

## DETERMINATION OF THE SOLAR GALACTOCENTRIC DISTANCE FROM THE KINEMATICS OF MASERS

A. T. Bajkova<sup>1</sup> and V. V. Bobylev<sup>1,2</sup>

<sup>1</sup> *Central (Pulkovo) Astronomical Observatory of RAS, 65/1 Pulkovskoye Ch., St. Petersburg, Russia; anisabajkova@rambler.ru*

<sup>2</sup> *Sobolev Astronomical Institute, St. Petersburg State University, Bibliotchnaya pl. 2, St. Petersburg, Russia; vbobylev@gao.spb.ru*

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**Abstract.** We determine the parameters of Galactic rotation and the solar galactocentric distance  $R_0$  by simultaneously solving Bottlinger’s kinematic equations using data for masers with known line-of-sight velocities and highly accurate VLBI trigonometric parallaxes and proper motions. Our sample includes 93 masers spanning the range of galactocentric distances  $R$  from 3 to 15 kpc. The inferred parameters are  $\Omega_0 = 29.7 \pm 0.5 \text{ km s}^{-1} \text{ kpc}^{-1}$ ,  $\Omega'_0 = -4.20 \pm 0.11 \text{ km s}^{-1} \text{ kpc}^{-2}$ ,  $\Omega''_0 = 0.730 \pm 0.029 \text{ km s}^{-1} \text{ kpc}^{-3}$ , and  $R_0 = 8.03 \pm 0.12 \text{ kpc}$ , implying a circular rotation velocity of  $V_0 = 238 \pm 6 \text{ km s}^{-1}$  at the solar distance  $R_0$ .

**Key words:** masers – Galaxy: kinematics and dynamics – Galaxy: solar galactocentric distance

### 1. INTRODUCTION

Both kinematic and geometric characteristics are essential for studying the Galaxy, and the solar galactocentric distance  $R_0$  is the most important among them. Various data are used to determine the parameters of Galactic rotation. These include the line-of-sight velocities of neutral and ionized hydrogen clouds with the distances estimated by the tangential point method (Clemens 1985; McClure-Griffiths & Dickey 2007; Levine et al. 2008), Cepheids with the distance scale based on the period–luminosity relation, open star clusters and OB associations with photometric distances (Mishurov & Zenina 1999; Rastorguev et al. 1999; Zabolotskikh et al. 2002; Bobylev et al. 2008; Mel’nik & Dambis 2009), and masers with VLBI trigonometric parallaxes (Reid et al. 2009a; McMillan & Binney 2010; Bobylev & Bajkova 2010; Bajkova & Bobylev 2012).

In kinematical analyses the solar galactocentric distance  $R_0$  is often assumed to be known, because not all of the kinematic data allow  $R_0$  to be reliably estimated. In turn, different methods of analysis (including the direct ones) yield different values for  $R_0$ .

Reid (1993) reviewed the  $R_0$  measurements made by that time using various methods. He subdivided all measurements into primary, secondary, and indirect ones and inferred the “best value” as a weighted mean of the measurements published over a period of 20 years:  $R_0 = 8.0 \pm 0.5 \text{ kpc}$ . Nikiforov (2004) proposed

a more complete three-dimensional classification, which took into account (1) the type of the method used to determine  $R_0$ , (2) the method used to determine the reference distances, and (3) the type of reference objects. With the main types of errors and correlations associated with the classes of measurements taken into account, Nikiforov derived what he called the “best value”,  $R_0 = 7.9 \pm 0.2$  kpc, by analyzing the results of various authors published between 1974 and 2003.

Foster & Cooper (2010) obtained the mean  $R_0 = 8.0 \pm 0.4$  kpc based on 52 results published between 1992 and 2010. Francis & Anderson (2013) reviewed 135 estimates of  $R_0$  published between 1918 and 2013. They concluded that the results obtained after 2000 give a mean value of  $R_0$  close to 8.0 kpc.

