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Research Article

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Biermann battery as a source of astrophysical magnetic fields

https://doi.org/10.1515/astro-2021-0017 Received Oct 31, 2021; accepted Nov 30, 2021

Abstract: A large number of galaxies have large-scale magnetic fields which are usually measured by the Faraday rotation of radio waves. Their origin is usually connected with the dynamo mechanism which is based on differential rotation of the interstellar medium and alpha-effect characterizing the helicity of the small-scale motions. However, it is necessary to have initial magnetic field which cannot be generated by the dynamo. One of the possible mechanisms is connected with the Biermann battery which acts because of different masses of protons and electrons passing from the central object. They produce circular currents which induce the vertical magnetic field. As for this field we can obtain the integral equation which can be solved by simulated annealing method which is widely used in different branches of mathematics

Keywords: magnetic field, galaxy, Biermann battery

1 Introduction

A wide range of galaxies have large-scale magnetic fields which have the typical strength of several microgauss (Arshakian et al. 2009). First observational confirmations of their existence were obtained several decades ago while studying cosmic rays (Bochkarev 2011). After that the magnetic fields have been studied using the synchrotron emission (Ginzburg 1959), and nowadays most of the research is done by the Faraday rotation mesurements of the polarization plane of the radiowaves. When such waves pass through the medium with regular magnetic field structures, the polarization plane rotates proportionally to the induction of the field and squared wavelength (Zeldovich et al. 1983). As for the Milky Way, most of the sources of the polarized can be associated with the pulsars. First research works were based on quite small amounts of the sources, but they established the basic features of the magnetic field in our Galaxy (Manchester 1973; Andreasyan and Makarov 1989; Han and Qiao 1994). Nowadays there are more than thousand pulsars (Andreasyan et al. 2016) which can be used to study the field. It is necessary to

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emphasize that the sources with large rotation measure $(RM > 200 \text{ rad m}^{-2})$ play the most important role for the magnetic field study (Andreasyan et al. 2020). It has been shown that the magnetic field of our Galaxy has so-called reversals which are connected with different directions of the field. As for another galaxies, there are large databases of the sources of different nature which allow us to study the magnetic field of more than one hundred objects (Opperman *et al.* 2012).

From the theoretical point of view, the magnetic field of galaxies is usually described by the so-called dynamo mechanism (Arshakian et al. 2009). It describes transition of the energy of the turbulent motions to the energy of the magnetic field (Sokoloff 2015). The basic drivers of the dynamo are connected with the differential rotation and alpha-effect. Differential rotation is based on changing angular velocity of the objects (which is smaller in the outer parts), produces the azimuthal magnetic field and makes it larger. Alpha-effect shows the helicity of the turbulent motions of the interstellar medium and describes the transition of the azimuthal magnetic field to the radial one. Both of these mechanisms compete with the turbulent diffusion which tries to destroy the large-scale structures of the magnetic field. If the dynamo-number (describing this effect) is large enough, we can say that the magnetic field will grow according to the exponential law (Arshakian et al. 2009). The rate of this growth can be calculated using eigenvalue problem (Mikhailov 2020) or numerically (Moss 1995; Mikhailov 2014). However, one of the most important problem is connected with the initial conditions. The dynamo mechanism should have some seed field and it

cannot describe the field growth from the zero one. The initial magnetic field can be explained using the small-scale dynamo which is connected with the properties of the turbulent motions. Unfortunately even this mechanism requires seed fields. So, we should take some principally different approach, which is not connected with the dynamo action.

One of the most perspective explanations of the magnetic field was supposed by Biermann in the middle of the previous century (Biermann and Schluter 1951; Harrison 1970). It is based on the flows from the center of the galaxy. It contains protons and electrons which interact with the rotating medium. They have principally different masses, so the electrons move with the velocity which is close to the surrounding medium, and protons "lag behind" according to their large mass (Andreasyan 1996). This effect produces non-zero circular currents which induce vertical magnetic fields. First estimates of the field produced by the Biermann mechanism were quite moderate, so this effect was undeservedly forgotten. Nowadays it is widely recognized as a basic source of the interstellar magnetic field. So it is quite necessary to give not only the typical value of the field, but to study its radial structure which can be quite useful for the next stages of the field evolution (Arshakian et al. 2009). The field generated by the Biermann mechanism can be the initial field for the small-scale dynamo, and after that the generated small-scale field can be the source for the initial field for the step connected with large-scale dynamo (Beck et al. 1996).

