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Applications

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Observer design for a single mast stacker crane

Beobachterentwurf für ein Hochregalbediengerät

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Abstract: This contribution is concerned with the design of observers for a single mast stacker crane, which is used, e. g., for storage and removal of loads in automated warehouses. As the mast of such stacker cranes is typically a lightweight construction, the system under consideration is described by ordinary as well as partial differential equations, i. e., the system exhibits a mixed finite-/infinite-dimensional character. We will present two different observer designs, an Extended Kalman Filter based on a finite-dimensional system approximation, using the Rayleigh-Ritz method and an approach exploiting the port-Hamiltonian system representation for the mixed finite-/infinite-dimensional scenario where in particular the observer-error system should be formulated in the port-Hamiltonian framework. The mixed-dimensional observer and the Kalman Filter are employed to estimate the deflection of the beam based on signals acquired by an inertial measurement unit at the beam tip. Such an approach considerably simplifies mechatronic integration as it renders strain-gauges at the base of the mast obsolete. Finally, measurement results demonstrate the capability of these approaches for monitoring and vibration-rejection purposes.

Keywords: infinite-dimensional systems, energy-based observer design, Extended Kalman Filter, single mast stacker crane

Zusammenfassung: Dieser Beitrag beschäftigt sich mit dem Entwurf von Beobachtern für ein Hochregalbediengerät in automatisierten Lagerhallen. Da es sich bei dem Mast typischerschweise um eine Leichtbaukonstruktion handelt, wird das betrachtete Modell von gewöhnlichen und partiellen Differentialgleichungen beschrieben, d. h. das Modell weist einen gemischt finite-/infinite-dimensionalen Charakter auf. Im Rahmen dieser Arbeit werden zwei verschiedene Beobachterentwürfe vorgestellt, ein Extended Kalman Filter, das auf einer finite-dimensionalen Systemapproximation basiert, unter Verwendung der Rayleigh-Ritz-Methode, und einen Ansatz, der die port-Hamiltonsche Systemdarstellung für das finite-/infinite-dimensionale Szenario ausnutzt, wobei insbesondere das Beobachterfehlersystem als port-Hamiltonsches System formuliert wird. Der finite-/infinite-dimensionale Beobachter sowie das Kalman Filter werden zur Schätzung der Balkenverformung herangezogen, wobei lokale Vibrationen durch einen an der Mastspritze angebrachten Beschleunigungssensor erfasst werden. Schlussendlich soll anhand von Messungen an einem realen Labormodell die Leistungsfähigkeit dieser Ansätze in Kombination mit einem Beschleunigungssensor für die Überwachung kritischer Größen und die Möglichkeit der Schwingungsunterdrückung demonstriert werden.

Schlagwörter: infinit-dimensionale Systeme, energiebasierter Beobachterentwurf, Extended Kalman Filter, Hochregalbediengerät

1 Introduction

Single mast stacker cranes (SMCs), also called rack feeders, are deployed in automated warehouses or logistic centers in order to move payloads to desired positions. From an economic point of view, to improve the productivity the intention is to decrease the access time of such stacker cranes. To this end, stacker cranes are typically build as lightweight constructions, consisting of driving units in fixed ducts and flexible mast structures. Therefore, a mathematical model of the SMC is characteristically governed by a combination of ordinary differential equations (ODEs) and partial differential equations (PDEs). However, as a consequence of the lightweight construction undesirable beam vibrations occur. This problem is of increasing relevance in practical applications as stacker cranes in auto-

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mated warehouses are ever growing in height due to floor space constraints. Therefore, the idea is to address the problem from a control engineering point of view. With regard to appropriate control methods, for the governing mathematical model it is necessary to distinguish between scenarios allowing for an active lifting unit or not. Note that an active lifting unit implies a more complex model, however, it allows to decrease the access time since the driving and lifting unit move simultaneously. For example, in [5] a flatness-based feedforward control for the scenario of an SMC with an inactive lifting unit – i. e., the lifting unit remains at a constant position – has been presented, where the flatness of the mixed-dimensional problem has been shown. In contrast, in [1] and [3] a flatness-based feedforward control for a finite-dimensional system approximation, which was obtained by the Rayleigh-Ritz method, has been derived, where an active lifting unit has been taken into account. Furthermore, in [3] a passivity-based controller has been proposed for the stabilization of the infinite-dimensional error system, whereas in [1, 2] a dynamic controller based on the so-called energy-Casimir method has been derived for that purpose.

In this contribution, instead of discussing a further control methodology for SMCs, we focus on the derivation of proper observation strategies. A well-known approach for linear systems governed by ODEs is the Kalman Filter. Moreover, extensions of the Kalman Filter, like the Extended Kalman Filter (EKF), have become widely used for the state estimation of nonlinear systems, see, e. g., [4]. Therefore, the EKF can also be applied for the SMC by considering its finite-dimensional approximation.

A further relevant research topic is the observer design for infinite-dimensional systems, see, e. g., [6], where an approach based on the so-called backstepping methodology has been presented. Furthermore, in [7] a dissipativity-based observer-design strategy has been discussed. However, note that compared to systems governed by ODEs, in the distributed-parameter scenario there is an enormous rise of complexity, as, besides others, the stability analysis of the infinite-dimensional observer-error systems requires special attention. In fact, functional analytic methods are used to show the well-posedness of the governing PDE system, see [8] exemplarily for a detailed closed-loop stability investigation of a gantry crane with a heavy chain. To ensure that the observer state indeed converges to the system state, additionally the asymptotic stability has to be verified, see, e. g., [9], where LaSalle's invariance principle for infinite-dimensional systems was used.

In this contribution, we exploit a certain port-Hamiltonian (pH) system representation for the observer design. It should be noted that for the infinite-dimensional case, in particular the so-called Stokes-Dirac scenario, see, e. g., [11], as well as an approach based on jet-bundle structures [13, 14] have turned out to be especially suitable. The main difference of these approaches is the choice of the states – energy variables versus derivative coordinates – see [10] for a detailed comparison by means of the Mindlin plate. Furthermore, in [12] the pH system representation based on Stokes-Dirac structures, where strain variables are exploited for mechanical systems, has been used for the observer design. The SMC can be modelled in a variational framework, and furthermore, since the beam deflection is of particular interest in our approach, we base our considerations on the jet-bundle scenario.

For the purpose of structure health monitoring and vibration rejection at least partial information about the current distortion of the beam is required. The mentioned controllers for the SMC presented in [1, 2] and [3] rely on the ability to measure the bending moment at the beam base by the use of a strain gauge. Integrating strain-gauges for directly measuring these quantities of interest can be challenging. Therefore, the proposed observer estimates the corresponding signals based on acceleration measurements acquired at the tip of the beam.