We have some experience in determining  $R_0$  by simultaneously solving Bottlinger’s kinematic equations for the Galactic rotation parameters. To this end, we used the data for open star clusters (Bobylev et al. 2007) distributed within about 4 kpc of the Sun. Clearly, masers located in regions of active star formation and distributed over a much broader region of the Galaxy are of great interest for this purpose. However, the first such analysis for a sample of 18 masers performed by McMillan & Binney (2010) showed the probable value of  $R_0$  to be within a fairly wide range, 6.7–8.9 kpc. Since then, the number of masers with measured trigonometric parallaxes has increased (Reid et al. 2014), allowing this range to be narrowed significantly.

The goal of this paper is to determine the parameters of Galactic rotation and the distance  $R_0$  from the data on masers with measured trigonometric parallaxes.

## 2. METHOD

Here, we use the rectangular Galactic coordinate system with the axes directed away from the observer toward the Galactic center ( $l=0^\circ$ ,  $b=0^\circ$ , the  $X$  axis), in the direction of Galactic rotation ( $l=90^\circ$ ,  $b=0^\circ$ , the  $Y$  axis), and toward the north Galactic pole ( $b=90^\circ$ , the  $Z$  axis).

The determination of the kinematic parameters consists in minimizing quadratic functional  $F$ ,

$$\min F = \sum_{j=1}^N [w_r^j (V_r^j - \hat{V}_r^j)]^2 + \sum_{j=1}^N [w_l^j (V_l^j - \hat{V}_l^j)]^2 + \sum_{j=1}^N [w_b^j (V_b^j - \hat{V}_b^j)]^2, \quad (1)$$

subject to the following constraints derived from Bottlinger’s formulas with the angular velocity of Galactic rotation  $\Omega$  expanded into a Taylor series up to the second order in  $r/R_0$ :

$$V_r = -u_\odot \cos b \cos l - v_\odot \cos b \sin l - w_\odot \sin b + R_0(R - R_0) \sin l \cos b \Omega'_0 + 0.5R_0(R - R_0)^2 \sin l \cos b \Omega''_0, \quad (2)$$

$$V_l = u_\odot \sin l - v_\odot \cos l + (R - R_0)(R_0 \cos l - r \cos b) \Omega'_0 + (R - R_0)^2(R_0 \cos l - r \cos b) 0.5 \Omega''_0 - r \Omega_0 \cos b, \quad (3)$$

$$V_b = u_\odot \cos l \sin b + v_\odot \sin l \sin b - w_\odot \cos b - R_0(R - R_0) \sin l \sin b \Omega'_0 - 0.5R_0(R - R_0)^2 \sin l \sin b \Omega''_0. \quad (4)$$

Here  $N$  is the number of objects used;  $j$  is the current object number;  $\hat{V}_r^j$ ,  $\hat{V}_l^j$ , and  $\hat{V}_b^j$  are the model values of the three-dimensional velocity field: the line-of-sight velocity and the proper motion velocity components in the  $l$  and  $b$  directions,

respectively;  $V_l = 4.74r\mu_l \cos b$ ,  $V_b = 4.74r\mu_b$  are the measured components of the velocity field (data), where 4.74 is the ratio of the number of kilometers in an astronomical unit to the number of seconds in a tropical year;  $w_r^j, w_l^j, w_b^j$  are the weight factors;  $r$  is the heliocentric distance of the star calculated via the measured parallax  $\pi$ ,  $r = 1/\pi$ ; the star's proper motion components  $\mu_l \cos b$  and  $\mu_b$  are in mas yr<sup>-1</sup> (milliarcseconds per year), the line-of-sight velocity  $V_r$  is in km s<sup>-1</sup>;  $u_\odot, v_\odot, w_\odot$  are the components of the reflex velocity of the bulk motion of the stellar group relative to the Sun (the velocity  $u$  is directed toward the Galactic center,  $v$  is in the direction of Galactic rotation,  $w$  is directed to the north Galactic pole);  $R_0$  is the galactocentric distance of the Sun, and  $R$  is the distance from the star to the Galactic rotation axis,

$$R^2 = r^2 \cos^2 b - 2R_0 r \cos b \cos l + R_0^2. \quad (5)$$

$\Omega_0$  is the angular velocity of rotation at the distance  $R_0$ ; the parameters  $\Omega'_0$  and  $\Omega''_0$  are the first and second derivatives of the angular velocity at the distance  $R_0$  with respect to  $R$ , respectively.

