

THE CONSEQUENCES OF ASSUMING $m=0$ FOR GLOBAL MODEL-FITTING

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Abstract. A recent re-analysis of Whole Earth Telescope observations of GD 358 obtained in 1990 suggest that asteroseismology of additional DBV white dwarfs can lead to independent constraints on the important, but poorly determined, $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ nuclear reaction rate. Data exist for several other DBV white dwarfs, but relatively few modes are detected and there is often no multiplet structure to aid in the identification of the spherical harmonic indices l, m . I use a new grid of one million DBV models covering a broad range of masses, temperatures, and surface helium layer masses to investigate the consequences of assuming $m=0$ for global model-fitting. I find that when the spherical degree is known and the rotation period is of order 1 day, the model-fitting procedure applied to modes with unknown m values will still correctly identify the *families* of possible solutions, and has a high probability of identifying the same globally optimal solution found when the m -value is known.

Key words: methods: numerical—stars: individual (GD 358)—stars: oscillations—stars: white dwarfs.

1. INTRODUCTION

More than a decade after the outstanding successes with PG 1159 and GD 358 (Winget et al. 1991, 1994), the Whole Earth Telescope (WET; Nather et al. 1990) continues to look for variable white dwarfs with rich pulsation spectra. Unfortunately we have learned that these stars are truly exceptional – either in the number and variety of their intrinsic variations, or in our ability (and luck) to detect so many modes. A more typical WET run tends to yield a smaller number of modes, not necessarily with consecutive radial overtone numbers (k), and with few (if any) triplets or quintuplets to simplify the identification of the spherical degree (ℓ) and azimuthal order

(m) of each mode. This observational difficulty limits our ability to find meaningful theoretical model-fits for other white dwarfs. Consequently, what we have learned from more recent runs has been limited when compared to the shining successes of the past.

2. DBV MODEL GRID

Recent work using the 1990 data for GD 358 has suggested that reliable measurements of the central ratio of carbon to oxygen can be derived from asteroseismological data, which can subsequently yield precise constraints on the rate of the important $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ nuclear reaction (Metcalfe et al. 2000, 2001, 2002). The analysis of additional DBV white dwarfs using the same model-fitting method can, in principle, provide independent constraints on the reaction rate under varying conditions of density and temperature. However, until recently none of the other DBV white dwarfs had a sufficient number of observed periods, and the mode identifications were not secure. This meant that, at best, only the mass, temperature, and surface helium layer mass could be reliably deduced from the limited data.

With a larger number of free parameters a genetic algorithm based approach is preferable, but the prospect of fitting many DBVs with only 3 parameters makes the calculation of a full grid of models more efficient in the long run. Using a 32-processor beowulf system at Aarhus University, the $\ell=1$, $m=0$ pulsation periods between 100 and 1000 seconds were calculated for one million carbon-core DBV models with masses between 0.45 and 0.95 $M_{\odot}$ (0.005 $M_{\odot}$ resolution), temperatures between 20 000 and 30 000 K (100 K resolution), and $\log(M_{\text{He}}/M_{\star})$ between -2.0 and -7.3 (0.05 dex resolution). The grid required nearly 4 GHz-CPU-months to complete, but it was finished on the cluster after only a few days. This model grid now makes it possible to map the entire 3-dimensional parameter-space for a given set of observed periods in about 30 seconds¹. Such a map for GD 358 in 1990 is shown in Figure 1.

3. MONTE CARLO SIMULATIONS

Single-site observations of the faint DBV white dwarf CBS 114 revealed a total of 7 independent pulsation modes, and the period spacing implies that they all have spherical degree $\ell=1$ (Handler,

¹A web interface to the model-fitting program that uses this grid is available online at <http://whitedwarf.org/research/fitwd/>

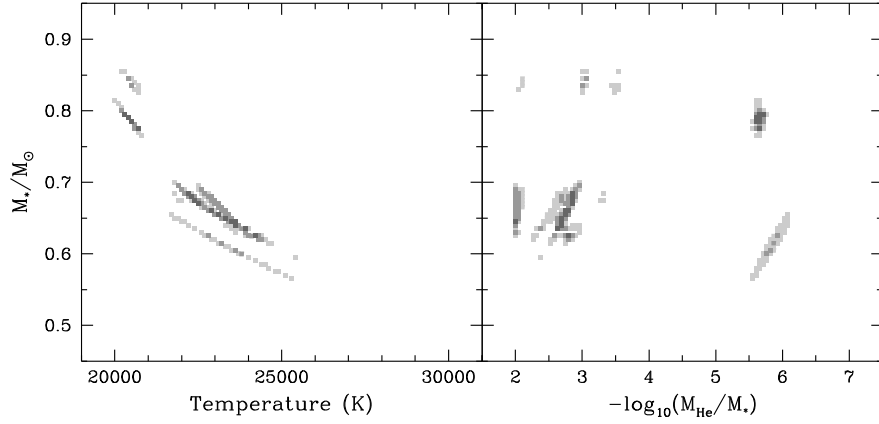


Fig. 1. Front and side views of the model-space for the pulsation periods observed in GD 358 showing the families of good matches to the observations. Each point in the left panel corresponds one-to-one with a point in the right panel. The shade of each point is an indication of how well the model matches the observations: root-mean-square period differences are less than 3 seconds (lightest), 2.75 seconds, and 2.5 seconds. High mass models can be ruled out based on the luminosity of GD 358 from its observed parallax.

