For Online Publication

Online Appendix A: Time-Varying Monitoring Cost

We set the monitoring cost to be time-invariant in the theory part. We find that the adverse growth effects of the borrowing constraint disappear as the economy grows. For the asymptotic irrelevance to growth, it is crucial to assume that the monitoring cost is fixed at a certain level. This is because the monitoring cost μ becomes relatively smaller as the borrower's net worth π_1 (the entrepreneurial profit) increases along with the growing economy. Then, from the financial intermediaries' perspective, the borrower's incentive to renege on his debt becomes negligible, endogensouly relaxing the incentive compatibility *IC*.

However, if the monitoring cost also increases along with economic growth, the asymptotic irrelevance would not hold for sure. This is because, if this were the case, the monitoring cost would not be negligible for the financial intermediaries even in the very long run unless the growth rate of the monitoring cost, g_{μ_t}, is *strictly* smaller than that of the borrower's profit, $g_{\pi_{i,t}}$. This means that the borrowing constraint may adversely affect growth *permanently* in this case.

To demonstrate this point more clearly, the monitoring cost is redefined as the cost depending on the wage per efficiency unit of labor w and the TFP level A: $\mu_t \equiv \mu\left(w_t, A_t\right)$. Then, it is likely that $\mu_w\left(w_t, A_t\right) > 0$ since monitoring requires labor service. Note that w remains constant in the stationary state (see Proposition 3), and thus, the growth rate of the monitoring cost g_μ will be solely determined by $\mu_A\left(w_t, A_t\right)$ in the stationary state. Now, suppose that $\mu_A < 0$, then the monitoring cost will diminish over time, i.e., $g_\mu < 1$, since the TFP level A is continually growing, while w is fixed on the balanced growth path.[1] More generally, we can summarize the negative growth effects with the time-varying monitoring cost as follows:

[1] One can interpret this case as an example where financial intermediation technology evolves *relatively* faster than an entrepreneur's defection technology. Refer to Laeven, Levine and Michalopoulos (2015) for the race between the financial intermediation and defection technologies and its implications on growth.

Proposition 6 ***Growth Effect with Time-Varying Monitoring Cost:*** *Suppose that* $\mu = \mu(w, A)$ *is continuously differentiable in* A*. Then, we have the following results:*

Case (1) $\boldsymbol{\mu_A(w, A) \leq 0}$*: The growth effects of the borrowing constraint are temporary.*

Case (2) $\boldsymbol{\mu_A(w, A) > 0}$*: (i) If* $\mu(w, \phi A) = \phi^n \mu(w, A)\ \forall \phi \geq 1$ *with* $n \in (0, 1)$*, then the growth effects of the borrowing constraint are temporary.*

(ii) If $\mu(w, \phi A) = \phi \mu(w, A)\ \forall \phi \geq 1$*, then the growth effects of the borrowing constraint are permanent.*

(iii) If $\mu(w, \phi A) = \phi^n \mu(w, A)\ \forall \phi \geq 1$ *with* $n > 1$*, then the economy stops growing in the long run, i.e.,* $g_{Y,t} \to 1$*.*

Case (1) of Proposition 6 indicates that the growth effects of the borrowing constraint are temporary if the monitoring cost never grows in the long run, i.e., $g_\mu \leq 1$. This is because the growth rate of the monitoring cost is smaller than the growth rate of the borrower's profit g_{π_i}; recall that g_{π_i} is larger than one. The borrowing constraint, however, can be still irrelevant to growth even when the monitoring cost continually grows. More precisely, this is Case (2-i) where the monitoring cost grows slower than the borrower's profit, i.e., $g_{\pi_i} > g_\mu > 1$, and hence, it becomes negligible in the long run. In contrast, if it grows as fast as the entrepreneurial profit, i.e., $g_\mu = g_{\pi_i}$, the negative growth effects are permanent, meaning the growth rate in the stationary state gets smaller as the borrowing constraint is more severe. This results, which corresponds to Case (2-ii), is quite intuitive since the monitoring cost will not be trivial any longer even in the long run when it grows as fast as the economy, resulting in the negative growth effects *permanently*. Finally, Case (2-iii) corresponds to the case where the lending rate diverges indefinitely if the monitoring cost increases at an even faster rate than the economy and, consequently, the borrower's net worth. As a result, no one will be willing to invest in R&D, which will ultimately halt the growth of the economy. Based on the results, we offer the following example of the time-varying monitoring cost and its growth implications:

Example: Linear Monitoring Costs

Suppose that the monitoring cost is linear in the efficiency unit of labor as follows:

$$\mu_t = w_t \ell_m (A_t)$$

where ℓ_m is the efficiency unit of labor required for monitoring, which is determined by the current TFP level such that $\ell_m : \mathbb{R}_{++} \to \mathbb{R}_+$. Assume that ℓ_m is twice continuously differentiable. Then, from Proposition 6, we have the following results:

Case (1) $\ell_m' (A_t) \leq 0$: The monitoring cost converges to zero, i.e., $\mu_t \to 0$, and therefore, the negative growth effects are temporary.

Case (2-i) $\ell_m' (A_t) > 0, \ell_m'' (A_t) < 0$: The gross growth rate of the monitoring cost converges to unity, i.e., $g_{\mu_t} \to 1$, since A_t increases to infinity. Hence, the negative growth effects are temporary.

(2-ii) $\ell_m' (A_t) > 0, \ell_m'' (A_t) = 0$: The gross growth rate of the monitoring cost is the same as the growth rate of A_t, rendering the negative growth effects permanently.

(2-iii) $\ell_m' (A_t) > 0, \ell_m'' (A_t) > 0$: Growth stops in the long run.

Which Case is Empirically Plausible?

Then, which one is the empirically plausible case in the real world? First, Case (2-iii) seems not the one because the interest rate spread increases forever if it is true (see the proof of Proposition 6) while, in the real world, the interest rate spread decreases and then stabilizes over time (see Figure 2).

Similarly, Case (2-ii) seems not empirically plausible either. If it is true, the theory predicts that the growth rate, the investment rate and the interest rate spread will be constant over time, respectively, once r_t is fixed; see the proofs of Proposition 4 and 6. This implies that the severity of the borrowing constraint is not endogenously relaxed, contradicting the actual transition features depicted in Figure 2. As shown in the figure, the averaged growth rate and investment rate increase along with a decreasing trend of the interest rate spread, reflecting the endogenous relaxation of the borrowing constraint. Therefore, this case does not support the asymptotic irrelevance property, which contradicts the empirical finding in Section 4; recall that δ turned out strictly positive, not zero.

Then, only Case (1) and Case (2-i) remain. Both cases are empirically plausible since they are consistent with the essential features of transition dynamics in reality. Therefore, we have assumed that the monitoring cost is fixed at a certain level over time, a specific example of Case (1) for a more straightforward exposition of the main idea without any change in the theoretical results.

