Methods

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Constrained stochastic predictive control of linear systems with uncertain communication

Stochastische prädiktive Regelung beschränkter linearer Systeme mit unsicherer Kommunikation

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Abstract: This paper proposes a scheme of model predictive control for single-loop networked control system (NCS) with probabilistically modeled communication channel and disturbances. Uncertainties of the communication network are projected onto a tailored probability for the satisfaction of state and input constraints. The proposed receding horizon control scheme uses a probabilistic terminal state and set to establish a balance between control performance and state probability distribution, while satisfying the given constraints. In addition to describing the control approach, its properties are discussed, and it is illustrated by an example.

Keywords: networked control, stochastic disturbances, constrained linear control, model predictive control, age of information

Zusammenfassung: Dieser Artikel schlägt ein Schema zur modellprädiktiven Regelung einschleifiger Regelkreise mit probabilistisch modelliertem Kommunikationskanal und Störung vor. Unsicherheiten des Kommunikationsnetzes werden auf eine angepasste Wahrscheinlichkeit zur Erfüllung von Zustands- und Eingangsbeschränkungen projiziert. Die Nutzung von probabilistischen Terminalmengen sowie -zuständen erlaubt die Einstellung eines Kompromisses zwischen der Regelgüte und der Zustandsverteilung bei Einhaltung der Beschränkungen. Der Beitrag stellt den Ansatz vor, diskutiert seine Eigenschaften und beschreibt die Anwendung anhand eines Beispiels.

Schlagwörter: vernetzte Regelung, stochastische Störungen, beschränkte lineare Regelung, modellprädiktive Regelung, Alter von Informationen

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1 Introduction

This article combines principles of model predictive control (MPC) and networked control systems (NCS) to address timing effects that arise from wireless components within control loops [1]. As wireless sensors have become widely available and are simple to integrate [2], they also contribute to enabling concepts of *Industry 4.0*, e. g., the control of mobile systems with stationary motion capture systems [3]. On the downside, the drawback of electromagnetic susceptibility (like packet loss and/or latency) arises if used in control systems. As imperfect wireless communication can be interpreted as a stochastic process, the behavior, control performance, and stability of the NCS depend on stochastic information flow. In case the communication delays are in the order of the dominant time constants of the plant dynamics, the consideration of the stochastic process within the controller synthesis is mandatory, while being omitted in the vast majority of methods proposed for NCS. Some approaches model the imperfections by considering a worst case delay [4, 5, 6] – however, the use of constant worst case delays within controller synthesis typically leads to overly conservative results and degraded performance. Own work in this direction aimed at the reduction of conservatism by use of variable communication delays within the controller synthesis [7, 8]. The key point there is to predict and robustly bound the delay of the communication network, and to consider the time-varying bounds in distributed MPC.

However, as all these approaches aim at robust satisfaction of constraints (while the underlying communication process is of probabilistic nature), the reduction of conservatism is limited. Therefore, the objective of this contribution is to model the NCS completely stochastic, and to use stochastic MPC (SMPC). In SMPC the probabilistic system behavior is explicitly described by its underlying probability distribution to directly address the probability to satisfy or violate associated constraints. For a brief overview of SMPC and its application to control systems without network structures, see [9]. For more explicit literature on SMPC with state and input constraints, the interested reader is referred to [10, 11]. In the context of NCS, the satisfaction of state constraints – especially in the probabilistic setting – is still a largely open issue. Thus, [12] has further investigated the stochastic problem setup and provides a stochastic optimal control scheme, which is able to handle both, uncertain communication and additive disturbances affecting the plant as stochastic processes. The uncertainty of when future sensor information will be received by the controller are projected onto a tailored likelihood to satisfy state and input constraints. The promising result is that a poorly performing communication link will affect the uncertainty, but not necessarily the performance of controlled behavior. In this article, the findings of [12] are used to develop an MPC scheme, in order to cope with larger horizons and to further reduce the conservatism. As usual for MPC, this implies the formulation and use of a terminal control law and set for the stochastic system to ensure recursive feasibility and stability in a probabilistic sense.

The article is structured as follows: Section 2 introduces notation and clarifies different mathematical aspects used throughout the article. Section 3 specifies the networked control system and the control problem. Section 4, as main part of the paper, details aspects of the stochastic communication channel, the computation of the terminal control law, and the stochastic MPC scheme including properties. The scheme is then applied in Sec. 5 to the example of an aerial vehicle, and conclusions are contained in Sec. 6.

2 Preliminaries

The symbol $s(k)$ denotes a discrete-time value of a signal $s(t) \in \mathbb{R}^{n_s}$ at time $t = k \cdot \Delta t$, with $k \in \mathbb{N}^+$, and constant time-step $\Delta t \in \mathbb{R}^+$. The value of the signal s for a future time-step $k+t, t \in \mathbb{N}^+$ as predicted in k is denoted by $s_{t|k}$.

A convex polytope for s, parameterized by a pair (C_s, b_s) of matrix and vector of appropriate dimension, is referenced by:

$$\mathbb{S} = \{s \mid C_s \cdot s \leq b_s\}.$$

An ellipsoidal set is defined by its center point and shape matrix $(\bar{s}, \mathcal{S})$:

$$\varepsilon(\bar{s}, \mathcal{S}) = \{s \mid (s-\bar{s})^\intercal \mathcal{S}^{-1}(s-\bar{s}) \leq 1\}.$$

A multivariate normal distribution of an n_s-dimensional random vector s with covariance $\mathcal{S} \in \mathbb{R}^{n_s \times n_s}$ and a mean-value $\bar{s} \in \mathbb{R}^{n_s}$ is denoted by $s \sim \mathcal{N}(\bar{s}, \mathcal{S})$. Correspondingly, the expectation of a value s is denoted by $\bar{s} = \mathbb{E}[s]$. An affine transformation of a normal distribution with matrix M and vector v is again a normal distribution according to:

$$M \cdot \mathcal{N}(\bar{s}, \mathcal{S}) + v = \mathcal{N}(M\bar{s} + v, M\mathcal{S}M^\intercal),$$

and the sum of two normally distributed random vectors $s_1 \sim \mathcal{N}(\bar{s}_1, \mathcal{S}_1)$ and $s_2 \sim \mathcal{N}(\bar{s}_2, \mathcal{S}_2)$ is also again a normal distribution:

$$s_1 + s_2 \sim \mathcal{N}(\bar{s}_1 + \bar{s}_2, \mathcal{S}_1 + \mathcal{S}_2).$$