The weight factors in functional (1) are assigned according to the following formulas (we drop the subscript  $j$  for legibility):

$$w_r = S_0 / \sqrt{S_0^2 + \sigma_{V_r}^2}, \quad w_l = \beta S_0 / \sqrt{S_0^2 + \sigma_{V_l}^2}, \quad w_b = \gamma S_0 / \sqrt{S_0^2 + \sigma_{V_b}^2}, \quad (6)$$

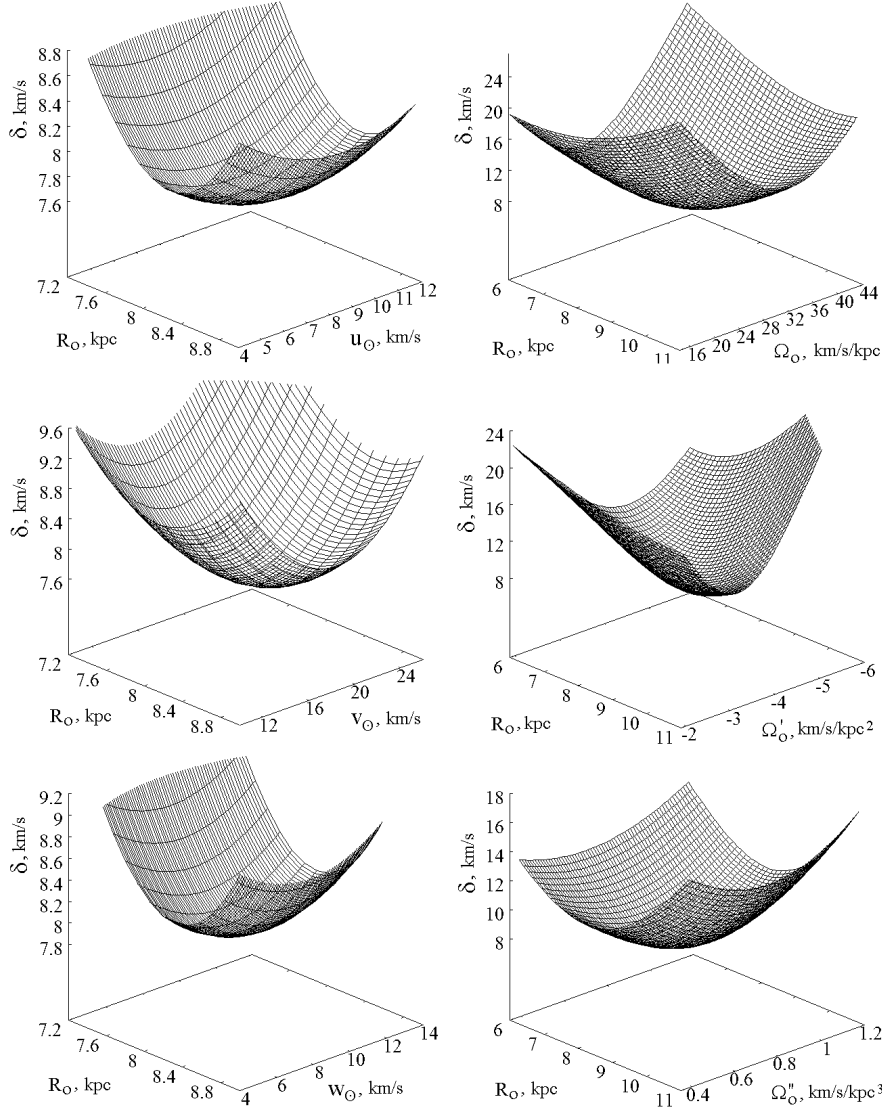
where  $S_0$  denotes the dispersion averaged over all observations — the “cosmic” scatter — taken to be 8 km s<sup>-1</sup>;  $\beta = \sigma_{V_r} / \sigma_{V_l}$  and  $\gamma = \sigma_{V_r} / \sigma_{V_b}$  are the scale factors, where  $\sigma_{V_r}, \sigma_{V_l}$  and  $\sigma_{V_b}$  denote the velocity dispersions along the line of sight, the Galactic longitude, and the Galactic latitude, respectively. The system of weights (6) is close to that used by Mishurov & Zenina (1999). We adopt  $\beta = \gamma = 1$  in accordance with Bobylev & Bajkova (2014).