The description of the magnetic field generated by the Biermann mechanism leads us to the Fredholm integral equation of the second kind (Mikhailov and Andreasyan 2021). The solution of this problem is connected with illposed problem which is widely known in mathematical physics. It can be solved using the Tikhonov regularization (Goncharsky *et al.* 1985; Tikhonov *et al.* 1995). After that the inversion of the regularized functional is used. The problem of inversion of matrices which should be done here is a quite "expensive" operation. However, nowadays the methods of machine learning are used for a lot of similar problems in different branches of mathematics and physics and are much more simple to be described and to be realized on computer (Shamin 2019).

Here we used the simulated annealing method which is widely used, mostly in problems of control sciences (van Laarhoven 1987; Granville *et al.* 1994; Shamin 2019). Here we have the integral equation which can be reduced to a problem of minimization of some functional. It can be done by the iteration algorythm which is connected with random perturbations of the approximations of the solution. One of the basic points is connected with using "bad" changes at some stages of the process.

Firstly in this paper we present the basic equations which describe the motion of particles and show the generation of the field by singular circular current. After that, we obtain the integral equation which describes the structure of the magnetic field (Mikhailov and Andreasyan 2021). Finally, we describe the simulated annealing method and the solution of the problem. We present the basic figures which show the results for the magnetic field radial structure and for the evolution of the iterative approximations.

2 The particles motion and equation for the magnetic field

If we describe the motion of the particle from the central part of the object, its motion can be described by the equaition (Mikhailov and Andreasyan 2021):

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{1}{m}\mathbf{f} + \frac{q}{mc}[\mathbf{v} \times \mathbf{B}],$$

where \mathbf{r} is the radius vector of the particle, , \mathbf{v} is the velocity, m is its mass, \mathbf{f} is the total force which is connected with the gravitation, interaction with the medium and the pressure of the radiation, \mathbf{B} is the magnetic field and q is the charge.

We shall assume that the radial velocity V is constant, and the typical processes in azimuthal direction are much faster than in the radial one. So the azimuthal part of the motion equation is the most important. As for the force we shall have the approximation (Mikhailov and Andreasyan 2021):

$$f_{\omega} = -mR\tau^{-1}(\omega - \Omega),$$

where R is the distance from the center, ω is the angular velocity of the particle and Ω is the angular velocity of the medium. As for the typical time of the interaction $\tau \sim m^2$ we have different values for protons and electrons.

The equation for the angular velocity will be (Mikhailov and Andreasyan 2021):

$$\frac{d\omega}{dt} + 2\frac{V}{R}\omega + \frac{R}{\tau}(\omega - \Omega) + \frac{qVB}{mc} = 0.$$

It can be solved as:

$$\omega(t) = \omega_q \left(1 - e^{-\frac{t}{\tau}(1+2V\tau/R)} \right);$$

where

$$\omega_q = \Omega \frac{1 - \frac{q\tau VB}{mcR\Omega}}{1 + 2V\tau/R}.$$

The angular velocity will become close to ω_q with typical timescale $\frac{\tau}{1+2V\tau/R}$ which is quite small for our problem.

So we can say that the particles will move with angular velocities (Mikhailov and Andreasyan 2021):

$$\omega = \omega_q \approx \Omega - \frac{V\tau}{R} \left(2\Omega + \frac{qB}{mc} \right);$$

where q = e for protons and q = -e for electrons.