Thus, the main contributions of this paper are as follows. In Section 3, we present two observers for the model of an SMC with active lifting unit, which is briefly introduced in Section 2. For the finite-dimensional system approximation of the SMC, which is obtained by the Rayleigh-Ritz method, we propose an Extended Kalman Filter. Furthermore, based on the pH system representation we derive a mixed finite-/infinite-dimensional observer. In Section 4 we provide an experimental validation of both observers on a lab demonstrator, where each observer is tested in two selected scenarios. In Scenario A we confine ourselves to an inactive lifting unit, whereas in Scenario B an active lifting unit is considered and furthermore, also a closed-loop experiment based on a damping injection controller is presented. Section 5 is dedicated to a sketch of the stability proof regarding the asymptotic behaviour of the observer-error system of the finite-/infinite-dimensional observer where for simplicity we restrict ourselves to Scenario A with an inactive lifting unit at the top.

2 Mathematical modelling

In this section we briefly recapitulate the derivation of the mixed finite-/infinite-dimensional model based on the variational principle together with its port-Hamiltonian representation, as well as a finite-dimensional system approximation obtained by means of the Rayleigh-Ritz method, see [1, 3] and [15] for more details. The considered physical model, see Fig. 1, consists of three main mechanical parts. The position of the rigid driving unit with mass m_w is denoted by x^1. The mast with the constant length L, the mass density ρA and the flexural rigidity EI is clamped onto the driving unit and meets the Euler-Bernoulli hypothesis. A lifting unit with the mass m_h, described by its vertical and horizontal position x^2 and x^3, respectively, is attached to the mast. Furthermore, the absolute position of the mast is denoted by w, and its spatial derivatives are represented with the subscript Y, where w_{YY} denotes the second spatial derivative of the deflection for instance. The horizontal position of the tip mass m_k at the beam end is specified by the coordinate x^4. The rotational moments of inertia of the beam, the tip mass and the lifting unit are not taken into account. Moreover, the electrical dynamics are assumed to be much faster than the mechanical ones and are therefore neglected. Hence, the forces F_1 and F_2, which are enforced by a subordinated control loop, serve directly as manipulated variables of the actuated driving and lifting unit, and are considered as control inputs. It should be noted that we use tensor notation, i. e., upper (lower) indices represent components of contravariant (covariant) vectors, with powers of variables enclosed in parentheses, e. g., $(x^1)^2$.

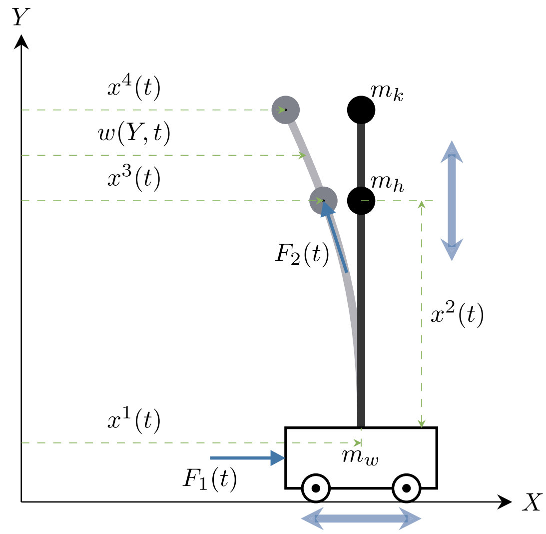

Figure 1: Schematic of the Single Mast Stacker Crane.

2.1 Finite-/infinite-dimensional model

Under the assumptions mentioned above the overall kinetic energy of the single mast stacker crane reads as

$$E_{kin} = \frac{1}{2}m_w(\dot{x}^1)^2 + \frac{1}{2}m_h\left((\dot{x}^3)^2 + (\dot{x}^2)^2\right) + \frac{1}{2}m_k(\dot{x}^4)^2 + \frac{1}{2}\int_0^L (\dot{w})^2 \rho A \mathrm{d}Y \tag{1}$$

while the overall potential energy is given by

$$E_{pot} = m_h g x^2 + \frac{1}{2}\int_0^L EI(w_{YY})^2 \mathrm{d}Y. \tag{2}$$

Therefore, the Lagrangian is considered to be of the form $L = E_{kin} - E_{pot} + E_{ext} = \bar{L}(x,\dot{x}) + \int_0^L l(\dot{w}, w_{YY})\mathrm{d}Y$ with the internal energies (1), (2) and external contributions $E_{ext} = x^1F_1 + x^2F_2$. Moreover, the Lagrangian functional is resulting in $\mathcal{L} = \int_{t_1}^{t_2} L + \lambda_i(t)\Theta^i \mathrm{d}t$ where the quantities λ_i correspond to the Lagrange multipliers with their associated mechanical constraints $\Theta^1 = x^3 - w|_{x^2}$ and $\Theta^2 = x^4 - w|_L$. Note that we use Einstein's summation convention to improve readability and $w|_{x^2}$ denotes the beam deflection at the lifting-unit position for instance. Furthermore, following the so-called extended Hamilton principle, i. e., solving the variational problem $\delta\mathcal{L} = 0$, one can obtain the equations of motion, which consists of the partial differential equation for the Euler-Bernoulli beam

$$\rho A\ddot{w} + EIw_{YYYY} = 0, \tag{3a}$$

the ordinary differential equations of the driving unit, the lifting unit and the tip mass

$$m_w\ddot{x}^1 + EI\, w_{YYY}|_0 = F_1 \tag{3b}$$

$$m_h\ddot{x}^2 + m_h g + m_h\ddot{x}^3\, w_Y|_{x^2} = F_2 \tag{3c}$$

$$EI\left(w_{YYY}|_{x^2_-} - w_{YYY}|_{x^2_+}\right) = m_h\ddot{x}^3 \tag{3d}$$

$$EI\, w_{YYY}|_L = m_k\ddot{x}^4, \tag{3e}$$

and are restricted to the kinematic boundary conditions and mechanical constraints

$$w|_0 = x^1 \qquad w_Y|_0 = 0 \tag{4a}$$

$$w|_{x^2} = x^3 \qquad w|_L = x^4 \tag{4b}$$

$$w_{YY}|_{x^2_-} - w_{YY}|_{x^2_+} = 0 \qquad w_{YY}|_L = 0. \tag{4c}$$

Here, the quantities x^2_+ and x^2_- denote the upper and the lower limit of the lifting-unit position x^2. The equations of motion (3) and (4) correspond to a mixed finite-/infinite-dimensional problem, where the Euler-Bernoulli beam equation (3a) is valid on the spatial domains $\mathcal{D}_1 = [0, x^2_-]$ and $\mathcal{D}_2 = [x^2_+, L]$. Obviously, $w|_{x^2_-} = w|_{x^2_+}$ and $w_Y|_{x^2_-} = w_Y|_{x^2_+}$ apply at the interface of the domains $\mathcal{D}_1$ and $\mathcal{D}_2$. Additionally, note that the conditions (4c) are a consequence of the fact that we neglected the rotational inertia of the tip mass, the beam and the lifting unit. Moreover, for practical applications, i. e., health monitoring, fault detection or advanced control purposes, several physical quantities such as the bending moment and the shear force are of great interest. It should be noted that in equation (3) and (4) the second and third spatial derivatives correspond to the bending moment M and the shear force Q evaluated at specific spatial positions $Y = h$, i. e.,

