#### **Contributions**

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# Knowledge licensing in a model of R&D-driven endogenous growth

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**Abstract:** I model knowledge (patent) licensing and evaluate intellectual property regulation in an endogenous growth framework where the engine of growth is in-house R&D performed by high-tech firms. I show that high-tech firms innovate more and economic growth is higher when there is knowledge licensing, and when intellectual property regulation facilitates excludability of knowledge, than when knowledge is not excludable and there are knowledge spillovers among high-tech firms. However, the number of high-tech firms is lower, and welfare is not necessarily higher when there is knowledge licensing than when there are knowledge spillovers.

**Keywords:** endogenous growth; in-house R&D; intellectual property regulation; knowledge licensing; welfare.

**JEL classification:** O31; O34; L16; L50; O41.

# 1 Introduction

A number of growth models treat private firms' intentional investments in R&D as the driver of long-run growth and welfare (e.g. Romer 1990; Aghion and Howitt 1992). These models assume that there are knowledge spillovers in the R&D process and that R&D builds on a pool of knowledge. In this sense, these growth models abstract from the role of knowledge (patent) licensing and from the details about the exchange of knowledge in the economy. Nevertheless, licensing and citing patents is common in high-tech industries and seems to play a significant role for innovation (e.g. Anand and Khanna 2000; Shapiro 2001; Arora and Gambardella 2010). Current high-profile examples include licensing agreements and patent citations among product market rivals such as Google, Microsoft, and Apple Inc.

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In this paper, I present an endogenous growth model where high-tech firms engage in in-house R&D which then drives long-run growth. High-tech firms have exclusive rights to their product type. In a high-tech firm, the innovation enhances firm/product-specific knowledge, which reduces the firm's marginal costs or increases the quality of its product. High-tech firms finance their R&D expenditures from operating profits. They set prices and compete strategically in their output market. My point of departure is that I model knowledge (patent) licensing among high-tech firms. The knowledge generated in a high-tech firm cannot be used for free, since intellectual property regulation facilitates its excludability (i.e. it favors patents). However, it can be licensed. Given that each high-tech firm produces a distinct type of good, for such a firm the knowledge of other high-tech firms is complementary to its own. If a high-tech firm licenses the knowledge of another, it can combine that knowledge with its own and improve its in-house R&D process, since the latter builds on the knowledge that the firm possesses.

I evaluate intellectual property regulation and contrast the inference from this setup to the inference from a setup where intellectual property regulation does not facilitate excludability and there are knowledge spillovers among high-tech firms. I also show how market concentration and the intensity of competition, as measured by the elasticity of substitution between high-tech goods, can matter for innovation in the high-tech industry and aggregate performance in both setups.

I show that innovation in the high-tech industry, economic growth, and welfare, are higher in case when there is knowledge licensing than when there are knowledge spillovers. Innovation and growth are higher when there is knowledge licensing because firms appropriate the benefits from their R&D more in this case. In turn, welfare is higher because these dynamic gains prevail over the static losses in terms of within-period output.

In both setups that I consider, innovation in the high-tech industry and economic growth increase with the number of high-tech firms and the intensity of competition. The drivers behind these results are the relative price distortions, which are due to price setting by high-tech firms. These distortions adversely affect the demand for high-tech goods. Given that high-tech firms interact strategically in the output market, a higher number of firms and more intensive competition imply lower mark-ups and lower distortions. This increases the demand for high-tech goods and implies higher output and investments in R&D in the high-tech industry.1

<sup>1</sup> O'Donoghue and Zweimüller (2004) have a similar result in a Schumpeterian growth model. Vives (2008) shows that such a result can also hold in partial equilibrium for various types of demand functions. The positive relation between innovation and competitive pressure is consistent with empirical findings of, for example, Blundell, Griffith, and van Reenen (1999).

Finally, I endogenize the number of high-tech firms, assuming cost-free entry. Again, innovation in the high-tech industry and economic growth are higher when there is knowledge licensing than when there are knowledge spillovers. This happens, however, at the expense of the number of high-tech firms (or the variety of high-tech goods). The number of high-tech firms is lower when there is knowledge licensing than when there are knowledge spillovers. Because of this, welfare is not necessarily higher when there is knowledge licensing.

I also show that increasing the intensity of competition reduces the number of firms in both setups. This has no effect, however, on innovation in the hightech industry and economic growth.

This paper is related to the endogenous growth literature (e.g. Romer 1990; Aghion and Howitt 1992; Smulders and van de Klundert 1995), where the positive growth of the economy on a balanced growth path is a result of technological and preference factors. In particular, it is related to studies which, in an endogenous growth framework, suggest how the aggregate performance can be affected by imperfect competition in an industry where the firms engage in in-house R&D (e.g. Peretto 1996; van de Klundert and Smulders 1997). It contributes to these streams of studies while showing how knowledge licensing in such an industry can affect innovation, economic growth, and welfare.

Several studies evaluate intellectual property regulation from the perspective of the duration of patents in the standard variety-expansion frameworks (e.g. Judd 1985; Futagami and Iwaisako 2007). There are also a number of studies that model knowledge and technology licensing in the standard Schumpeterian growth framework and show how intellectual property regulation and international technology licensing can affect innovation and growth e.g. (e.g. Yang and Maskus 2001; O'Donoghue and Zweimüller 2004; Tanaka, Iwaisako, and Futagami 2007). In these studies, licensing happens between incumbents and entrants given that in the standard Schumpeterian growth framework incumbents have no incentives to innovate. Licensing does not explicitly aid the R&D process and licenses are essentially permits for production. In order to maintain incentives for licensing, these studies assume that either licensors and licensees (incumbents and entrants) collude in the product market, or licensees can access a larger market (e.g. one of the countries bans FDI). The share in collective profits and licensing fees compensate incumbents' loss of the product market (and costs of technology transfer), and are either exogenous or exogenously determined by intellectual property regulation. In contrast, this paper has a non-tournament framework where incumbents engage in in-house R&D, which improves their productivity or the quality of their product. Incumbents innovate because it enables the stealing of market share, and licensing happens among incumbents. Firms have incentive to license knowledge from other firms because that aids their R&D process. The intellectual property regulation affects the excludability of knowledge and its market structure. License fees are determined by the structure of the market for knowledge and supply and demand conditions. To that end, the framework and analysis of this paper can be thought of as complementary.