The level-curves of a Gaussian probability density function are ellipsoidal, such that the δ-confidence ellipsoid of a normal distribution with rank n and with probability $\delta \in [0, 1]$ is defined by:

$$\mathcal{S}^{[\delta]} = \varepsilon\left(\bar{s}, c^{[\delta]}\mathcal{S}\right),$$

using a scaling factor $c^{[\delta]} = F^{-1}_{\chi^2}(\delta, n)$, in which F_{χ^2} denotes the cumulative distribution function of a χ^2-distribution. Throughout the paper, the mean value of a distribution coincides with the center point of the confidence ellipsoid, and the shape matrix is equal to the covariance matrix of the distribution. Thus, the same notation is used for these quantities.

The probability for an event $\mathcal{A}$ is referred to by $\mathbb{P}(\mathcal{A})$, and if this probability is specific for the time-step k, it is denoted by $\mathbb{P}_k(\mathcal{A})$. Likewise, the conditioned expectation for a signal $s_{t|k}$ is denoted by $\mathbb{E}_k[s_{t|k}] =: \bar{s}_{t|k}$, since prediction and expectation rely on the same time-step k throughout the paper.

Finally, the symbol $\|s\|_Q^2 = s^\intercal \cdot Q \cdot s$ denotes a weighted 2-norm of the vector s with weight Q.

3 Considered class of networked control systems

The class of systems under consideration in this paper represent single-loop feedback structures with wireless communication network in the signal link between plant and controller, see Fig. 1a. This setting can be motivated, e. g., in the context of controlling aerial vehicles, as explained in Sec. 5. The plant is modeled by discrete-time linear systems with probabilistic additive disturbances:

$$x(k+1) = Ax(k) + Bu(k) + Ew(k), \tag{1}$$
$$w(k) \sim \mathcal{N}(0, \mathcal{W}(k)),$$

with state vector $x \in \mathbb{R}^{n_x}$, input vector $u \in \mathbb{R}^{n_u}$, and a vector of disturbances $w \in \mathbb{R}^{n_w}$. The additive disturbances cannot be measured and are assumed to be normally distributed with zero-mean and covariance matrix $\mathcal{W}$, and to be independent, identically distributed (i. i. d.).

Assumption 1. The stochastic disturbance process may vary over time, but is assumed to be known with an upper bound $\hat{\mathcal{W}}$ of the covariance: $\mathcal{W}(k) \preccurlyeq \hat{\mathcal{W}}$.

The states $x(k)$ and inputs $u(k)$ have to satisfy polytopic chance-constraints with likelihoods δ_x, and δ_u respectively (both typically chosen close to 1):

$$\mathbb{P}\left(x(k) \in \mathbb{X}_k\right) \geq \delta_x, \quad \mathbb{X}_k = \{x \mid C_{x_k} \cdot x \leq b_{x_k}\}, \tag{2a}$$

$$\mathbb{P}\left(u(k) \in \mathbb{U}\right) \geq \delta_u, \quad \mathbb{U} = \{u \mid C_u \cdot u \leq b_u\}. \tag{2b}$$

The admissible sets $\mathbb{X}_k$ and $\mathbb{U}$ are assumed to be convex, compact, and to contain the origin in their interior for all k.

The controller and plant are statically connected in wired form, but sensor and controller are loosely connected by wireless communication. The communication network is established as a simple one-link channel and consists of a sender S, a receiver R, and the link-probability $p(k)$, see Fig. 1a. The link probability can be understood as an abstract representation of the network characteristics (or protocol), as, e. g., resending information in case of failed transmission, or activation/deactivation of the channel by a network controller, but it could also be interpreted as the probability that measured sensor data is available. However, for assuming a time-varying link-probability, the communication network is modeled by a Markov chain [13] in each time-step k, as shown in Fig. 1b.

Assumption 2. The link-probability $p(k)$ may vary over time, but is assumed to be known (by measurement, or possibly according to a predictive network control scheme, see [14]). Furthermore, possible dropouts of information at times k and $k+t$ are i. i. d. for all $\{k, t\} \in \mathbb{N}_+$.

In time-step k, the current state $x(k)$ is available to S. The information is broadcast to R, and thus available to the controller with probability $p(k)$. If information is not received, the information remains with S. Then, the last information received by the controller is outdated, such that the age of the currently available state information increments. Similar to the previous work in [8, 12], the following definition is proposed:

Definition 1. *The quantity $a(k) \in \mathbb{N}_{\geq 0}$ denotes the age of the newest state information $x(t), t \in \mathbb{N}_{\leq k}$ available to the controller (short: AoI), and it quantifies the difference between the time-instances of sending and using the information $x(t)$.*

If $x(k+1)$ is received, the AoI is reset to zero, otherwise it is incremented again.

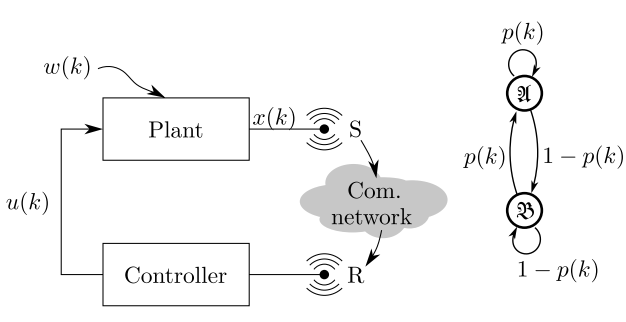
The main control objective for this networked control system is to steer the states to the origin with acceptable costs for the input, i. e., to find a control sequence $\{u(k)\}_{k=0}^{\infty}$ which minimizes the infinite horizon cost function:

$$J_\infty = \sum_{k=0}^{\infty} l(x(k), u(k))$$

subject to the state and input constraints, and the uncertain AoI respectively. The stage costs are formulated in the following common form:

$$l(x, u) = \|x\|_{Q_x}^2 + \|u\|_{Q_u}^2 \tag{3}$$

with symmetric and positive semi-definite weighting matrix $Q_x = Q_x^\intercal \geq 0$ for the states, and definite weighting matrix $Q_u = Q_u^\intercal > 0$ for the inputs.