$$M|_h = EIw_{YY}|_h, \quad Q|_h = -EIw_{YYY}|_h. \tag{5}$$

2.2 Port-Hamiltonian representation

In this section the finite-/infinite-dimensional problem, given in Section 2.1, is represented as a port-Hamiltonian system. For this purpose, we introduce by means of regular Legendre mappings the momenta $p_1 = m_w\dot{x}^1$, $p_2 = m_h\dot{x}^2$, $p_3 = m_h\dot{x}^3$, $p_4 = m_k\dot{x}^4$ and $p_b = \rho A\dot{w}$. Subsequently, the Hamiltonian of the SMC can be written as

$$\mathscr{H} = \underbrace{\frac{1}{2}\frac{(p_1)^2}{m_w} + \frac{1}{2}\frac{(p_2)^2 + (p_3)^2}{m_h} + \frac{1}{2}\frac{(p_4)^2}{m_k}}_{H_f} + \int_0^L \underbrace{\frac{1}{2}\frac{1}{\rho A}(p_b)^2 + \frac{1}{2}EI(w_{YY})^2}_{\mathcal{H}_i} \mathrm{d}Y. \tag{6}$$

The partial derivative of the finite-dimensional part with the Hamiltonian H_f and state $x_f = [x^1, x^2, x^3, x^4, p_1, p_2, p_3, p_4]^T$ yields $\partial_{x_f} H_f = [0, m_h g, 0, 0, \frac{p_1}{m_w}, \frac{p_2}{m_h}, \frac{p_3}{m_h}, \frac{p_4}{m_k}]^T$, where $\partial_{x_f} = \partial/\partial_{x_f}$. Contrary, the infinite-dimensional part in the port-Hamiltonian representation can be stated with $x_i = [w, p_b]^T$, where its corresponding variational derivative reads $\delta_{x_i}\mathcal{H}_i = [\delta_w \mathcal{H}_i, \delta_{p_b}\mathcal{H}_i]^T = [EIw_{YYYY}, \frac{p_b}{\rho A}]^T$ as

$$\delta_w \mathcal{H}_i = \partial_w \mathcal{H}_i - d_Y(\partial_w^Y \mathcal{H}_i) + d_Y(d_Y(\partial_w^{YY}\mathcal{H}_i))$$

and $\delta_{p_b}\mathcal{H}_i = \partial_{p_b}\mathcal{H}_i$ with the total derivative

$$d_Y = \partial_Y + w_Y\partial_w + w_{YY}\partial_w^Y + w_{YYY}\partial_w^{YY} + w_{YYYY}\partial_w^{YYY}$$

with $\partial_w^{YY} = \partial/\partial w_{YY}$. Therefore, the mixed-dimensional model represented as pH system can be written as

$$\underbrace{\begin{bmatrix}\dot{x}_f \\ \dot{x}_i\end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} J_f & 0 \\ 0 & \mathcal{J}_i\end{bmatrix}}_{\mathcal{J}} \underbrace{\begin{bmatrix}\partial_{x_f} H_f \\ \delta_{x_i}\mathcal{H}_i\end{bmatrix}}_{\delta\mathcal{H}} + \underbrace{\begin{bmatrix} G_F \\ 0\end{bmatrix}}_{\mathcal{G}_F} F + \underbrace{\begin{bmatrix} G_Q \\ 0\end{bmatrix}}_{\mathcal{G}_Q} Q. \tag{7}$$

Here, the canonical skew-symmetric matrix J_f and the internal input matrix G_Q, which reads as

$$J_f = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix},$$

$$G_Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -w_Y|_{x^2} & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

describe the internal power flow of the finite-dimensional part and allows to incorporate the constraint forces $Q = [Q|_L, Q|_{x^2}, Q|_0]^T = [EIw_{YYY}|_L, EI(w_{YYY}|_{x^2_-} - w_{YYY}|_{x^2_+}), -EIw_{YYY}|_0]^T$, respectively. Furthermore, the internal power flow of the infinite-dimensional part is represented by the canonical skew-symmetric operator $\mathcal{J}_i$ and the external inputs $F = [F_1, F_2]^T$ can be included by means of the input mapping G_F.

$$\mathcal{J}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad G_F = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T$$

Consequently, the output of the port-Hamiltonian system (7) collocated to the external input F_1 and F_2 reads as

$$y_e = \begin{bmatrix} y_{e,1} \\ y_{e,2}\end{bmatrix} = \mathcal{G}_F^T \delta\mathcal{H} = \begin{bmatrix}\partial_{p_1} H_f \\ \partial_{p_2} H_f\end{bmatrix} = \begin{bmatrix} p_1(m_w)^{-1} \\ p_2(m_h)^{-1}\end{bmatrix}, \tag{8}$$

whereas those of the constraint forces are given by $y_{\partial,1} = \partial_{p_1} H_f = \frac{p_1}{m_w}$, $y_{\partial,2} = \partial_{p_2} H_f = \frac{p_2}{m_h}$, $y_{\partial,3} = \partial_{p_3} H_f = \frac{p_3}{m_h}$ and $y_{\partial,4} = \partial_{p_4} H_f = \frac{p_4}{m_k}$. Moreover, by means of integration by parts it can be shown that the Hamiltonian functional (6) evolves along solutions of (7) according to $\dot{\mathscr{H}} = \frac{p_1}{m_w}F_1 + \frac{p_2}{m_h}F_2$, see [15] for a detailed computation.

Remark 1. *Note that due to the expression $-w_Y|_{x^2}$ in the matrix G_Q it becomes obvious that the system possesses a nonlinear characteristic regarding the coupling expression in the principle of the conversation of momentum* (3c).

2.3 Finite-dimensional approximation

A finite-dimensional model can be obtained by approximating the deformation of the beam $w(Y,t)$ by appropriate ansatz functions. In fact, the beam deflection $w(Y,t)$ is approximated by the first-order Rayleigh-Ritz ansatz

$$w^*(Y,t) = x^1(t) + \Phi_1(Y)\bar{q}^1(t) \tag{9}$$

with a spatial ansatz function $\Phi_1(Y)$ and the generalised coordinate $\bar{q}^1$ as a solution of the Euler-Bernoulli equation (3a) fulfilling the kinematic boundary conditions and dynamical restrictions at the beam tip. By substituting the ansatz (9) into the energies (1), (2), and using Einstein's sums convention, one can obtain the Lagrangian as

$$L(q,\dot{q}) = \bar{L}(q,\dot{q}) + \int_0^L l(\dot{w}^*, w^*_{YY})\mathrm{d}Y \tag{10a}$$

$$= \frac{1}{2}M_{\alpha\beta}(q)\dot{q}^\alpha\dot{q}^\beta - V(q) \tag{10b}$$

with $\alpha,\beta = 1,\dots,3$, the generalized coordinates $q = [x^1,\bar{q}^1,x^2]$ and the symmetric mass matrix $M = [M_{\alpha\beta}]$. As a result, the Euler-Lagrange equations of the model of the finite-dimensional approximation of the single mast stacker crane can be written as