There is also a large body of firm- and industry-level studies that analyze the implications of patent licensing, patent consortia or pools, and knowledge exchange among firms on innovation and market conduct (e.g. Gallini and Winter 1985; Katz and Shapiro 1985; Faulí-Oller and Sandonís 2002; Arora and Fosfuri 2003). This paper analyzes such issues at the aggregate level in a dynamic general equilibrium framework, which assumes an undistorted market for knowledge/patents. This assumption allows for tractable inference. In turn, the dynamic general equilibrium framework endogenizes the growth rate of the economy and the effect of knowledge licensing on, for example, the interest rate which affects the incentives to perform R&D. Licensing in this paper *ceteris paribus* motivates R&D. This implies a higher growth rate and higher rate of interest which reduces the incentives to perform R&D.

The next section introduces the model. Section 3 analyzes the features of dynamic equilibrium. Section 4 concludes. The proofs of the results are offered at the end of the paper.

## 2 The model

#### 2.1 Households

The economy is populated by a continuum of identical and infinitely lived households of mass one. The representative household is endowed with a fixed amount of labor L. It inelastically supplies its labor to firms which produce final goods and to high-tech firms. The household has a standard CIES utility function with an inter-temporal substitution parameter  $1/\theta$  and discounts the future streams of utility with rate  $\rho$  ( $\theta$ ,  $\rho$ >0). The utility gains are from the consumption of amount C of final goods. The lifetime utility of the household is

$$U = \int_{0}^{+\infty} \frac{C_t^{1-\theta} - 1}{1 - \theta} \exp(-\rho t) dt.$$

The household maximizes its lifetime utility subject to a budget constraint:

$$\dot{A} = rA + wL - C, \tag{1}$$

where A is the household's asset holdings [A(0)>0], r and w are the market returns on its asset holdings and labor supply.

The rule that follows from the household's optimal problem is the standard **Euler equation:** 

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho).$$

This, together with the budget constraint, describes the paths of the household's consumption and assets.

### 2.2 Final goods

Final goods, Y, are homogeneous. The household's demand for final goods is served by a representative producer. The production of final goods requires labor and X, which is a CES composite of high-tech goods  $\{x_i\}$  with an elasticity of substitution  $\varepsilon$  ( $\varepsilon$ >1).

The production of the final goods has a Cobb-Douglas technology and is given by

$$Y = X^{\sigma} L_{\nu}^{1-\sigma}, \tag{2}$$

where

$$X = \left(\sum_{i=1}^{N} X_{i}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},\tag{3}$$

 $L_v$  is the share of the labor force employed in final goods production, N is the number of high-tech goods, and  $1>\sigma>0$ .

For ease of exposition, the problem of the representative final goods producer is divided into two steps. In the first step, the representative producer decides on the optimal combination of  $L_v$  and X in Y and in the second step it decides on the optimal amounts of high-tech goods *x* in *X*.

Therefore, in the first step the representative producer solves the following problem:

$$\max_{L_Y,X}\{Y-wL_Y-P_XX\},$$

**<sup>2</sup>** I allow *N* to be real number in order to avoid complications arising from integer constraints.

where  $P_X$  is the private marginal value of X and Y is the numeraire. The optimal rules that follow from this problem describe the final goods producer's demand for labor and the optimal amount of X in the production of Y:

$$wL_{v} = (1 - \sigma)Y, \tag{4}$$

$$P_{\mathbf{y}}X = \sigma Y.$$
 (5)

In the second step, the producer solves:

$$\max_{\{x_i\}_{i=1}^N} \left\{ P_X X - \sum_{i=1}^N p_{x_i} x_i \right\},$$

where  $p_x$  is the price of x. This implies that the demand for a high-tech good j (j=1,...,N) is given by

$$x_{j} = X \left( \frac{P_{X}}{p_{x_{j}}} \right)^{\varepsilon}. \tag{6}$$

From this expression follow two equilibrium conditions:

$$P_{X}X = \sum_{i=1}^{N} p_{x_{i}} x_{i}, \tag{7}$$

$$P_{X} = \left(\sum_{i=1}^{N} p_{X_{i}}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$
(8)

where (7) implies that there is no waste and (8) implies that  $P_x$  is an index of  $p_x$ .

## 2.3 High-tech goods

Each high-tech firm owns a design of distinct high-tech good x, which it produces. The production of a high-tech good requires labor input  $L_x$ . The production function of a high-tech good x is given by

$$x = \lambda L_{x},$$
 (9)

where  $\lambda$  measures the producer's knowledge of the production process or the quality of the high-tech good. This knowledge is firm/product-specific since each high-tech firm produces a distinct good.

High-tech firms are price setters in their output market and discount their future profit streams  $\pi$  with the market interest rate r. I assume that high-tech firms cannot collude in the output market.

#### 2.3.1 In-house R&D

High-tech firms can engage in R&D for accumulating knowledge and increasing  $\lambda$ . This can be interpreted as a process innovation that increases productivity (the firms are able to produce more of x), or as a quality upgrade (the firms are able to produce the same amount of higher quality x). Knowledge is not a rival input so that, potentially, it can be used at the same time in multiple places/firms.

In this section, I offer two different settings for the R&D process. The differences stem from how knowledge is exchanged among high-tech firms. In both cases, I assume that the firms cannot collude and stop innovating (e.g. because of antitrust regulation or non-sustainability of collusion).3

Hereafter, when appropriate, for ease of exposition, I describe the properties of the high-tech industry, taking as an example a high-tech firm j. In order to improve its knowledge  $\lambda_i$ , the firm needs to hire researchers/labor  $L_r$ . Researchers use the current knowledge of the firm in order to create better knowledge.