Online Appendix B: Proofs

Proof of Lemma 1:

Let us suppress the time subscript for convenience. First, from the first-order conditions for (k_0, ℓ_0), one can notice that $\pi_0 > 0$ for any given price vector, $(r, w) \in \mathbb{R}^2_{++}$, since $A > 0$. Then, $k_0 > 0$ and $\ell_0 > 0$, so that the non-negative constraints for (k_0, ℓ_0) are not binding. Also, the first-order conditions for (k_0, ℓ_0) and (k_1, ℓ_1) imply that $k_1 = \gamma k_0$ and $\ell_1 = \gamma \ell_0$, and therefore, $\pi_1 = \gamma \pi_0$. Since $x'(z) < 0$, $\lim_{z \to z_L} x(z) = \infty$ and $\lim_{z \to z_H} x(z) = 0$, there is a unique interior solution, $z^* \in (z_L, z_H)$, which solves $p\pi_1 - \pi_0 = (p\gamma - 1)\pi_0 = p(1+i)x(z^*)$ for any given $i \geq 0$ if, and only if, $p\gamma - 1 > 0$, or equivalently, $p\gamma > 1$. ■

Proof of Lemma 2:

Let us suppress the time subscript for convenience. Since the objective function is linear in the choice variables, the optimal lending rate $1 + i$ should be minimized while satisfying both PC and IC. Note that $\pi_1 - [1 + i(z)]\, x(z) \geq \pi_0 \ \forall z \in [z^*, z_H]$ and $\pi_0 > 0$ since $A > 0$. This implies that $\pi_1 > 0$. Also, $1 + i(z) \geq 1 + r - \delta$ should be satisfied, so that $1 + i(z) > 0$ since $1 + r - \delta > 0$. Then, we can replace PC and IC with the following inequality:

$$\frac{p}{(1-p)\mu} x(z)[1 + i(z)] - \frac{1 + r - \delta}{(1-p)\mu} x(z) \geq \frac{x(z)}{\pi_1}[1 + i(z)] \qquad \text{(B.1)}$$

It is easily shown that if inequality (B.1) is satisfied, there exists a $\eta(z)$ satisfying IC. More specifically, if the LHS is equal to the RHS, there exists a unique $\eta(z)$ such that IC holds with equality. Rearranging inequality (B.1) yields:

$$[p\pi_1 - (1-p)\mu][1 + i(z)] \geq \pi_1(1 + r - \delta) \qquad \text{(B.2)}$$

Hence, we must assume that $p\pi_1 - (1-p)\mu > 0$ as in Assumption 2 since both the RHS of inequality (B.2) and $1 + i(z)$ are strictly positive. In reverse, once $1 + i(z) \geq 1 + r - \delta$ is satisfied, Assumption 2 must be satisfied; recall that $\pi_1 > 0$. Then, to minimize $(1 + i)$, inequality (B.2) should hold with equality, resulting in equation (1). Since inequality (B.2) holds with equality, IC also holds with equality, yielding equation (2). Then, $\eta \geq 0 \; \forall z \in [z_L, z_H]$ by Assumption 2. Finally, we must check whether or not $1 \geq \eta \; \forall z \in [z^*, z_H]$. First note that $\eta(z) = [1 + i(z)] x(z) / \pi_1$ from equations (1) and (2). Since $p\{\pi_1 - [1 + i(z)] x(z)\} \geq \pi_0 \; \forall z \in [z^*, z_H]$ and $\pi_0 > 0$, $1 > \eta$. In sum, $\eta \in [0, 1) \; \forall z \in [z^*, z_H]$ as desired. ■

Proof of Proposition 1:

Note first that $k_{1,t} = \gamma k_{0,t}$ and $l_{1,t} = \gamma l_{0,t}$. Then, from the law of large numbers, we have:

$$K_t \equiv \int k_{i,t}(z) \, dF(z) = \left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right] k_{0,t}$$
$$L_t \equiv \int \ell_t(z) \, dF(z) = \left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right] \ell_{0,t}$$

Hence,

$$\left(K_t^{\alpha} L_t^{1-\alpha}\right)^v = \left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right]^v \left(k_{0,t}^{\alpha} \ell_{0,t}^{1-\alpha}\right)^v \tag{B.3}$$

Since $Y_t \equiv \int y_t(z) \, dF(z) = \left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right] A_{t-1}^{1-v} \left(k_{0,t}^{\alpha} \ell_{0,t}^{1-\alpha}\right)^v$, equation (B.3) yields:

$$Y_t \equiv \left(\left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right] A_{t-1}\right)^{1-v} \left(K_t^{\alpha} L_t^{1-\alpha}\right)^v$$

Finally, from the perfect competition in the factor markets, we have:

$$w_t = v(1 - \alpha) A_{t-1}^{1-v} k_{0,t}^{v\alpha} \ell_{0,t}^{v(1-\alpha)} \ell_{0,t}^{-1}$$

and observe that:

$$\begin{aligned} \frac{\partial Y_t}{\partial L_t} &= v(1 - \alpha) \left[p\gamma \left\{1 - F\left(z_{t-1}^*\right)\right\} + F\left(z_{t-1}^*\right)\right]^{1-v} A_{t-1}^{1-v} \left(K_t^{\alpha} L_t^{1-\alpha}\right)^v L_t^{-1} \\ &= v(1 - \alpha) A_{t-1}^{1-v} k_{0,t}^{v\alpha} \ell_{0,t}^{v(1-\alpha)} \ell_{0,t}^{-1} = w_t \end{aligned}$$

where we use equation (B.3) for the second equality. Note also that $\partial Y_t / \partial L_t = v(1 - \alpha) Y_t / L_t$. The same argument can be applied to the remaining part. ■

Proof of Corollary 1:

Since π_1 is decreasing in μ (see (iii) of Proposition 2), the interest rate spread, $\Delta \equiv p\pi_1/\left[p\pi_1 - (1-p)\,\mu\right]$, is increasing in μ. ■

Proof of Proposition 2:

Without loss of generality, we let $L_0 = 1$ for simplicity. Since r_t is fixed at r in a steady state, from equation (12), we have:

$$r = \alpha v \left[A_{-1}\psi\left(z^*\right)\right]^{1-v} k^{\alpha v - 1}$$

Then, the capital per efficiency unit of labor in the steady state, k, is given by:

$$k = \left(\frac{\alpha v}{r}\right)^{\frac{1}{1-\alpha v}} \left[A_0 \psi\left(z^*\right)\right]^{\frac{1-v}{1-\alpha v}} \tag{B.4}$$

From the entrepreneur's profit maximization problem, we have the following fundamental equation that determines z^* for any given (A_0, r):

$$\left(\frac{p\gamma - 1}{p\gamma}\right)\left[p\gamma\left(1-v\right)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}} A_0^{\frac{1-v}{1-\alpha v}} \left[\psi\left(z^*\right)\right]^{\frac{-v(1-\alpha)}{1-\alpha v}} - (1-p)\,\mu\right] = (1 + r - \delta)\, x\left(z^*, A_0\right) \tag{B.5}$$

The RHS is continuous at any $z \in [z_L, z_H]$ while strictly decreasing from infinity to zero; recall that $x_z(z, A) < 0$, $\lim_{z \to z_L} x(z, A) = \infty$ and $\lim_{z \to z_H} x(z, A) = 0$. Meanwhile, $\psi'(z^*) < 0$ and $\psi(z_L) = p\gamma > 1$ and $\psi(z_H) = 1$, implying that the LHS is continuous at any $z \in [z_L, z_H]$ while strictly increasing in z^*. Now, note that $z^* > z_L$ since $\lim_{z \to z_L} x(z, A) = \infty$. Meanwhile, $z^* \leq z_H$ if, and only if,

$$p\gamma\left(1-v\right)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}} A_0^{\frac{1-v}{1-\alpha v}} \left[\psi\left(z^*\right)\right]^{\frac{-v(1-\alpha)}{1-\alpha v}} = p\pi_1\big|_{z^* = z_H} \geq (1-p)\,\mu$$

