The Government in SNA-compliant DSGE models
Technical Appendix

(Not intended for publication)

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This Technical Appendix reports additional details on the quantitative results presented in Section 4 of the main text. Section 1 provides details on the model specification, Section 2 summarizes the system of equilibrium equations used in the simulations, Section 3 reports the calibration of the parameters and fiscal policy instruments.

1 Model specification

The model used for simulations is a perfect-foresight neoclassical growth model augmented with a SNA-compliant government sector as specified in the main text. In the following, we report the main modeling choices and the functional forms used.

Households

The instantaneous households utility function is given by

$$U(\widetilde{C}_t, L_t) = \gamma \log \widetilde{C}_t + (1-\gamma)\log(1-L_t)$$

where time endowment has been normalized to one, and labor L_t is split between private labor $L_{p,t}$, and public labor $L_{g,t}$, holding $L_t = L_{p,t} + L_{g,t}$. Total consumption $\widetilde{C}_t$ is a composite of both private goods purchased by households, $C_{pp,t}$, public-produced goods, $Y_{g,t}$,

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and public-provided goods, $C_{pg,t}$. We assume that all three goods are perfectly substitute. Thus, total consumption is defined as $\widetilde{C}_t = C_{pp,t} + C_{pg,t} + C_{gg,t}$.

Household's budget constraint is given by,

$$(1+\tau_t^c)C_{pp,t} + C_{pg,t} + C_{gg,t} + I_{p,t} + B_{t+1} - B_t = \\ (1-\tau_t^l)[w_{p,t}L_{p,t} + w_{g,t}L_{g,t}] + (1-\tau_t^k)r_t K_{p,t-1} - \delta_p \tau_t^k K_{p,t} + Z_t + (1-\tau_t^\pi)\Pi_t$$

where τ_t^c is the consumption tax, τ_t^l is the labor income tax, τ_t^k is the capital income tax, τ_t^π is a profits (corporate) tax, $I_{p,t}$ is private investment, $K_{p,t}$ is private capital, $w_{p,t}$ is private wage, $w_{g,t}$ is public wage, r_t is private capital returns, δ_p is the private capital depreciation rate, Z_t is government lump-sum transfers, and Π_t are profits. As usual, the accumulation process of private capital is given by $K_{p,t+1} = (1-\delta_p)K_{p,t} + I_{p,t}$.

Firms

We assume that private production is a Cobb–Douglas function with constant returns to scale. Goods are sold in a perfectly competitive market. All public capital is used by private firms. The following production function accommodates these assumptions

$$Y_{p,t} = A_{p,t} K_{p,t}^{\alpha_p} K_{g,t}^{\alpha_g} L_{p,t}^{1-\alpha_p-\alpha_g}$$

where $A_{p,t}$ is total factor productivity in the private production sector, and $K_{g,t}$ is public capital. Parameters α_p, α_g represent the elasticity of output with respect to private capital and public capital, respectively. Public labor market is modeled following de-Córdoba et al. (2012a).

Given that firms use public capital as an input at no cost, profits are positive and equal to

$$\Pi_t = A_{p,t} K_{p,t}^{\alpha_p} K_{g,t}^{\alpha_g} L_{p,t}^{1-\alpha_p-\alpha_g} - (1+\tau_t^s)w_{p,t}L_{p,t} - r_t K_{p,t}$$

where τ_t^s is the social security contributions rate which it is assumed to be paid by the firms.

Government

Total government spending G_t is defined as a fraction θ of GDP_t, i.e.

$$\frac{G_t}{GDP_t} = \theta$$

As specified in the paper, the components of total government spending are:

$$G_t = (1+\tau_t^s)w_{g,t}L_{g,t} + I_{g,t} + Z_t + C_{pg,t} + C_{gi,t} + r_t^B B_t$$

where τ_t^s are social security contributions paid by firms, $w_{g,t}$ is the public wage rate and $L_{g,t}$ is public labor. Transfers in kind are defined $C_{pg,t}$, whereas social benefits and transfers other than in kind are defined Z_t. Intermediate consumption $C_{gi,t}$ includes all production expenses bear by the government, and $I_{g,t}$ denotes gross capital formation in the public sector (public investment). Finally, debt service is labeled $r_t^B B_t$, in which r_t^B is the implicit interest rate on the stock of public debt B_t. We assume that public debt is sold to international investors, thus having no effect on the interest rate of the model economy.