**Knowledge licensing:** This is the benchmark setup, and I call it S.1. Knowledge in this setup can be licensed. If high-tech firm *j* decides to license knowledge from other high-tech firms, its researchers combine that knowledge with the knowledge available in the firm, in order to produce new knowledge. The knowledge available in the firm is an essential input in the R&D of the firm. Moreover, it is the only essential input. This implies that the high-tech firm does not need to acquire knowledge from other firms in order to advance its own. However, it needs to have its own knowledge in order to build on it. This is in line with that high-tech firms produce distinct goods.

I assume that the in-house R&D process is given by

$$\dot{\lambda}_{j} = \xi \left[ \sum_{i=1}^{N} (u_{i,j} \lambda_{i})^{\alpha} \right] \lambda_{j}^{1-\alpha} L_{r_{j}}, \tag{10}$$

where  $\xi$  is an exogenous efficiency level ( $\xi$ >0),  $u_{i,j}$  is the share of knowledge of firm *i* that firm *j* licenses,  $u_{i,j}=1$ , and  $1>\alpha>0$ .

Intellectual property regulation facilitates excludability of knowledge and grants bargaining power to the licensors in the sense that they can make a 'take it or leave it' offer to licensees. In turn, license contracts do not allow for sub-licensing.

<sup>3</sup> Online Appendix E.1 offers a setup where firms cooperate in R&D and compete in the product market.

<sup>4</sup> I also need to assume that patent infringements are detectable. This seem to be a plausible assumption given the recent history of the high number of patent infringement lawsuits in hightech industries.

It can be shown that in (10) the elasticity of substitution between the different types of knowledge that the high-tech firm licenses is equal to  $1/(1-\alpha)$ . It can also be shown that the elasticity of substitution between the high-tech firm's knowledge and any knowledge that it licenses is lower than  $1/(1-\alpha)$ . This restates the importance of the firm's knowledge for its R&D process.

In this R&D process, the productivity of researchers increases linearly with knowledge licensed from an additional high-tech firm because of summation. Such a formulation can be justified if there are significant complementarities among the knowledge of high-tech firms. Further, it might seem brave to assume that R&D in a single firm can have non-decreasing returns.<sup>6</sup> This assumption allows the focus to be on the effect of market structure of the high-tech industry on innovation in that industry through competitive pressure. It can be relaxed by setting  $u_{i,j}$ =0 in square brackets in (10). In such a case, in this model knowledge licensing (or exchange of knowledge) is a necessary condition to ensure non-decreasing returns to R&D and positive growth in the long-run (for further details, see online Appendix E.2). The R&D process (10) can also be viewed as a simplification leading to tractable results. It ensures that there is a balanced growth path, for example.<sup>7</sup>

One way to think about this setup is that each high-tech firm can license the patented knowledge of other firms in order to generate its own patented knowledge, which helps to improve its production or output. The firm does not use the knowledge that it licensed directly in the production of its good, because that knowledge needs to be combined with its own knowledge, and that requires investments in terms of hiring researchers and time. The latter seems plausible for technologically sophisticated (e.g. high-tech) goods.

**Knowledge spillovers:** In this setup, S.2, there are knowledge spillovers among high-tech firms. In high-tech firm *j*, the researchers combine the knowledge that spills over from other high-tech firms with the knowledge available in the firm,

<sup>5</sup> Rivera-Batiz and Romer (1991) and Grossman and Helpman (1995) have a similar additive structure for knowledge in the R&D process in the context of knowledge spillovers among countries. Peretto (1998a,b) has a similar structure in the context of knowledge spillovers within an industry.

<sup>6</sup> In this respect, an example of a high-tech firm might be IBM, which started with tabulating machines a century ago and reached the point of producing supercomputers.

<sup>7</sup> This formulation of the R&D process leads to scale effects which are contentious (Jones 1995, 2005; Jones and Romer 2010). The mechanisms behind the main results of this paper are based on factor allocations. Therefore, as shown in online Appendix E.5 and online Appendix E.6, the results might not generalize in the "second generation" growth models, but can generalize in the "third generation" models. I maintain the current framework for its analytical simplicity.

while generating new knowledge. In order to maintain symmetry, I also assume that the researchers do not fully internalize the use of the current knowledge available in the firm and have external benefits from it. Similar to the previous setup, this assumption allows the focus to be on the effect of market structure of the high-tech industry on innovation through competitive pressure.

The R&D process is given by

$$\dot{\lambda}_{j} = \xi \tilde{\Lambda} \lambda_{j}^{1-\alpha} L_{r_{i}}, \qquad (11)$$

where I assume that in equilibrium  $\tilde{\Lambda}$  is equal to

$$\tilde{\Lambda} = \sum_{i=1}^{N} \lambda_i^{\alpha}.$$
 (12)

An interpretation for this case is that intellectual property regulation does not enforce excludability and firms cannot maintain secrecy. Another possible interpretation is that there is a market for knowledge, and intellectual property regulation grants the bargaining power to the potential licensees, so that they have the right to make a 'take it or leave it' offer. The licensees under this assumption receive the knowledge at no cost (i.e. there are spillovers) if the supply of knowledge is not elastic. The supply is necessarily inelastic if licensors do not have trade-offs associated with licensing knowledge. It seems natural to assume that once knowledge is created its supply entails virtually no costs. There would then be no trade-offs if licensors do not take into account that the knowledge they license is used for stealing business: the licensees use it in order to reduce their prices and steal market share. In this line of literature, it is common to assume that the originators of knowledge spillovers do not internalize the effect of spillovers on others' R&D and production processes. In the frames of this model, this assumption is necessary in order to give such a market-based interpretation to knowledge spillovers. The choice of the interpretation is a matter of taste.

In practice, licensing and spillovers tend to coexist in high-tech industries. These setups are then polar cases. This sharpens the comparison of inference (see online Appendix E.3 for a setup where there is knowledge licensing and spillovers).