Each pair of particles is connected with a circular current (Mikhailov and Andreasyan 2021):

$$I=\frac{e\omega_p-e\omega_e}{2\pi},$$

where

$$\omega_p = \Omega - \frac{V\tau_p}{R} \left(2\Omega + \frac{eB}{m_p c} \right)$$

corresponds to protons and

$$\omega_e = \Omega - \frac{V\tau_e}{R} \left(2\Omega - \frac{eB}{m_e c} \right)$$

corresponds to electrons. It can be shown that taking into account the typical values of the parameters of the particles,

$$I \approx -\frac{eV\tau_p}{\pi R} \left(\Omega + \frac{eB}{2m_pc}\right).$$

Each circular current generates the axisymmetric magnetic field at the distance r from the center (Mikhailov and Andreasyan 2021):

$$b=\frac{I}{cR}\Phi(r/R),$$

where Φ is the function which is defined as:

$$\Phi(\alpha) = \int_{0}^{2\pi} \frac{1 - \alpha \cos \varphi}{1 - 2\alpha \cos \varphi + \alpha^2} d\varphi.$$

If we have the density of these particles n(R), each differentially thin layer $[R, R + dR] \times [-h, h] \times [0, 2\pi]$ (in cylindrical coordinates, h is the half-thickness of the disc) will produce tthe field:

$$dB_1(r) = \frac{4\pi In(R)h}{c}\Phi(r/R)dR.$$

The magnetic field B will make the particles of the main part of the medium move with angular velocity $\pm \frac{qB}{2mqc}$. Each proton - electron pair will produce the currrent

$$I = -\frac{e^2B}{4\pi c} \left(\frac{1}{m_e} + \frac{1}{m_p} \right) \approx -\frac{e^2B}{4\pi m_e c}.$$

These currents will produce extra magnetic field

$$dB_2(r) = -\frac{N(R)he^2}{m_ec^2}B(R)\Phi(r/R)dR.$$

The magnetic field will be:

$$dB(r) = dB_1(r) + dB_2(r).$$

Integrating both parts we will obtain the equation from the inner radius R_{min} to the outer one R_{max} , we shall obtain (Mikhailov and Andreasvan 2021):

$$B(r) = -\int_{R_{min}}^{R_{max}} \frac{4\pi In(R)h}{c} \Phi(r/R) dR -$$

$$-\int_{R_{min}}^{R_{max}} \frac{N(R)he^2}{m_e c^2} B(R) \Phi(r/R) dR.$$

If we assume that $n(r)=rac{n_0R_{min}^2}{r^2}$, $N(r)=rac{N_0R_{min}}{r}$, $au_p(R)\sim$ R and measure distances in dimensionless units of R_{max} , the equation will have the standard form for the Fredholm integral equation of the second kind:

$$B(r) = \lambda \int_{R_{min}}^{1} K(r, R) + F(R);$$

where

$$K(r,R) = -\frac{1}{R}\Phi(r/R),$$

$$F(R) = -\int_{R_{min}}^{1} \frac{1}{R^2}\Phi(r/R)dR;$$

$$\lambda = \frac{N_0 e^2 h R_{min}}{m_e c^2}.$$

The field is measured in units of $\frac{4n_0R_{min}heV\tau_p\Omega}{cR_{max}}$. The field can be found by minimizing the functional:

$$U[B] = \int_{R_{min}}^{1} \left(\int_{R_{min}}^{1} K(r,R)B(r)dR + \frac{1}{\lambda}F(R) - \frac{1}{\lambda}B(r) \right)^{2} dr.$$

3 Simulated annealing method

The integral equation can be solved using the simulated annealing method, which is one of the most simple methods of machine learning and widely used in control sciences (van Laarhoven 1987). The previous works showed (Mikhailov and Andreasyan 2021) that the typical magnetic field can be approximated as:

$$B(r) = A \frac{r - R_{min}}{r^2} \left(1 + \frac{D}{r} \right)$$

The zero approximation can be used taking $A_0 = D_0 =$ 0, so

$$B_0(r)=0$$
.