(a) Overall system **(b)** Com. network

Figure 1: Structure of the networked control system and the corresponding Markov chain modeling the communication network.

4 SMPC based on stochastic optimization

To cope with the challenges of control under uncertainty of the reception of data and the satisfaction of chance constraints, the following control scheme is proposed: In each time-step k in which information is received (i. e., $a(k) = 0$), the controller solves a *finite-time* stochastic optimal control problem with horizon H. The solution of the optimal control problem is a control sequence $\{u_{t|k}\}_{t=0}^{H-1}$, which has to guarantee the satisfaction of the constraints (2) with respect to the upcoming communication link probabilities $\{p(k+t)\}_{t=1}^{H-1}$. Then, as usual for MPC, the first control input is applied to the system, i. e., $u(k) = u_{0|k}$. In case no information is received, i. e., $a(k) > 0$, the controller solves no optimization problem, but applies the subsequent control input of the optimal control law computed in the previous optimization. This was accomplished at time $k - a(k)$, thus the input is chosen according to $u(k) = u_{a(k)|k-a(k)}$. Note that the use of a previously computed control input is relatively unusual in predictive control, but here it is used to avoid accounting for uncertain initial states within the optimization problem, discussed later on.

This scheme is repeated recursively, thus the case of an infinite horizon control, as mentioned in Sec. 3, would be approximated. However, due to the properties of the probabilistic communication link, the possibility exists that the communication link fails for more than H subsequent time-steps, i. e., $\mathbb{P}(a(k) \geq H) > 0$. In this case, no control law computed before is available. This problem is immanent to the considered setting, and cannot be healed by any closed-loop control strategy (while its probability of occurrence can, of course, be reduced by increasing H within finite-horizon MPC). Consequently, closed loop control may not be possible, if the lack of measured information persists permanently, while open-loop control remains to be an option (obvious from Fig. 1a). Thus, the following assumption needs to be stated:

Assumption 3. A fallback strategy, like stalling or shutting-down operation, exists, which can be applied in the case that the communication link fails for H or more subsequent time-steps, to ensure safety of operation.

Obviously, the age of information (AoI) is a crucial value in operating the NCS and for the success of the predictive control policy. Hence, this quantity is detailed in the following subsection.

4.1 Modelling the age of information

According to the one-link communication network in Fig. 1, the AoI depends on the behavior of the communication link, and its transmission probability. In each time-step, the transmission succeeds if $x(k)$ is made available to the controller, implying according to Def. 1 that $a(k) = 0$. If the transmission fails, the newest information previously sent to the controller remains the same as before. But since this information is older by one time-step, $a(k) := a(k-1)+1$ results. According to this scheme, the behavior of the AoI is modeled (and becomes therefore predictable) according to a discrete-time Markov chain with states $\sigma_j(k+t)$ implying $a(k+t) = j$, and with probabilities $q(k+t) = 1 - p(k+t)$, as shown in Fig. 2.

With an initial distribution $\mu(k)$ (determined by the current AoI), the distribution for future values of the AoI results to:

$$\mu_{t|k} = \left(\prod_{r=1}^{t} P(r) \right) \cdot \mu(k) \tag{4}$$

with time-varying transition probability matrix $P(k)$. For the example of Fig. 2, $P(k)$ is given by:

$$P(k) = \begin{bmatrix} p(k) & p(k) & p(k) & 1 \\ q(k) & 0 & 0 & 0 \\ 0 & q(k) & 0 & 0 \\ 0 & 0 & q(k) & 0 \end{bmatrix}.$$

The probability for a specific value of the AoI in time-step $k+t$ predicted in k is given by:

$$\mathbb{P}_k\left(a(k+t) = j\right) = \mu_{t|k}[j], \tag{5}$$

where $\mu[j]$ denotes the $(j+1)$-th entry of the vector μ.

4.2 AoI-dependent constraints

The predicted evolution of the NCS is subject to two types of uncertainties, the additive disturbances and the AoI. While the disturbances may lead to violation of the state constraints, the AoI may prevent necessary control actions. To minimize the uncertainty of prediction (and thus to enhance the control performance), a control law is synthesized which compensates the additive disturbances. To obtain a synthesis problem with admissible solution, a scheme of disturbance feedback is used. Since the applicability of the control law depends on the AoI, the constraints are formulated depending on this quantity. In any time-step, in which the controller receives new information, i. e., $a(k) = 0$, the predictive scheme requires to solve the stochastic optimal control problem. The solution $\{u_{t|k}\}_{t=0}^{H-1}$ has to satisfy input and state constraints (2), despite the uncertainty arising from the AoI. This raises the

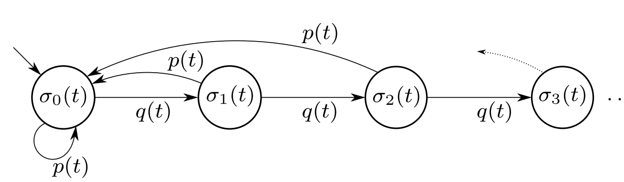


Figure 2: Example of a Markov chain to model the AoI for 3 subsequent time-steps with $k = 0$ and $a(0) = 0$.

question of how the feedback policies should be chosen, given the situation that it is unclear whether the future states are available to the controller. Therefore, uncertainties of the communication network are projected onto a tailored probability for the satisfaction of state and input constraints.

However, the availability of the states is predictable according to the AoI with (5): $x(k+r)$ is available to the controller in time-step $k+t, t \geq r$, if $a(k+t) \leq t-r$ holds. Now assume for a moment, that if $x(k+r)$ is available, also all prior states $\{x(k+t)\}_{t=0}^{r}$ are available to the controller. Then, also all affecting disturbances $\{w(k+t)\}_{t=0}^{r-1}$ (except of the last) could be reconstructed by the controller according to (1).