Metcalfe & Wood 2002). None of the modes exhibited multiplet structure, so the identification of the m -value was not possible. In 1990, triplets were observed in 9 of the 11 identified modes for GD 358, so we can use this data to determine empirically the consequences of making a wrong assumption about the m -values.

One hundred random data sets were created, drawing the period for each k from the observed $m=(-1,0,+1)$ triplets at random. Since no triplets were observed for $k=12$ and $k=18$, these periods were included unaltered into each simulated data set. The model parameters that yielded the closest match to each set of simulated periods were determined by comparison with the entire grid of models described in Section 2, and the root-mean-square period residuals were calculated.

4. RESULTS AND DISCUSSION

The globally optimal models for each simulated data set are shown in Figure 2, where the darkness of the point indicates the quality of the fit. The families of models from Figure 1 are outlined for reference. In nearly every case, the data sets created from random m -values lead to optimal models that fall within the same families of

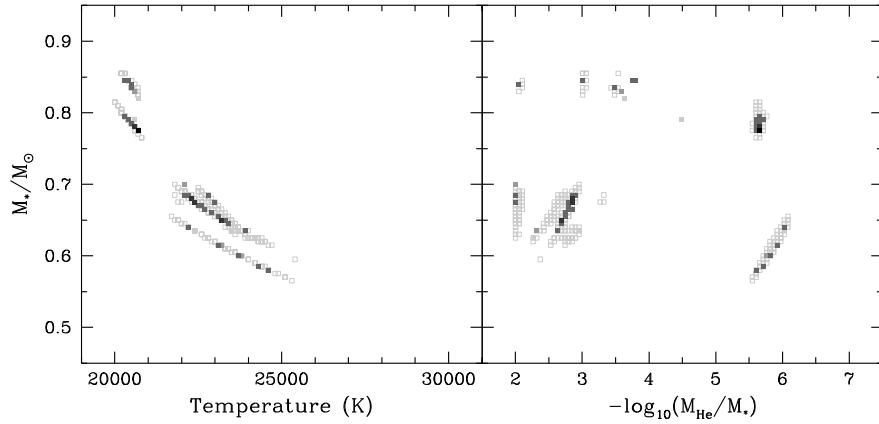


Fig. 2. The distribution of globally optimal model parameters for the 100 simulated data sets created using random m -values from the observed modes in GD 358. The families of models from the $m=0$ case in Figure 2 are shown in outline for comparison. Again the shade indicates root-mean-square period differences less than 3 seconds (lightest), 2.75, 2.5, 1.75, and 1.5 seconds (darkest).

models found when all of the modes are $m=0$. The basic picture of the model-space that we derive from modes with unknown m -value is very similar to what we see when they are all known to be $m=0$. The *particular* set of model parameters that turns out to be optimal may shift slightly, or may even fall into a different family of models when the m -values are unknown. Furthermore, the optimal models fall into the various families in rough proportion to the average quality of the fits in each region. In more than 80 percent of the simulations, the optimal model fell near the two darkest areas in Figure 1. Thus, we can think of the inclusion of modes with non-zero values of m as a perturbation on the shape of the model-space: the general terrain is determined by the spacing of the modes regardless of their m -value, and small local features on that terrain are determined by the correct identification of m .

The rotation period of GD 358 at the surface is 0.9 days (Winget et al. 1994), so these results are only applicable to white dwarfs with comparably slow rotation. But it is encouraging that without a secure identification of m , we can still be confident that our overall picture of the model-space is sound. We can map out the possibilities, and to the degree that the optimal model is unique we can maximize the probability of finding an optimal solution very near to what it would be if we were certain that all of the modes were $m=0$.

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REFERENCES

- Handler G., Metcalfe T.S., Wood M.A. 2002, MNRAS, in press
Metcalfe T.S. et al. 2000, ApJ, 545, 974
Metcalfe T.S. et al. 2001, ApJ, 557, 1021
Metcalfe T.S. et al. 2002, ApJ, 573, 803
Nather R.E. et al. 1990, ApJ, 361, 309
Winget D.E. et al. 1991, ApJ, 378, 326
Winget D.E. et al. 1994, ApJ, 430, 839