$$M_{\alpha\beta}(q)\ddot{q}^\beta + C_\alpha(q,\dot{q}) = Q_\alpha \tag{11}$$

where the mass matrix, with the ansatz abbreviation $\tilde{\Phi}_1 = \Phi_1(x^2)$ and its spatial derivations, is represented by

$$M = \begin{bmatrix} m_{11} & m_{12} + m_h\tilde{\Phi}_1 & m_h\partial_Y\tilde{\Phi}_1\bar{q}^1 \\ \cdot & m_{22} + m_h\tilde{\Phi}_1^2 & m_h\tilde{\Phi}_1\partial_Y\tilde{\Phi}_1\bar{q}^1 \\ \text{sym.} & \cdot & m_h + m_h(\partial_Y\tilde{\Phi}_1\bar{q}^1)^2 \end{bmatrix} \tag{12}$$

with the constant mass abbreviations $m_{11} = \rho AL + m_h + m_k + m_w$, $m_{12} = \rho A\int_0^L \Phi_1(Y)\mathrm{d}Y + m_k\Phi_1(L)$ and $m_{22} = \rho A\int_0^L (\Phi_1(Y))^2\mathrm{d}Y + m_k(\Phi_1(L))^2$. In this approximation, the vector $C = [C_\alpha]$ and the input matrix $G = [G_{\alpha\xi}]$, where $Q_\alpha = G_{\alpha\xi}u^\xi$, are given by

$$C = \begin{bmatrix} m_h(\dot{x}^2)^2\partial_{YY}\tilde{\Phi}_1\bar{q}^1 + 2m_h\dot{x}^2\partial_Y\tilde{\Phi}_1\dot{\bar{q}}^1 \\ EIq^1\int_0^L(\partial_{YY}\Phi_1)^2\mathrm{d}Y + \tilde{\Phi}_1C_1 \\ C_1\partial_Y\tilde{\Phi}_1\dot{\bar{q}}^1 + m_hg \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Therefore, with $\bar{M}$ denoting the inverse of the mass matrix (12), the state-space system representation of the approximated model can be written as

$$\underbrace{\begin{bmatrix} \dot{q}^\alpha \\ \dot{v}^\alpha \end{bmatrix}}_{\dot{x}^\alpha} = \underbrace{\begin{bmatrix} v^\alpha \\ -\bar{M}^{\alpha\beta}(q)C_\beta(q,v) + \bar{M}^{\alpha\beta}(q)G_{\beta\xi}u^\xi \end{bmatrix}}_{f^\alpha(x)}. \tag{13}$$

At this point, only the determination of the ansatz function $\Phi_1(Y)$ is remaining. As mentioned before, a proper ansatz for the deflection must satisfy the equation of the Euler-Bernoulli beam, as well as the kinematic boundary conditions and, as far as possible, also the dynamical restrictions. Therefore, our intention is to construct solutions for the partial differential equation of the beam via separation of variables. In this way, the resulting eigenfunctions of the infinite-dimensional problem might be chosen as ansatz functions. An additional challenge arises from the fact that the eigenfunction problem in the general case depends on the lifting position x^2. This in turn means that we have to solve nonlinear equations depending on the state of the lifting unit in order to determine the eigenfunction, which implies significant computing efforts. Therefore, we use as ansatz function the first eigenmode of the Euler-Bernoulli beam, where the lifting unit is neglected, which results in a problem that is independent of x^2. This simplification was justified by numerical investigations, see [15]. Considering the spatial domain $\mathcal{D} = [0,L]$, the fundamental solution has the form

$$\Phi_1 = A\sin(\gamma Y) + B\cos(\gamma Y) + C\sinh(\gamma Y) + D\cosh(\gamma Y) \tag{14}$$

with $\gamma^2 = \omega\sqrt{\frac{\rho A}{EI}}$. The remaining task is the determination of the coefficients $[K^j] = [A,B,C,D]$ as well as the eigenfrequency ω. This can be achieved by substituting the ansatz (14) in the adapted boundary constraints (3e) and (4) to obtain a system of equations $G_{kj}(\omega)K^j = 0$, $k = 1,\dots,4$. By numerically solving the nonlinear equation $\det(G(\omega)) = 0$ for the first eigenfrequency ω, we obtain a non trivial solution for K and therefore a proper ansatz function $\Phi_1(Y)$.

3 Observer design

The focus in this section is on the design of two observers for the SMC. To begin with, we briefly present an Extended Kalman Filter for the finite-dimensional approximated model. Afterwards, in Section 3.2 a pH observer based on the mixed finite-/infinite-dimensional characteristics of the SMC is presented.

3.1 Extended Kalman Filter

This section is dedicated to the observer design based on the finite-dimensional system approximation (11). As mentioned, the aim is to design an algorithm in order to estimate the shear force and the bending moment at the bottom of the flexible beam, respectively, which are of particular importance with regard to vibration suppression. If the ansatz function is used for the deflections in (5), the problem corresponds to estimate the Ritz variable $\bar{q}^1$, see

$$M|_0 = EI\partial_{YY}\Phi_1(0)\bar{q}^1, \; Q|_0 = -EI\partial_{YYY}\Phi_1(0)\bar{q}^1. \tag{15}$$

With the objective to do state estimation without measuring the bending moment by the use of a strain gauge, our effort is to develop an observation strategy with the acceleration $a|_L$ of the beam tip as alternative measurement unit. Therefore, the idea is to abstract the velocity $\dot{\bar{q}}^1$ by the given measurements including the acceleration and utilize it as input for the observer. This issue will be explained in detail in Section 4. For a finite-dimensional observer design, we rely on a nonlinear version of the well-known Kalman Filter, namely the Extended Kalman Filter. Originally developed for linear systems, the Kalman Filter assumes that the probability density function of the state and the noises are Gaussian distributed. The crucial advantage in this assumption consists in the fact that the linear equations obtain the Gaussian distribution of the state, which can be fully described by its two first stochastic moments, namely the expectation $\hat{x}$ and the variance P, $x \sim (\hat{x}, P)$. Therefore, instead of transforming the whole probability density function from one time step to the next and inferring measurements, the Kalman filter restricts the prediction and measurement update to the first two stochastic moments. For this purpose we consider a discretization of the system (13) as

$$x_{k+1} = f_k(x_k, u_k, w_k) \tag{16a}$$
$$y_k = h_k(x_k, v_k) \tag{16b}$$

with the time index k, the system state $x_k = [x_k^1, \bar{q}_k^1, x_k^2, \dot{x}_k^1, \dot{\bar{q}}_k^1, \dot{x}_k^2]^T \in \mathbb{R}^6$, the output quantities $y_k = [x_k^1, \dot{\bar{q}}_k^1, x_k^2]^T \in \mathbb{R}^3$, the input $u_k = [F_1, F_2]^T \in \mathbb{R}^2$ as well as the process and measurement noise $w_k \in \mathbb{R}^6$ and $v_k \in \mathbb{R}^3$, respectively. In the following, we denote an estimation of the state with $\hat{x}_k$ and the *a posteriori* and *a priori estimation* with $\hat{x}_k^+$ and $\hat{x}_k^-$. These terms are of particular importance, because the Taylor series expansions of the equations (16) along these states corresponds to the linearised system