After that we will take small random perturbations ΔA and ΔD constructing the next approximation:

$$A_{n+1} = A_n + \Delta A$$
;

$$D_{n+1}=D_n+\Delta D.$$

If $U[B_{n+1}] < U[B_n]$, we shall pass to the next iteraction. If $U[B_{n+1}] > U[B_n]$, we will return to A_n and D_n with probability

$$p=1-\exp\left(-\frac{U[B_{n+1}]-U[B_n]}{T_n}\right).$$

However, with probability

$$q = \exp\left(-\frac{U[B_{n+1}] - U[B_n]}{T_n}\right)$$

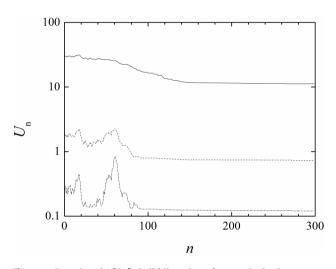


Figure 1. Functional $U[B_n]$. Solid line show $\lambda = 5$, dashed one $-\lambda = 20$, dot-dashed one $-\lambda = 50$.

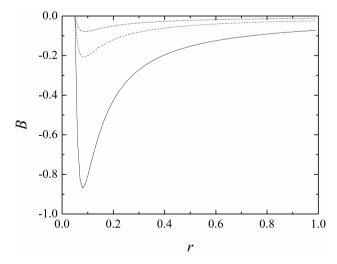


Figure 2. Magnetic field structure. Solid line show $\lambda = 5$, dashed one $-\lambda = 20$, dot-dashed one $-\lambda = 50$.

we should take A_{n+1} and D_{n+1} . As for the "temperature" T_n we have the law:

$$T_{n+1} = 0.9T_n$$
.

Figure 1 shows the values of the functional and its evolution for different iteractions. In the ideal case it could become zero, but it comes to some minimal value. It is connected with the inaccuracy of the algebraic model of the magnetic field. Figure 2 describes the magnetic field for different λ . We can see that the magnetic field will enlarge for smaller values of λ .

4 Conclusion

In this paper, we have studied the process of the magnetic field generation using the Biermann mechanism. The structure of the field is described by the integral equation. It is solved using the simulated anneaing method which can be associated with simplest methods of artificial intelligence and machine learning. It is more fast and effective than the classical methods of solving integral equation. However, the results are nearly the same (Mikhailov and Andreasyan 2021).

The magnetic field generated by the Biermann battery mechanism is quite small, and depending on the type of the object, we can obtain the fields which have the typical magnitudes of $10^{-27...17}$ G (Mikhailov and Andreasyan 2021). So, it is necessary to stress that if we take in as an initial magnetic field for the mean field dynamo, it won't be possible to have the values of 10^{-6} G that are measured in observations. Also the magnetic field produced by this mechanism is oriented vertically and its projection to the eigenfunction of the mean field dynamo operator is close to zero. So it is quite useful to combine the mean field dynamo mechanism and the small-scale dynamo which is based on turbulent effects. The typical timescale for the small-scale dynamo in galaxies is about 10⁷ years. So, if the field generated by Biermann battery mechanism will be the initial field for the turbulent dynamo, it will reach the equipartition value of 10⁻⁶ G during times of less than 10⁹ years (Beck et al. 1996). This field will have random orientation, but the number of turbulent cells will be finite and has the order of 10⁴. So the mean value of the magnetic field will be non-zero. According to simple statistical laws it will have the order of 10^{-8} G. It can be the inititial field for the mean field dynamo, which has the typical timescale of 10⁹ years and can describe the growth of the magnetic field, which will have the regular component of 10⁻⁶ G during several Gyr.

This appoach can also be quite interesting to study the magnetic fields of another objects, such as accretion discs which surround black holes (Shakura and Sunvaev 1973) or can be associated with eruptive stars (Andreasyan et al. 2021). Previously it has been shown that the magnetohydrodynamical processes in such objects can be desribed by the mechanisms which are nearly the same as the ones for the galactic discs (Moss et al. 2016; Boneva et al. 2021). So we can apologize that the Biermann battery can be the source of the magnetic field and it can play even a more important role than for the galaxies. Of course, we should take into account different spatial lenghtscales. However, out model which uses the dimensionless variables can be simply adopted for such cases.

Acknowledgment: The work was partially supported by by Russian Ministry of Science and Higher Education (agreement No. 075-15-2019-1621).

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

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