Fiscal revenues are given by:

$$T_t = \tau^c C_t + (\tau^l + \tau^s)(w_{p,t} L_{p,t} + w_{g,t} L_{g,t}) + \tau^k K_{p,t}(r_t - \delta_p) + \tau^\pi \Pi_t$$

Given the government budget constraint, we have

$$G_t - T_t = \Delta B_t$$

On the other hand, non-interest (primary) government expenditures, $G_{prim,t}$, is defined as

$$G_{prim,t} = G_t - r_t^B B_t$$

The exogenous components of total government spending are defined as

$$C_{pg,t} = \theta_{pg} G_t$$

$$I_{g,t} = \theta_{ig} G_t$$

$$C_{gi,t} = \theta_{gi} G_t$$

$$(1 + \tau_t^{ss}) w_{g,t} L_{g,t} = \theta_{wl} G_t$$

$$Z_t = \theta_{zg} G_t$$

where $\theta_{pg} + \theta_{ig} + \theta_{gi} + \theta_{wl} + \theta_{zg} = 1 - r_t^B B_t / G_t$.

Finally, we assume that all public labor is used in public production, which according to SNA rules is defined as

$$Y_{g,t} = C_{gi,t} + (1 + \tau_t^s) w_{g,t} L_{gg,t}$$

The accumulation process of public capital is given by $K_{g,t+1} = (1 - \delta_g) K_{g,t} + I_{g,t}$ where δ_g is the public capital depreciation rate, and $I_{g,t}$ is public investment.

2 Model equations

The collection of the model's first order conditions, market clearing and resource constraints is as follows.

$$\frac{\gamma}{C_{pp,t} + C_{pg,t} + C_{gg,t}} - \lambda_t (1 + \tau_t^c) = 0 \tag{1}$$

$$\frac{1 - \gamma}{1 - L_{p,t} - L_{g,t}} - \lambda_t (1 - \tau_t^l) w_{p,t} = 0 \tag{2}$$

$$\beta \left[\lambda_{t+1} \left(1 + (1 - \tau_{t+1}^k)(r_{t+1} - \delta_p) \right) \right] - \lambda_t = 0 \tag{3}$$

$$Y_t - A_{p,t} K_{p,t}^{\alpha_p} K_{g,t}^{\alpha_g} L_{p,t}^{1 - \alpha_p - \alpha_g} = 0 \tag{4}$$

$$r_t - \alpha_p A_{p,t} K_{p,t}^{\alpha_p - 1} K_{g,t}^{\alpha_g} L_{p,t}^{1 - \alpha_p - \alpha_g} = 0 \tag{5}$$

$$(1+\tau_t^s)w_{p,t} - (1-\alpha_p-\alpha_g)A_t K_{p,t}^{\alpha_p} K_{g,t}^{\alpha_g} L_{p,t}^{-\alpha_p-\alpha_g} = 0 \tag{6}$$

$$\Pi_t - A_{p,t} K_{p,t}^{\alpha_p} K_{g,t}^{\alpha_g} L_{p,t}^{1-\alpha_p-\alpha_g} - (1+\tau_t^s)w_{p,t}L_{p,t} - r_t K_{p,t} = 0 \tag{7}$$

$$K_{p,t+1} - (1-\delta_p)K_{p,t} - I_{p,t} = 0 \tag{8}$$

$$K_{g,t+1} - (1-\delta_g)K_{g,t} - I_{g,t} = 0 \tag{9}$$

$$G_t - (1+\tau_t^s)w_{g,t}L_{g,t} - I_{g,t} - Z_t - C_{pg,t} - C_{gi,t} - r_t^B B_t = 0 \tag{10}$$