Similar to  $\lambda$ , the design of a high-tech good can be viewed as knowledge/ patent. It needs to be assumed that (at least for some time) the knowledge about the design of high-tech goods cannot be used by other firms without appropriate compensation, in order to guarantee that high-tech firms have incentives to innovate. Any high-tech firm, nevertheless, could sell the design of its good at market value: the discounted sum of profit streams. Therefore, the assumption that intellectual property regulation enforces excludability of knowledge  $\lambda$  and grants the bargaining power to the licensors seems to be more consistent in such a setup.<sup>8</sup>

#### 2.3.2 Optimal problem

The revenues of high-tech firm j are gathered from the supply of its good  $(x_j)$  and when there is knowledge licensing from the supply of its knowledge  $(u_{j,i}\lambda_j; \forall i\neq j)$ . The costs are the labor compensations and license fees, case when there is knowledge licensing. The profits of high-tech firm j are given by

$$\pi_{j} = p_{x_{j}} x_{j} - w(L_{x_{j}} + L_{r_{j}}) + \left[ \sum_{i=1, i \neq j}^{N} p_{u_{j,i}\lambda_{j}}(u_{j,i}\lambda_{j}) - \sum_{i=1, i \neq j}^{N} p_{u_{i,j}\lambda_{i}}(u_{i,j}\lambda_{i}) \right], \tag{13}$$

where the term in square brackets appears when there is knowledge licensing, and  $p_{u_i,\lambda_i}$  and  $p_{u_i,\lambda_i}$  are the license fees for  $u_{j,\lambda_j}$  and  $u_{i,j}\lambda_i$ .

The high-tech firm maximizes the present discounted value  $V_j$  of its profit streams. For brevity, I assume that high-tech firms choose the prices of their products and in that sense engage in a generalized type of Bertrand competition see for Cournot competition (see Jerbashian 2014 for Cournot competition). Therefore, the problem of high-tech firm j is

$$V_{j}(\overline{t}) = \max_{p_{x_{j}}, L_{r_{j}}, \{u_{j,i}, u_{i,j}\}_{i=1;(i\neq j)}^{N}} \left\{ \int_{\overline{t}}^{+\infty} \pi_{j}(t) \exp\left[-\int_{\overline{t}}^{t} r(s) ds\right] dt \right\}$$
s.t.
(6), (9), (13), and either (10) or (11),

where  $\overline{t}$  is the entry date and  $\lambda_i(\overline{t}) > 0$  is given.

The solution of the optimal problem implies that the supply of high-tech good x, and the demand for labor for R&D are given by

$$w = \lambda_j p_{x_j} \left( 1 - \frac{1}{e_j} \right), \tag{14}$$

$$w = q_{\lambda_j} \frac{\dot{\lambda}_j}{L_{r_j}},\tag{15}$$

**<sup>8</sup>** It might be insightful to assesses the impact of intellectual property regulation which affects both the excludability of  $\lambda$  and the duration of rights/patent on x. This is left for future research.

where  $e_i$  is the elasticity of substitution between high-tech goods perceived by the high-tech firm and  $\,q_{\lambda}\,$  is the shadow value of knowledge accumulation.

It can be shown that

$$e_{j} = \varepsilon - \left[ \frac{(\varepsilon - 1)p_{x_{j}}^{1-\varepsilon}}{\sum_{i=1}^{N} p_{x_{i}}^{1-\varepsilon}} \right]. \tag{16}$$

The term in square brackets in (16) measures the extent of strategic interactions among high-tech firms, which create a wedge between e and the actual elasticity of substitution  $\varepsilon$ . Therefore, it measures some of the distortions in the economy which stem from imperfect competition with a finite number of hightech firms. The term in square brackets and these distortions tend to zero when the number of firms increases.

In the case when there is knowledge licensing, the returns on R&D are given by

$$\frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left(\frac{e_j - 1}{e_j} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} + \sum_{i=1, i \neq j}^{N} \frac{p_{u_{j,i} \lambda_j}}{q_{\lambda_j}} u_{j,i}\right), \tag{17}$$

where the first term in brackets is the benefit from accumulating knowledge in terms of increased output. The second term is the benefit in terms of higher amount of knowledge available for subsequent R&D:

$$\frac{\partial \dot{\lambda}_{j}}{\partial \lambda_{j}} = \xi \left[ 1 + (1 - \alpha) \sum_{i=1, i \neq j}^{N} \left( \frac{u_{i,j} \lambda_{i}}{\lambda_{j}} \right)^{\alpha} \right] L_{r_{j}}.$$
 (18)

The third term is the benefit in terms of increased amount of knowledge that can be licensed.

The demand for, and the supply of knowledge in this case are given by

$$p_{u_{i,j}\lambda_i} = q_{\lambda_j} \xi \alpha \left( \frac{\lambda_j}{u_{i,j}\lambda_i} \right)^{1-\alpha} L_{r_j}, \ \forall i \neq j, \tag{19}$$

$$u_{i,j}=1, \ \forall i\neq j, \tag{20}$$

which means that the firm has a downward sloping demand for knowledge and licenses (supplies) all its knowledge.

In the case when there are knowledge spillovers among high-tech firms, the returns on R&D are given by (17) but

$$p_{u_{i,i}\lambda_i} = 0, \forall i,$$
 (21)

and

$$\frac{\partial \dot{\lambda}_{j}}{\partial \lambda_{j}} = \xi (1 - \alpha) \left[ \sum_{i=1}^{N} \left( \frac{\lambda_{i}}{\lambda_{j}} \right)^{\alpha} \right] L_{r_{j}}. \tag{22}$$

The first expression means that the licensees receive knowledge (patents) for free. In turn, there is a difference between (18) and (22) because when there is knowledge licensing the returns to R&D are fully appropriated within high-tech firms.

The expression for the price of knowledge (19) indicates that the licensees pay a fixed fee for it. The fee is equal to their marginal valuation of the knowledge that they acquire. This valuation includes all future benefits from using that knowledge for augmenting their current knowledge. Therefore, the licensors appropriate all the benefit from licensing knowledge (i.e. they make the 'take it or leave it' offer). With a continuous accumulation of knowledge, as given by (10), at each and every instant the licensees acquire new knowledge at a fixed fee.

