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# OBSERVING roAp STARS WITH WET: A PRIMER

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**Abstract.** We give an extensive primer on roAp stars – introducing them, putting them in context and explaining terminology and jargon, and giving a thorough discussion of what is known and not known about them. This provides a good understanding of the kind of science WET could extract from these stars. We also discuss the many potential pitfalls and problems in high-precision photometry. Finally, we suggest a WET campaign for the roAp star HR 1217.

**Key words:** stars: interiors, oscillations

## 1. CHEMICALLY PECULIAR STARS OF THE UPPER MAIN SEQUENCE

On and near the main sequence for $T_{\rm eff} > 6600$ K there is a plethora of spectrally peculiar stars and photometric variable stars with a bewildering confusion of names. There are Ap, Bp, CP and Am stars; there are classical Am stars, marginal Am stars and hot Am stars; there are roAp stars and noAp stars; there are magnetic peculiar stars and non-magnetic peculiar stars; He-strong stars, He-weak stars; Si stars, SrTi stars, SrCrEu stars, HgMn stars, PGa stars; $\lambda$ Boo stars; stars with strong metals, stars with weak metals; pulsating peculiar stars, non-pulsating peculiar stars; pulsating normal stars; non-pulsating normal stars; $\delta$ Sct stars, $\delta$ Del stars and $\rho$ Pup stars; $\gamma$ Dor stars, SPB stars, $\beta$ Cephei stars; $\gamma$ Cas stars, $\lambda$ Eri stars, $\alpha$ Cyg stars; sharp-lined and broad-lined stars, some of which are peculiar and some of which are not. There are pre-main sequence Ae and Be stars, collectively called HAeBe stars; there are Oe and Be stars which are not pre-main sequence stars.

What a mess! And some of the mess is partially the result of people having previously said, "What a mess!", then trying to clean up the mess by "simplifying" the nomenclature by introducing new names, which not by everyone adopted, thus adding to the mess. We have contributed to this mess ourselves, so we do not intend to mess-up further by attempting a clean-up here. What we will do is to provide a partial guide through the morass.

Table 1 shows some of the sub-groups of chemically peculiar (CP) stars as a function of temperature. All of them are on or near the main sequence. These stars all show spectral peculiarities. Members of the magnetic group are known to have global magnetic fields which are roughly dipolar with strengths of hundreds to tens of thousands of G. (The global magnetic field of the Sun is about half a G; the magnetic field in sunspots is about 1500 G.) It is possible that a few members of the "non-magnetic" group may have magnetic fields, but it is clear that the vast majority do not. Mathys & Lanz (1990) discuss the detection of a 2 kG field in the hot Am star $o$ Pegasi. The magnetic stars are mostly known as Ap stars (for A peculiar; "Ap" is pronounced " A-pee"), but many of the "Ap" stars are B stars, so sometimes those are called Bp stars. However, note that there is a group, the HgMn stars, which are called Ap, even though they lack magnetic fields and they are B stars! The A stars which have peculiar spectra, but not the same peculiarities as the Ap stars, and are non-magnetic, are Am stars (for A metallic-lined stars; "Am" is always pronounced "A-em").

**Table 1.** Magnetic and non-magnetic peculiar stars of the upper main sequence.

| $T_{\rm eff}$ (K) | Magnetic stars | Non-magnetic stars |
|---|---|---|
| 7000–10000 | Ap SrCrEu<br>A3–F0 | Am, $\lambda$ Boo<br>A0–A1 |
| 10000–14000 | Ap Si<br>B8–A2 | Ap HgMn<br>B6–B9 |
| 13000–18000 | He-weak Si, SrTi<br>B3–B7 | He-weak PGa<br>B4–B5 |
| 18000–22000 | He-strong<br>B1–B2 | |

Am stars are given three spectral classifications: one based on the Balmer lines, which give a good measure of the effective temperature; one based on the Ca II K-line which, because of its weakness relative to normal stars, gives an earlier spectral type; and one based on the metal lines which, because of the enhanced strength of the metal lines, gives a later spectral type. "Classical Am" stars have K-line and metal line spectral types that differ by 5 or more spectral subclasses; this difference in "marginal Am" stars is less than 5 subtypes. (Marginal Am stars are designated "Am:", a potentially confusing notation in written text because of the use of a colon as part of the classification; "Am:" is always inelegantly pronounced "A-em-colon"; if you do not like that, then call them "marginal Am stars".) Because the classification criteria are harder to diagnose for the hotter Am stars, the classical Am stars have H-line types that lie between A3 and F1. It was later recognized at higher dispersion that the Am phenomenon continues to A0 (Sirius is an Am star), so Am stars with H-line spectral types between A0 and A3 are known as "hot Am" stars.

Evolved Am stars with luminosity classes IV and III are classified as $\delta$ Del stars in the Michigan Spectral Catalogues (Houk & Cowley 1975; Houk 1978, 1982; Houk & Smith-Moore 1988) and many other publications. However, Kurtz (1976) and Gray & Garrison (1989) found the $\delta$ Del class to be highly inhomogeneous. Because of this, Gray & Garrison (1989) re-classified the luminous, late-A to mid-F, evolved Am stars as $\rho$ Pup stars, after the prototype. They recommend that the $\delta$ Del classification be dropped, but not everyone knows about, or chooses to follow that recommendation.

Note that all of the spectral types for the chemically peculiar stars are defined by spectral standard stars and do not depend on the physical characteristics, such as the magnetic field, which we have pointed out. Their classifications are phenomenologically defined by their spectra alone.

To rationalize this situation Preston (1974) introduced the following nomenclature:

- CP $\equiv$ chemically peculiar star of the upper main sequence,
- CP1 $\equiv$ Am stars,
- CP2 $\equiv$ magnetic Ap, Bp stars,
- CP3 $\equiv$ HgMn stars, and
- CP4 $\equiv$ He-weak B stars.

This system is widely used – principally, but not exclusively, in Europe – whereas North Americans seem to stick to the older Ap-Bp terminology. A good reference on this subject is Wolff's monograph on the A stars (Wolff 1983). See particularly her Table 1 which lists the observed properties of the CP stars. More recent reviews and contributions covering the field can be found in the conference proceedings edited by Dworetsky et al. (1993). For particular groups, see Smith (1971) for Am stars, Adelman (1973) for Ap SrCrEu stars, and Takada-Hidai (1991) and White et al. (1976) for HgMn stars. These papers show in detail the difficulties of the problem of the spectral peculiarities.