Hence, the probability that a disturbance $w(k+r)$ is available to the controller in $k+t$ results with (5) to:

$$\mathbb{P}_k\left(r < t - a(k+t)\right) = \sum_{j=0}^{t-r-1} \mu_{t|k}[j] =: p_{t,r|k},$$

which follows from (4) with $\mu(k) = [1, 0, \ldots, 0]^\top$ as $a(k) = 0$. With the probabilities $p_{t,r|k}$, the likelihoods $\{\delta_u, \delta_x\}$ for satisfying the input and state constraints are separated into: the likelihood that the control policy from optimization in k is applicable in $k+t$: $\alpha_{t|k} \in [\max\{\delta_u, \delta_x\}, 1]$, and the tailored likelihoods to satisfy the input and state constraints: $\{\gamma_{t|k}^{(u)}, \gamma_{t|k}^{(x)}\}$, such that:

$$\delta_u = \gamma_{t|k}^{(u)} \cdot \alpha_{t|k}, \quad \delta_x = \gamma_{t|k}^{(x)} \cdot \prod_{r=1}^{t-1} \alpha_{r|k}. \tag{6}$$

Note, that the choice of the $\alpha_{t|k}$ allows for some degrees of freedom, but is proposed to be as in [12]:

$$\begin{aligned} \alpha_{t|k} &= \min_{r \in \mathbb{N}_{\geq 0}} \{p_{t,r|k}, 1\}, \qquad (7)\\ \text{s.t.:} \quad & p_{t,r|k} > \max\{\delta_x, \delta_u\},\\ & \prod_{r=1}^{t-1} \alpha_{r|k} \geq \delta_x \quad \forall\, t \geq H. \end{aligned}$$

Finally, the disturbance feedback policy used for the optimization results to:

$$u_{t|k} = V_{t|k} \cdot x(k) + \sum_{r=0}^{\hat{r}_{t|k}} M_{t,r|k} \cdot w(k+r), \tag{8}$$

with feedback matrices $V \in \mathbb{R}^{n_u \times n_x}$ and $M \in \mathbb{R}^{n_u \times n_w}$, and $\hat{r}_{t|k}$ as the minimizer of (7). Note that (8) is equivalent to a linear state-feedback [15], and could easily extended to an affine control law [16]. However, this is not necessary here, since the NCS is controlled to the origin.

In summary, the uncertain AoI requires the computation of the probabilities $\{\mu_{t|k}\}_{t=1}^{H-1}$ to choose the values $\{\alpha_{t|k}, \hat{r}_{t|k}\}_{t=1}^{H-1}$, and to tighten the state and input constraints with (6) according to:

$$\mathbb{P}_k\left(x_{t|k} \in \mathbb{X}_{k+t}\right) \geq \gamma_{t|k}^{(x)}, \quad \forall\, t = \{1, \ldots, H\}, \tag{9a}$$

$$\mathbb{P}_k\left(u_{t|k} \in \mathbb{U}\right) \geq \gamma_{t|k}^{(u)}, \quad \forall\, t = \{0, \ldots, H-1\}. \tag{9b}$$

Finally, the uncertainty of the AoI is cast into a variation of the control law, and the state and input constraints, such that a common stochastic optimal control problem can be formulated. In order to adapt the optimal control problem into one of predictive control, the next section clarifies how to specify the terminal conditions for the NCS with uncertain AoI.

Note the following important point: in each time-step k, in which new information arrives, an optimization over the control law (8) is initiated. Then, only the state $x(k)$ is needed to compute the input $u_{0|k}$ (which has to be applied to the NCS), since $w(k+r)\ \forall r \geq 0$ is never available in k. In the first succeeding time-step $k+t$ for which the feedback of disturbance $w(k)$ (the first disturbance affecting the system after reception of $x(k)$) was expected in k, new information is received with probability $\alpha_{t|k}$. In case this information is received (and $w(k)$ could be used within $u_{t|k}$), a new optimization is started and the reconstruction of $w(k)$ is not necessary anymore. In case the expected information is not available, the input $u_{t|k}$ has to be applied without the expected feedback of disturbance $w(k)$. This may violate the state and input constraints, but is scheduled according to the tightened probabilities $\gamma_{t|k}^{(u)}$, and $\gamma_{t|k}^{(x)}$. Hence, the assumption to know all prior states and disturbances (which was made above), is not necessary within the predictive control scheme, but allows the prediction of an admissible control sequence according to (8).

4.3 The terminal system and covariance

The control strategy aims at recursively solving the stochastic optimal control problem to obtain a predictive control scheme. In deterministic constrained predictive control, stability is usually guaranteed by i.) the use of an invariant set as *terminal constraint* for the last state of the prediction horizon, and ii.) by the use of a Lyapunov-based terminal cost term in the objective function. Similarly to the control of delayed systems, the terminal set and cost term have to be modified for the NCS to account for the uncertain AoI. In addition and according to the additive and probabilistic disturbance $w(k)$, the use of an invariant covariance is introduced to ensure recursive feasibility. In order to formulate terminal covariance and terminal set, a terminal system is introduced, motivated by the findings in [4].

Terminal system

With delay-time $\tau \in \mathbb{N}_{>0}$, the state vector of the terminal system $\xi \in \mathbb{R}^{n_\xi}$ with $n_\xi = (\tau + 1) \cdot n_x$ is introduced to consist of multiple state vectors of the NCS for different time instances:

$$\xi_{H|k}^{\top} = \left[x_{H-\tau|k}^{\top} \ \cdots \ x_{H-2|k}^{\top} \ x_{H-1|k}^{\top} \ x_{H|k}^{\top}\right].$$

The dynamics of the terminal system is chosen to:

$$\xi_{t+1|k} = A_\xi \cdot \xi_{t|k} + B_\xi \cdot \phi_{t|k} + E_\xi \cdot w(k+t) \tag{10}$$

for $t \geq H$ and with matrices:

$$A_\xi = \begin{bmatrix} 0_{\tau \cdot n_x \times n_x} & I_{\tau \cdot n_x} \\ 0_{n_x \times \tau \cdot n_x} & A \end{bmatrix}, B_\xi = \begin{bmatrix} 0_{\tau \cdot n_x \times n_u} \\ B \end{bmatrix}, E_\xi = \begin{bmatrix} 0_{\tau \cdot n_x \times n_w} \\ E \end{bmatrix},$$

and the input vector $\phi \in \mathbb{R}^{n_u}$. Obviously, (10) comprises and extends the dynamics (1), if $\phi_{t|k} := u_{t|k}$ is chosen.