From (19) it follows that  $p_{u_{j,i}\lambda_{j}}$  declines with  $\lambda_{j}$ . It is clear from (17) that I have assumed that the firm treats  $p_{u_{j,i}\lambda_{j}}$  as exogenous and does not take into account this effect while accumulating knowledge. In this sense, I focus on a perfect market for knowledge where the price of knowledge is equal to its marginal product and the licensors appropriate all benefits.

In the frames of this model the assumption that the licensors of knowledge do not take into account that their knowledge is used for stealing business amounts to assuming that firm j takes  $q_{\lambda_i}$  as exogenous for any i different than j. This is in line with assuming that it takes  $p_{u_j,\lambda_j}$  as exogenous (see online Appendix E.4 for the case when it takes into account  $p_{u_i,\lambda_j}$ ).

Finally, in equilibrium there is no difference if high-tech firms license their knowledge in return to wealth transfer or knowledge of other firms. Therefore, knowledge licensing among high-tech firms can also be thought to resemble patent consortia and cross-licensing.<sup>9</sup>

#### 2.3.3 Firm entry

I focus on two regimes of entry into the high-tech industry. In the first regime there are exogenous barriers to entry (i.e. there is no entry) and all firms in the

<sup>9</sup> At this level of abstraction, the license fees can also be thought to represent patent citations.

market are assumed to have entered at *t*=0. In the second regime there are no barriers to entry. Moreover, entry entails no costs see for a setup with endogenous sunk costs (see for a setup with endogenous sunk costs Jerbashian 2014).

# 3 Features of the dynamic equilibrium

I restrict the attention to a symmetric equilibrium in the high-tech industry, and denote the growth rate of a variable Z by  $g_z$ . For subsequent analysis it is useful to define an indicator function  $I_{s,\lambda}^1$  as

$$I_{S,2}^1 = \begin{cases} 1 \text{ for S.2,} \\ 0 \text{ otherwise.} \end{cases}$$

From (10), (11), and (12) it follows that the growth rate of knowledge/productivity in both setups can be written as

$$g_{\lambda} = \xi N L_{r}$$
 (23)

The rate of return on knowledge accumulation can be derived from the optimal rules of the high-tech firm (14), (15), and (17)–(22). It is given by

$$g_{q_{\lambda}} = r - g_{\lambda} \left( \frac{L_{x}}{L_{r}} + 1 - \alpha I_{S.2}^{1} \right). \tag{24}$$

This expression determines the allocation of labor to R&D in a high-tech firm relative to the allocation of labor to production. This ratio does not (explicitly) depend on competitive pressure in the high-tech industry because high-tech firms decide on the division of labor between production and R&D internally and  $L_{x}$  and  $L_{x}$  are paid the same wage.

From the high-tech firm's demand for labor for production (14), the representative final goods producer's optimal rules (4)–(5), and the relation between  $P_x X$  and  $p_x X$  (7) follows a relationship between  $NL_x$  and  $L_x$ :

$$NL_{x} = \frac{\sigma}{1 - \sigma} bL_{y}, \qquad (25)$$

where

$$b = \frac{e-1}{\rho},\tag{26}$$

$$e = \varepsilon - \frac{\varepsilon - 1}{N}.\tag{27}$$

This relationship takes into account the effect of price setting by high-tech firms. According to (26) and (27),  $L_y/NL_x$  declines with the number of firms N and  $\varepsilon$ . This is because increasing N and  $\varepsilon$  reduces mark-ups and the relative price of x, which increases  $NL_x$ . Meanwhile, final goods producers substitute X for  $L_y$  which reduces  $L_y$ .

The relationship between  $NL_x$  and  $L_y$  (25) together with the labor market clearing condition,

$$L = L_v + N(L_v + L_v), \tag{28}$$

implies a relationship between NL, and NL:

$$NL_{v} = D(L - NL_{v}). \tag{29}$$

In this expression, *D* is equal to

$$D = \frac{\sigma(e-1)}{e-\sigma}$$
.

It measures the effect of competitive pressures in the high-tech industry on allocations of labor force.

In the final goods market it must be that

$$Y = C.^{10}$$
 (30)

# 3.1 Entry regime 1: barriers to entry

I take N>1 and allow profits  $\pi$  in (13) to be negative. This is needed in order to characterize the behavior of growth rates and allocations for any N>1 and  $\varepsilon$ . It can be supported by subsidies, for example.

Let the consumers be sufficiently patient so that  $\theta \ge 1$ , which is a standard stability condition in multi-sector endogenous growth models and seems to be the empirically relevant case. Moreover, let the following parameter restriction hold for any N:

**<sup>10</sup>** The main innovation of this paper is that it models knowledge licensing among high-tech firms. In the remainder of the text, I highlight the main innovation and its implications. The remainder of the properties of the framework can be found well characterized in Smulders and van de Klundert (1995), van de Klundert and Smulders (1997), Peretto (1998a,b), Peretto and Smulders (2002), amongst others.

$$\xi DL > \rho$$
. (31)

This inequality ensures that the inter-temporal benefit from allocating labor force to R&D outweighs its cost. It is a necessary condition for having an interior solution for labor force allocation to R&D. The following proposition offers equilibrium labor force allocations and growth rates. I use NE superscript to denote the case when there is no entry.