$$x_{k+1} = F_k x_k + L_k w_k + \tilde{u}_k \tag{17a}$$
$$y_k = H_k x_k + M_k v_k \tag{17b}$$

with the Jacobi matrices $F_k = \frac{\partial f_k}{\partial x}\big|_{\hat{x}_k^+}$, $L_k = \frac{\partial f_k}{\partial w}\big|_{\hat{x}_k^+}$, $H_k = \frac{\partial h_k}{\partial x}\big|_{\hat{x}_k^-}$, $M_k = \frac{\partial h_k}{\partial v}\big|_{\hat{x}_k^-}$ and the input $\tilde{u}_k = f_k(\hat{x}_k^+, u_k, 0) - F_k\hat{x}_k^+$. Under the assumption of mean-free Gaussian process and measurement noise, i. e., $w_k \sim (0, Q_k)$ and $v_k \sim (0, R_k)$, the prediction step reads as

$$P_{k+1}^- = F_k P_k^+ F_k^T + L_k Q_k L_k^T \tag{18a}$$
$$\hat{x}_{k+1}^- = f_k(\hat{x}_k, u_k, 0) \tag{18b}$$

where the measurement update can be written as, see [4],

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \tag{19a}$$
$$\hat{x}_k^+ = \hat{x}_k^- + K_k\left(y_k - h_k(\hat{x}_k^-, 0)\right) \tag{19b}$$
$$P_k^+ = P_k^- - K_k H_k P_k^-. \tag{19c}$$

3.2 Infinite-dimensional observer

The underlying idea is to exploit the port-Hamiltonian system representation and its collocated outputs for the observer design, by using a copy of the plant extended by an observer-correction term $\hat{K}$, where it is assumed that the driving unit position x^1, the lifting unit position x^2 and the velocity of the beam tip $\dot{w}|_L = \frac{p_4}{m_k}$ are available as measurement quantities. If the system were linear, the observer-error system could be formulated in the pH framework in a straightforward manner, however as stated in Remark 1 the overall model exhibits a nonlinear characteristics. By exploiting the fact that x^2 is available as measurement quantity, we immediately obtain a linear system with the reduced observer state $\hat{x} = [\hat{x}^1, \hat{x}^3, \hat{x}^4, \hat{p}_1, \hat{p}_3, \hat{p}_4, \hat{w}, \hat{p}_b]^T$.

Remark 2. *It should be stressed, that due to the measurement x^2 we omit the corresponding balance of linear momentum* (3c). *However, as* (3d) *remains, the coupling of the active lifting unit and the beam is still valid.*

The observer-correction terms $\hat{K} = [\hat{k}^1, \hat{k}^2]^T$ are introduced such that on the one hand, $\hat{k}^2$ is collocated to the velocity of the beam tip in order to impose a dissipative behaviour on the observer error, and on the other hand, $\hat{k}^1$ can be used to overcome steady-state errors of the driving unit position. As a result of the chosen structure of the observer, which reads as

$$\begin{bmatrix} \dot{\hat{x}}_f \\ \dot{\hat{x}}_i \end{bmatrix} = \begin{bmatrix} J_f & 0 \\ 0 & \mathcal{J}_i \end{bmatrix} \begin{bmatrix} \partial_{\hat{x}_f}\hat{H}_f \\ \delta_{\hat{x}_i}\hat{\mathcal{H}}_i \end{bmatrix} + \begin{bmatrix} G_F \\ 0 \end{bmatrix} F_1 + \begin{bmatrix} G_Q \\ 0 \end{bmatrix} \hat{Q} + \begin{bmatrix} G_K \\ 0 \end{bmatrix} \hat{K}$$

with

$$J_f = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad G_Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

Table 1: SMC laboratory model and observer parameter.

Symbol	Value	Symbol	Value	Symbol	Value	Symbol	Value
m_w	13.1 kg	L	0.53 m	m_k	0.32 kg	α	3000
m_h	0.86 kg	EI	14.97 N m^2	ρA	2.1 kg/m	β	20

the constraint shear forces $\hat{Q} = [\hat{Q}|_L, \hat{Q}|_{x^2}, \hat{Q}|_0]^T = [EI\hat{w}_{YYY}|_L, EI(\hat{w}_{YYY}|_{x^2_-} - \hat{w}_{YYY}|_{x^2_+}), -EI\hat{w}_{YYY}|_0]^T$, the input matrix $G_F = [0,0,0,1,0,0]^T$ and

$$\mathcal{J}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad G_K^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

the error system with state $\tilde{x} = [\tilde{x}^1, \tilde{x}^3, \tilde{x}^4, \tilde{p}_1, \tilde{p}_3, \tilde{p}_4, \tilde{w}, \tilde{p}_b]$, i. e., $\tilde{x} = [x^1 - \hat{x}^1, x^3 - \hat{x}^3, x^4 - \hat{x}^4, p_1 - \hat{p}_1, p_3 - \hat{p}_3, p_4 - \hat{p}_4, w - \hat{w}, p_b - \hat{p}_b]$, can be deduced as

$$\begin{bmatrix} \dot{\tilde{x}}_f \\ \dot{\tilde{x}}_i \end{bmatrix} = \begin{bmatrix} J_f & 0 \\ 0 & \mathcal{J}_i \end{bmatrix} \begin{bmatrix} \partial_{\tilde{x}_f} \tilde{H}_f \\ \delta_{\tilde{x}_i} \tilde{\mathcal{H}}_i \end{bmatrix} + \begin{bmatrix} G_Q \\ 0 \end{bmatrix} \tilde{Q} - \begin{bmatrix} G_K \\ 0 \end{bmatrix} \hat{K} \tag{20}$$

where the outputs (21) are collocated to the observer-correction terms $\hat{k}^1$ and $\hat{k}^2$

$$\begin{bmatrix} \tilde{y}^1_{\hat{k}} \\ \tilde{y}^2_{\hat{k}} \end{bmatrix} = -\begin{bmatrix} G_K^T & 0 \end{bmatrix} \begin{bmatrix} \partial_{\tilde{x}_f} \tilde{H}_f \\ \delta_{\tilde{x}_i} \tilde{\mathcal{H}}_i \end{bmatrix} = \begin{bmatrix} -\dot{\tilde{w}}|_0 \\ -\dot{\tilde{w}}|_L \end{bmatrix}. \tag{21}$$

Subsequently, we show that by a proper choice of $\hat{K}$, it suffices to use the beam tip velocity $\dot{w}|_L$ and the driving-unit position $x^1 = w|_0$ regarding the error-injection terms for the observer. For the design of the observer we utilize a Lyapunov functional that consists of the error Hamiltonian $\tilde{\mathscr{H}} = \frac{1}{2}\frac{1}{m_w}(\tilde{p}_1)^2 + \frac{1}{2}\frac{1}{m_h}(\tilde{p}_3)^2 + \frac{1}{2}\frac{1}{m_k}(\tilde{p}_4)^2 + \int_0^L \frac{1}{2}\frac{1}{\rho A}(\tilde{p}_b)^2 + \frac{1}{2}EI(\tilde{w}_{YY})^2 \mathrm{d}Y$ extended by a term depending on the error of the deflection at the bottom of the mast, i. e., we have

$$\tilde{\mathscr{H}}_e = \tilde{\mathscr{H}} + \frac{\alpha}{2}(\tilde{w}|_0)^2. \tag{22}$$