The above list is not exhaustive for chemically peculiar stars of the upper main sequence. There are also the $\lambda$ Bootis stars which have H-line types between A0 and F0, a Ca II K-line type of A0, or slightly later, and *weak* metallic lines, particularly Mg II 4481 Å. Abundance analyses show marked *under*-abundances of the Fe-peak elements, with lighter elements essentially normal. Many $\lambda$ Boo stars are pulsating $\delta$ Sct stars. See Paunzen et al. (1999) and Martinez et al. (1998a) for discussions of these stars.

## 2. PULSATING VARIABLE STARS OF THE UPPER MAIN SEQUENCE

At this juncture, it is instructive to review the nomenclature associated with the different types of pulsating stars found along the upper main sequence:

- The $\delta$ Sct stars are H-core-burning, main sequence and post-main sequence stars which lie in the instability strip, which ranges from A2 to F0 on the main sequence and from A3 to F5 at luminosity class III. They pulsate primarily in low-overtone radial and non-radial *p*-modes with periods between about 30 min and 6 h. Some $\delta$ Sct stars may also pulsate in *g*-modes.
- The rapidly oscillating Ap (roAp; sometimes pronounced "r-o-A-p", sometimes "row-ap") stars are mid-A to early-F main sequence Ap SrCrEu, CP2 stars which pulsate in high-overtone *p*-modes with periods in the range 5–16 min and amplitudes $\leq$ 0.016 mag (generally much less). They mostly lie within the $\delta$ Sct instability strip, but a few of them are cooler. It is not clear whether the He II $\kappa$-mechanism drives the pulsation in these stars, or not. Many CP2 Ap stars do not show roAp pulsations (Martinez & Kurtz 1994b); Mathys et al. (1996) have dubbed these stars noAp (non-oscillating) Ap stars. It is still not clear whether magnetic stars can be $\delta$ Sct stars, although

the evidence suggests that a few may be. This is discussed in detail in section 5 of this paper.

- The Herbig Ae and Be stars, known collectively as HAeBe stars, are pre-main sequence A and B stars which show emission lines from strong stellar winds and from cocoons of remnant gas from which they collapsed. These are massive counterparts of the T Tauri stars. Two Herbig Ae stars are now known to be $\delta$ Sct stars.
- The $\gamma$ Dor stars are multi-periodic, non-radial $g$-mode main sequence pulsators with periods in the range 0.3–3 d. A study of 70 $\gamma$ Dor candidates from the *Hipparcos* catalogue shows that they are confined to an instability strip which partially overlaps the $\delta$ Sct stars in the HR Diagram; they range in temperature from 7200–7700 K on the main sequence, and 6900–7500 for $\log g \approx 4$ (Handler 1999).
- The Slowly Pulsating B (SPB) stars (also known as 53 Per stars) are multi-periodic, non-radial $g$-mode main sequence pulsators with periods in the range 0.6–3 d and amplitudes typically less than a few hundredths of a magnitude. They lie in a narrow instability box in the HR Diagram which ranges from B2 to B9, thus they do not overlap with the $\delta$ Sct stars (Waelkens et al. 1998).
- The $\beta$ Cephei stars are giant and sub-giant, $p$-mode pulsators with periods in the range 2–7 h. Some are singly periodic, some are multi-periodic; they pulsate in both radial and non-radial modes of low degree, $\ell \leq 3$, and low overtone, $n$. Their photometric amplitudes are $\leq 0.1$ mag, except for the star BW Vul, about which there is an extensive literature.
- Even hotter than the $\beta$ Cep stars there are (possibly) pulsating O stars known as $\alpha$ Cygni variables. Their periods are typically of the order of 1 to 2 months, and their amplitudes are less than a few tenths of a magnitude. They are thought to be pulsating in $g$-modes, or so-called "strange modes" – modes associated with a sound-speed inversion, caused by a density inversion, caused by an opacity bump, most likely from Fe, H and/or He (Glatzel 1998).
- There was one report of rapid pulsations with a period of 627 seconds driven by the $\epsilon$-mechanism in the Wolf-Rayet star HD 96548 (WR40) (Blecha et al. 1992), but this could not be confirmed (Martinez et al. 1994). Discovery of such pulsations in 30–40 $M_\odot$ WR stars would be tremendously exciting, since they

are almost-bare helium cores of massive stars that shed their H in $10^{-5}$-$M_{\odot}$/yr winds as they try to evolve to giants, but run up against the Eddington limit instead. This causes them to return repeatedly to a new, higher $\mu$ main sequence where CNO cycle products show in the spectra of WN stars, and $3\alpha$ products show in the spectra of WC stars. Finding pulsations in these stars would provide important constraints on their internal structure.

- Amongst the B stars there are emission-line Be stars which have periodic variations with periods between 0.3 and 3 d and amplitudes between 0.01 and 0.3 mag. These are known as $\lambda$ Eri stars. It is contentious as to whether their variability is caused by $g$-mode pulsation, or by some rotational effect (see Balona 1998).

To round out this discussion of the nomenclature of the pulsators of the upper main sequence, it is useful to be aware of the non-pulsating variables, too. They include the following kinds of stars:

- eruptive Luminous Blue Variables (LBVs), also called S Doradus stars, with variations of about 0.2 mag on time-scales of weeks to months. If there is thought to be periodicity in the variations, then these are called $\alpha$ Cyg stars as discussed above. LBVs also have eruptions of 0.5–2 mag on a time-scale of years to decades, and giant eruptions on the time-scale of a millennium, $\eta$ Car being the best known, most spectacular example of this (Humphreys & Davidson 1994). Thus, finding pulsation amid all the other, higher amplitude, non-periodic variations is a challenge – one similar to that in the HAeBe stars.
- WR stars which show non-periodic variations with amplitudes of the order of 0.02 mag caused by the variable winds and possibly by rotation.
- non-periodic, long-time-scale light variable Oe and Be stars known as $\gamma$ Cas stars which are in addition to the periodic Be stars ($\lambda$ Eri stars) mentioned previously. They range from spectral type O6 to B9 and through luminosity classes V to III. Their light variations are caused by variable winds (Harmanec 1994).

