Research Article

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Modified Jeans instability and Friedmann equation from generalized Maxwellian distribution

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Abstract: We study Jeans instability with generalized Maxwellian distribution. The results reveal two significant features of the modified Jeans instability. First, the Jeans wavelength of the system covers the original λ_J when $k = 1$. Second, as k approaches 0, the modified Jeans wavelength approaches infinity. This means that the system is always gravitationally stable. Furthermore, we examine the implications of the modified Maxwellian distribution on the Friedmann equation. Our analysis suggests that the effective gravitational constant should incorporate the contribution of temperature T in order to describe the system dynamics.

Keywords: Jeans instability, generalized Maxwellian distribution, modified Jeans wavelength, Friedmann equation, effective gravitational constant

1 Introduction

It is known that the formation of galaxies and large-scale structures in the universe through gravitational collapse is considered one of the most important processes in astrophysics (Sandoval-Villalbazo and Sagaceta-Mejia 2020). The Jeans instability of interstellar gas leads to the formation of star when the internal gas pressure is not sufficient to prevent gravitational collapse of the matter (Sharma *et al.*, 2015). The dynamical stability of the self-gravitating system can be described by the following four equations, equation of continuity, Euler's equation, Poisson's equation, and the equation of state of the gas (Jiulin 2004):

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla P - \nabla\varphi, \tag{2}$$

$$\nabla^2\varphi = 4\pi G\rho, \tag{3}$$

$$P = \frac{kT}{\mu m_{\mathrm{H}}}\rho, \tag{4}$$

where ρ is the mass density, $\mathbf{v}$ is the fluid velocity from Euler's equation and the equation of continuity, P is the pressure of fluid, T is the temperature of the matter, G is the Newton gravitational constant, φ is the gravitational potential, μ is the mean molecular weight, and m_H is the atomic mass of hydrogen (Jiulin 2004). Based on the small perturbation of the above four equations, the famous Jeans wavelength can be written as (Abreu *et al.*, 2016)

$$\lambda_J = \sqrt{\frac{\pi k_B T}{\mu m_H G \rho_0}}. \tag{5}$$

The Jeans wavelength λ_J means that if the wavelength of density fluctuation is greater than critical wavelength, the system will become gravitationally unstable.

In recent years, a number of studies about Jean instability have been carried out. For example, the Hall current on the self-gravitational instability of rotating plasma has been investigated by Prajapati *et al.*, (2010). They found that the Hall parameter affects only the longitudinal mode of propagation and it has no effect on the transverse mode of propagation. Kinetic treatment of the Jeans gravitational instability with collisions is presented (Trigger *et al.*, 2004). It was shown that collisions do not affect the Jeans instability criterion. Modified Jeans instability criterion for magnetized systems is discussed by Lundin *et al.*, (2008), and it was found that the intrinsic magnetization of the plasma can enhance the Jeans instability and modify the structure of the instability spectra. For other important work, please refer the studies

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by Chavanis (2002), Herrera and Santos (1994), Gumrukcuoglu *et al.*, (2016), Roshan and Abbassi (2014), Arbuzova *et al.*, (2014), Harko (2019), Nazari *et al.*, (2017), Tsiklauri (1998), Kremer *et al.*, (2018), and Capozziello *et al.*, (2012).

However, it has been shown that systems with the long range gravitational interaction may be nonextensive (Jiulin 2004). Thus, the Boltzmann–Gibbs statistical mechanics may not be appropriate for this gravitational system. For example, the Jeans length in the context of the Kaniadakis statistics has been studied by Abreu *et al.*, (2016), and they found that the self-gravitating system is stable as $\kappa \to 2/3$. The q-nonextensive velocity distribution function for free particles was studied by Jiulin (2004). The effects of nonextensive statistics on big bang nucleosynthesis are also examined by Bertulani *et al.*, (2013), Hou *et al.*, (2017), Kusakabe *et al.*, (2019), and Jang *et al.*, (2021). Moreover, these nonextensive generalizations of the Maxwell velocity distributions have been used to study various other physics problems (Lima *et al.*, 2001, Biro and Molnar 2012, Pereira *et al.*, 2009, Osada and Wilk 2008, Jiulin 2004). Motivated by this, we focus on the Jeans instability from generalized Maxwellian distribution f_k which was introduced by Voda (2009), where k is the shape parameter. Specifically, it is necessary to know how k shapes the Jeans wavelength, which in turn affects the stability of the system. Besides, we also study the modified Friedmann equation from generalized Maxwellian distribution to investigate how the parameter k modifies cosmic expansion.