The errors of the velocities  $V_l$  and  $V_b$  are calculated with the equation

$$\sigma_{(V_l, V_b)} = 4.74r \sqrt{\mu_{l,b}^2 \left( \frac{\sigma_r}{r} \right)^2 + \sigma_{\mu_{l,b}}^2}. \quad (7)$$

We minimize functional (1) subject to Eqs. (2)–(4) by numerically determining the seven unknown parameters,  $u_\odot, v_\odot, w_\odot, \Omega_0, \Omega'_0, \Omega''_0$ , and  $R_0$ , from a necessary condition for the existence of an extremum. A sufficient condition for the existence of an extremum in a particular domain is the positive definiteness of the Hessian matrix composed of the elements  $\{a_{i,j}\} = d^2 F / dx_i dx_j$ , where  $x_i (i = 1, \dots, 7)$  denote the required parameters, everywhere in this domain. We calculated the Hessian matrix in a wide domain of parameters or, more specifically,  $\pm 50\%$  of the nominal values of the parameters.

Our analysis of the Hessian matrix for both cases of weighting showed it to be positive definite, suggesting the existence of a global minimum in this domain and, as a consequence, the uniqueness of the solution. We illustrate this in Fig. 1, which shows the dependence of the square root of residual functional  $F$  on two parameters: (1)  $R_0$  and (2)  $u_\odot, v_\odot, w_\odot, \Omega_0, \Omega'_0$ , and  $\Omega''_0$  with the remaining parameters fixed at the values corresponding to the resulting solution. The graphs show a well-defined global minimum in a wide domain of parameter space. In the case



**Fig. 1.** Graphical representation of the dependence of the square root of residual functional  $\delta = \sqrt{F}$  on two parameters: (1)  $R_0$  and (2)  $u_\odot, v_\odot, w_\odot, \Omega_0, \Omega'_0$  and  $\Omega''_0$  with the remaining parameters fixed at the values corresponding to solution (8).

of unit weight factors, the Hessian matrix is also positive definite far beyond this domain. However, as will be shown below, the adopted weighting improved the accuracy of the solutions obtained.

We estimated the errors of the sought-for parameters by running 100 Monte Carlo simulations. With this number of simulations the mean parameter values virtually coincide with the parameters inferred from the initial data without adding any measurement errors.

### 3. DATA

We compiled published data for Galactic masers including the coordinates, line-of-sight velocities, proper motions, and VLBI trigonometric parallaxes with errors that are less than 10% on the average. These masers are associated with very young objects, mostly massive protostars located in the regions of active star formation.

Trigonometric parallaxes and proper motions of masers were measured within the framework of several projects. One of them is the Japanese VERA (VLBI Exploration of Radio Astrometry) campaign dedicated to observing  $\text{H}_2\text{O}$  masers at 22.2 GHz (Hirota et al. 2007) and a number of SiO masers (which are very few among young objects) at 43 GHz (Kim et al. 2008).

VLBI observations of methanol ( $\text{CH}_3\text{OH}$ , 6.7 and 12.2 GHz) and  $\text{H}_2\text{O}$  masers are performed in the USA (Reid et al. 2009a). Similar observations are also carried out by the European VLBI network (Rygl et al. 2010), which includes three Russian antennas at Svetloe, Zelenchukskaya, and Badary. These two programs are within the scope of the BeSSeL project<sup>1</sup> (Bar and Spiral Structure Legacy Survey, Brunthaler et al. 2011).