The inequality is satisfied by the assumption that $p\pi_1 - (1-p)\,\mu > 0$ in any equilibrium to guarantee $i \geq r - \delta$. Hence, $z^* \in (z_L, z_H)$. Finally, the LHS shifts downward when μ increases, so that z^* increases in μ, so does ψ. Then, both k and y decrease in μ, respectively, from equations (B.4) and (8). Since $Y = yL$, Y is also decreasing in μ. This means that $\hat{y} = Y/\hat{N}$ is also decreasing in μ. Finally, from equations (9) and (10), π_i $(i = 1, 2)$ is decreasing in μ. ■

Proof of Proposition 3:

We let $\varphi_t \equiv (A_t/L_t)^{1-v}$ for convenience. First, from equation (12), we have:

$$r_t = \alpha v \varphi_t k_t^{\alpha v - 1}$$

Hence, k_t is given by:

$$k_t = \left(\frac{av\varphi_t}{r_t} \right)^{\frac{1}{1-\alpha v}} \tag{B.6}$$

First note that $\varphi_t \equiv (A_0/L_0)^{1-v} \equiv \varphi$ where φ is a constant due to the assumption that $g_{L,t} = g_{A,t}$ $\forall t \geq 0$ for stationary growth. Then, $g_k = 1$ since $g_r = 1$ in the stationary state; see Definition 3. To show $g_w = 1$, observe equation (11) to have:

$$w_t = (1 - \alpha) v \varphi k_t^{av}$$

which means that $g_w = 1$ in the long run since $g_k = 1$, and this is consistent with the definition of long-run growth steady state. Also, from equation (8), $y = \varphi k^{\alpha v}$, and therefore, $g_y = 1$, proving (ii). To prove (iii), suppose that $g_{z^*} = 1$; we will verify this later. Then, $g_A = \psi(z^*)$ from equation (13), and hence, $g_L = \psi(z^*)$. Now, recall that $Y_t \equiv y_t L_t$, implying that $g_Y = g_L = \psi(z^*)$. To prove (iv), note that:

$$\pi_{0,t} = (1 - v) \varphi_t \left[\psi\left(z_{t-1}^*\right) \right]^{-1} L_t k_t^{av} = (1 - v) \varphi_t L_{t-1} k_t^{av}$$

which is obtained from equation (9). Hence, $g_{\pi_i} = \psi(z^*)$ $(i = 1, 2)$ since $\pi_{1,t} = \gamma \pi_{0,t}$ from equation (10). Now, we will check that $g_i = 1$ in the long run. From equations (1) and (2), we have:

$$g_{(1+i)} = \frac{(1+i)'}{1+i} = \frac{p\gamma\pi_0'}{p\gamma\pi_0} \frac{p\gamma\pi_0 - (1-p)\mu}{p\gamma\pi_0' - (1-p)\mu} = \psi(z^*) \frac{p\gamma\pi_0 - (1-p)\mu}{p\gamma\pi_0' - (1-p)\mu} \simeq \psi(z^*) \frac{1}{\psi(z^*)} = 1$$

where the prime denotes variables in the following period. The last approximation holds for large enough π_1, and it must hold in the long-run if π_1 grows over time. We will prove later that this is true in the long run by showing that $\psi(z^*) > 1$; recall that the growth rate of π_i is equal to $\psi(z^*)$ $(i = 1, 2)$. Hence, asymptotically, $1+i = 1+i'$ that implies $g_i = 1$, and therefore, $g_\Delta = 1$ from equation (1). We should prove that $g_{z^*} = 1$ too. From the entrepreneur's problem, the skill threshold z^* is given by:

$$(p\gamma - 1)\,\pi_0 = p\,(1+i)\,x\,(z^*, A)$$

which implies that:

$$\frac{(1+i)'}{1+i} = \frac{\psi\,(z^*)\,x\,(z^*, A)}{x\,(z'^*, A')} = \frac{\psi\,(z^*)\,x\,(z^*, A)}{x\,(z'^*, \psi\,(z^*)\,A)}$$

since $A' = \psi(z^*)\,A$ and $g_{\pi_0} = \psi(z^*)$ in the steady state. Since $g_i = 1$ in the long run, we have:

$$x\,(z'^*, \psi\,(z^*)\,A) = \psi\,(z^*)\,x\,(z^*, A)$$

Since $x(z, A)$ is linear in A while strictly decreasing in z, z'^* should be equal to z^*, implying that $g_{z^*} = 1$ in the long run. Now, we will prove the uniqueness of the balanced growth path. First note that $1 + i = (1 + r - \delta)/p$ in the asymptotic stationary state, which means that $\lim_{t\to\infty} \Delta_t = 1$. The asymptotic threshold z^* is, therefore, determined by the following equation from the entrepreneur's profit maximization problem:

$$(p\gamma - 1)\,\pi_0' = (1 + r - \delta)\,x\,(z^*, A)$$

From equation (8), knowing that $g_{L,t} = g_{A,t}$ $\forall t$, $\pi_0' = (1-v)\,\varphi L k^{\alpha v}$. Then, from equation (9), we have:

$$(p\gamma - 1)\,(1-v)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}}\left(\frac{A_{-1}}{L_{-1}}\right)^{\frac{1-v}{1-\alpha v}} L = (1 + r - \delta)\,x\,(z^*, A)$$

Finally, using the linearity of $x(z, A)$ in A and $g_{L,t} = g_{A,t} = \psi(z_t^*)$, we have:

$$(p\gamma - 1)\,(1-v)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}}\left(\frac{A_{-1}}{L_{-1}}\right)^{\frac{1-v}{1-\alpha v}} = (1 + r - \delta)\,x\,(z^*, A_{-1}) \qquad \text{(B.7)}$$

equation (B.7) is similar to equation (B.5) but different in that the monitoring cost

μ is irrelevant to the determination of z^*. Then, it immediately follows that $z^* > z_L$ since $\lim_{z \to z_L} x(z, A_t) = \infty$. Also, z^* is below its upper bound, i.e., $z^* < z_H$. This is because the LHS of equation (B.7) is $p\pi_1 - (1-p)\mu$ where $\mu = 0$, while $p\pi_1 - (1-p)\mu > 0$ for guaranteeing $i \geq r - \delta$. Hence, z^* is uniquely pinned down, and it is interior of $[z_L, z_H]$. That is, $z^* \in (z_L, z_H)\ \forall (A_0, L_0, r)$. Finally, it is straightforward to show that $\psi(z^*) > 1$ since the p.d.f. of skill distribution, $F'(\cdot)$ satisfies $F'(x) > 0$ $\forall x \in [z_L, z_H]$. ■

Proof of Corollary 2:

The asymptotic skill threshold z^* is determined by equation (B.7). Note that z^* is independent of μ since equation (B.7) does not include μ. Note now that the gross growth rate $\psi(z_t^*)$ is solely dependent on z_t^*, the asymptotic growth rate is independent of μ. ■

Proof of Corollary 3:

On the transition phase, z_t^* is determined by the following equation, which is analogous to (B.5):

$$\left(\frac{p\gamma - 1}{p\gamma}\right)\left[p\gamma(1-v)\left(\frac{\alpha v}{r_{t+1}}\right)^{\frac{\alpha v}{1-\alpha v}}\left(\frac{A_t}{L_{t+1}}\right)^{\frac{1-v}{1-\alpha v}} L_{t+1}\left[\psi(z_t^*)\right]^{\frac{-v(1-\alpha)}{1-\alpha v}} - (1-p)\mu\right]$$
$$= (1 + r_{t+1} - \delta)\, x(z_t^*, A_t)$$

which is rewritten as follows:

$$\left(\frac{p\gamma - 1}{p\gamma}\right)\left[p\gamma(1-v)\left(\frac{\alpha v}{r_{t+1}}\right)^{\frac{\alpha v}{1-\alpha v}}\left(\frac{A_{-1}}{L_{-1}}\right)^{\frac{1-v}{1-\alpha v}} L_t - (1-p)\mu\right] = (1 + r_{t+1} - \delta)\, x(z_t^*, A_t)$$

since $g_A = \psi(z_t^*)$, and $g_{L,t} = g_{A,t}$. From Lemma 1, z_t^* is uniquely pinned down given any vector of (A_t, L_t, r_{t+1}). Then, one can notice that an increase in μ shifts the LHS downward, so that z_t^* is increasing in μ, implying that the gross growth rate in period t, $\psi(z_t^*)$, is decreasing in μ. Meanwhile, we have the following equation from (B.6):

$$k_{t+1} = \left(\frac{\alpha v \varphi}{r_{t+1}}\right)^{\frac{1}{1-\alpha v}} \tag{B.8}$$

which implies that k_{t+1} is independent of μ. Also, from equations (9) and (10), $\pi_{i,t+1}$ $(i = 0, 1)$ is decreasing in z_t^* and hence, decreasing in μ as well; recall that z_t^* is increasing in μ. This proves (iii). Therefore, Δ_t is increasing in μ by equation (1). From Proposition 1, $g_{Y,t} = \psi(z_t^*)$, so that $g_{Y,t}$ is decreasing in μ. Hence, $g_{\hat{y},t}$ is also decreasing in μ since it is monotone increasing in $g_{Y,t}$. ■

Proof of Proposition 4:

(i) The asymptotic convergence is trivial from the proof of Proposition 3. To prove the decreasing property, one can use the fact from the proof of Corollary 3 that z_t^* is determined by the following equation for all $t \geq 0$:

$$\left(\frac{p\gamma - 1}{p\gamma}\right)\left[p\gamma(1-v)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}}\left(L_t^{v(1-\alpha)}A_t^{1-v}\right)^{\frac{1}{1-\alpha v}} - (1-p)\mu\right] = (1 + r - \delta)\, x(z_t^*, A_t)$$

where we let r_t be constant at r $\forall t \geq 0$. Then, we have:

$$\begin{aligned}
\psi(z_t^*)\, x\left(z_{t+1}^*, A_t\right) &= x\left(z_{t+1}^*, A_{t+1}\right) \\
&= (1 + r - \delta)^{-1}\left(\frac{p\gamma - 1}{p\gamma}\right)\left[p\gamma(1-v)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}}\left(L_{t+1}^{v(1-\alpha)}A_{t+1}^{1-v}\right)^{\frac{1}{1-\alpha v}} - (1-p)\mu\right] \\
&\equiv R^{-1}X_1\left[C_2\left(L_{t+1}^{v(1-\alpha)}A_{t+1}^{1-v}\right)^{\frac{1}{1-\alpha v}} - (1-p)\mu\right]
\end{aligned}$$

where $R \equiv 1 + r - \delta$, $C_1 \equiv (p\gamma - 1)/(p\gamma)$ and $C_2 \equiv p\gamma(1-v)$. Hence, we obtain:

$$\begin{aligned}
x\left(z_{t+1}^*, A_t\right) &= \{\psi(z_t^*)\}^{-1} R^{-1}C_1\left[C_2\left(L_{t+1}^{v(1-\alpha)}A_{t+1}^{1-v}\right)^{\frac{1}{1-\alpha v}} - (1-p)\mu\right] \\
&= \{\psi(z_t^*)\}^{-1} R^{-1}C_1\psi(z_t^*)\left[C_2\left(L_t^{v(1-\alpha)}A_t^{1-v}\right)^{\frac{1}{1-\alpha v}} - \{\psi(z_t^*)\}^{-1}(1-p)\mu\right] \\
&> R^{-1}C_1\left[C_2\left(L_t^{v(1-\alpha)}A_t^{1-v}\right)^{\frac{1}{1-\alpha v}} - (1-p)\mu\right] \\
&= x(z_t^*, A_t)
\end{aligned}$$

where the second equality holds since $g_{L,t} = g_{A,t} = \psi(z_t^*)$. Then, we have:

$$x\left(z_{t+1}^*, A_t\right) > x(z_t^*, A_t) \quad \forall t \geq 0$$

which implies $z^*_{t+1} < z^*_t \ \forall t \geq 0$.

(ii): This is straightforward from (i) of Proposition 4. The convergence holds by the continuity of ψ in z^*.

(iii): Note that, given any constant r over time, $g_{k,t} = 1 \ \forall t \geq 0$ from equation (B.8). Hence, let us denote k_t by $k \ \forall t \geq 0$. Then, we have:

$$\begin{aligned}
\iota_t \equiv \frac{I_t}{Y_t} &\equiv \frac{K_t - (1-\delta) K_{t-1}}{Y_t} \\
&= \frac{L_t}{Y_t}\left(1 - \frac{1-\delta}{\psi\left(z^*_{t-1}\right)}\right) k \\
&= \left(\frac{A_{-1}}{L_{-1}}\right)^{v-1}\left(1 - \frac{1-\delta}{\psi\left(z^*_{t-1}\right)}\right) k^{1-\alpha v} \\
&< \left(\frac{A_{-1}}{L_{-1}}\right)^{v-1}\left(1 - \frac{1-\delta}{\psi\left(z^*_{t}\right)}\right) k^{1-\alpha v} = \iota_{t+1}
\end{aligned}$$

where we use $L_{t-1}/L_t = 1/\psi\left(z^*_{t-1}\right)$, and $k_t = k_{t-1} = k$ in the second equality. The third equality holds since $Y_t/L_t = (A_t/L_t)^{1-v} k_t^{\alpha v}$ from Proposition 1 with the assumption that $g_{A,t} = g_{L,t} \ \forall t \geq 0$. Now, note that $1 - (1-\delta)/\psi\left(z^*_{t-1}\right) > 0 \ \forall t \geq 0$ since $\psi\left(z^*_{t-1}\right) > 1 \ \forall t \geq 0$; from Lemma 1, z^*_t is an interior solution $\forall t \geq 0$, implying that $\psi\left(z^*_t\right) > 1 \ \forall t \geq 0$ since $f(z) > 0 \ \forall z$ where $F'(z)$ is the p.d.f. of the skill z. Also, we know from part (i) of Proposition 4 that z^*_t decreases over time, and therefore, $\psi\left(z^*_t\right) > \psi\left(z^*_{t-1}\right)$, proving that $\psi\left(z^*_t\right)$ increases $\forall t \geq 0$. The convergence is obvious by the continuity of ι in z^*; recall that $\psi(z)$ is continuous at any z.