Now consider a delayed control law (which is later called *terminal control law*):

$$\phi_{t|k} = K_\tau \cdot x_{t-\tau|k} =: K_\xi \cdot \xi_{t|k} \tag{11}$$

with $K_\tau \in \mathbb{R}^{n_u \times n_x}$ such that the terminal system has a stable closed loop matrix $A_{\xi,cl} = (A_\xi + B_\xi K_\xi)$. Note that this control law is applicable as long as the AoI does not exceed the value τ, what is assumed temporarily.

Given the probabilistic disturbance $w(k)$ with zero mean, the closed loop behavior of the terminal state is predictable with $\xi_{t|k} \sim \mathcal{N}(\bar{\xi}_{t|k})\Xi_{t|k}$ with mean value $\bar{\xi}_{t|k}$ and covariance $\Xi_{t|k}$ evolving according to:

$$\bar{\xi}_{t+1|k} = A_{\xi,cl}\bar{\xi}_{t|k}, \tag{12a}$$

$$\Xi_{t+1|k} = A_{\xi,cl}\Xi_{t|k}A_{\xi,cl}^{\top} + E_\xi \mathcal{W}(k+t)E_\xi^{\top}. \tag{12b}$$

Due to the structure of the terminal system, the rank of the underlying distribution is limited to n_x, regardless of the dimension of n_ξ, or τ respectively. Thus, an ellipsoid with confidence δ for containing the predicted state $\xi_{t|k}$ is introduced and denoted by $\Xi_{t|k}^{[\delta]} = \varepsilon\left(\bar{\xi}_{t|k}, c^{[\delta]} \cdot \Xi_{t|k}\right)$, with a scaling factor $c^{[\delta]} = F_{\chi^2}^{-1}(\delta, n_x)$.

Invariant covariance and probabilistic invariant set

By applying (12) recursively, the evolution of the terminal state is obtained. With stable closed-loop matrix $A_{\xi,cl}$ and for $t \to \infty$, the expected value $\xi_{t|k}$ converges to zero. With respect to the upper bound $\hat{\mathcal{W}} \succcurlyeq \mathcal{W}(k)$ from Asm. 1, an invariant covariance can be obtained, similar to the work in [17].

Definition 2. *A matrix $\Sigma_\xi = \Sigma_\xi^{\top} \succcurlyeq 0$ is called invariant covariance for system* (10) *with control law* (11) *and upper bound $\hat{\mathcal{W}} \succcurlyeq \mathcal{W}(k)$, if it satisfies the Lyapunov equation:*

$$\Sigma_\xi = A_{\xi,cl}\Sigma_\xi A_{\xi,cl}^{\top} + E_\xi \hat{\mathcal{W}} E_\xi^{\top}.$$

Corollary 1. *If Σ_ξ is an invariant covariance, the following holds:*

$$\Xi_{t+1|k} \preccurlyeq \Sigma_\xi, \quad \forall\ \Xi_{t|k} \preccurlyeq \Sigma_\xi, \quad \forall\ \mathcal{W}(k) \preccurlyeq \hat{\mathcal{W}}.$$

Obviously from Def. 2 and Corollary 1, the predicted terminal state converges to a neighborhood around the origin, i. e., $\xi_{\infty|k} \sim \mathcal{N}(0, \Sigma_\xi)$.

Next, the satisfaction of the constraints (2) is considered for the terminal system: From Corollary 1 follows directly that the terminal system is held within an ellipsoid of δ_x-confidence:

Corollary 2. *Given an invariant covariance Σ_ξ, the probability $\delta_x \in [0, 1]$, and a corresponding confidence ellipsoid $\Sigma_\xi^{[\delta_x]} = \varepsilon\left(0, c^{[\delta_x]} \cdot \Sigma_\xi\right)$ with scaling factor $c^{[\delta_x]} = F_{\chi^2}^{-1}(\delta_x, n_x)$, the relation:*

$$\Xi_{t|k}^{[\delta_x]} \subseteq \Sigma_\xi^{[\delta_x]} \tag{13}$$

implies for all $r \geq t$ that:

$$\mathbb{P}_k\left(\xi(k+r) \in \Sigma_\xi^{[\delta_x]}\right) \geq \delta_x.$$

In consequence, satisfaction of (2a) can be guaranteed, if the confidence ellipsoid lies within the admissible state-space, i. e., $\Sigma_\xi^{[\delta_x]} \subseteq \mathbb{X}_{k+t-\tau} \times \ldots \times \mathbb{X}_{k+t}$, and thus if the *ellipsoid-in-ellipsoid* constraint (13) is satisfied. In general, an *ellipsoid-in-ellipsoid* problem can be solved according to the S-procedure proposed in [18] – the solution may be difficult, however, e. g., if the center points of the ellipsoids differ. In [19], a general *ellipsoid-in-ellipsoid* constraint was reformulated with respect to the linearization of a shape-matrix with heuristically chosen linearization point. The use of heuristics is not appropriate for the purpose of recursive feasibility in this paper. Another possibility is to use polytopic under-approximations $\check{\Sigma}_\xi^{[\delta_x]} \subset \Sigma_\xi^{[\delta_x]}$, leading to an *ellipsoid-in-polytope* constraint. This is simple to formulate, but the computation of $\check{\Sigma}_\xi^{[\delta_x]}$ is in general only feasible for small dimensions n_ξ. Since the uncertainties arising from the AoI typically lead to terminal systems of higher dimension, polytopic computations are also not applicable here. Therefore, *ellipsoid-in-ellipsoid* constraints with equivalent center-points are used here:

Proposition 1. *Given two ellipsoids $E_1 \sim \varepsilon(q, \mathcal{E}_1)$ and $E_2 \sim \varepsilon(q, \mathcal{E}_2)$ with $q \in \mathbb{R}^{n_q}$, then $E_1 \subseteq E_2$ iff:*

$$\mathcal{E}_2 - \mathcal{E}_1 \succcurlyeq 0.$$

Hence, constraint (13) is reformulated to:

$$\bar{\xi}_{t|k} = 0, \tag{14a}$$

$$\Sigma_\xi - \Xi_{t|k} \succcurlyeq 0. \tag{14b}$$

Since the terminal system is used within the optimization problem for the end of the prediction horizon, the conservatism introduced by (14a) reduces if the prediction horizon increases. To likewise satisfy the input constraint (2b) with use of (11) and the terminal state $\xi_{t|k}$, the δ_u-confidence ellipsoid for the input $\phi_{t|k} \sim \mathcal{N}(\bar{\phi}_{t|k}, \Phi_{t|k})$ has to lie within the admissible input set, i. e.:

$$\Phi_{t|k}^{[\delta_u]} = \varepsilon\left(K_\xi \bar{\xi}_{t|k}, c^{[\delta_u]} \cdot K_\xi \Xi_{t|k} K_\xi^\intercal\right) \subseteq \mathbb{U}. \tag{15}$$

If the constraints (14) are satisfied, (15) is satisfied too, if:

$$\varepsilon\left(0, c^{[\delta_u]} \cdot K_\xi \Sigma_\xi K_\xi^\intercal\right) \subseteq \mathbb{U} \tag{16}$$

is guaranteed. Note that the satisfaction of (16) can be proven offline with determination of K_ξ and Σ_ξ.

4.4 Constrained stochastic optimization

Before the optimization problem is stated, the existence of a terminal control law, which complies with the restrictions above, is assumed.

Assumption 4. For the terminal system (10) with delay-time τ and the communication link probabilities $\{p(k+t)\}_{t=1}^{\infty}$, there exist a linear control law (11), a terminal weight P_ξ and a probabilistic terminal set $\Sigma_\xi^{[\delta_x]}$ such that the following holds:

1. The delay-time τ is chosen to satisfy the threshold:

$$\mathbb{P}\,(a(k) \le \tau) \ge \delta_\tau, \quad \forall\, k \in \mathbb{N}_{\ge 0} \tag{17}$$

2. The terminal set is the δ_x-confidence ellipsoid of the invariant covariance Σ_ξ for the closed loop system:

$$\xi(k+1) = A_{\xi,cl}\xi(k) + E_\xi w(k), \quad w(k) \sim \mathcal{N}(0, \hat{\mathcal{W}}).$$

3. The terminal set is a subset of the state admissible set, i. e., $\Sigma_\xi^{[\delta_x]} \subseteq \mathbb{X}_{k-\tau} \times \ldots \times \mathbb{X}_k, \quad \forall\, k \in \mathbb{N}_{\ge 0}$.
4. The terminal control law $u(k) = K_\xi \cdot \xi(k)$ satisfies the probabilistic input constraint according to (16).
5. The weighting matrix P_ξ of the terminal cost term solves a Lyapunov function in the sense that:

$$\|\xi(k+1)\|_{P_\xi}^2 - \|\xi(k)\|_{P_\xi}^2 + \|\xi(k)\|_{Q_\xi}^2 \preccurlyeq 0$$

holds for the weights of the stage cost function (3):

$$Q_\xi = \operatorname{diag}\left(K_\xi Q_u K_\xi^\intercal, 0, \ldots, 0, Q_x\right).$$

With these assumptions, a common stochastic finite-horizon cost-function is used with the terminal weight P_ξ and the stage cost (3):

$$J_H(k) = \mathbb{E}_k\left[\|\xi_{H|k}\|_{P_\xi}^2 + \sum_{t=0}^{H-1} l(x_{t|k}, u_{t|k})\right] \tag{18}$$

The optimization problem to be solved in each k with $a(k) = 0$ is eventually:

$$J_H^\star(k) = \min_{\{V_{t|k}, M_{t,r|k}\}_{\{r,t\}=0}^{\{\bar{r}_{t|k}, H-1\}}} J_H(k) \tag{19a}$$

$$\text{s. t.:} \quad x_{t+1|k} = Ax_{t|k} + Bu_{t|k} + Ew(k+t),$$
$$u_{t|k} \quad \text{satisfies (8)},$$
$$\gamma_{t|k}^{(x)}, \gamma_{t|k}^{(u)} \quad \text{given by (9a), (9b)}, \tag{19b}$$
$$\xi_{H|k}^\intercal = [x_{H-\tau|k}^\intercal \ \cdots \ x_{H|k}^\intercal]$$
$$\bar{\xi}_{H|k} = 0, \quad \Sigma_\xi - \Xi_{H|k} \succcurlyeq 0. \tag{19c}$$

Note, that reformulation of (18) into a quadratic cost-functional is well-known in literature, e. g., cf. [10, 20], and that [15] already provides a method to reformulate (19b) into an LMI, such that (19) results in an SDP. The optimal solution $\{V_{t|k}^\star, M_{t,r|k}^\star\}_{\{t,r\}=0}^{H-1}$ together with (8) provides admissible control inputs for the next H time-steps.

As the NCS is affected by an unbounded additive disturbance, and thus the state may become unbounded, too, the optimization problem (19) may also become infeasible. Therefore, and similar to the findings in [20], the use of positive slack variables is proposed to soften both types of constraints (19b) according to [21]. As discussed in [20], this simply augments the set of feasible initial states to $\mathbb{R}^{n_x}$, and recursive feasibility of (19) can be ensured.

Theorem 1. *If* (19) *is feasible in* $k = 0$ *without slack variables, the proposed control scheme guarantees the satisfaction of constraints* (2) *for all* $k > 0$ *at least with probability* δ_τ *in each* k.