**Proposition 1:** In decentralized equilibrium in both S.1 and S.2 setups, the economy makes a discrete jump to a balanced growth path where labor force allocations and growth rates of final output and knowledge/productivity are given by

$$NL_{r}^{NE} = \frac{1}{\xi} \frac{\xi DL - \rho}{(\theta - 1)\sigma + \alpha I_{S,2}^{1} + D},$$
(32)

$$NL_x^{NE} = D(L - NL_x^{NE}), \tag{33}$$

$$L_{Y}^{NE} = L - NL_{X}^{NE} - NL_{Y}^{NE}, (34)$$

and

$$g_{Y}^{NE} = \sigma g_{\lambda}^{NE}, \tag{35}$$

$$g_{\lambda}^{NE} = \frac{\xi DL - \rho}{(\theta - 1)\sigma + \alpha I_{s, 2}^{1} + D}.$$
 (36)

The interaction between  $\alpha$  and  $I_{s,2}^1$  measures the extent of non-appropriated returns on R&D when there are knowledge spillovers compared to when there is knowledge licensing. Clearly, the growth rates of knowledge and final output,  $g_{\lambda}^{NE}$  and  $g_{Y}^{NE}$ , decline with  $\alpha I_{s,2}^{1}$ . Therefore, high-tech firms innovate more and the economy grows at a higher rate when there is knowledge licensing:  $g_{\lambda}^{NE,S.1} > g_{\lambda}^{NE,S.2}$  and  $g_{\nu}^{NE,S.1} > g_{\nu}^{NE,S.2}$ .

Total (consumer) welfare can be found using the household's life-time utility function, (30), and the fact that the economy is always on a balanced growth path. A monotonic transformation of the welfare function, which preserves only the relevant terms, is given by

$$\overline{U}^{NE} = -\left[N^{\frac{\sigma}{\varepsilon-1}}(NL_{x}^{NE})^{\sigma}(L_{Y}^{NE})^{1-\sigma}\right]^{-(\theta-1)}\frac{1}{(\theta-1)\sigma g_{1}^{NE} + \rho}.$$

**Proposition 2:** Welfare is higher when there is knowledge licensing than when there are knowledge spillovers.

In a given instant, *ceteris paribus* the final output is lower when there is knowledge licensing than when there are knowledge spillovers according to (33) and (34). This result then holds because welfare increases with the growth rate of final output,  $g_{\nu}$ , which is proportional to the growth rate of knowledge/productivity,  $g_{\lambda}$ .<sup>11</sup> For brevity, hereafter I solely discuss the results for  $g_{\lambda}$  while keeping in mind that  $g_{\nu}$  is proportional to it.

**Corollary 1:** In both S.1 and S.2 setups, the growth rate of knowledge/productivity  $g_{\lambda}$  increases with the elasticity of substitution between high-tech goods  $\varepsilon$  and the number of high-tech firms N. Moreover, it is a concave function of  $\varepsilon$  and N.

The driver behind these results are the relative price distortions, which are due to price setting by high-tech firms. These distortions increase the demand for labor in final goods production. Increasing the elasticity of substitution and/ or the number of firms reduces these distortions and motivates final goods producers to substitute (a basket of) high-tech goods for labor. Higher demand for high-tech goods and higher amount of available labor increase the incentives of high-tech firms to conduct R&D in line with (24). This increases  $g_3$ .

The next section compares these decentralized equilibrium results with the first best allocations and growth rates.

#### 3.1.1 Social optimum

The hypothetical Social Planner selects the paths of quantities so as to maximize the lifetime utility of the household. The Social Planner solves the following problem:

$$\max_{L_x,L_r} \left\{ \int_0^{+\infty} \frac{C_t^{1-\theta} - 1}{1 - \theta} \exp(-\rho t) dt \right\}$$

s.t.

$$C = \left(N^{\frac{\varepsilon}{\varepsilon - 1}} \lambda L_{x}\right)^{\sigma} [L - N(L_{x} + L_{r})]^{1 - \sigma}, \tag{37}$$

$$\dot{\lambda} = \xi \lambda N L_{r},$$

$$\lambda(0) > 0.$$
(38)

<sup>11</sup> Online Appendix E.5 shows that in a second generation growth model welfare can be lower when there is knowledge licensing than when there are knowledge spillovers. This holds because the growth rate of final output in such a model does not depend on innovation incentives and allocations.

The following proposition offers the Social Planner's allocations and growth rates, where SP superscript is used to make a distinction between these outcomes and the decentralized equilibrium outcomes.

**Proposition 3:** The Social Planner chooses labor force allocations so that the economy, where there is "no entry", makes a discrete jump to a balanced growth path, where

$$NL_{r}^{NE,SP} = \frac{1}{\xi} \frac{\xi \sigma L - \rho}{\theta \sigma},$$
(39)

$$NL_{x}^{NE,SP} = \sigma(L - NL_{y}^{NE,SP}), \tag{40}$$

$$L_{Y}^{NE,SP} = L - NL_{X}^{NE,SP} - NL_{Y}^{NE,SP}$$

$$\tag{41}$$

and

$$g_{Y}^{NE,SP} = \sigma g_{\lambda}^{NE,SP}, \tag{42}$$

$$g_{\lambda}^{NE,SP} = \frac{\xi \sigma L - \rho}{\theta \sigma}.$$
 (43)

The Social Planner necessarily innovates if  $\xi \sigma L > \rho$ , which holds as long as (31) holds since  $\sigma > D$ . As in decentralized equilibrium, this inequality states that the benefit from R&D outweighs its cost.

Corollary 2: In decentralized equilibrium, the economy innovates less than is socially optimal and therefore grows at a lower rate:  $g_i^{NE,SP} > g_i^{NE,S.1}$ . Moreover, it fails to have socially optimal labor force allocations.

The drivers behind these results are the relative price distortions and knowledge spillovers in the S.2 case. Due to these distortions final goods producers substitute labor for high-tech goods, which lowers the output of high-tech firms and the number of researchers that these firms hire. The spillovers in R&D have an effect of similar direction. If such spillovers are present then high-tech firms do not fully internalize the returns on R&D, which reduces their incentives to invest in R&D.

The deviation of D from  $\sigma$  summarizes the differences among socially optimal and decentralized equilibrium growth rates and labor force allocations which stem from the relative price distortions. It is easy to notice that for sufficiently high N

$$\lim_{s\to +\infty} D=\sigma$$
.