(iv): Note first that:

$$\rho_{t+1} \equiv \frac{X_{t+1}}{Y_{t+1}} > \frac{X_t}{Y_t} \equiv \rho_t \Leftrightarrow g_{X,t} \equiv \frac{X_{t+1}}{X_t} > \frac{Y_{t+1}}{Y_t} \equiv g_{Y,t}$$

From Proposition 1 with the result from the proof of (iii) of Proposition 4 such that $g_{k,t} = 1 \ \forall t \geq 0$ under a constant interest rate r, one can show that: $g_{Y,t} = \psi\left(z^*_t\right)$ $\forall t \geq 0$. Meanwhile, we also have:

$$g_{X,t} \equiv \frac{X_{t+1}}{X_t} = \frac{\int_{z^*_{t+1}}^{z^H} x\left(z, A_{t+1}\right) dF\left(z\right)}{\int_{z^*_t}^{z^H} x\left(z, A_t\right) dF\left(z\right)} = \frac{\psi\left(z^*_t\right) \int_{z^*_{t+1}}^{z^H} x\left(z, A_t\right) dF\left(z\right)}{\int_{z^*_t}^{z^H} x\left(z, A_t\right) dF\left(z\right)} > \psi\left(z^*_t\right) = g_{Y,t}$$

where the third equality comes from the linearity of $x(z, A)$, and the last inequality comes from $z_{t+1}^{*} < z_t^{*}$ from the part (i). Hence, $\rho_{t+1} > \rho_t \ \forall t$. The convergence holds by the continuity of ρ at any z.

(v): From equation (13) and $g_{A,t} = g_{L,t} = \psi\left(z_{t-1}^{*}\right)$, we have:

$$y_t = \frac{A_{-1}}{L_{-1}} k^{\alpha v}$$

Notice first from the proof of (iii) of Proposition 4 that k is time-invariant when r is time-invariant. Hence, y_t is also time-invariant. Also, from Proposition 1, $w_t = v(1-\alpha) Y_t/L_t = v(1-\alpha) y_t$, implying that w_t is also time-invariant. Then, the saving, which can be defined by the saving function, $s_t \equiv s(r_{t+1}, w_t)$, is time-invariant too. Knowing it all, from equation (5), we obtain:

$$\theta_t \equiv \frac{B_t^f}{Y_t} = k + \rho_t - \frac{s(k)}{y\psi\left(z_{t-1}^{*}\right)} < k + \rho_{t+1} - \frac{s(k)}{y\psi\left(z_t^{*}\right)} \equiv \frac{B_{t+1}^f}{Y_{t+1}} = \theta_{t+1}$$

where $\rho_t \equiv X_t/Y_t$. We know from (ii) and (iv) of Proposition 4 that both ρ_t and $\psi\left(z_{t-1}^{*}\right)$ are increasing over time, converging to their respective asymptotic values. Hence, $\theta_t \equiv B_t^f/Y_t$ is increasing $\forall t \geq 0$. The convergence is trivial by the continuity.

■

Proof of Proposition 5:

One can solve for the threshold z_t^{*} by the following equation:

$$(p\gamma - 1)\, \mathbb{E}_t\left[\pi_{0,t+1}\right] = (1 + r_{t+1} - \delta)\, x\left(z_t^{*}, A_t\right) \mathbb{E}_t\left[\Delta_{t+1}\right] \tag{B.9}$$

where $\pi_1\left(s^{t+1}\right) = \gamma(1-v)\left(\frac{\alpha v}{r_{t+1}}\right)^{\frac{\alpha v}{1-\alpha v}} \left[L_{t+1}^{-1}\psi\left(z_t^{*}\right)\right]^{\frac{-v(1-\alpha)}{1-\alpha v}} \tilde{A}_t^{\frac{1-v}{1-\alpha v}}$ and $\pi_0\left(s^{t+1}\right) = \gamma^{-1}\pi_1\left(s^{t+1}\right)$ from equations (9), (10) and (12). Then, equation (B.9) is rewritten as follows:

$$\left(\frac{p\gamma - 1}{p\gamma}\right) \frac{\mathbb{E}_t\left[\tilde{A}_t^{\frac{1-v}{1-\alpha v}}\right]}{\mathbb{E}_t\left[\frac{\tilde{A}_t^{\frac{1-v}{1-\alpha v}}}{p\gamma(1-v)\left(\frac{\alpha v}{r_{t+1}}\right)^{\frac{\alpha v}{1-\alpha v}}\tilde{A}_t^{\frac{1-v}{1-\alpha v}} - (1-p)\mu}\right]} = (1 + r_{t+1} - \delta)\, x\left(z_t^{*}, A_t\right) \tag{B.10}$$

First, from equation (B.10), the equilibrium threshold z_t^* is uniquely pinned down as an interior point on $[z_L, z_H]$ since the LHS of equation (B.10) is strictly positive and independent of z. Meanwhile, the RHS of equation (B.10) is strictly decreasing in z from infinity to zero. Note also that $\pi_{0,t+1}$ is strictly concave in $\tilde{A}_t$, and therefore, the LHS of equation (B.9) is strictly decreasing in mean-preserving spread of the random shock s. Now, suppose that there is no borrowing constraint, i.e., $\mu = 0$, and hence, $\Delta_{t+1} = 1$. Then, the RHS of equation (B.9) is equal to $(1 + r_{t+1} - \delta)\, x\,(z_t^*, A_t)$. In this situation, z_t^* increases along with mean-preserving spread, and therefore, the mean growth rate $\psi\,(z_t^*)$ decreases with mean-preserving spreaB. Hence, the mean growth rate of Y_t also decreases as the volatility increases; recall that $g_{Y_{t+1}}$ is strongly monotonic in $\psi\,(z_t^*)$ by Corollary 3. In other words, growth decreases by higher volatility even without the borrowing constraint. Next, to prove the amplification effect, note that the wedge, $\Delta_{t+1} \equiv \frac{p\pi_{1,t+1}}{p\pi_{1,t+1}-(1-p)\mu}$, is strictly convex and decreasing in $\pi_{1,t+1}$, while $\pi_{1,t+1}$ is strictly concave in $\tilde{A}_t$. This implies that $\frac{p\pi_{1,t+1}}{p\pi_{1,t+1}-(1-p)\mu}$ is strictly convex in $\tilde{A}_t$. Hence, the RHS of equation (B.9) increases with mean-preserving spread. Hence, z_t^* increases more compared to the case without the borrowing constraint. ■

Proof of Proposition 6:

Case 1) The case that $\mu_A\,(w, A) = 0$ is equivalent with the baseline case that the monitoring cost is fixed over time, i.e., $g_{\mu,t} = 1\ \forall t$. This is because wage w_t is constant in the stationary state. Hence, the growth effects disappear asymptotically in this case. Also, $\mu_A\,(w, A) < 0$ implies that μ decreases in each period in the steady state since $\psi\,(z^*) > 1$ implies that A continuously increases. Hence, $g_\mu < 1$, and μ converges to its lower bound, 0; recall that μ is assumed to be greater than zero. Hence, the economy asymptotically boils down to the standard growth model without the borrowing constraint, so that the growth effects disappear in the long run.