Proof. For $k+1$, consider the following solution candidate:

$$\{u_{t|k+1}\}_{t=0}^{H-1} = \{u_{1|k}, \ldots, u_{H-1|k}, (K_\xi \xi_{H|k})\}. \tag{20}$$

Each predicted input $\{u_{r|k}\}_{r=1}^{H-1}$ is applicable to the NCS with probability $\alpha_{r|k}$, and the $\gamma_{r|k}^{(u)}$-confidence ellipsoids of the input lie within $\mathbb{U}$. Hence, the probability that each input lies within the input admissible set $\mathbb{U}$ is at least δ_u. For $k+H$, an admissible input can be predicted with respect to the terminal control law $u_{H-1|k+1} = K_\xi \cdot \xi_{H|k}$. The use of the terminal control law implies the satisfaction of the input at least with probability δ_u, cf. Asm. 4. 4. Eventually, the input constraint (2b) is satisfied by (20), under the condition of an applicable terminal control law, which is given at least with probability δ_τ, see Asm. 4. 1. The satisfaction of the state constraints follows analogously. Note, that possible communication dropouts are stochastically independent, such that $\alpha_{t-1|k+1} := \alpha_{t|k}$ holds recursively (cf. Asm. 2), and satisfaction of constraints (2) for all times $k+t,\ \forall t \in \mathbb{N}_{>0}$ follows directly by induction over k. □

Obviously, constraint satisfaction of the predictive control scheme depends on the threshold δ_τ for the AoI and the underlying probability of the communication link. In general, the link probability does not depend on the control system and is therefore a quantity solely depending on the communication network. However, Asm. 4. 1 applies to the communication network and is in general guaranteed with values δ_τ close to one, see, e. g., [22] for variable transmission power of wireless networks.

Since the disturbance $w(k)$ is stochastically distributed with unbounded support, the controlled NCS results in a perturbed Markov process. Stability of a Markov process (in the sense of mean-square stability) implies a bounded average for the expected step costs [23, Prop. 1. iv)], which is often used to show stability in stochastic model predictive control schemes, e. g., [24, 25, 26, 10]. With Asm. 4. 5, problem (19) is a common stochastic control problem with tailored control law and likelihoods to satisfy the constraints. In addition, the existence of an admissible solution-candidate is given by (20). Therefore, the decrease of expected costs can be shown analogously to the references above, and mean-square stability is obtained, but conditioned by δ_τ.

5 Numerical simulation

This section describes the application of the proposed method to the example of an unmanned aerial vehicle (UAV) moving in a constrained area. Usually, the control of UAVs is twofold: i) an inner control loop stabilizes the UAV in hovering mode, and ii) an outer control loop determines the path to follow. While the inner loop controls states which are locally measurable, as rotor speeds (in the case of drones) or the yaw, roll, and pitch angles, the outer loop controls the position and height in the 3-dimensional air-space. Therefore, the outer control loop is responsible for safe operation with respect to the UAV's environment, i. e., physical and virtual obstacles like buildings or no flight zones in urban areas. In contrast to the inner loop, the states of the outer loop, i. e., the exact position in 3D, is often not locally measurable because of an unmeasurable drift of the gyros used for stabilization, uneven terrain (e. g., if cameras or infrared sensors are used), or varying air pressure. Hence, the position is often measured by GPS for outdoor motion, and via motion tracking systems in case of indoor motion. Both methods have the drawback that the control loop relies on these external signals. In motion tracking systems, the position of the UAV is preprocessed by a stationary unit, and afterwards transmitted to the UVA via a communication link (commonly via wireless LAN). In contrast, GPS is based on the run-time measurement of electromagnetic satellite-signals and relies therefore on possibly faulty connectivity to satellites. However, in both cases the availability of the exact position measurements for the outer UAV control loop can be modeled by the communication link in Fig. 1b, and its transmission probability. Here, the UAV motion is modeled in a two-dimensional space with integrator dynamics, and with an underlying inner control loop, as proposed in [11]. The dynamics (1) is specified by:

$$A = \begin{bmatrix} \tilde{A} & \\ & \tilde{A} \end{bmatrix}, \quad B = \begin{bmatrix} \tilde{B} & \\ & \tilde{B} \end{bmatrix}, \quad E = \begin{bmatrix} \tilde{E} & \\ & \tilde{E} \end{bmatrix},$$
$$\tilde{A} = \begin{bmatrix} 1 & 0.79 \\ 0 & 0.61 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0.21 \\ 0.39 \end{bmatrix}, \quad \tilde{E} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},$$

with state vector $x^\top = \left[p_x^\top, v_x^\top, p_y^\top, v_y^\top\right]$, input vector $u^\top = \left[a_x^\top, a_y^\top\right]$, and disturbances $w^\top = \left[w_x^\top, w_y^\top\right]$. Here, $\{p_x, p_y\}$ are the positions, $\{v_x, v_y\}$ the velocities, $\{a_x, a_y\}$ the acceleration, and $\{w_x, w_y\}$ the disturbances in x- and y-direction. In contrast to [11], only one disturbance per direction is considered, modeling a wind force, and affecting both the position and the velocity of the UAV. The disturbance is modeled as a Gaussian random variable, and the communication link is considered to transmit with constant probability:

$$p(k) = 0.7, \quad \mathcal{W}(k) \preccurlyeq \hat{\mathcal{W}} = \begin{bmatrix} 0.6 & \\ & 0.6 \end{bmatrix} \quad \forall\, k.$$

Position and velocity are constrained according to (2a) with likelihood $\delta_x = 0.95$ and the admissible set:

$$\mathbb{X}_k = \left\{ x \,\middle|\, \begin{array}{l} -30 \le p_x \le 50 \\ -8 \le v_x \le 7 \\ -25 \le p_y \le 20 \\ -8 \le v_y \le 8 \end{array} \right\}, \quad \forall\, k.$$

Similarly, the input is constrained according to (2b) with likelihood $\delta_u = 0.9$ and the admissible set $\mathbb{U} = \{u |\ \|u\|_\infty \le 7.5\}$. Caused by the uncertain AoI, the threshold δ_τ is selected to 99 %. With respect to the constant link probability and according to (17), the threshold results in $\delta_\tau \approx 99.19\,\%$ with delay-length $\tau = 4$ for the terminal system and the control law. Note, that $\tau = 4$ implies $H \ge 4$, and that the expected value of the last 4 predicted states have to lie in the origin (cf. (19c)).