This article is organized as follows. In Section 2, we introduce generalized Maxwellian distribution and derive the modified Jeans wavelength. We explore how the parameter k affects the stability of self-gravitating systems and the Jeans wavelength. In Section 3, we focus on the modified Friedmann equation. Specifically, we study the impact of parameter k on the cosmic expansion. Further discussion and conclusion are given in Section 4.

2 Generalized Maxwellian distribution and modified Jeans wavelength

The traditional method to study the Jeans instability is to perturb the four equations first and then solve the equations. Finally, we can obtain the Jeans wavelength. However, the disadvantage of this method is that it requires complex calculation. The main reason is that the Euler equation in the set of four equations is nonlinear. Therefore, in this work, we plan to use Verlinde's framework (Verlinde 2011) to study the Jeans wavelength. In this framework, gravity is considered an entropy force. In other words, we can study gravity through statistical mechanics or thermodynamics. Let us start with the generalized Maxwellian velocity distribution for free particles (Huang and Chen 2016)

$$f_k(v) = \frac{k}{2^{k/2}\left(\sqrt{\frac{k_BT}{m}}\right)^{2+1/k}\Gamma(1+k/2)} v^{2k}\exp\left(-\frac{v^{2k}}{2\left(\sqrt{\frac{k_BT}{m}}\right)^{2}}\right), \tag{6}$$

where v is the velocity of the particle, where Γ is the Gamma function and k represents the parameter which characterizes nonextensibility. This distribution was introduced as an alternative to the idealized Gaussian or Maxwell distributions, which may not accurately represent real-world scenarios. It is not hard to find that Eq. (6) is the traditional Maxwellian velocity distribution when $k = 1$. Note that we suggest a very strong modification of the conventional framework of canonical thermodynamics and assume that the distribution does not take the standard form $\sim \exp(-E/T)$. This assumption may initially appear incorrect. Nevertheless, related research approaches can be found in a similar context, as evidenced by Jiulin (2004).

With the generalized Maxwellian velocity distribution in hand, according to the standard thermodynamic relationship, the average value of the square of velocity can be written as

$$\langle v^2\rangle_k = \frac{\int_0^\infty f_k v^2 \mathrm{d}v}{\int_0^\infty f_k \mathrm{d}v} = \frac{2^{\frac{1}{k}}\Gamma\left[1+\frac{3}{2k}\right]}{3\Gamma\left[1+\frac{1}{2k}\right]}\left(\frac{k_BT}{m}\right)^{1/k}. \tag{7}$$

The dimension of $\langle v^2\rangle$ appears to vary with the parameter k, which raises concerns about its physical interpretation. However, the results in the study by Sheykhi (2020) suggest that the dimension of a quantity F also varies with parameter β, which indicates that the dimension of physical quantities may exhibit variability depending on the specific function used. Then, the equipartition theorem of energy is modified as

$$E_k = \frac{1}{2}Nm\langle v^2\rangle_k. \tag{8}$$

Combining Eqs (7) and (8), we can find that

$$E_k = \frac{2^{\frac{1}{k}}\Gamma\left[1+\frac{3}{2k}\right]}{6\Gamma\left[1+\frac{1}{2k}\right]}\left(\frac{k_BT}{m}\right)^{1/k-1} Nk_BT. \tag{9}$$

This is the final form of the modified equipartition theorem. In order to derive gravity from thermodynamics, we need to study it in Verlinde's framework. One can do this as follows. First, the number of bits N is proportional to the holographic screen area A, which can be written as $N = \frac{A}{\ell_P^2}$. On the other hand, the total energy on the holographic screen is $E_k = Mc^2$. Thus, we can have the expression

$$Mc^2 = \frac{2^{\frac{1}{k}}\Gamma\left[1 + \frac{3}{2k}\right]}{6\Gamma\left[1 + \frac{1}{2k}\right]}\left(\frac{k_BT}{m}\right)^{1/k-1} Nk_BT. \tag{10}$$

Next in order to obtain the gravitational acceleration, we need to consider the Unruh effect (Takagi 1986, Unruh 1976), which is given by

$$k_BT = \frac{1}{2\pi}\frac{\hbar a}{c}. \tag{11}$$

Finally, the gravitational acceleration formula is given by

$$a = \frac{3\Gamma\left[1 + \frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1 + \frac{3}{2k}\right]} G\frac{M}{r^2}. \tag{12}$$

Thus, we can define the effective gravitational constant as

$$G_k = \frac{3\Gamma\left[1 + \frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1 + \frac{3}{2k}\right]} G. \tag{13}$$