## 3. THE A-TYPE MAGNETIC CP STARS

This paper is mostly concerned with the A-type CP stars which lie within the $\delta$ Sct instability strip. It is useful for understanding the problems presented by these stars to have in mind some of their physical characteristics. These CP stars:

- have strongly anomalous abundances with overabundances of heavy elements up to a factor of $10^5$, and underabundances of some light elements up to a factor of $10^{-2}$;
- are young and near the main sequence: Am stars are known in the Ori Ic Association ($t \approx 1 - 3 \times 10^6$ yr); Ap stars are found in NGC 2516 ($t \approx 1.1 \times 10^8$ yr);
- have abundance anomalies that are confined to a thin surface layer;
- rotate slowly; $v \sin i \leq 125$ km s$^{-1}$;
- have magnetic fields that have been detected and measured in: Ap SrCrEu stars, Ap Si stars, Ap He-weak, Si, SrTi stars, and Ap He-strong stars (as seen in Table 1);
- have magnetic fields that vary on a time-scale of d to decades;
- have spectra that vary synchronously with the magnetic variations;
- have mean brightnesses (ignoring short-period pulsation) that vary synchronously with the magnetic and spectrum variations.

In general, there is an exclusion between the CP stars and the $\delta$ Sct stars. But the roAp stars are CP stars that do pulsate, and there are a few CP $\delta$ Scuti stars. To understand why they are important it is useful to know more about them.

### *3.1. The oblique rotator model*

To understand the observations of the magnetic CP stars, it is useful to have the well-established oblique rotator model in mind. The oblique rotator model explains the magnetic variations, the spectrum variations and the mean light variations seen in CP stars. Fig. 1 shows the geometry. The line-of-sight to the Earth is to the right. The inclination of the rotation pole is $i$, the obliquity of the magnetic axis is $\beta$, and the variable angle between the magnetic pole and the line-of-sight is $\alpha$. As the star rotates, the magnetic field is seen from varying aspect, so its effective strength is observed to be variable. Because the stars have anomalous abundance patches at their magnetic poles, the spectrum also varies with rotation as does the mean brightness – both in phase with the magnetic variations. For a centred dipolar magnetic field:

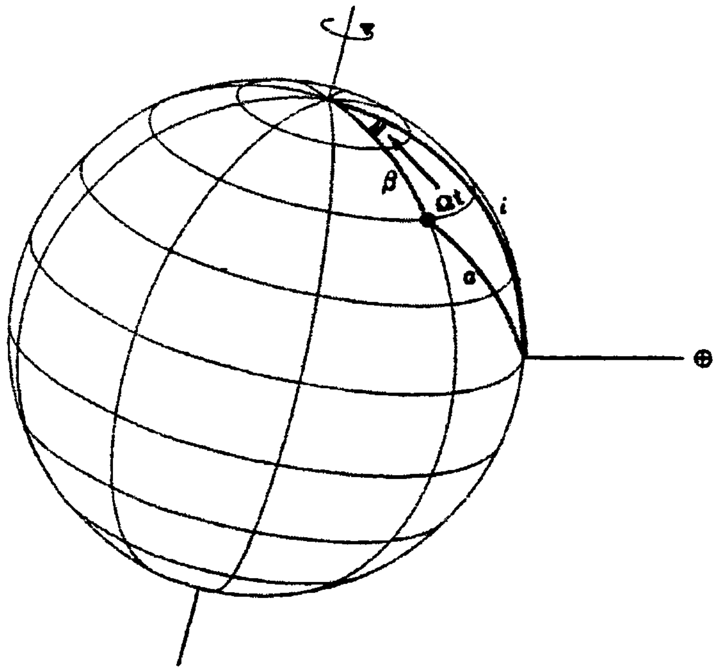


**Fig. 1.** The geometry of the oblique rotator model.

$$H_{\text{eff}} = \frac{1}{20}\frac{15+\mu}{3-\mu}H_p\left(\cos i\cos\beta + \sin i\sin\beta\cos\Omega t\right) \qquad (1)$$

where $H_{\rm p}$ is the polar magnetic field strength, $i$ and $\beta$ are the rotational inclination and magnetic obliquity respectively, $\mu$ is the limb-darkening coefficient, and $\Omega$ is the rotation frequency. It can be seen from Eq. (1) that dipolar magnetic fields give rise to magnetic variations which are sinusoidal with amplitudes proportional to $\sin i \sin\beta$ and zero points proportional to $\cos i \cos\beta$, and that this immediately constrains the values of $i$ and $\beta$.

### *3.2. Magnetic fields in CP stars*

The first detection of a stellar magnetic field was by Hale, when in 1908 he published his detection of Zeeman splitting in spectral lines in sunspots. The first detection of a magnetic field in a star other than the Sun was in 1947 when Babcock discovered a large, *variable* field in the Ap star 78 Vir. Magnetic fields are now measured in magnetic CP stars, magnetic white dwarfs (MG fields), and solar-type stars with spotty fields. In addition, fields are inferred in pulsars, polars (magnetic cataclysmic variables) and magnetars (neutron stars with petaGauss, $10^{15}$ G, fields!).

In a magnetic CP star the magnetic field varies on a time-scale of days to decades, the spectrum varies synchronously with the magnetic variations, and the mean brightness (ignoring short-period pulsation which we will discuss later) varies synchronously with the magnetic and spectrum variations. There are some recent excellent reviews of magnetic fields in stars: see Mathys (1989) and Landstreet (1992).

Early observations were photographic. Usually, they measured the effective, or longitudinal field:

$$\langle H_z \rangle = H_{\text{eff}} = \frac{3}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} H_z\,(\phi, \varphi) \cos^2\theta \sin\theta \, d\theta \ . \qquad (2)$$

Typical errors per measurement for the photographic technique were $\pm\,300 - 500$ G. Excellent measurements on bright stars gave $\pm\,100 - 200$ G. At first, the photographic measurements often gave non-sinusoidal magnetic curves. Then, photoelectric magnetic measurements led to more sinusoidal magnetic curves. Fig. 2 shows Borra & Landstreet's (1980) photoelectric magnetic curve for $\beta$ CrB (a well-observed Ap SrCrEu star) with Preston's older photographic curve over-laid on it for comparison.

The photoelectric technique was developed to its highest precision using a multiplexing technique and 230 spectral lines by Borra et al. (1981) who argued persuasively that they reached a precision of $\pm 1$ G! (We recommend the reading of this paper for the excellent discussion of the assessment of experimental errors alone.) They even found probable fields of $\sim 30$ G in the Cepheid variables Polaris and $\delta$ Cephei itself. Fig. 3 shows magnetic field observations of Borra et al. of $\beta$ CrB with errors of $\pm 20$ G with the older photoelectric curve overlaid. This again shows that as the observations became more precise, the magnetic curve became more sinusoidal.