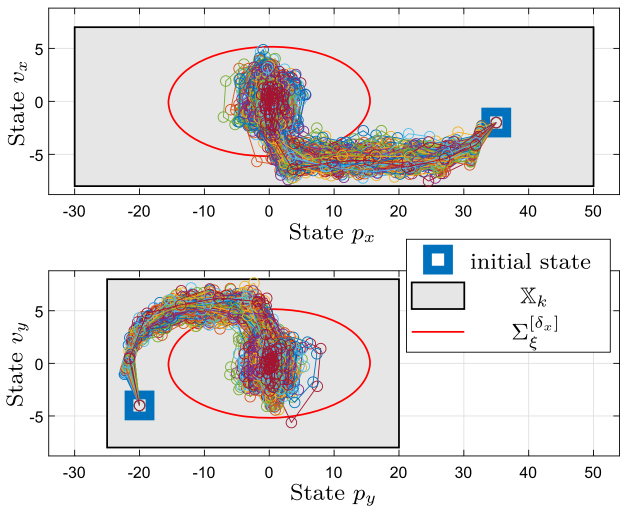


Figure 3: Simulation results for 200 Monte-Carlo simulations with the initial state marked as blue square, the admissible state space as light gray box, and the projection of the terminal set onto the corresponding direction as red ellipsoid.

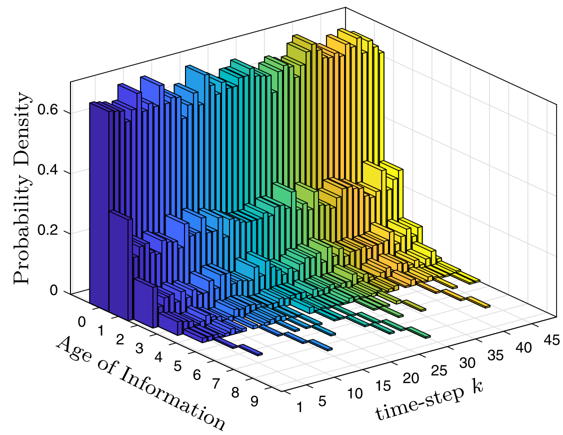


Figure 4: Distribution of the Age of Information for the Monte-Carlo simulations and each time-step of the simulation time.

The feedback matrix of the terminal control law K_τ is chosen to:

$$K_\tau = \begin{bmatrix} -0.058 & -1.172 & 0 & 0 \\ 0 & 0 & -0.058 & -1.172 \end{bmatrix}$$

satisfying the assumptions 4.3 and 4.4.

The prediction horizon is chosen to $H = 12$, and the stage cost function from [12] is used with:

$$Q_x = \begin{bmatrix} \tilde{Q} & \\ & \tilde{Q} \end{bmatrix}, \; \tilde{Q} = \begin{bmatrix} 10 & \\ & 0.1 \end{bmatrix}, \; Q_u = \begin{bmatrix} 0.1 & \\ & 0.1 \end{bmatrix}, \; \mathcal{S} = 1.$$

Starting from an initial state $x_0 = [35, -2, -20, -4]^\intercal$ (blue square), Fig. 3 shows 200 Monte Carlo simulations with 50 time-steps each in both directions of the state space, where each color denotes a different simulation. Note, that in none of these simulations the use slack variables was needed. For simulation, the covariance of the disturbance is chosen to $\mathcal{W}(k) = \hat{\mathcal{W}}$. All simulations show that system trajectories converge into a neighborhood of the origin. The neighborhood is significantly smaller than the 95 %-confidence ellipsoid of Σ_ξ, which is identical to the region for which the control law (11) is defined.

Figure 4 shows the distribution of the AoI values for each time-step k of the simulations. Even if the probability is small, the maximum value for the AoI is 8, thus larger than $\tau = 4$. Note, that the control strategy only guarantees probabilistic constraint satisfaction and mean-square stability as long as $a(k) \le \tau$. By a solution of (20) without the use of slack variables, it provides a suitable control input as long as $a(k) < H$. Here, suitability means that a possible violation of the state and/or input constraints with an aged control input $u(k) = u_{k|k-t}$, $\tau \le t < H$ is considered in the optimal control problem, together with the likelihoods $\{\gamma_k^{(u)}, \gamma_k^{(x)}\}$. Since aged control inputs are rarely used (according to Fig. 4 only with probability around 40 % in each time-step), the satisfaction of constraints is much higher than $\{\delta_u, \delta_x\}$. Here, the admissible state space is never left over all simulations.

6 Conclusion

This article has introduced a model predictive control scheme for constrained linear systems with uncertain availability of state information. The proposed method is based on a recently suggested optimal control scheme for linear systems with uncertain communication, which projects the uncertainty of available information onto a tailored likelihood to satisfy state and input constraints. Here, solving such a stochastic optimal control problem in receding horizon fashion implies the use of terminal constraints to guarantee recursive feasibility and stability of the control scheme. Due to the uncertain communication, the use of a state-augmented terminal system with delayed control law is proposed. In contrast to existing methods, which use reachable sets as terminal sets, the terminal set here is derived with respect to the confidence ellipsoid of an invariant covariance. This approach scales better with the dimension of the system and the communication link probability, compared to the use of polytopic sets.

The simulation results show that the proposed predictive control scheme is able to steer a networked control system with high (and specified probability) into the target, while the AoI changes probabilistically. Although the control scheme is based on probabilistic constraint satisfaction, the system state remains in the admissible state space. At the same time, the state trajectory does not require considerable safety margins to the state constraints, thus introducing only little conservatism. When approaching the target state, the proposed control scheme keeps the system state in a much tighter neighborhood than approaches which are based solely on a worst case delay of the communication link.

It is worth noting, that the computational complexity of the predictive control does not significantly grow with the complexity of the communication network between sensor and controller, or if the threshold δ_τ (and therefore the dimension of the terminal system) is increased.

This control scheme opens the field to distributed predictive control systems using collectively a single communication network – this is matter of future work of the authors.

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