We adopt the initial data for 103 masers from Reid et al. (2014).

### 4. RESULTS

Minimizing functional (1) with weights (6) for the three-dimensional maser velocity field of the sample of 103 masers with respect to seven unknowns yields the following parameters:

$$\begin{aligned} (u_\odot, v_\odot, w_\odot) &= (5.20, 17.47, 7.73) \pm (0.74, 0.72, 0.32) \text{ km s}^{-1}, \\ \Omega_0 &= 29.74 \pm 0.45 \text{ km s}^{-1} \text{ kpc}^{-1}, \\ \Omega'_0 &= -4.20 \pm 0.11 \text{ km s}^{-1} \text{ kpc}^{-2}, \\ \Omega''_0 &= 0.730 \pm 0.029 \text{ km s}^{-1} \text{ kpc}^{-3}, \\ R_0 &= 8.03 \pm 0.12 \text{ kpc}, \\ \sigma_0 &= 10.59 \text{ km s}^{-1}, \\ N_\star &= 93. \end{aligned} \tag{8}$$

Note that, in this case, ten sources (G000.67-00.03, G010.47+00.02, G010.62-00.38, G023.70-00.19, G025.70+00.04, G027.36-00.16, G009.62+00.19, G012.02-00.03, G078.12+03.63, G168.06+00.82) were rejected according to the  $3\sigma$  criterion. From this solution, the linear rotation velocity at the solar distance  $R_0$  is  $V_0 = 238 \pm 6 \text{ km s}^{-1}$  and the Oort constants  $A = 0.5R_0\Omega'_0$  and  $B = \Omega_0 + 0.5R_0\Omega'_0$  are equal to  $A = -16.86 \pm 0.45 \text{ km s}^{-1} \text{ kpc}^{-1}$  and  $B = 12.88 \pm 0.63 \text{ km s}^{-1} \text{ kpc}^{-1}$ .

As a clear illustration of the uniqueness of the solution obtained (i.e., the existence of a global minimum of functional  $F$  in a wide range of sought for parameters), Fig. 1 presents the two-dimensional dependence of the residuals  $\delta = \sqrt{F}$  (see Eq. (1)) on (1)  $R_0$  and (2) one of the parameters  $u_\odot, v_\odot, w_\odot, \Omega_0, \Omega'_0$ , and  $\Omega''_0$  with the remaining parameters fixed at the values corresponding to solution (8).

Figure 2 shows the Galactic rotation curve computed with parameters (8) including  $R_0 = 8.03 \text{ kpc}$ ; when calculating the boundaries of the confidence region, we took into account the 0.12 kpc uncertainty in the inferred value of  $R_0$ .

<sup>1</sup> <http://www3.mpifr-bonn.mpg.de/staff/abrunthaler/BeSSeL/index.shtml>

