/d\lambda > 0$.

For the unconditional average, one can apply the law of nested expectations to get

$$E[\gamma\chi] = E[\chi E[\gamma|\lambda,\phi]] > E[\chi].$$

Similarly, applying the law of nested expectations to the variance leads to the following sequence of events:

$$\begin{aligned}
\sigma^2_{\gamma\chi} &= E\left[\gamma^2\chi^2\right] - E[\gamma\chi]^2 \\
&= E\left[\chi^2 E\left[\gamma^2|\lambda,\phi\right]\right] - E\left[\chi E[\gamma|\lambda,\phi]\right]^2 \\
&= E\left[\chi^2\left(E\left[\gamma^2|\lambda,\phi\right] - E[\gamma|\lambda,\phi]^2 + E[\gamma|\lambda,\phi]^2\right)\right] - E\left[\chi E[\gamma|\lambda,\phi]\right]^2 \\
&= E\left[\chi^2\left(E\left[\gamma^2|\lambda,\phi\right] - E[\gamma|\lambda,\phi]^2\right)\right] + \left(E\left[\chi^2 E[\gamma|\lambda,\phi]^2\right] - E\left[\chi E[\gamma|\lambda,\phi]\right]^2\right) \\
&\quad > E\left[\chi^2\sigma^2_{\gamma|\lambda,\phi}\right] + \sigma^2_\chi > \sigma^2_\chi
\end{aligned}$$

The last result is because $E[\gamma|\lambda,\phi] > 1$ and $dE[\gamma|\lambda]/d\lambda > 0$. ■

Proof of Proposition 3: An exogenous reduction from E to E' forces the value of A to be lower (note that price markups from a CES utility are unaffected by A, hence, the denominator in eq. [2] does not change with varying E). Hence, $V_S(\phi_o|E') < V_S(\phi_o|E)$. Since the free entry condition of eq. [23] must hold under any circumstance, then the endogenous values need to adapt in the direction of achieving equality again. The least that is required for this purpose is that $\rho' > \rho$, otherwise cutoff productivities have to stay put as equalities eqs [14] and [16] are not affected by a change in A. Through eq. [14], an increase in ρ can only be supported by an increase in $\underline{\lambda}$, that is, in the new equilibrium $\underline{\lambda}' > \underline{\lambda}$ and by correspondence $\bar{\phi}'_o < \bar{\phi}_o$. Now, through eq. [16], a drop in $\bar{\phi}_o$ has to be supported by a drop in $\underline{\phi}_o$, that is, $\underline{\phi}'_o < \underline{\phi}_o$ in the new equilibrium and by correspondence $\bar{\lambda}' > \bar{\lambda}$. From eq. [22], a drop in $\underline{\phi}_o$ coincides with a drop in $\phi^*(\lambda)$ for all λ. The mass of outsourcing producers is the area covered by $\phi^*(\lambda)$, and it is smaller now. ■

Proof of Proposition 4: In this case, one has $V_S(\phi_o|K > 0) < V_S(\phi_o|K = 0)$. With falling expected profits, the free entry condition [23] will be violated. The rest of the reasoning is identical to that of Proposition 3. ■

Proof of Proposition 5: Start by assuming

$$(\lambda', \phi', \phi'_o, \underline{\phi}'_o, \bar{\phi}'_o, \underline{\lambda}', \bar{\lambda}') = \tau(\lambda, \phi, \phi_o, \underline{\phi}_o, \bar{\phi}_o, \underline{\lambda}, \bar{\lambda}).$$

Let $\rho' = \rho$, then eqs [14], [16] and $\phi^*(\lambda)$ will be unaffected due to the uniform scaling of productivities on both sides of the equalities. However, using eq. [13], one gets

$$V_S^\tau(\phi_o|\rho) = \pi_S(\tau\phi_o, \tau\underline{\lambda}) = \tau^{\frac{\alpha}{1-\alpha}} V_S(\phi_o|\rho) > V_S(\phi_o|\rho) \qquad \text{for } a > 1,$$

where $V_S^\tau(\cdot)$ is the option value with scaled productivities. The increase in the option value violates the free-entry condition [23]. As a result, in the new equilibrium it is required that $\rho' < \rho$. The same line of argument as in Proposition 3 evinces that in this case $\underline{\lambda}' < \tau\underline{\lambda}$, $\bar{\phi}_o' > \tau\bar{\phi}_o$ which in turn leads to $\bar{\lambda}' < \tau\bar{\lambda}$ and $\underline{\phi}_o' > \tau\underline{\phi}_o$. From eq. [22], a more than proportional increase in $\underline{\phi}_o$ coincides with an increase in $\phi^*(\lambda)$ for all λ, in turn, with an increase in the mass of outsourcing firms. ■

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