Remark 3. *Note that the necessity of the additional term in* (22) *turns out within detailed well-posedness investigations. If we omit this term, we are not able to detect steady state errors since we have no information about the error deflection. Furthermore, the equivalence of the energy norm $||\chi||_{\mathcal{X}}$, where $\tilde{\mathscr{H}}_e = \frac{1}{2}||\chi||^2_{\mathcal{X}}$ can be used as a Lyapunov functional with regard to stability investigations, and the natural norm $||\chi||_n$ of an appropriate function space $\mathcal{X}$, where $\chi \in \mathcal{X}$, cannot be shown. This fact implies that the observer-error system would not be well-posed. To overcome this, we incorporate the term $\frac{\alpha}{2}(\tilde{w}|_0)^2$.*

If we investigate the formal change of the Lyapunov functional (22), which corresponds to the collocated energy ports and the derivation of the incorporated term of Remark 3, $\dot{\tilde{\mathscr{H}}}_e = -\frac{\tilde{p}_1}{m_w}\hat{k}^1 - \frac{\tilde{p}_4}{m_k}\hat{k}^2 + \alpha\frac{\tilde{p}_1}{m_w}\tilde{w}|_0$, we find that the choice $\hat{k}^1 = \alpha\tilde{w}|_0$ and $\hat{k}^2 = \beta\frac{\tilde{p}_4}{m_k} = \beta\dot{\tilde{w}}|_L$ with $0 < \beta \in \mathbb{R}$, renders the evolution of (22) along solutions of the error system (20) to $\dot{\tilde{\mathscr{H}}}_e = -\beta\left(\frac{\tilde{p}_4}{m_k}\right)^2 \leq 0$, and therefore the energy functional (22) does not increase. It should be stressed, that this finding does not yet imply that the observer-error system is asymptotically stable. Instead, the well-posedness and the asymptotic stability in the light of LaSalle's invariance principle has to be investigated. However, we want to emphasize that this proof of the asymptotic stability is not trivial, since for LaSalle's invariance principle the solution of two mixed-dimensional systems with time variant domains $[0, x^2_-(t)]$ and $[x^2_+(t), L]$ must be determined. We therefore confine ourselves with the non-increasing energy functional and show the applicability of the observer by means of experiments on the laboratory model and present a more detailed stability investigation only for a special scenario in Section 5.

4 Experimental results

In the following, the application of both observers are validated by measurement results obtained from a laboratory model of a single mast stacker crane, where the parameters are stated in Table 1. It should be mentioned that several control strategies for equilibrium point stabilization as well as for trajectory tracking purposes have been successfully implemented for the finite and mixed-dimensional problem, see, e. g., [1] and [3]. Exemplarily, an IDA-PBC law from [3] is given by

$$F_{1,e} = -c_1 x^1_e - \alpha_2 \dot{x}^1_e - \frac{1-\alpha_1}{\alpha_1} Q_e \tag{23a}$$

$$F_{2,e} = -c_2 x^2_e - \alpha_1\alpha_3 \dot{x}^2_e \tag{23b}$$

with the error terms $x^1_e = x^1 - x^1_d$, $x^2_e = x^2 - x^2_d$ and $Q_e = Q - Q_d$, where the index d represents reference trajectories, and $c_i(\xi)\xi > 0$ for $\xi \neq 0$, $i = 1,2$ and $c_i(0) = 0$ as well as $\alpha_1, \alpha_2\alpha_3 > 0$ are degrees of freedom. Therefore, the knowledge of the shear force and the bending moment is not only beneficial for structural health monitoring applications, but also for advanced control strategies, such as, e. g., vibration rejection, see (23), where the shear force $Q = EIw_{YYY}|_0$ is required. As mentioned in Section 3.1, the finite-dimensional scenario give us a direct proportional relation between the Ritz coordinate $\bar{q}^1$ and the bending moment, which is measurable by a strain gauge, as well as the shear force Q, see (15). Moreover, the Ritz coordinate is also associated with the relative deflection $w_{rel}(Y,t) = w(Y,t) - x^1(t) = \Phi_1(Y)\bar{q}^1(t)$, see (9), which furthermore give us a relation between the deflection of the beam at a specific position Y and the shear force. The laboratory model possesses a strain gauge in order to measure the bending moment and the shear force, respectively, although, it is not a favoured and common tool. Instead, IMUs for measuring the acceleration at the tip of the beam may be a viable alternative for high bay racks with unguided masts. Therefore, an obvious intention is to apply acceleration measurements for the abstraction of the shear force and the bending moment. It is well known that double integration of noisy acceleration signals is not advisable. It should be mentioned that there exists methods, such as complementary filters, see, e. g., [19], to determine the position from acceleration measurements. However, for the sensor configuration of the laboratory model under consideration it turned out that this approach is not suitable.

In fact, our intention is to exploit filtered measurements of the beam tip acceleration $a|_L$ of the laboratory model as well as the positions x^1, x^2 of the driving unit and lifting unit as input signals for the Extended Kalman Filter and the energy-based infinite-dimensional observer, where the velocity of the beam tip is obtained according to the discrete transfer function

$$\dot{w}_{rel}|_L(z) = \frac{1 - e^{(-w_o T_a)}}{z - e^{(-w_o T_a)}} \frac{T_a}{z - e^{(-w_u T_a)}} a|_L(z) - \dot{x}^1(z)$$

with the cutoff frequencies $w_o = 200\,\mathrm{s}^{-1}$ and $w_u = 1\,\mathrm{s}^{-1}$. Furthermore, it should be noted that with respect to the infinite-dimensional observer the shear force and the bending moment can be approximately obtained by the use of the Finite-Difference method for (5). For comparison and validation of the two introduced observers from Section 3 we present the results from two experiments.

Scenario A (inactive lifting unit): In the first experiment, we propose a scenario of an SMC with inactive lifting unit moving horizontally from $x^1 = 0.1\,\mathrm{m}$ to $x^1 = 0.25\,\mathrm{m}$, while the lifting unit remained constant at the beam tip, i. e., $x^2 = L$. For this purpose, a PD control law was used for reference tracking of the driving unit, where the desired trajectory $x_d^1(t)$ stems from a double integration of a trapezoidal acceleration profile. Figure 2 shows a comparison between the two proposed observers and measurements by means of the driving unit position and the beam tip deflection, where the subscripts EKF and pH indicate the observer quantities of the Extended Kalman Filter and the port-Hamiltonian observer, respectively. Note that the bending moment measured by the strain gauge serves as reference measurement for the beam deflection, where the relation between the bending moment and beam deflection stems from the Ritz ansatz function as explained at the beginning of this section. Furthermore, it should be noted that the (not very sophisticated) chosen trajectory planning was used on purpose, although we are aware of the fact that there exists proper methods like flatness based trajectory planning, see [3, 5, 1]. However, our main intention is to verify the capability of the observers and therefore we chose trajectories such that the eigenfrequencies of the model are stimulated.