This equality holds because for sufficiently high N the limiting case  $\varepsilon = +\infty$  would imply perfect competition in the high-tech industry. According to (32)–(36) and (39)–(43), socially optimal and decentralized equilibrium allocations and growth rates would then coincide when there is knowledge licensing. This is because high-tech firms fully appropriate the benefits from accumulation of knowledge when there is knowledge licensing and there are no distortions in the market for knowledge.<sup>12</sup>

In such a limiting case, however, in decentralized equilibrium high-tech firms make no profits from the sale of their goods and have no market incentives to innovate. In this respect, if there are no subsidies that keep the profits of high-tech firms non-negative, the positive relationship between innovation and  $\varepsilon$  holds, as long as high-tech firms have sufficient profits to cover the costs of R&D. Profits of high-tech firms and  $\varepsilon$  are inversely related. Once profits net of R&D expenditures are equal to zero, increasing  $\varepsilon$  further reduces innovation to zero. The relationship between intensity of product market competition and innovation then resembles an "inverted-U" shape because of the concavity of the relationship between innovation and  $\varepsilon$  and this discontinuity. Such a relation is consistent with Schumpeter's argument that firms need to be sufficiently large in order to innovate. Moreover, it is in line with the empirical findings of Aghion et al. (2005), and provides an alternative explanation for those findings.

# 3.2 Entry regime 2: cost-free entry

From (13), (14), and (24) it follows that the profits of a high-tech firm are given by

$$\pi = wL_{x}\overline{\pi}$$
,

where

$$\overline{\pi} = \frac{1}{e-1} - \frac{g_{\lambda}}{r - g_{q_{\lambda}} - (1 - \alpha I_{s.2}^1) g_{\lambda}}.$$

The economy is on a balanced growth path for a given N according to Proposition 1. On the balanced growth path, it is straightforward to show that  $\bar{\pi}$  is constant and declines with N. In order to have a meaningful equilibrium, I assume that the parameters are such that there exists  $N \in (1, +\infty)$  where  $\bar{\pi} = 0$  in both S.1 and S.2 setups (e.g.  $\varepsilon$  is sufficiently high).

**<sup>12</sup>** At the limit when  $\sigma$ =1, allocations and growth in the decentralized equilibrium are welfare maximizing when there is knowledge licensing since there are no relative price distortions.

The condition that endogenizes the number of high-tech firms is

$$\bar{\pi}$$
=0 (44)

because entry into the high-tech industry entails no costs.

The number of high-tech firms, N, makes a discrete jump to the balanced growth path equilibrium level at t=0 given these assumptions and the above mentioned properties of  $\bar{\pi}$ . Therefore, in decentralized equilibrium with costfree entry the economy is on a balanced growth path for any t>0, where N=0. Labor force allocations and growth rate of knowledge/productivity are given by (32)–(34) and (36), where N is endogenous. It is determined from (27) and the following equation:

$$e = \frac{\xi \sigma L[1 + \alpha I_{S,2}^1 + (\theta - 1)\sigma]}{\xi \sigma L - \rho}.$$
(45)

In both S.1 and S.2 setups, there is unique *N* which satisfies this equation.<sup>13</sup> It is straightforward to show that the equilibrium number of firms declines with  $\varepsilon$ . This is because higher  $\varepsilon$  implies lower mark-ups, which reduces  $\bar{\pi}$  for a given N. The growth rate of knowledge  $g_i$  does not depend on  $\varepsilon$  and N since the right-hand side of (45) does not depend on them. Therefore, higher  $\varepsilon$  entails no dynamic gains but static losses in terms of *N* in this setup. These losses reduce welfare because of love-for-variety specification.14

**Proposition 4:** The growth rate of knowledge/productivity is higher in case when there is knowledge licensing than when there are knowledge spillovers among hightech firms:  $g_1^{CFE,S.1} > g_1^{CFE,S.2}$ . However, there are fewer high-tech firms when there is knowledge licensing then when there are knowledge spillovers among these firms:  $N^{CFE,S.1} < N^{CFE,S.2}$ 

The latter result holds because R&D investments are fixed costs. High-tech firms invest more in R&D when there is knowledge licensing. The number of firms in equilibrium declines with these costs.

The reduction in the number of firms weakens competitive pressures and distorts allocations in this framework. It also implies a lower variety of goods.

**<sup>13</sup>** If at t=0 the number of high-tech firms is higher than the number determined by  $\bar{\pi}=0$  and  $\varepsilon$ -1- $\alpha$ -( $\theta$ -1) $\sigma$ >0 then high-tech firms will exit the market till this condition is satisfied. Considering such a setup, or exit of high-tech firms instead of entry, can support the zero entry costs assumption.

<sup>14</sup> Clearly, there are no welfare losses because of higher  $\varepsilon$  if there is no love-for-variety. Moreover, the result that e in (45) does not depend on N and  $\varepsilon$  can be repealed if, for example, high-tech firms have fixed operating costs.

Therefore, it implies lower welfare because of love-for-variety in (2), and the following proposition holds.

**Proposition 5:** Depending on model parameters, welfare when there is knowledge licensing can be higher or lower than welfare when there are knowledge spillovers.

- Welfare is lower when there is knowledge licensing than when there are knowledge spillovers if  $\alpha$  is close to 0.

The love-for-variety specification plays an essential role in these results. It is shown in the proof of Proposition 5 that it is the sole decisive factor for this welfare comparison if  $\alpha$  is close to zero. Welfare then is lower when there is knowledge licensing than when there are knowledge spillovers because the number of high-tech firms and goods is lower. Further, eliminating the love-for-variety by dividing

*Y* to  $N^{\frac{\circ}{\varepsilon-1}}$  and setting  $\theta=1$  it can be shown that welfare is necessarily higher when there is knowledge licensing for any  $\alpha>0$ . This result is confirmed for  $\theta>1$  using numerical methods.