This looks like a good case for purely centred dipolar magnetic fields – and it is for $\beta$ CrB – but in many CP stars that is far from true. While it became apparent with the higher precision magnetic observations that some stars which previously appeared to have non-sinusoidal variations, in fact had sinusoidal ones, other stars were

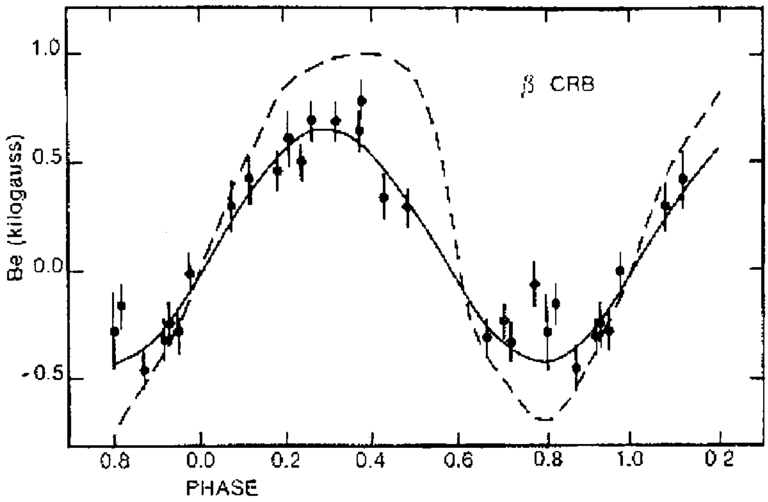


**Fig. 2.** Photoelectric magnetic curve for $\beta$ CrB with an older photographic curve (dashed line) shown. From Borra & Landstreet (1980).

clearly shown to truly have non-sinusoidal magnetic variations. To illustrate the complexity of the problem, let us consider some strongly non-sinusoidal magnetic curves. Landstreet (1988) found for 53 Cam that he could best model the magnetic field with co-linear axisymmetric components of a dipole with a polar strength of $-16\,300$ G, a quadrupole with a strength of $-7300$ G, and an octupole with a strength of $+4900$ G, all varying with $P_{\rm rot} = 8.03$ d. An even more extreme case is the He-strong star HD 37776 (Thompson & Landstreet 1985) for which the quadrupole field dominates. Fig. 4 shows the magnetic curve and a depiction of the magnetic configuration for this star.

A new technique now being used is the observation of broad-band linear polarization (BBLP) in CP stars. For saturated spectral lines the $\pi$ components of the Zeeman pattern saturate before the $\sigma$ components leaving a net *linear* polarization which can be detected through broad-band filters. Leroy and his co-workers have been measuring BBLP in Ap SrCrEu stars with strong magnetic fields, $B_{\rm eff} \geq 1000$ G (e.g. Leroy 1995; Leroy et al. 1993, 1995). These observations constrain the magnetic field differently from the longitudinal measurements. In some cases the rotational inclination, $i$, the magnetic obliquity, $\beta$, and the polar magnetic field strength, $H_{\rm p}$, can all be uniquely determined. Leroy et al. describe complex

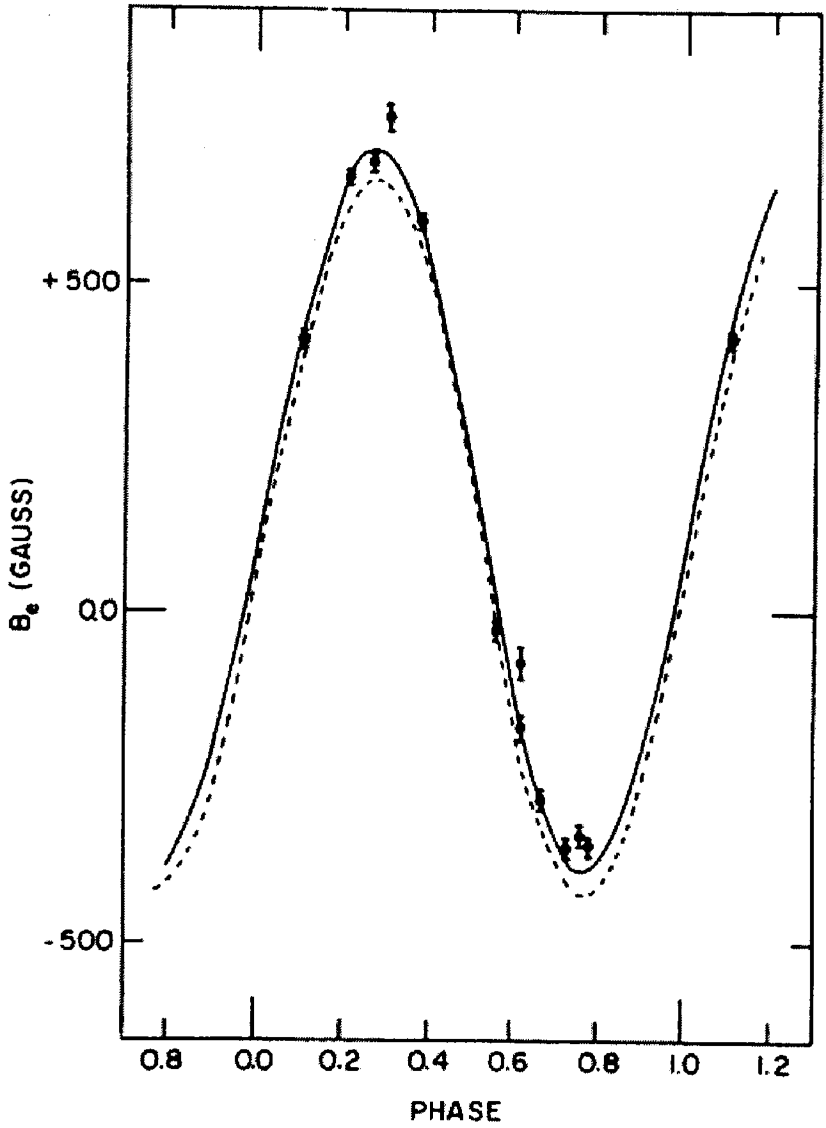


**Fig. 3.** The "ultimate" photoelectric magnetic curve for $\beta$ CrB with errors of only $\pm 20$ G. The dotted curve is the photoelectric curve shown in Fig. 2. From Borra et al. (1981).