**Table 1.** Kinematic parameters inferred from an analysis of the three-dimensional velocity field.

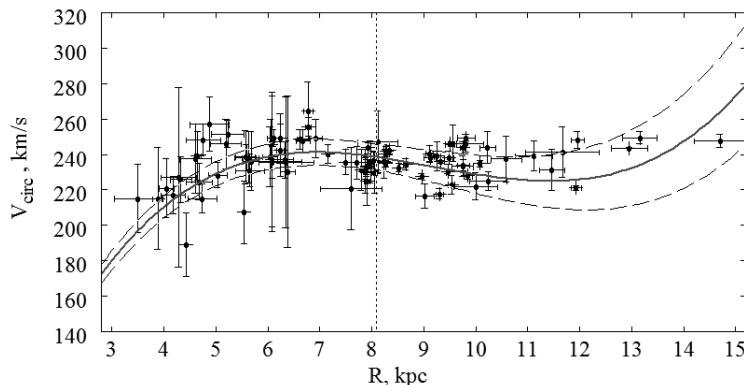
Parameters	All masers	$4 < R < 12$ kpc	$e_\pi/\pi < 12\%$	Outliers excluded
$u_\odot$ , km s <sup>-1</sup>	$6.85 \pm 0.75$	$5.65 \pm 0.72$	$6.04 \pm 0.77$	$7.83 \pm 0.79$
$v_\odot$ , km s <sup>-1</sup>	$14.31 \pm 0.65$	$15.48 \pm 0.73$	$14.23 \pm 0.81$	$13.25 \pm 0.75$
$w_\odot$ , km s <sup>-1</sup>	$7.74 \pm 0.35$	$8.45 \pm 0.38$	$8.24 \pm 0.42$	$9.18 \pm 0.43$
$\Omega_0$ , km s <sup>-1</sup> kpc <sup>-1</sup>	$29.55 \pm 0.45$	$29.49 \pm 0.43$	$29.76 \pm 0.48$	$29.39 \pm 0.46$
$\Omega'_0$ , km s <sup>-1</sup> kpc <sup>-2</sup>	$-3.86 \pm 0.08$	$-4.36 \pm 0.11$	$-4.05 \pm 0.12$	$-3.76 \pm 0.10$
$\Omega''_0$ , km s <sup>-1</sup> kpc <sup>-3</sup>	$0.59 \pm 0.02$	$0.95 \pm 0.05$	$0.68 \pm 0.03$	$0.56 \pm 0.02$
$R_0$ , kpc	$8.25 \pm 0.41$	$7.84 \pm 0.13$	$8.10 \pm 0.13$	$8.46 \pm 0.12$
$\sigma_0$ , km s <sup>-1</sup>	12.49	9.97	9.60	8.86
$N_\star$	101	88	78	80

We also obtained several other solutions by applying various cuts to the data (see Table 1). The results shown might be of some interest. In the second column of the table we present the solution based on nearly all masers ( $N = 101$ ) with only two sources (G000.67-00.03, G010.47+00.02) rejected because of unreliable velocities. The third and fourth columns present the solutions obtained for subsamples with limits on galactocentric distance  $R$  and fractional parallax error  $e_\pi/\pi$ . The first solution has a substantial error of unit weight,  $\sigma_0$ , which is significantly smaller in other cases. We therefore admit the first solution to be the least reliable. In the last column we give the results obtained when neglecting 23 masers that were flagged as outliers by Reid et al. (2014). As is evident from the table, there are no substantial differences between the three solutions. In our opinion, solution (8) based on the largest maser sample ( $N = 103$ ) is of greatest interest.

## 5. DISCUSSION

Our inferred parameters of the Galactic rotation curve (8) agree well with the results of the analyses of such young Galactic disk objects as OB associations,  $\Omega_0 = 31 \pm 1$  km s<sup>-1</sup> kpc<sup>-1</sup> (Mel'nik & Dambis 2009), blue supergiants,  $\Omega_0 = 29.6 \pm 1.6$  km s<sup>-1</sup> kpc<sup>-1</sup> and  $\Omega'_0 = -4.76 \pm 0.32$  km s<sup>-1</sup> kpc<sup>-2</sup> (Zabolotskikh et al. 2002), or distant OB3 stars ( $R_0 = 8$  kpc),  $\Omega_0 = 31.9 \pm 1.1$  km s<sup>-1</sup> kpc<sup>-1</sup>,  $\Omega'_0 = -4.30 \pm 0.16$  km s<sup>-1</sup> kpc<sup>-2</sup> and  $\Omega''_0 = 1.05 \pm 0.35$  km s<sup>-1</sup> kpc<sup>-3</sup> (Bobylev & Bajkova 2013). Solution (8) agrees well with local circular velocity estimates of  $V_0 = 254 \pm 16$  km s<sup>-1</sup> for  $R_0 = 8.4$  kpc (Reid et al. 2009a) and  $V_0 = 244 \pm 13$  km s<sup>-1</sup> for  $R_0 = 8.2$  kpc (Bovy et al. 2009) determined from a sample of 18 masers. Note also the study by Irrgang et al. (2013), who proposed three Galactic potential models based on the data for hydrogen clouds and masers, which yields a velocity  $V_0$  of about 240 km s<sup>-1</sup> and a solar galactocentric distance of  $R_0 \approx 8.3$  kpc.