Case 2) (i) In this case, it is trivial to show that $g_{\mu,t} \in (1, g_{\pi_{i,t}})\ \forall t \geq 0$. We now let $\varepsilon_t \equiv g_{\pi_{1,t}}/g_{\mu,t} > 1\ \forall t \geq 0$. Then, $\forall M > 0$, $\exists T > 0$ such that $\pi_{1,t}/\mu_{t+1} > M\ \forall t > T$ since one can choose any $T > 0$ satisfying that $\underline{\varepsilon}^{T+1} > M\mu_0/\pi_{1,0}$ where $\underline{\varepsilon} \leq \min\{\varepsilon_1, \varepsilon_2, \cdots \varepsilon_{T+1}\}$. Then, by picking an arbitrary large M such that $\mu_{t+1}/\pi_{1,t} = 1/M \simeq 0$, we have the following result for large enough $t > T$ satisfying that $\mu_{t+1}/\pi_{1,t} \simeq 0$:

$$g_{(1+i)} = \frac{p\pi_1'}{p\pi_1}\frac{p\pi_1 - (1-p)\,\mu}{p\pi_1' - (1-p)\,\mu'} = \psi\left(z^*\right)\frac{p - (1-p)\,\mu/\pi_1}{p\pi_1'/\pi_1 - (1-p)\,\mu'/\pi_1} \simeq \psi\left(z^*\right)\frac{1}{\psi\left(z^*\right)} = 1$$

which implies $g_i = 1$ asymptotically. Then, we can prove $g_{z^*} = 1$ by using the same method applied to the proof of Proposition 3. Finally, $\mu_{t+1}/\pi_{1,t+1} \simeq 0$ for any $t > T$ since $\mu_{t+1}/\pi_{1,t} \simeq 0\ \forall t > T$ and $\pi_{1,t+1} > \pi_{1,t}$. Hence, the determination of the asymptotic threshold z^* is irrelevant to the monitoring cost.

Case 2) (ii) In this case, it is straightforward that $g_{\pi_1,t} = g_{\mu,t}\ \forall t \geq 0$. Hence, the equation that determines z^* becomes different from equation (B.7), and it is given by:

$$\frac{p\gamma - 1}{p\gamma}\left[p\gamma\left(1-v\right)\left(\frac{\alpha v}{r}\right)^{\frac{\alpha v}{1-\alpha v}}\left(\frac{A_{-1}}{L_{-1}}\right)^{\frac{1-v}{1-\alpha v}} - (1-p)\,\mu_0\right] = (1 + r - \delta)\,x\left(z^*, A_{-1}\right)$$

where z^* is decreasing in the initial monitoring cost μ_0. Hence, the initial monitoring cost reduces the long-term growth rate $\psi\left(z^*\right)$ in the steady state.

Case 2) (iii) In this case, $g_{\mu,t} > g_{\pi_{i,t}} = \psi\left(z_t^*\right) > 1$. Hence, $1 + i$ grows faster than π_1 over time while letting that Assumption 2 is still satisfied. Then, the equilibrium threshold $z_t^* \in \left(z_L, z_H\right)$ must decrease over time, converging to its minimum value, z_L. This, in turn, implies that the growth rate of the economy converges to $\psi\left(z_L\right) = 1$, meaning no-growth in the stationary state. ■

Online Appendix C: The Closed Economy

Switching to recursive notation where primes denote next period variables, we can rewrite the equation that determines the threshold of the entrepreneurial ability z^* as follows:

$$p\left[\pi_1' - (1+i')\,x\,(z^*)\right] = \pi_0'$$

Since $\pi_1 = \gamma\pi_0$ and $1+i = (1+r-\delta)\,\Delta/p$, the equation is rewritten as follows:

$$\frac{p\gamma - 1}{p\gamma}\left\{p\gamma\pi_0' - (1-p)\,\mu\right\} = (1+r'-\delta)\,x\,(z_t^*)$$

where we use $\Delta \equiv p\gamma\pi_0/\left[p\gamma\pi_0 - (1-p)\,\mu\right]$. From equations (8) and (11), we can define π_0' and r' as follows: $\pi_0' \equiv \pi\,(z^*, k')$; $r' \equiv r\,(z^*, k')$. Then, we have:

$$\frac{p\gamma - 1}{p\gamma}\left\{p\gamma\pi\,(z^*, k') - (1-p)\,\mu\right\} = \left\{1 + r\,(z^*, k') - \delta\right\} x\,(z^*)$$

From (8), it is obvious that $\pi_z > 0$ and $\pi_{k'} > 0$. This implies that γ needs to be sufficiently large since the condition that $p\gamma\pi\,(z, k') - (1-p)\,\mu > 0$ should be satisfied at any equilibrium, or equivalently, $i \geq r + \delta$ for any given z^* and $k' > 0$. Hence, suppose that γ is sufficiently large, so that $p\pi\,(z_L, \underline{k}) - (1-p)\,\mu = 0$ for $\underline{k} \approx 0$. Then, one can easily verify that, for any $k' \geq \underline{k} \approx 0$, $z^* \in [z_L, z_H]$ is uniquely determined since the RHS is decreasing in z^* from infinity to zero (note that $r_z < 0$), while the LHS is increasing in z^* under the assumption, $p\pi\,(z^*, k') - (1-p)\,\mu > 0$ in any equilibrium. Hence, we can define $z^* \equiv z\,(k'; \mu)$, which is straightforward that $z_k < 0$ and $z_\mu > 0$.

To pin down the capital investment in equilibrium, we can rewrite the capital market clearing condition, given by equation (5), by letting $B_t^f = 0$. Then, we have:

$$k' + \frac{X\,(z\,(k'; \mu))}{L} = s\,(z\,(k; \mu)\,, z\,(k'; \mu)\,, k, k')$$

where we use $w \equiv w\,(z\,(k; \mu)\,, k)$ from equation (10), $r' \equiv r\,(z\,(k'; \mu)\,, k')$ from equation (11) and $s \equiv s(w, r')$. The LHS is the demand for funds and the RHS is the supply of funds. Now, let us confine our attention on a steady state. Then, the capital market clearing condition is rewritten as follows:

$$k + \frac{X\left(z\left(k;\mu\right)\right)}{L} = s\left(z\left(k;\mu\right), k\right)$$

Note first that $dX\left(z\left(k;\mu\right)\right)/dk > 0$ since $z_k < 0$ and $x_z < 0$. Now let us specify the R&D investment cost function $x(z, A)$ as follows:

$$x\left(z, A\right) \equiv \zeta A\left(\frac{1}{z - z_L} - \frac{1}{z_H - z_L}\right)$$

where $\zeta > 0$ measures the efficiency in the R&D investment. It is straightforward that the cost function satisfies all of the assumptions on $x\left(z, A\right)$: $x_z\left(z, A\right) < 0$, $\lim_{z\to z_L} x\left(z, A\right) = \infty$, $\lim_{z\to z_H} x\left(z, A\right) = 0$ and $x_A\left(z, A\right) > 0$. With this cost function, one can show that $\lim_{k\to 0} dX\left(z\left(k;\mu\right)\right)/dk = 0$,[2] and $d^2X\left(z\left(k;\mu\right)\right)/\left(dk\right)^2 > 0$.[3]

Similarly, $ds\left(z\left(k;\mu\right), k\right)/dk > 0$ with the log utility function $u\left(c\right) = \ln c$ since, with the log utility function, $s\left(z\left(k;\mu\right), k\right) = Q\psi\left(z\left(k;\mu\right)\right)k^{\alpha v}$ where Q is a positive constant.[4] Then, it is easy to check that $\lim_{k\to 0} ds/dk = \infty$.[5] Hence, a sufficient