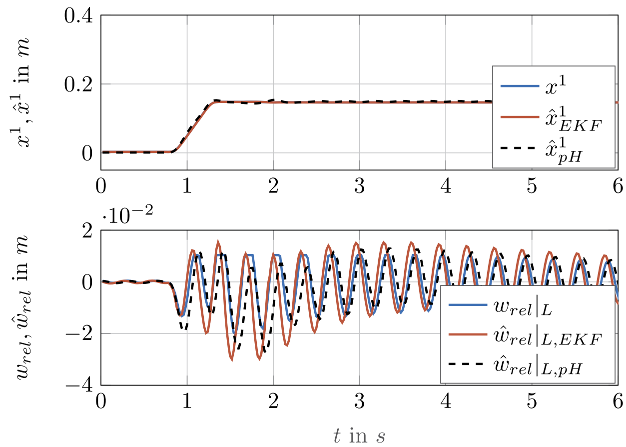

Figure 2: Scenario A: Comparison of the EKF and the pH observer in a open-loop scenario with inactive lifting unit at the beam tip.

Scenario B (active lifting unit): In the second experiment, we demonstrate the capability of both observers for an active lifting unit, where the driving unit as well as the lifting unit were driven simultaneously from $x^1 = 0.1\,\mathrm{m}$ and $x^2 = 0.1\,\mathrm{m}$ to $x^1 = 0.3\,\mathrm{m}$ and $x^2 = 0.15\,\mathrm{m}$ using the same PD trajectory planning as in Scenario A. Furthermore, in order to show the applicability of the proposed observers for vibration rejection purposes, the control law (23) is activated at the time $t = 5\,\mathrm{s}$ where the shear force stems from the Extended Kalman Filter and the infinite-dimensional observer, respectively. It should be stressed that the vibration rejection control law could have been enabled much earlier, but our purpose was primarily to highlight the behavior of the observers. A slight difference in the trajectories is due to two separate closed-loop recordings on the real laboratory model.

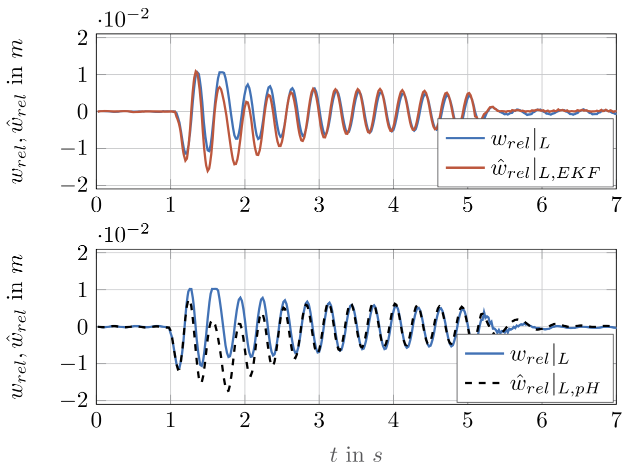

Figure 3: Scenario B: Comparison of the EKF and the pH observer in a closed-loop vibration rejection scenario with active lifting.

Remark 4. *In general, spillover phenomena in early-lumping approaches can lead to drastic problems, e. g., the excitation of high-frequency natural oscillations that can provoke closed-loop instability, see* [15]. *However, for the EKF, which is based on the finite-dimensional model, no such effects turned out to be crucial.*

5 Stability investigations

The aim in this section is to present a sketch of the well-posedness and stability investigations of the mixed-dimensional observer, where we confine ourselves to Scenario A, where the lifting unit is located at the top, i. e., $x^2 = L$. Note that the stability proof for a similar problem treated within the Stokes-Dirac scenario is presented in [20]. As a consequence, the PDE (3a) is valid on the domain $\mathcal{D} = [0, L]$, the (error) momentum $\tilde{p}_4$ can be introduced as $\tilde{p}_4 = (m_h + m_k)\dot{\tilde{x}}^4$, and therefore, the (error) principle of momentum (3e) for the tip mass reads

$$(m_h + m_k)\ddot{\tilde{x}}^4 = \dot{\tilde{p}}_4 = EI\tilde{w}_{YYY}|_L - \beta\dot{\tilde{w}}|_L. \tag{24}$$

The error principle unit for the driving unit follows to

$$(m_w)\ddot{\tilde{x}}^1 = \dot{\tilde{p}}_1 = -EI\tilde{w}_{YYY}|_0 - \alpha\tilde{w}|_0, \tag{25}$$

whereas those of the lifting unit can be omitted.

5.1 Well-posedness investigations

First, the observer-error dynamics are reformulated as an abstract Cauchy problem, which allows to apply the variant of the Lumer-Phillips theorem according to [16, Theorem 1.2.4]. Therefore, we consider the state space $\mathcal{X} = H_C^2(0, L) \times L^2(0, L) \times \mathbb{R} \times \mathbb{R}$, where $H_C^2 = \{\tilde{w} \in H^2(0, L)| \ \tilde{w}_Y|_0 = 0\}$ because of the boundary condition (4a), and the state vector $\chi = [\chi_1, \chi_2, \chi_3, \chi_4]^T = [\tilde{w}, \tilde{p}_b, \tilde{p}_1, \tilde{p}_4]^T$. Note that $H^l(0, L)$ denotes a Sobolev space of functions whose derivatives up to order l are square integrable. Thus, the state space $\mathcal{X}$ is equipped with the natural (standard) norm

$$\|\chi\|_n^2 = \langle \tilde{w}, \tilde{w} \rangle_{H^2} + \langle \tilde{p}_b, \tilde{p}_b \rangle_{L^2} + (\tilde{p}_1)^2 + (\tilde{p}_4)^2 \tag{26}$$

with $\langle \tilde{w}, \tilde{w} \rangle_{H^2} = \langle \tilde{w}, \tilde{w} \rangle_{L^2} + \langle \tilde{w}_Y, \tilde{w}_Y \rangle_{L^2} + \langle \tilde{w}_{YY}, \tilde{w}_{YY} \rangle_{L^2}$. However, for our purposes it is of advantage to introduce the (energy) norm

$$\begin{aligned}\|\chi\|_{\mathcal{X}}^2 &= EI \langle \tilde{w}_{YY}, \tilde{w}_{YY} \rangle_{L^2} + \frac{1}{\rho A} \langle \tilde{p}_b, \tilde{p}_b \rangle_{L^2} \\ &\quad + \alpha(\tilde{w}|_0)^2 + \frac{1}{m_w} (\tilde{p}_1)^2 + \frac{1}{m_h + m_k} (\tilde{p}_4)^2,\end{aligned} \tag{27}$$