$$w_{g,t} = w_{p,t} \tag{11}$$

$$G_t = \theta GDP_t \tag{12}$$

$$C_{pg,t} - \theta_{pg} G_t = 0 \tag{13}$$

$$I_{g,t} - \theta_{ig} G_t = 0 \tag{14}$$

$$C_{gi,t} - \theta_{gi} G_t = 0 \tag{15}$$

$$(1+\tau_t^s)w_{g,t}L_{g,t} - \theta_{wl} G_t = 0 \tag{16}$$

$$Z_t - \theta_{zg} G_t = 0 \tag{17}$$

$$T_t - \begin{pmatrix} \tau_t^c C_{p,t} + \tau_t^l (w_{p,t}L_{p,t} + w_{g,t}L_{g,t}) + \tau_t^k (r_t - \delta_{K_p})K_{p,t-1} \\ +\tau_t^s (w_{p,t}L_{p,t} + w_{g,t}L_{g,t}) + \tau_t^\pi \Pi_t \end{pmatrix} = 0 \tag{18}$$

$$G_t + (1+r_t^B)B_t - (T_t + B_{t+1}) = 0 \tag{19}$$

$$L_t - L_{p,t} - L_{g,t} = 0 \tag{20}$$

$$\begin{gathered}(1+\tau_t^c)C_{pp,t} + C_{pg,t} + C_{gg,t} + I_{p,t} + B_{t+1} - B_t - (1-\tau_t^l)[w_{p,t}L_{p,t} + w_{g,t}L_{g,t}] \\ -(1-\tau_t^k)r_t K_{p,t-1} + \delta_p \tau_t^k K_{p,t} - Z_t - (1-\tau_t^\pi)\Pi_t = 0\end{gathered} \tag{21}$$

$$Y_{g,t} = C_{gi,t} + (1+\tau_t^s)w_{g,t}L_{gg,t} \tag{22}$$

$$GDP_t = C_{pp,t} + C_{pg,t} + I_{p,t} + I_{g,t} + Y_{g,t} \tag{23}$$

The set of equations is completed with the usual transversality condition $\lim_{t\to\infty} \beta^t \lambda_t K_t = 0$.

3 Calibration

Table 1 presents the calibration of deep parameters and Table 2 presents the calibration of the fiscal policy parameters. For the calibration of government spending, we use data for France as described in Section 3.2 of the main text (details on data sources in Section 2.2). For the calibration of fiscal income, we use data for France as described in Section 2.3 of the main text.

Table 1: Calibration: Deep parameters

Description	**Parameter**	**Value**	**Target**
Discount factor	β	0.96	Subjective discount rate 4.1%
Preference parm	γ	0.45	Labor fraction 33% of time endow.
Private capital depreciation rate	δ_p	0.07	Annual depreciation 7%
Public capital depreciation rate	δ_g	0.04	Annual depreciation 4%
Cobb–Douglas technology parm	α_p	0.27	Labor share of income 65%
Cobb–Douglas technology parm	α_g	0.08	Public/private capital ratio
Public/private wage premium	w_g/w_p	1.00	OECD wages data*
Private aggregate productivity	$\overline{A}_p$	1.00	Normalization

The wage premium is calibrated using the estimations for France as reported in De-Córdoba et al. (2012b).

Table 2: Calibration: Policy Vector

Description	**Parameter**	**Value**	**Target**
Total Government spending	G_t	56.60%	TLYCG (% GDP)
Primary spending	$G_{prim,t}$	54.71%	TLYCG - D4 (% GDP)
Intermediate consumption	θ_{gi}	0.0887	P2 (% G)
Transfers in kind	θ_{pg}	0.1062	D63 (% G)
Gross capital formation	θ_{ig}	0.0615	P5+K2+D9 (% G)
Compensation to employees	θ_{wl}	0.2208	D1 (% G)
Transfers other than in kind	θ_{zg}	0.4844	D62 + D3+ D7 (% G)
Capital income tax	τ^k	0.1179	Capital income tax (effective)
Labor income tax	τ^l	0.3620	Personal income tax (effective)
Social Security tax	τ^s	0.3581	Social Security contributions (effective)
Consumption tax	τ^c	0.2000	Taxes on consumption (effective)
Profits tax	τ^π	0.2800	Corporate tax (statutory)

Spending codes from OECD database >> *Annual National Accounts* >> *General Government Accounts* >> *11. Government expenditures by function (COFOG).*
Effective tax rates from OECD database >> *Tax* >> *Tax policy analysis* >> *OECD Tax Database*

References

[1] De-Córdoba, G., Pérez, J. and J. L. Torres (2012a): "Public and private sector wages interactions in a general equilibrium model", *Public Choice*, 150(1), 309-326.

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