[2] From the entrepreneur's problem, we have:

$$\begin{aligned}\frac{dz}{dk} &= \frac{r_k\left(z\left(k\right), k\right)x\left(z\left(k\right)\right) - p\pi_k\left(z\left(k\right), k\right)}{p\pi_k\left(z\left(k\right), k\right) - \left[r_z\left(z\left(k\right), k\right)x\left(z\left(k\right)\right) + \left\{1 + r\left(z\left(k\right), k\right) - \delta\right\}dx\left(z\left(k\right)\right)/dz\left(k\right)\right]} \\ &= \frac{C_1k^{\alpha v-2}x\left(z\left(k\right)\right) - C_2k^{\alpha v-1}}{C_2k^{\alpha v-1} - \left[C_3k^{\alpha v-1}x\left(z\left(k\right)\right) + C_4k^{av} + C_5\right]}\end{aligned}$$

where C_i $(i = 1, 2, \cdots, 5)$ is a constant. Then,

$$\frac{dX}{dk} = \frac{dX}{dz}\frac{dz}{dk} = \frac{C_2k^{\alpha v-1}x\left(z\left(k\right)\right) - C_1k^{\alpha v-2}\left[x\left(z\left(k\right)\right)\right]^2}{C_2k^{\alpha v-1} - \left[C_3k^{\alpha v-1}x\left(z\left(k\right)\right) + C_4k^{av} + C_5\right]}$$

since $dX\left(z\right)/dz = -x\left(z\right)$. Hence, given any $z\left(k\right)$, if $x\left(z\left(k\right)\right)$ satisfies that $x\left(z\left(k\right)\right) \geq k$ $\forall k \in [0, \varepsilon]$, then $\lim_{k\to 0} dX\left(z\left(k\right)\right)/dk = 0$. This condition is easily satisfied. Since $x\left(z, A\right) \equiv \zeta A\{1/\left(z - z_L\right) - 1/\left(z_H - z_L\right)\}$ where $\zeta > 0$ and $A > 0$, the condition requires:

$$z\left(k\right) - z_L \leq \frac{1}{k/A + 1/\left(z_H - z_L\right)} \quad \forall k \in [0, \varepsilon]$$

Note that, for any $k \approx 0$, $1/\left[k/A + 1/\left(z_H - z_L\right)\right] \approx z_H - z_L = \max\{z\left(k\right) - z_L\} \geq z\left(k\right) - z_L$ as desired.

[3] $d^2X/\left(dk\right)^2 = -[z_{kk}\left(k;\mu\right)]^2\{x\left(z\right) + x'\left(z\right)\}$. Hence, $d^2X/\left(dk\right)^2 > 0$ if, and only if, $-x'\left(z\right) > x\left(z\right)$, which always holds with the cost function, $x\left(z\right) \equiv \zeta A\{1/\left(z - z_L\right) - 1/\left(z_H - z_L\right)\}$.

[4] Q is equal to $\left(1 - \beta\right)\left(1 - \alpha\right)v\left(A/L\right)^{1-v} > 0$.

[5] $s'\left(k\right) = Q\left[\psi'\left(k\right)k^{\alpha v} + \alpha v\psi\left(k\right)k^{av-1}\right]$ where $F'\left(z\right)$ is the p.d.f. of the skill/ability z. Since $\psi'\left(k\right) = -\left(p\gamma - 1\right)f\left(z\right)z_k > 0$ $\forall$ $k > 0$ (recall that $F'\left(z\right) > 0$ $\forall$ $z \in [z_L, z_h]$), $\lim_{k\to 0}\psi'\left(k\right)k^{\alpha v} \equiv$

condition for the uniqueness of $k^* > \underline{k} \approx 0$ is that $d^2 s\left(z\left(k;\mu\right),k\right)/\left(dk\right)^2 < 0$, which is, unfortunately, not trivial to verify. However, if the c.d.f. of the skill, given by $F\left(z\right)$, is assumed to be highly concave, so that the p.d.f. $F'(z)$ is decreasing in z fast, i.e., $F''\left(z\right) \ll 0$, it is likely to have $d^2 s\left(z\left(k;\mu\right),k\right)/\left(dk\right)^2 < 0$, and then, k^* is uniquely determined.[6]

Suppose now that k^* is uniquely pinned down for analytical simplicity. Then, we have:

$$\frac{dk^*}{d\mu} = \frac{z_\mu\left(k^*\right)\left\{\frac{X'(z^*)}{L} + s_z\left(z^*,k^*\right)\right\}}{1 - \left\{s_z\left(z^*,k^*\right)z_k\left(k^*\right) + s_k\left(z^*,k^*\right) - \frac{X'(z^*)}{L}\right\}}$$

where $s_z\left(z^*,k^*\right) < 0$ with the log utility function, so that the numerator is negative. Hence, $dk^*/d\mu > 0$ if, and only if, the denominator is negative. Since $X'\left(z^*\right) < 0$, the denominator is negative when $ds\left(k^*\right)/dk = s_z\left(z^*,k^*\right)z_k\left(k^*\right) + s_k\left(z^*,k^*\right) > 0$ is sufficiently large. One possible case is that k^* is sufficiently small, so that the marginal product of capital is sufficiently large; recall that we have assumed $d^2 s\left(z\left(k;\mu\right),k\right)/\left(dk\right)^2 < 0$ for the uniqueness of k^*.

This implies that an increase in the monitoring cost may result in a huge increase in the supply of funds (the domestic saving) so does the capital stock k. The equilibrium threshold z^*, then, decreases due to the fact that $z_k\left(k;\mu\right) < 0$, and this leads to an increase in the output. Therefore, the negative scale effect of the borrowing constraint presented in Proposition 1 is ambiguous *a priori* in the closed economy.

$\xi \geq 0$. Hence, $\lim_{k\to 0} s'\left(k\right) = Q\xi + Q\lim_{k\to 0}\alpha v\psi\left(k\right)k^{\alpha v-1} = \infty$.

[6]Note that $s''\left(k\right) = Q\left[\psi''\left(k\right)k^{\alpha v} + 2\alpha v\psi'\left(k\right)k^{av-1} - \alpha v\left(1-\alpha v\right)\psi\left(k\right)k^{\alpha v-2}\right]$ where $\psi'\left(k\right) = -\left(p\gamma - 1\right)F'\left(z\right)z_k > 0$ and $\psi''\left(k\right) = -\left(p\gamma-1\right)\left[F''\left(z\right)z_k + F'\left(z\right)z_{kk}\right]$, the sign of which is not determined *a priori*. However, if $F''\left(z\right) \ll 0$, then it is likely to have $\psi''\left(k\right) \ll 0$ so is $s''\left(k\right)$.

Online Appendix D: The Exogenous Borrowing Constraint

We examine the exogenous borrowing constraint, which has been widely employed in the prior research, in order to underscore its drawbacks in comparison to the endogenous borrowing constraint. The significance of employing the endogenous borrowing constraint in growth analysis will be elucidated through this comparison. Prevalent in the prior research, the exogenous borrowing constraint is described as follows:

$$b_t \leq \lambda a_t$$

where a and b denote asset and debt, respectively, and λ is exogenously given.[7]

Within our conceptual framework, each potential borrower (an entrepreneur) commences life devoid of any assets, and therefore, $a_t = 0$ for all $t \geq 0$. We, thus, assume that an entrepreneurs born in period t is endowed one unit of labor as a worker. He, then, earns labor income in the first period of his life. The efficiency unit of labor supplied by entrepreneurs is denoted by L_t^e. Recall that we have assumed $g_{L,t} = g_{A,t}$ for the stationary growth. Similarly, we assume that $g_{L^e,t}(= g_{L,t}) = g_{A,t}$. Then, $g_{a,t} = g_{L^e,t}$ since the total population of entrepreneurs born in period t is given by unity $\forall t \geq 0$.