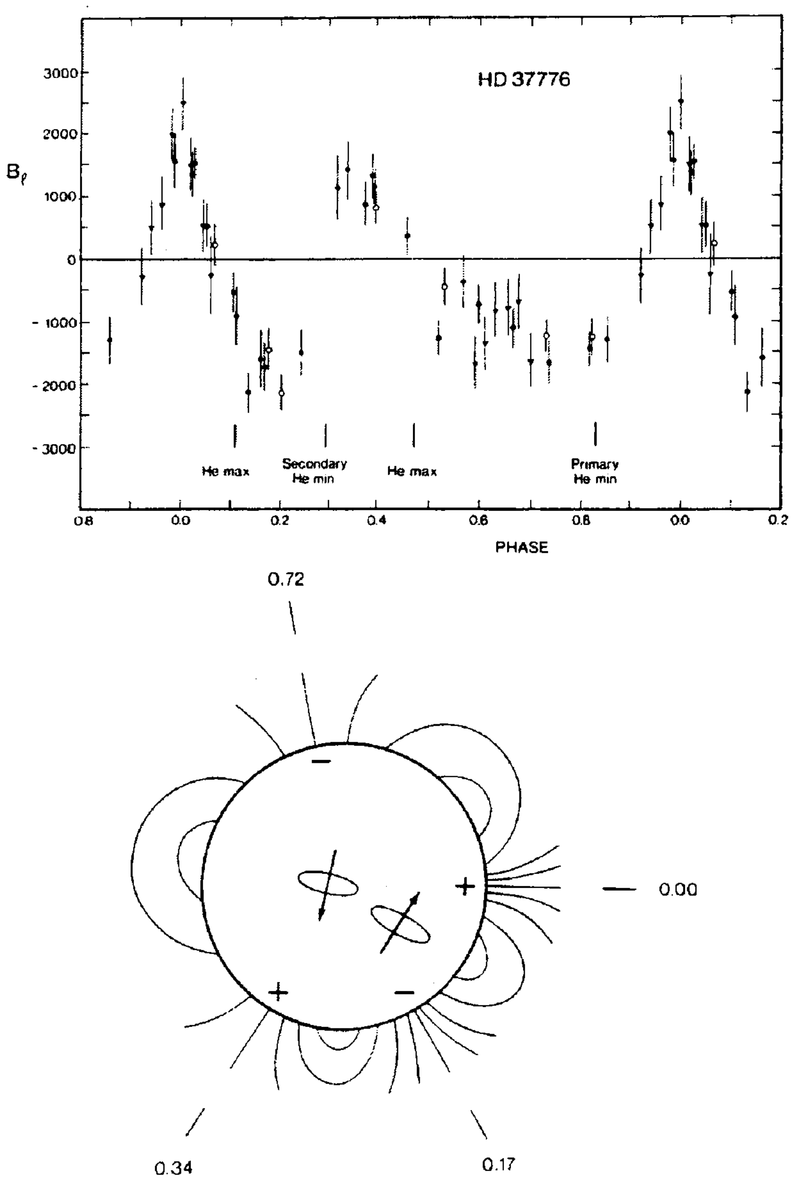


**Fig. 4.** This is the magnetic curve (above) and a conjectured magnetic field configuration (below) for HD 37776, a He-strong star in which the quadrupole field dominates. From Thomson & Landstreet (1985).

field geometries in terms of spherical harmonic series. Their linear polarization results suggest that often multipolar fits are better than dipolar fits.

To explain these more complicated variations earlier work on a generalized oblique pulsator model was developed with decentred, axisymmetric magnetic fields (Landstreet 1970) and with completely decentred dipole fields (Stift 1975). Recently, multipolar magnetic fields have been elegantly modeled using spherical tensor calculus by Bagnulo et al. (1996).

Tangled magnetic fields are now detected in solar type stars from broadening of magnetically sensitive lines, and from linear polarization. Traditionally, magnetic fields were thought not to be present in Am, HgMn and He-weak PGa stars. Mathys & Lanz (1990), however, found a 1.8 kG (probably tangled) field in the hot Am star o Peg. This magnetic field was confirmed by Takeda (1993), who also suggested a technique for separating magnetic and micro-turbulent line broadening. Mathys & Hubrig (1995) have also found magnetic fields in two HgMn stars: $\chi$ Lupi with $B_{\rm eff} = -274$ G and 74 Aquarii with a quadratic field, $H_{\rm q} = \left\langle H^2 + H_{\rm z}^2 \right\rangle^{1/2} = 3.6\,$kG. Am stars have high microturbulent line-broadening. Is this partly magnetic broadening? If the "non-magnetic" CP stars have tangled fields, what differentiates them from the "magnetic" CP stars with dipolar fields? How are they related to the solar-type magnetic stars? All these questions are still to be solved.

### *3.3. Patchy abundances: spots and rings*

Magnetic Ap stars have patchy abundance distributions associated with the magnetic poles. This is what gives rise to their spectrum variability with rotation. Fig. 5 shows how the equivalent widths and radial velocities vary in $\alpha^2$ CVn over its 5.5-d rotation period (Pyper 1969). The radial velocity is variable because the spot approaches and recedes with the rotation of the star.

The patchy abundance distributions in CP stars give rise to asymmetries in the line profiles. The maximum entropy method is now used to model the observed line profiles in terms of abundance patches in a technique called Doppler imaging (see Hatzes et al. 1989; Wehlau & Rice 1993; Hatzes 1993).