where we have the equivalence $\mathscr{H}_e = \frac{1}{2}\|\chi\|_{\mathcal{X}}^2$. Since we are able to find constants $c_1, c_2 > 0$ such that $c_1 \|\chi\|_n^2 \leq \|\chi\|_{\mathcal{X}}^2 \leq c_2 \|\chi\|_n^2$ is fulfilled, the energy norm (27) is equivalent to the natural norm (26), cf. Remark 3, and consequently, $\mathcal{X}$ serves as a proper Hilbert space equipped with the inner product $\langle \chi, \bar{\chi} \rangle_{\mathcal{X}} = \|\chi\|_{\mathcal{X}}^2$. Next, to reformulate the error dynamics as an abstract Cauchy problem of the form $\dot{\chi}(t) = \mathcal{A}\chi(t), \chi(0) = \chi_0 \in \mathcal{X}$, we introduce the linear operator $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{X} \to \mathcal{X}$ with

$$\mathcal{A} : \begin{bmatrix} \tilde{w} \\ \tilde{p}_b \\ \tilde{p}_1 \\ \tilde{p}_4 \end{bmatrix} \to \begin{bmatrix} \frac{1}{\rho A}\tilde{p}_b \\ -EI\tilde{w}_{YYYY} \\ -EI\tilde{w}_{YYY}|_0 - \alpha\tilde{w}|_0 \\ EI\tilde{w}_{YYY}|_L - \beta\dot{\tilde{w}}|_L \end{bmatrix},$$

where $\mathcal{D}(\mathcal{A})$ denotes the (dense) domain of $\mathcal{A}$

$$\begin{aligned}\mathcal{D}(\mathcal{A}) = \Big\{\chi \in \mathcal{X} | \tilde{w} \in (H^4(0, L) \cap H_C^2(0, L)), \tilde{p}_b \in H_C^2(0, L), \\ \tilde{p}^1 = \frac{m_w}{\rho A}\tilde{p}_b|_0, \tilde{p}^4 = \frac{m_h + m_k}{\rho A}\tilde{p}_b|_L, EI\tilde{w}_{YY}|_L = 0\Big\}.\end{aligned}$$

Regarding the Lumer-Phillips theorem [16, Theorem 1.2.4] it has to be shown that the domain $\mathcal{D}(\mathcal{A})$ is dense in $\mathcal{X}$, the operator $\mathcal{A}$ is dissipative and $0 \in \rho(\mathcal{A})$, the resolvent set of $\mathcal{A}$. Similar to [8, Proof of Lemma 2.2], it can be shown that $\mathcal{D}(\mathcal{A})$ is dense in $\mathcal{X}$. Furthermore, the equivalence $\mathscr{H}_e = \frac{1}{2}\|\eta\|_{\mathcal{X}}^2$ implies $\langle \chi, \mathcal{A}\chi \rangle_{\mathcal{X}} = -\beta(\dot{\tilde{w}}|_L)^2$, and therefore, it follows that the operator $\mathcal{A}$ is dissipative. To prove that $0 \in \rho(\mathcal{A})$, it can be shown that for every $\bar{\chi} = [\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3, \bar{\chi}_4]^T \in \mathcal{X}$ we can uniquely solve $\mathcal{A}\chi = \bar{\chi}$ for $\chi = [\tilde{w}, \tilde{p}_b, \tilde{p}_1, \tilde{p}_4]^T \in \mathcal{D}(\mathcal{A})$, and verify that $\mathcal{A}^{-1}$ maps bounded set in $\mathcal{X}$ into bounded set of $\mathcal{N} = (H^4(0, L) \cap H_C^2(0, L)) \times H_C^2(0, L) \times \mathbb{R} \times \mathbb{R}$,

i. e., $\mathcal{A}^{-1}$ exists and is bounded, and thus, 0 cannot be an eigenvalue of $\mathcal{A}$. Consequently, we are able to apply the variant of the Lumer-Phillips theorem according to [16, Theorem 1.2.4], and thus, the linear operator $\mathcal{A}$ is the infinitesimal generator of a C_0-semigroup of contractions. Moreover, to ensure that the observer state converges to the system state, in the following the asymptotic stability of the observer error is verified.

5.2 Asymptotic stability

Next, regarding the investigation of the asymptotic stability of the observer error we use LaSalle's invariance principle for infinite-dimensional systems, see [17, Theorem 3.64, 3.65], which can be applied since the solution trajectories are precompact. This follows from the fact that the boundedness of $\mathcal{A}^{-1}$ implies that $\mathcal{A}^{-1}$ is also compact, see [17, p. 201] and [18, Remark 4.2]. Thus, we investigate the set $\mathcal{S} = \{\chi \in \mathcal{X} | \dot{\mathscr{H}}_e = 0\}$, where we consequently have $\dot{\tilde{w}}|_L = 0$, and therefore, $\ddot{\tilde{w}}|_L = 0$ has to hold. Since for that case from (24) we find $EI\tilde{w}_{YYY}|_L = 0$, it follows that the dynamics in the set $\mathcal{S}$ are restricted to $\dot{\tilde{w}} = \frac{1}{\rho A}\tilde{p}_b$, $\dot{\tilde{p}}_b = -EI\tilde{w}_{YYYY}$, $\dot{\tilde{p}}_1 = -EI\tilde{w}_{YYY}|_0 - \alpha\tilde{w}|_0$ together with the boundary conditions $\tilde{w}_Y|_0 = 0$, $EI\tilde{w}_{YY} = 0$, $EI\tilde{w}_{YYY} = 0$. In light of LaSalle's invariance principle, the objective is to verify that the only possible solution in $\mathcal{S}$ is the trivial one. An investigation of the eigenvectors and eigenfunctions allows to introduce the ansatz function (14), which identically fulfills the boundary conditions and can be used for the ansatz $\tilde{w}(Y,t) = \tilde{x}^1 + \Phi_k(Y)q^k(t)$. Due to the fact that we have infinitely many imaginary eigenvalues, we are able to introduce the ansatz $\dot{\tilde{w}}(Y,t) = \dot{\tilde{x}}^1 + \sum_{k=1}^{\infty}(a_k\cos(\lambda_k t) + b_k\sin(\lambda_k t))\Phi_k$ for the beam velocity. A careful investigation of $\dot{\tilde{w}}(L,t) = 0$ allows to conclude that this is only possible for $a_k = b_k = 0\ \forall k$. That is, we have verified that the only possible solution is the trivial one, which concludes the proof of the asymptotic stability of the observer-error system in the scenario of an inactive lifting unit.

6 Conclusion

In this contribution, two different observer strategies for a single mast stacker crane have been presented. First, based on the finite-dimensional system approximation an EKF has been proposed. Further, considering the mixed-dimensional character of the system, an observer has been derived by means of a port-Hamiltonian system representation. The collocated observer inputs were chosen such that the energy functional of the observer-error is non-increasing. A sketch of well-posedness and asymptotic stability investigations is given for the special scenario with inactive lifting mass at the beam tip. Both observers were verified in experiments on a laboratory model, where an acceleration measurement of the beam tip has been exploited. Finally, we want to stress that the estimated quantities can be successfully used for vibration rejection.

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