

A.1 Appendix

The extended dynamic model consists of 33 endogenous variables (expressed in log deviations from the steady state): $\rho_t, \mu_t, d_t, w_t, L_t, N_{E,t}, N_t, v_t, C_t, Y_t^C, P_t, \pi_t, i_t, \pi_t^{cpi}, r_t, v_t^E, Y_t, Y_t^T, L_t^E, L_t^C, l_t, y_t, \lambda_t, \gamma_t, Y_t^n, L_t^N, L_t^T, O_t, C_t^N, P_t^N, size_t, Pr_t, C_t^T$.

There are six exogenous AR(1) processes $\{Z_t^F, Z_t^n, g_t, \zeta_t, \Xi_t, f_{Et}\}$, where $\varepsilon_i \sim \mathcal{N}(0, \sigma_i)$ represent the innovations with $i = \{F, n, g, \zeta, \Xi, f_E\}$.

The corresponding system of 33 equations (in log-linear form) is given by:

(1) Pricing
$$\rho_t = \mu_t + \lambda_t \quad (1)$$

(2) NKPC
$$\pi_t = \beta(1 - \delta)\pi_{t+1} - \frac{\mu_t(\theta^F - 1)}{\kappa} + \zeta_t \quad (2)$$

(3) Product variety effect
$$\rho_t = N_t \frac{1}{\theta^F - 1} \quad (3)$$

(4) Profits - Formal sector
$$d_t = \mu_t(\theta^F - 1) + Y_t^C - N_t \quad (4)$$

(5) Free-entry condition
$$v_t = f_{Et} + w_t - Z_t^F + \kappa_e(N_{Et} - N_{Et-1}) \quad (5)$$

(6) Number of formal firms
$$N_t = (1 - \delta)N_{t-1} + \delta N_{Et-1} \quad (6)$$

(7) Formal consumption choice
$$\gamma_t = -C_t^T + \frac{1}{\theta}(C_t^T - C_t) \quad (7)$$

(8) Formal labor choice
$$w_t = \beta w_{t+1} + \frac{\varphi_w - 1}{\kappa_w} \left(\frac{\bar{L}^T}{\bar{O}\varphi} L_t^T - \gamma_t - w_t \right) \quad (8)$$

(9) Informal labor choice
$$\frac{\bar{L}^T}{\bar{O}\varphi} L_t^T = \gamma_t + P_t^N - P_t + Z_t^n - \frac{\psi \bar{P}r}{(1 - \psi \bar{P}r)} Pr_t \quad (9)$$

(10) Intertemporal optimality condition (shares - Formal sector)
$$\gamma_t = \gamma_{t+1} + (1 - \delta) \frac{v_{t+1}}{1 + r_t} - v_t + \frac{(\delta + r_t) d_{t+1}}{1 + r_t} \quad (10)$$

(11) Intertemporal optimality condition (bonds - Formal sector)
$$\gamma_t = \gamma_{t+1} + i_t - \pi^{cpi}_{t+1} \quad (11)$$

(12) Aggregate accounting
$$C_t + \frac{\delta}{\theta^F(\delta + r_t)}(v_t + N_{Et}) + \frac{\bar{g}Y}{C}(g_t + Y_t) = \frac{w}{C}(w_t + L_t) + \frac{1}{\theta^F}(N_t + d_t) + \frac{\psi \bar{P}r \bar{L}^N}{\frac{P}{P^N} C_{ss}}(Pr_t + P_t^N + Y_{nt} - P_t) \quad (12)$$

(13) CPI inflation
$$\pi^{cpi}_t = \pi_t - \rho_t + \rho_{t-1} \quad (13)$$

(14) Monetary policy rule
$$i_t = 0.8 i_{t-1} + 0.3 \pi_{t+1} + \Xi_t \quad (14)$$

(15) Informal output
$$Y_t^n = Z_t^n + L_t^N \quad (15)$$

(16) Consumption output - Formal sector
$$Y_t^C = C_t \quad (16)$$

(17) Formal consumption
$$C_t = C_t^T - \theta(P_t - P_t^T) \quad (17)$$

(18) Total price level
$$P_t^T = \left(\frac{\bar{P}}{\bar{P}^T} \right)^{1-\theta} P_t + \left(\frac{\bar{P}}{\bar{P}^T} \right)^{1-\theta} P_t^N \quad (18)$$

(19) Relative size of informal sector

$$size_t = L^N_t - L^T_t \quad (19)$$

(20) Real interest rate

$$r_t = i_{t-1} - \pi^{cpi}_t \quad (20)$$

(21) Investment in new firms

$$v_t^E = N_{Et} + v_t \quad (21)$$

(22) GDP - Formal sector

$$\frac{Y}{C} (Y_t (1 - \bar{g}) - \bar{g} g_t) = C_t + \frac{\delta}{\theta^F (\delta + r_t)} (v_t + N_{Et}) + \left(\frac{\bar{P}}{\bar{P}^N} \right)^\theta C^N_t \quad (22)$$

(23) GDP (Total)

$$Y^T_t = \frac{w}{C} (w_t + L_t) + \frac{1}{\theta^F} (N_t + d_t) + \frac{\bar{P}^N}{\bar{P}} (P^N_t - P_t + Y_t^n) \quad (23)$$

(24) Labor demanded by entering firms

$$L^E_t = f_{Et} + N_{Et} - Z_t^F \quad (24)$$

(25) Current labor demand - Formal sector

$$L^C_t = N_t + l_t \quad (25)$$

(26) Firm-level labor demand

$$l_t = C_t - N_t - \rho_t - Z_t^F \quad (26)$$

(27) Intertemporal optimality - Informal consumption

$$\gamma_t + P^N_t - P_t = \left(-C^T_t \right) + \frac{1}{\theta} \left(C^T_t - C^N_t \right) \quad (27)$$

(28) Total labor

$$L^T_t \bar{L} = L_t + \frac{\bar{L}^N}{\bar{L}} L^N_t \quad (28)$$

(29) Out-of-the-labor-force

$$\bar{O}O_t = -L_t - \frac{\bar{L}^N}{\bar{L}} L^N_t \quad (29)$$

(30) Firm-level output - Formal sector

$$y_t = Z_t^F + l_t \quad (30)$$

(31) Firm's shadow-value of output

$$\lambda_t = w_t - Z_t^F + i_t \frac{\iota r_{ss}}{1 + \iota r_{ss}} \quad (31)$$

(32) Producer price inflation - Formal sector

$$\pi_t = P_t - P_{t-1} \quad (32)$$

(33) Probability of getting caught - Informal sector

$$Pr_t = w_t + \frac{L_t}{w_{ss}} + \frac{\psi \bar{L}^N}{1 - \psi \bar{L}^N} Y_t^n \quad (33)$$