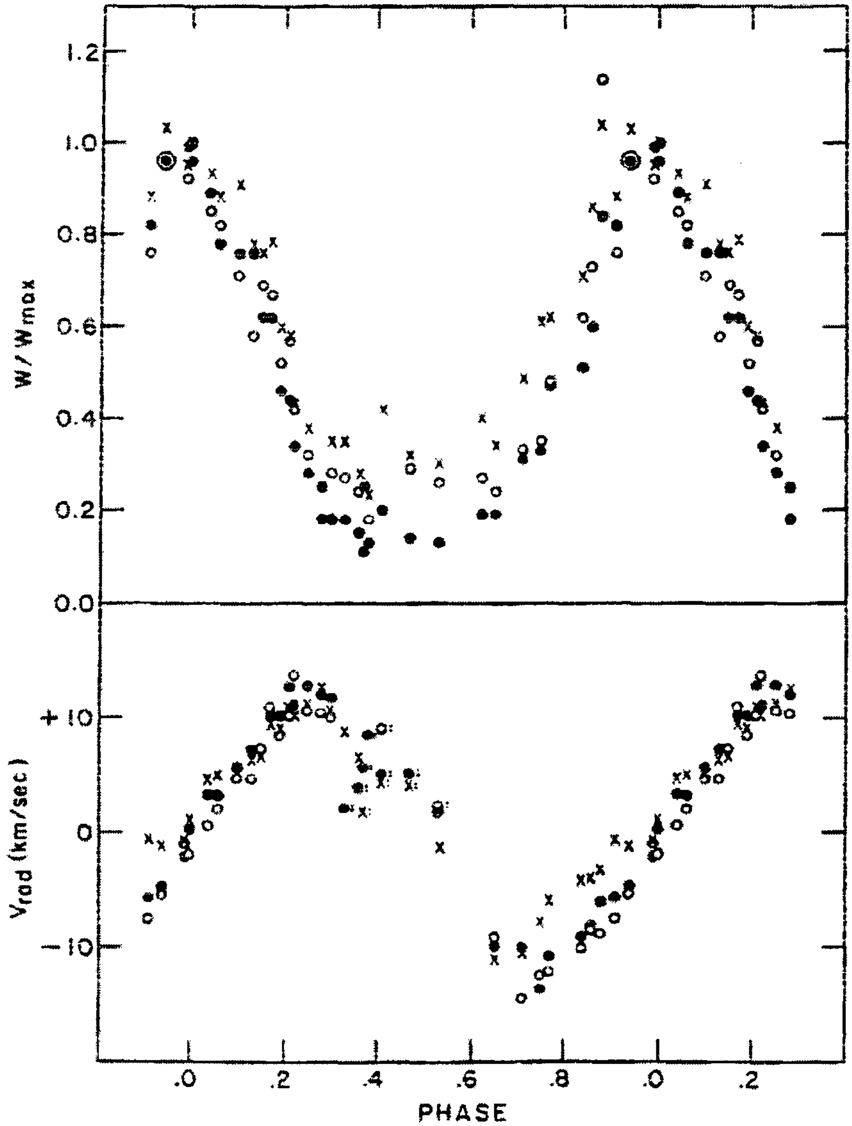


**Fig. 5.** The rotational variation of the equivalent widths of the Eu II and Dy II lines in $\alpha^2$ CVn (top) and the radial velocity variations (bottom). From Pyper (1969).

### *3.4. Mean light variations*

As already mentioned, mean light variations are observed in most magnetic CP stars. We call these "mean light" variations to distinguish them from the rapid oscillations that occur in the roAp stars. The rapid oscillations have periods in the range of 5 to 16 min, whereas the mean light variations occur on a time-scale of days, or longer. They are rotationally modulated light variations associated with the abundance spots or rings that occur at the magnetic poles in CP stars, as was discussed in the last section.

Fig. 6 shows a rotational mean light curve for the roAp star HD 6532. This star has a rotation period of 1.944973 d; both magnetic poles have spots, and both poles are seen over the rotation cycle. Because of the values of the rotational inclination and magnetic obliquity, one magnetic pole (hence one spot) is viewed closer to the line-of-sight than the other. Even if both spots are the same, this explains why there are two dips of unequal depth in the light curve: the spots are seen from different aspect.

The star is at minimum in $U$ and $B$, and at maximum in $V$, $R$, and $I$, when it is at magnetic maximum. The standard explanation for this is that the rare earths or lanthanides, which collect in the spots at the poles, have many absorption lines in the ultraviolet and blue, hence the star appears dimmer in those filters when the pole is near the line of sight. The line blocking by those absorption lines thus increases the temperature gradient, and at the depth viewed in $V$, $R$, and $I$ more radiation gets out. In effect, the radiation is redistributed from shorter wavelengths to longer wavelengths by line absorption in the spots (Wolff & Wolff 1971).

## 4. THE DIFFUSION HYPOTHESIS

There is no universally accepted model that explains all of the observed characteristics of the CP stars. Models involving anomalous atmospheres (Abt & Morgan 1976; Hardorp 1977), nucleosynthesis and dredging (Fowler et al. 1965), surface spallation (Searle & Sargent 1968), magnetic accretion (Havnes & Conti 1971), planetesimal impacts (Kumar et al. 1989), binary mass transfer (van den Heuvel 1968a,b), and element separation (diffusion) have all been tried with varying success. The most successful model, the *diffusion hypothesis*, is the working model of choice for the majority of investigators.