Let us look into the Jeans wavelength. If we assume that all other physical quantities in Eq. (5) except the gravitational constant do not change, then the modified Jeans wavelength is

$$\begin{aligned}\lambda_c^k &= \sqrt{\frac{3\Gamma\left[1 + \frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1 + \frac{3}{2k}\right]}\frac{\pi k_BT}{\mu m_H G\rho_0}} \\ &= \sqrt{\frac{3\Gamma\left[1 + \frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1 + \frac{3}{2k}\right]}}\lambda_J.\end{aligned} \tag{14}$$

It is known that if the wavelength of density fluctuation is greater than the Jeans wavelength, the system will become gravitationally unstable. Through a simple analysis of the modified Jeans wavelength, we find that it has two features. One is that the Jeans wavelength of the system is the original λ_J when $k = 1$. The other one is that the modified Jeans wavelength approaches infinity, namely, $\lambda_c^k \to \infty$ when $k \to 0$. This means that the modified Jeans wavelength is infinite. In other words, the system is always gravitationally stable. The system is always gravitationally stable because when k approaches 0, on the one hand, the modified Jeans length is infinite; on the other hand, if the system size is larger than Jeans length, the system will become unstable. None of them is greater than infinity. So, the system is always gravitationally stable.

3 Modified Friedmann equation

In Section 2, we mainly studied the Jeans instability, which tells us how the stars in the universe are formed. Then, it is natural to study how the universe formed and evolved. In order to answer this question, we need to study how to derive the Friedmann equation from thermodynamics. One can do this as follows. First, the Newtonian acceleration can be expressed as (Table 1, Abreu *et al.*, 2018):

$$\ddot{R} = \ddot{a}(t)r = \frac{3\Gamma\left[1 + \frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1 + \frac{3}{2k}\right]} G\frac{M}{R^2}, \tag{15}$$

Table 1: Comparison of standard Maxwellian and generalized Maxwellian distributions

Quantity	Standard Maxwellian	Generalized Maxwellian	Ratio $\left(\frac{\textbf{Generalized Maxwellian}}{\textbf{Standard Maxwellian}}\right)$
Average square of velocity ($\langle v^2\rangle$)	$\langle v^2\rangle$	$\frac{2^{\frac{1}{k}}\Gamma\left[1+\frac{3}{2k}\right]}{3\Gamma\left[1+\frac{1}{2k}\right]}\left(\frac{k_BT}{m}\right)^{\frac{1}{k}}\langle v^2\rangle$	$\frac{2^{\frac{1}{k}}\Gamma\left[1+\frac{3}{2k}\right]}{3\Gamma\left[1+\frac{1}{2k}\right]}\left(\frac{k_BT}{m}\right)^{\frac{1}{k}}$
Newton's gravitational constant (G)	G	$\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}-1}\Gamma\left[1+\frac{3}{2k}\right]}G$	$\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}-1}\Gamma\left[1+\frac{3}{2k}\right]}$
Jeans wavelength (λ_J)	λ_J	$\sqrt{\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}-1}\Gamma\left[1+\frac{3}{2k}\right]}}\lambda_J$	$\sqrt{\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}1}\Gamma\left[1+\frac{3}{2k}\right]}}$
Friedmann equation	$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$	$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}1}\Gamma\left[1+\frac{3}{2k}\right]}\left(\rho + \frac{3p}{c^2}\right)$	$\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{\frac{1}{k}1}\Gamma\left[1+\frac{3}{2k}\right]}$

where R is the physical radius which can be written as $R(t,r) = a(t)r$. $a(t)$ is the scale factor of the Friedmann–Robertson–Walker (FRW) metric, t is the time, and r is the radial comoving coordinate (Abreu *et al.*, 2018). Next the gravitational mass is assumed to be given by the well-known Tolman-Komar mass (Cai *et al.*, 2010, Li and Pang 2010).

$$M = \frac{4\pi}{3}a^3r^3\left(\rho + \frac{3p}{c^2}\right). \tag{16}$$

One can find that M is proportional to the scale function a. And r in the equation can be eliminated by using $r = \frac{c}{H}$. After substituting Eq. (16) in Eq. (15) and dividing both sides of the equation by a, we can arrive at the modified Friedmann equation

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\left(\rho + \frac{3p}{c^2}\right). \tag{17}$$