For simplicity, we assume that the asset a_t is *illiquid* in period t, and hence, cannot be invested in R&D in period t. Then, the exogenous borrowing constraint is rewritten as follows:

$$x\left(z, A_t\right) \leq \lambda a_t$$

and one can easily prove that the skill threshold under this exogenous borrowing constraint, say $\hat{z}_t$, is uniquely determined for all $t \geq 0$.[8]

Then, the exogenous borrowing constraint does not affect the economy as long as

[7] The exogenous borrowing constraint can be interpreted as a collateral constraint. Suppose that a financial intermediary can seize a borrower's asset a if the borrower renege on his debt b. Suppose also that a borrower can abscond with a fraction of $1/\lambda$ of the loan b. Then the financial intermediary must lend asset only to the extent that no borrower can defect: $b/\lambda \leq a$.

[8] The determination of the threshold $\hat{z}_t$ is the same as in the case with the endogenous borrowing constraint, except that the borrowing limit is exogenously given by λa_t and the gross lending rate $1 + i_t$ is equal to the fair insurance rate $\left(1 + r_t - \delta\right)/p$. To see this more clearly, set $\mu = 0$ in the financial market clearing condition given by equation (3).

the following condition holds:

$$x(z^*, A_t) \leq \lambda a_t$$

where z^* represents the asymptotic level of z_t^*, or alternatively, the skill threshold necessary for achieving the first-best optimum. The more intriguing scenario arises when the exogenous borrowing constraint is binding, i.e., $x(z^*, A_t) > \lambda a_t$, leading to $\hat{z}_t > z^*$, indicating the impossibility of attaining the first-best allocation.[9]

Similar to the argument used in Section 3, one can easily show that the growth rates of GDP and TFP are the same under the exogenous borrowing constraint, and the growth rate is equal to $\psi(\hat{z})$, where the threshold $\hat{z}$ increases as λ decreases. Thus, the severity of the borrowing constraint is measured by $1/\lambda$, which is exogenously given at a constant level $\forall t \geq 0$.

Limitations of the Exogenous Borrowing Constraint

First note that we cannot define and describe the interest rate spread with the exogenous borrowing constraint, since $1 + i_t = (1 + r_t - \delta)/p$ by the financial market clearing condition; see equation (3), letting $\mu = 0$.

It is also clear that the amplifying effect (the indirect growth effect) of the borrowing constraint is not captured. This is because the tightness of the borrowing constraint, as measured by $1/\lambda$, does not change whether aggregate uncertainty rises or falls.

Third, note that $\hat{z}_{t+1} = \hat{z}_t$ if the growth rate of a_t is equal to the growth rate of $x(\hat{z}_t, A_t)$. For comparison, assume that $x(\hat{z}, A)$ is linear in A as in the baseline case. Then the growth rate of R&D costs becomes equal to $g_{A,t}$, which is also equal to the growth rate of the entrepreneurial asset a_t. Thus, under the exogenous borrowing constraint, $\hat{z}_t$ is time-invariant given any constant interest rate r. This in turn implies that the negative growth effect is permanent since the skill threshold $\hat{z}_t$ is time-invariant even if the economy grows continuously.

The persistence of the growth effect is reminiscent of the well-known conclusion from the previous literature on growth with the exogenous borrowing constraint,

[9] This is because any entrepreneur with $z \in [z^*, \hat{z}_t)$ cannot invest in R&D even when the investment is profitable.

where the tightness of the borrowing constraint ($1/\lambda$) is arbitrarily given and fixed at a certain level. This is inconsistent with the empirical evidence supporting the asymptotic irrelevance of the borrowing constraint provided in Section 4.

Also, under the exogenous borrowing constraint, convergence is instantaneous, and therefore, the speed of convergence is irrelevant to the financial factor. In fact, the investment rate is fixed over time, which is inconsistent with the actual transition dynamics in the real world, as illustrated in Figure 2. Similarly, under the exogenous borrowing constraint, the maximum loan-to-value (LTV) ratio or the LTV ratio for a borrowing constrained individual is arbitrarily set and fixed, which seems clearly unrealistic. In contrast, the maximal LTV ratio, say $\lambda_t^* \equiv x\left(z_t^*, A_t\right)/a_t$, is endogenously determined under the endogenous borrowing constraint. Recall that the skill threshold z_t^* increases with the monitoring cost. Then, the maximal LTV ratio λ_t^* is monotone decreasing in μ since λ_t^* is monotone decreasing in z_t^*.[10] Consequently, $1/\lambda_t^*$, which is endogenously determined in equilibrium, can be interpreted as the *implied* tightness of the borrowing constraint.

Note also that the implied tightness $1/\lambda_t^*$ is time-varying along with various economic circumstances, while the economy converges to the stationary growth steady state. In particular, λ_t^* increases during the transition phase, converging to its asymptotic level, i.e., $\lambda_t^* \uparrow \lambda^*$.[11] Since the borrowing constraint is binding tightly at the beginning of economic development, the maximal LTV ratio, λ_t^*, is low, but it increases over time as the economy grows. Hence, less able entrepreneurs can finance funds for productivity-enhancing investments along with the endogenous relaxation of the borrowing constraint, and this causes an increasing trend of the investment rate consistent with what we observe from data.[12]

[10]This is simply because $\partial x_t/\partial z < 0$.

[11]This is easily verifiable:

$$\frac{\lambda_{t+1}^*}{\lambda_t^*} = \frac{x\left(z_{t+1}^*, A_{t+1}\right)}{x\left(z_t^*, A_t\right)}\frac{a_t}{a_{t+1}} = \frac{x\left(z_{t+1}^*, A_t\right)}{x\left(z_t^*, A_t\right)}\frac{\psi\left(z_t^*\right)}{\psi\left(z_t^*\right)} = \frac{x\left(z_{t+1}^*, A_t\right)}{x\left(z_t^*, A_t\right)} > 1$$

where the inequality comes from $z_t^* > z_{t+1}^*$ (see (i) of Proposition 4). The convergence holds by the continuity of λ_t^* in z_t^*. Obviously, this is consistent with the endogenous relaxation, represented by $\lim_{t\to\infty}\tilde{\Delta}_t = -\log p > 0$ (see Figure 1).

[12]It is worth noting that the increasing investment rate is obtainable under the exogenous borrowing constraint. For example, if an entrepreneur is allowed to self-finance through his own saving,

Aggregate LTV Ratio

We now think of aggregate LTV ratio, say Λ_t, which is defined by:

$$\Lambda_t \equiv \frac{\int_{\hat{z}_t}^{z_H} x_t(z)\, dF(z)}{\int a_t dF(z)} = \frac{X_t}{a_t} = \frac{X_t}{Y_t}\frac{Y_t}{a_t} = \rho_t \frac{Y_0}{a_0}.$$

where ρ_t is the R&D expenditure rate in period t as defined in Proposition 4. In the economy with the exogenous borrowing constraint, the aggregate LTV ratio is fixed over time, i.e., $\Lambda_t = \Lambda$, as long as the interest rate r is constant. This is because the threshold $\hat{z}_t$ does not change. However, under the endogenous borrowing constraint, the aggregate LTV ratio, say Λ_t^*, increases and converges to its steady state value Λ^* as ρ_t increases and converges to its steady state value; see part (iv) of Proposition 4.

he will be eventually able to conduct a profitable R&D project by accumulating necessary funds. This results in the realistic trend of the investment rate, which is increasing along with economic development; see Buera and Shin (2013) for details.

References

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