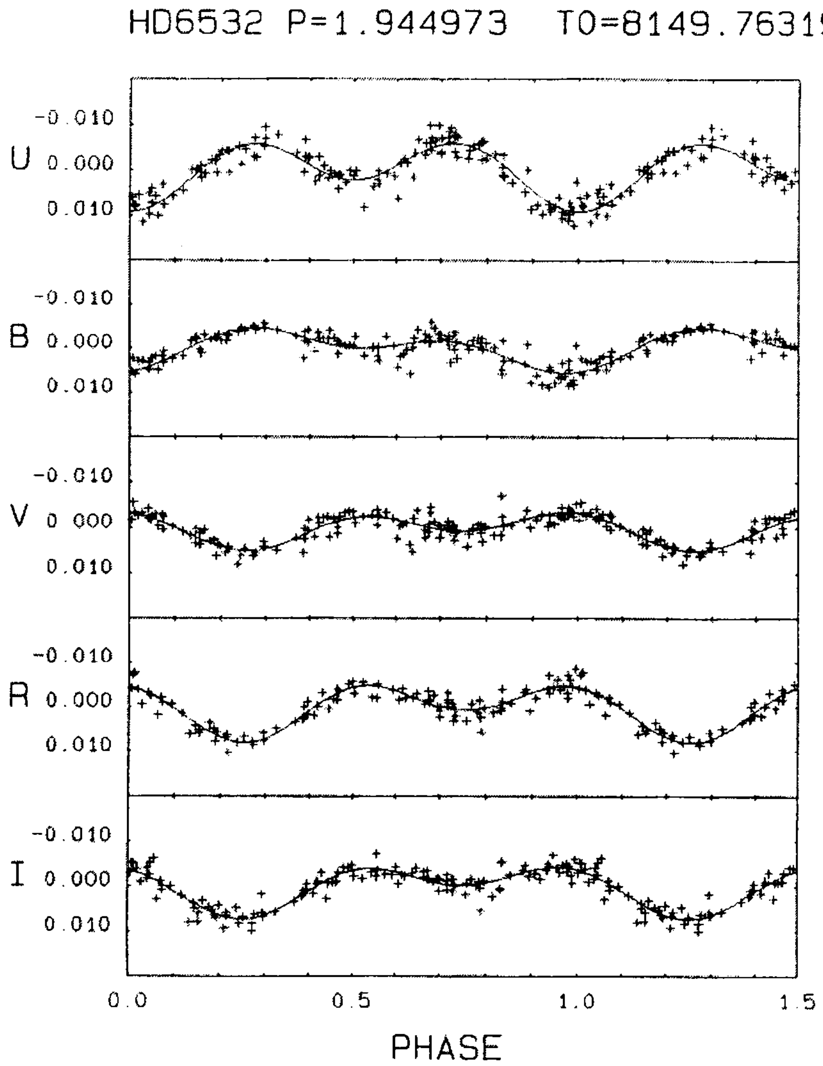


**Fig. 6.** The mean light curve of the roAp star HD 6532. From Kurtz et al. (1996).

Diffusion was developed by Praderie (1967), Schatzman (1969) and Michaud (1970) to explain the Ap stars. See Vauclair & Vauclair (1982) and Michaud & Proffitt (1994) reviews. The idea is simple in principle: if there are layers in a star which are stable against turbulent mixing, then elements heavier than H will tend to sink gravitationally, unless they have many absorption lines near the lo-

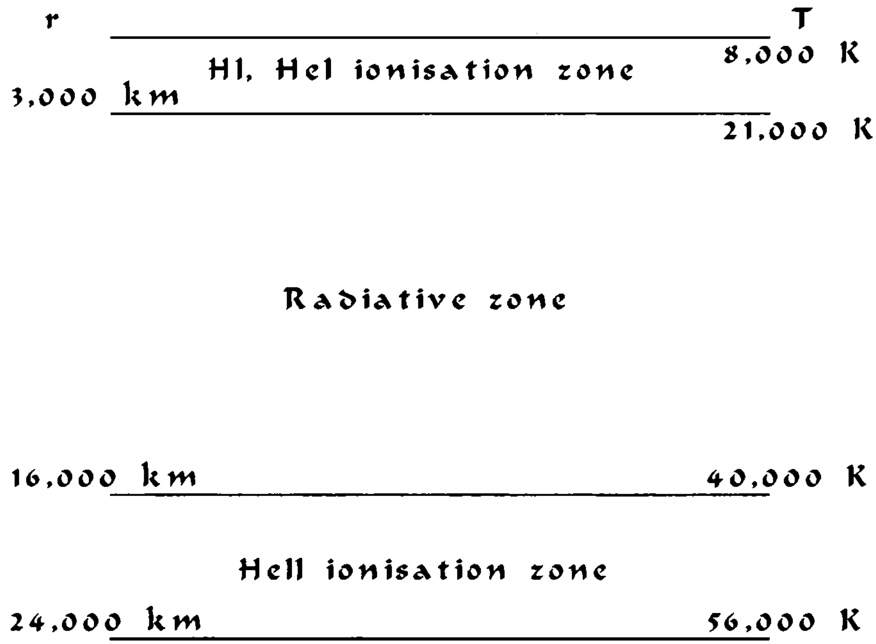


**Fig. 7.** This is a schematic diagram of the outer atmosphere of a $T_{\text{eff}} = 8000\,\text{K}$, $R = 2\,R_\odot$ A star. In the diffusion hypothesis radiative levitation causes ions with many absorption lines near flux maximum to rise to the surface from the stable radiative zone, while ions with few lines, or ions which are very abundant experience a net downward force and sink.

cal flux maximum, in which case the asymmetry in the intensity of the flux arising from the temperature gradient will mean that ions absorb more radiation from below than above, hence are radiatively driven towards the surface. These two competing effects can cause some elements to sink and other to rise, thus producing peculiar atmospheric abundances.

In the CP stars the diffusion hypothesis accounts qualitatively for:

**overabundances of the Fe-peak, rare earth and lanthanide elements:**

These elements, which are principally in their neutral and first-ionized states, have many absorption lines near the flux maximum at the temperature of the radiative zone shown in Fig. 7, hence they are driven upwards to the surface layer.

**underabundances of Ca, Sc, C, He in Am stars:**

In the radiative zone shown in Fig. 7 the principal ionization states of Ca and Sc are Ca III and Sc IV, which leave each of these elements, numbers 20 and 21, with 18 electrons so that they are in closed electronic shell configurations similar to Ar. With tightly bound electrons the absorption lines have high excitation potentials, hence lie in the UV where there is insufficient flux to levitate these ions. Thus they sink and are deficient.

**isotopic ratios of Hg in HgMn stars:**

There are seven observable isotopes of Hg in the HgMn stars – 196, 198, 199, 200, 201, 202, and 204 – which show drastically different abundances to the terrestrial mixture, and which show large variation from star to star. Radiative levitation preferentially lifts the less abundant isotopes to the surface. Because their lines are less saturated, they absorb more flux per ion than the more abundant isotopes, and thus the isotopic ratios are altered.

**ages of CP stars:**

Calculations indicate that diffusion can produce observable anomalies in $10^6$ yr, the age of the youngest Am stars.

**disappearance of anomalies in red giants with CP precursors:**

All models of CP stars can account for this, because it is known that the abundance anomalies must be confined to a thin atmospheric layer. One strong demonstration of this is the overabundance of some rare earths and lanthanides in Ap stars by a factor of $10^4 - 10^5$. These stars constitute no more than one in a hundred thousand stars. Hence, if the abundance anomalies were global, the Ap stars would contain almost all the rare earths and lanthanides in the universe, a clear absurdity. In all models, the growing depth of the surface convection layer with age, as stars move to the right in the HR diagram towards the zone of completely convective stars, leads to mixing away of the surface anomalies.