Individual independent methods give  $R_0$  estimates with typical errors of 10–15%. Here we point out some important measurements. Feast et al. (2008) obtained an estimate of  $R_0 = 7.64 \pm 0.21$  kpc based on Population II Cepheids and RR Lyr type variables in the Galactic bulge and improved calibrations derived from *Hipparcos* data and 2MASS photometry. Gillessen et al. (2009) derived the estimate  $R_0 = 8.33 \pm 0.35$  kpc based on an analysis of stellar orbits about the supermassive black hole at the Galactic center (the method of dynamical parallaxes). According to VLBI measurements, the radio source Sgr A\* has a proper motion



**Fig. 2.** Galactic rotation curve computed with parameters (8) (thick line); the thin lines mark the  $1\sigma$  confidence region; the vertical straight line indicates the position of the Sun.

of  $6.379 \pm 0.026 \text{ mas yr}^{-1}$  relative to extragalactic sources (Reid & Brunthaler 2004). Based on this estimate, Schönrich (2012) found  $R_0 = 8.27 \pm 0.29 \text{ kpc}$  and  $V_0 = 238 \pm 9 \text{ km s}^{-1}$ . There are two  $\text{H}_2\text{O}$  maser sources (Sgr B2) in the immediate vicinity of the Galactic center, where the radio source Sgr A\* is located. Based on their direct trigonometric VLBI measurements, Reid et al. (2009b) obtained a distance estimate of  $R_0 = 7.9^{+0.8}_{-0.7} \text{ kpc}$ .

Bobylyev & Bajkova (2014) used the data for 73 masers to obtain a kinematic estimate of  $R_0 = 8.3 \pm 0.2 \text{ kpc}$  and  $V_0 = 241 \pm 7 \text{ km s}^{-1}$ . Based on 80 maser sources, Reid et al. (2014) obtained a kinematic estimate of  $R_0 = 8.34 \pm 0.16 \text{ kpc}$  and  $V_0 = 240 \pm 8 \text{ km s}^{-1}$ . Thus our kinematic estimate of  $R_0 = 8.03 \pm 0.12 \text{ kpc}$  agrees well with the known measurements and is superior to them in terms of accuracy.

## 6. CONCLUSIONS

We compiled a sample of masers with published line-of-sight velocities and highly accurate VLBI trigonometric parallaxes and proper motions. We used these data to solve Bottlinger’s kinematic equations for the parameters of Galactic rotation ( $\Omega_0$  and its derivatives), the solar galactocentric distance ( $R_0$ ), and the components of the bulk velocity of the sample relative to the Sun ( $u_\odot, v_\odot, w_\odot$ ). We estimated the unknown parameters by minimizing the quadratic functional equal to the sum of the weighted squared residuals between the measured and model velocities. We found the solutions for the cases of both three- and two-dimensional velocity fields for various sets of sought-for parameters with various weighting methods applied. The most reliable solution found — Eqs. (8) — is the one based on an analysis of the three-dimensional maser velocity field for seven sought-for parameters ( $u_\odot, v_\odot, w_\odot, \Omega_0, \Omega'_0, \Omega''_0$ , and  $R_0$ ) corresponding to the global minimum of the functional. The resulting linear Galactic rotation velocity at the solar distance  $R_0$  is  $V_0 = 238 \pm 6 \text{ km s}^{-1}$ . The solar galactocentric distance  $R_0$  is the most important and debatable parameter. Our value  $R_0 = 8.03 \pm 0.12 \text{ kpc}$  agrees well with the most recent estimates and even surpasses them in terms of accuracy.

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