Note that the presence of the generalized Maxwellian distribution has a relatively minor effect on the energy density and pressure. The main reason is that for a given number of particles, the energy density and pressure in the system are solely related to the temperature T, while temperature T and the shape parameter k remain two independent variables. It is not difficult to find that Eq. (17) covers the usual Friedmann equation $\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$ when $k = 1$. By using $\ddot{a}/a = \dot{H} + H^2$, one can find that

$$\dot{H} + H^2 = \frac{4\pi}{3}G\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\left(\rho + \frac{3p}{c^2}\right), \tag{18}$$

where H is the Hubble constant. Now, the main problem is how to interpret the modified Friedmann equation. First, it is clear that the modified Friedmann equation is derived from thermodynamics, not from Einstein's field equation. Since we work in the framework of thermodynamics, the effective gravitational constant of the equation should include the contribution of some thermodynamic quantities, such as temperature. Obviously, in the modified Friedmann equation, we find that there is not only the contribution of k but also the contribution of temperature. To facilitate a clear understanding of how the obtained results vary with parameter k, we have summarized the key results such as the Jeans wavelength and effective gravitational constant in Table I.

Following Lambiase *et al.*, (2023), we investigated inflation within the context of slow rolling, with the premise that a scalar field (ϕ) propels this progression. Then, we derive the modified Friedmann equation under conditions of slow rolling, which is expressed as $H^2 \approx \frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\frac{8\pi G}{3}V(\phi)$. Thus, our analysis unveils that this inflation incorporates the influence of temperature as well. In addition to inflation, the presence of dark energy also plays a crucial role in the cosmic evolution. The condition for dark energy mandates that $\omega = \frac{P}{\rho} < 0$. However, from Eq. (18), we ascertain that the effective energy density $\rho_{\rm eff}$ and pressure $P_{\rm eff}$ are expressed as

$$\rho_{\rm eff} = \frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\rho, \tag{19}$$

$$P_{\rm eff} = \frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}P. \tag{20}$$

Thus, we find that the effective $\omega_{\rm eff} = \omega = \frac{P}{\rho}$. This implies that generalized Maxwellian distribution does not significantly affect ω. Next, we consider the Hubble tension within the framework of the generalized Maxwellian distribution. According to Jusufi and Sheykhi (2023), the first Friedmann equation under this framework is given as follows:

$$H^2 + \frac{K}{a^2} = \frac{8\pi}{3}\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\rho, \tag{21}$$

where K is the curvature constant. Now, if we set $K = 0$ and define $H_0^2 = 8\pi\rho/3$ (where H_0 represents the unmodified Hubble parameter), then we can rewrite Eq. (21) as

$$H = H_0\left(\frac{3\Gamma\left[1+\frac{1}{2k}\right]}{2^{\frac{1}{k}}\left(\frac{m}{k_BT}\right)^{1/k-1}\Gamma\left[1+\frac{3}{2k}\right]}\right)^{1/2}. \tag{22}$$

The current question is how to interpret Eq. (22). Prior to delving into this, it is essential to first grasp the concept of Hubble tension. It refers to the disparity in the measurements of the Hubble constant obtained through different methods. One method involves measuring the early cosmic microwave background radiation, with the Planck collaboration group providing a measurement value denoted as $H = 67.40 \pm 0.50$ km s^{-1} Mpc^{-1} (Planck *et al.*, 2020, Verma *et al.*, 2021, Jusufi and Sheykhi 2023). Another method involves observing supernovae and galaxies with the Hubble Space Telescope, providing a measurement value denoted as $H_0 = 74.03 \pm 1.42$ km s^{-1} Mpc^{-1}

(Riess *et al.*, 2019, Jusufi and Sheykhi 2023). The Hubble constant reported by Planck collaboration corresponds to H in Eq. (22), while the Hubble Space Telescope (HST) measurement corresponds to H_0 in the same equation. This distinction is based on Jusufi and Sheykhi (2023). Eq. (22) suggests that the difference between these measurements may arise from the contribution of coefficients preceding H_0 (unmodified Hubble parameter). Physically, this could be attributed to the contribution originating from the generalized Maxwellian distribution. This offers a potential explanation for understanding the issue of Hubble tension.

4 Conclusions

In this work, we first study the Jeans stability under the condition of generalized Maxwellian velocity distribution. The modified Jeans wavelength is derived from Verlinde's framework, without requiring the solution of a set of four equations. We find that the modified Jeans wavelength has two characteristics: the Jeans wavelength of the system is the original λ_J when $k = 1$ and the system is always gravitationally stable when $k \to 0$. Next we study the modified Friedmann equation, which is also derived from thermodynamics rather than from solving Einstein's field equation. The results show that the contribution of temperature T should be considered in effective gravitational constant.

However, how to understand parameter k in the generalized Maxwellian velocity distribution is still an open question. Additionally, to strengthen the validity of our theoretical derivations, it is essential to acquire more compelling experimental data. We will leave it to our future work.

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