Similarly to the case when there is no entry, the decentralized equilibrium fails to have socially optimal allocations and growth rates. Given that entry entails no costs, it follows from Proposition 3 that the Social Planner selects labor force allocations and N such that the economy makes a discrete jump to a balanced growth path. On this path labor force allocations and growth rate of knowledge  $g_{\lambda}$  are given by (39)–(41) and (43) and  $N=+\infty$ . <sup>15</sup>

Using (36), (45), and (43) it can be shown that the economy invests in R&D less than is socially optimal in both S.1 and S.2 setups in decentralized equilibrium with cost-free (endogenous) entry into the high-tech industry. Therefore, it grows at a lower than socially optimal rate. Further, it fails to have a socially optimal number of high-tech firms.

## 3.3 Policy inference

In this section, I offer policies that if implemented in decentralized equilibrium lead to the first best outcome. I assume that there is knowledge licensing in decentralized equilibrium. This can amount to assuming that intellectual

**<sup>15</sup>** In order to solve the Social Planner's optimal control problem with first order conditions C needs to be rescaled so that  $C < +\infty$  at t = 0 (i.e. C needs to be divided to  $N^{\frac{\sigma}{\varepsilon - 1}}$ ).

property rights regulation enforces excludability of knowledge and gives the bargaining power to the licensors. In this respect, such an action is one of the necessary policy instruments for increasing welfare in decentralized equilibrium in this model.

I assume that the set of policy instruments includes constant marginal transfers on the purchases of high-tech goods,  $\tau$ . It also includes lump-sum transfers to high-tech firms,  $T_{\sigma}$ , and to households, T. The latter balances government expenditures.

From the final goods producer's problem it follows that under such a policy (6) and (7) need to be rewritten as

$$x_{j} = X \left[ \frac{P_{X}}{(1 - \tau_{X}) p_{x_{j}}} \right]^{\varepsilon},$$

$$P_{X} X = (1 - \tau_{X}) \sum_{i=1}^{N} p_{x_{i}} X_{i}.$$

In turn, T and  $T_{\pi}$  need to be added to the budget constraint of the household (1) and the profit function of high-tech firm *j* (13), correspondingly.

It can be shown that, in symmetric equilibrium, labor force allocations are given by (32)–(34) where  $I_{s,2}^1 = 0$  and

$$D = \left[ (1 - \tau_{x}) \frac{1 - \sigma}{\sigma} \frac{1}{b} + 1 \right]^{-1}.$$
 (46)

Therefore, it is sufficient to equate labor force allocations to R&D and hightech goods production to their socially optimal counterparts in order to obtain socially optimal allocations and growth rates. Such an outcome can be achieved subsidizing the purchases of high-tech goods and setting  $\tau_{_{\rm X}}$ =1/e, which equates D in (46) to  $\sigma$ .

In this setup, it is enough to subsidize the demand for high-tech goods because the returns on knowledge accumulation are fully appropriated when there is knowledge licensing. Clearly, this result stems from the rather idealized setting for the market of knowledge. Nevertheless, it implies that subsidies to inhouse R&D, which are commonly suggested in similar growth models (e.g. van de Klundert and Smulders 1997), can be complemented or replaced with intellectual property rights regulation which enforces excludability of knowledge and gives the bargaining power to the licensors.

Although under this policy labor force allocations and growth rate of knowledge in decentralized equilibrium are equal to their socially optimal counterparts, welfare is not. In decentralized equilibrium, there is a lower number of high-tech firms/goods under the assumption that  $\bar{\pi}=0$  at  $N<+\infty$ .\^{16} The policy instrument which can correct for this is  $T_\pi$ . It is straightforward to show that it is sufficient to set  $T_\pi=wL_\chi\tau_\pi$ , where  $\tau_\pi$  is such that for any finite N the profits of firms are greater than zero but for  $N=+\infty$  profits are zero.

**Corollary 3:**  $\tau_{z}$  is a subsidy and it is given by

$$\tau_{\pi} = \frac{(\varepsilon - 1)\frac{1}{\sigma}(\xi \sigma L - \rho) - [(\theta - 1)\xi \sigma L + \rho]}{(\varepsilon - 1)[(\theta - 1)\xi \sigma L + \rho]}.$$

This implies that entry into the high-tech industry needs to be subsidized. Such subsidies are in the spirit of welfare improving R&D subsidies in model of Romer (1990) to the extent that entry can be thought to be a result of R&D that generates new types of goods.

## 4 Conclusions

The model presented in this paper incorporates knowledge (patent) licensing into a stylized endogenous growth framework, where the engine of growth is high-tech firms' in-house R&D. The inference from this model suggests that, if there is knowledge licensing, high-tech firms innovate more and economic growth and welfare are higher than when there are knowledge spillovers among these firms. The results also suggest that innovation in the high-tech industry, as well as economic growth, increase with the intensity of competition and the number of firms in that industry.

If entry is endogenous and entails no costs, innovation in the high-tech industry and economic growth are again higher when there is knowledge licensing. However, this happens at the expense of a lower number of high-tech firms. Welfare then is not necessarily higher when there is knowledge licensing than when there are knowledge spillovers because of this and love-for-variety. If there is no love-for-variety, then knowledge licensing delivers higher welfare.

Increasing the intensity of competition reduces the number of high-tech firms. However, it does not affect allocations, innovation in the high-tech industry, and economic growth. Therefore, if there is no love-for-variety, increasing the intensity of competition reduces welfare in this setup.

**<sup>16</sup>** Clearly, the number of firms in decentralized equilibrium is equal to the socially optimal number of firms if  $\bar{\pi}$ =0 at N=+ $\infty$ .

Taken together, these results suggest that intellectual property regulation which facilitates excludability of knowledge and motivates knowledge licensing increases the rate of economic growth. It also increases welfare if the number of firms is fixed and/or there is no love-for-variety. It can reduce welfare when the number of firms is endogenous and there is love-for-variety.

A policy consisting of four instruments can be sufficient for achieving the first best outcome in decentralized equilibrium. The policy gives the bargaining power in the market for knowledge to the licensors so that they appropriate all the benefit from their R&D. Further, it offsets the relative price distortions with subsidies on the purchases of high-tech goods. Finally, it subsidizes entry into the high-tech industry and uses non-distorting taxes to cover all these subsidies.

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