**slow rotation in Am and Ap stars:**

A-type stars may show equatorial rotational velocities up to $v_{\rm eq} \sin i \approx 250$–$300\,{\rm km\,s^{-1}}$, whereas Am stars have $v_{\rm eq} \sin i \leq 125$ km s$^{-1}$ and Ap stars have $v_{\rm eq} \sin i \leq 100$ km s$^{-1}$, although for the latter $v_{\rm eq} \sin i \leq 10$ km s$^{-1}$ is not unusual. In a rotating star gravitational equipotential surfaces are not surfaces of constant temperature and pressure, because of the non-spherical shape of the star. Rotation generates meridional circulation currents which flow faster with higher $v_{\rm eq} \sin i$. At some value of $v_{\rm eq} \sin i$ they become turbulent; that value is calculated to be about 50–100 km s$^{-1}$, in good agreement with the observations. The resultant turbulent mixing inhibits diffusion: see Baglin (1972) and Vauclair (1976, 1977) for discussions of this.

**binary nature of Am stars:**

Nearly 100 % of Am stars are binary stars with $1 \leq P_{\rm orbital} \leq 10$ d. Tidal synchronism locks the rotational and orbital periods and slows the rotation. A 2-R$_\odot$ star rotating with a period of 1 day has $v_{\rm eq} sini \approx 100$ km s$^{-1}$. Binary stars with $P_{orbital} < 1$ d rotate too quickly; those with $P_{orbital} > 10$ d do not become rotationally synchronized, thus tending to rotate too quickly, since there is no effective braking mechanism.

**magnetic braking of Ap stars:**

The very low rotational velocities of the Ap stars are considered to be the result of magnetic braking.

**concentration of elements in spots near the magnetic poles in Ap stars:**

Ap stars ave "spots" or "rings" of overabundant elements near their magnetic poles. Diffusion is thought to concentrate certain elements in these regions, perhaps because quadrupole magnetic field components may produce magnetic loops arranged in rings which halt radiative levitation and trap rising elements.

**near-exclusion of CP and $\delta$ Sct stars:**

About 30 % of the stars in the lower instability strip (where it crosses the main sequence among the A and early F stars) are $\delta$ Sct stars which pulsate with amplitudes ranging from a few mmag to nearly a mag. Most of the non-pulsating stars in this part of the instability strip are Am and Ap stars. Only a few Am and Ap stars pulsate, and it is a strength of diffusion theory that it can explain this. In stable A- and early F-star atmospheres helium settles gravitationally. This shuts off the $\kappa$-mechanism operating in the He II ionization zone which drives $\delta$ Sct pulsation. The general exclusion between the pulsators and the Am and Ap stars avoids the necessity of explaining how stars with surface radial velocity variations of km s$^{-1}$ can be stable to turbulence at the level of fractions of a cm s$^{-1}$, which is typical of calculated diffusion velocities (Kurtz 1998).

There are problems for the diffusion hypothesis in CP stars, however.

- Diffusion is at best semi-quantitative. Calculated diffusion velocities are very low – only $10^{-4}$ to 1 cm $s^{-1}$. Hence stars must be stable to turbulence at that level in the diffusive layers.
- Pulsation in stars such as HD 40765 (Kurtz et al. 1995) must be laminar, i.e. it must not mix away the anomalies. See section 5.2 below.
- An odd-even effect in abundances is apparent in Ap and Am stars. Opponents of the diffusion hypothesis argue that this indicates a partial role for nucleosynthesis. Proponents argue that diffusion does not erase completely the initial abundance patterns.

Thus, while the diffusion hypothesis is the most popular working model for the CP stars, it must be remembered that it has some weaknesses which are yet to be dealt with. That diffusion acts in CP stars is reasonably accepted; whether it is sufficient to explain all the peculiarities, or whether other mechanisms are working as well, is still to be determined.

Diffusion can also be important in other stars.

- The gravitational settling of He is an important part of the "Standard Solar Model" (Bahcall & Pinsonneault 1995) because it changes the internal solar temperature gradient enough to decrease the calculated neutrino production rate by 7%. Helium and some metal settling have also been included in calculations of Li and Be mixing in the Sun (Richard et al. 1996; Vauclair & Richard 1998).
- Gravitational settling of He in older stars converts gravitational potential energy to thermal energy, increasing core temperatures and nuclear reaction rates, hence shortening stellar lifetimes. This could possibly reduce globular cluster ages by 1–4 Gyr, a significant amount in the on-going cosmological conflict of low Hubble age, $H_0^{-1}$, and high stellar age from globular cluster evolutionary tracks (Stringfellow et al. 1983; Chaboyer et al. 1996).
- Gravitational settling also partially accounts for the Li-gap in F stars (Boesgaard & Tripicco 1986). Li is very sensitive to the depth of the convection zones in stars, since $Li^6$ fuses at $2 \times 10^6$ K via $Li^6 + H^1 \rightarrow He^4 + He^3$, and $Li^7$ fuses at $2.4 \times 10^6$ K via $Li^7 + H^1 \rightarrow 2He^4$.

# 5. PULSATION IN PECULIAR STARS: HISTORY AND THE CURRENT PROBLEMS

## *5.1. Pulsation in magnetic peculiar stars*

In a study of the incidence of $\delta$ Sct pulsation in both field and cluster stars, Breger (1970) found that the *classical* Am stars do not pulsate, but the evolved Am stars (then called $\delta$ Del stars, now called $\rho$ Pup stars) may pulsate. He discussed two hypotheses: (1) the mechanism which produces the abundance anomalies inhibits pulsation, and/or (2) the pulsation generates turbulence that mixes away the abundance anomalies, which are confined to a thin surface layer. In a further study (Breger 1972), he preferred the first explanation in terms of the diffusion hypothesis. As discussed in the last section, for a star with a sufficiently stable atmosphere, radiative levitation and gravitational settling produce the observed atmospheric abundance anomalies in the CP stars. Since He settles from the He II ionization zone (see Fig. 7), and since $\delta$ Sct pulsation is driven by the $\kappa$-mechanism in this zone, pulsation is inhibited in CP stars. This is expected to be true for both magnetic and non-magnetic CP stars. Furthermore, *a priori* there was an expectation that the magnetic fields of the Ap stars should stabilize them against pulsation, too.

There were observations which seemed to contradict these expectations, however. The star 21 Comae (HR 4766, HD 108945) is an A3p Sr(Cr) (CP2) star. Variability on many time-scales has been reported for this star: starting in the early 1950s and continuing, periods between 5 min and 11 d have been reported. Of particular interest are reports of periods around 30 min (Percy 1973, 1975; Aslanov et al. 1978; Weiss et al. 1980; Santagati et al. 1989), since this would be possible for an early, A3, $\delta$ Sct star. That is, 21 Com seemed to be a magnetic Ap star with $\delta$ Sct pulsation, thus confounding the theoretical expectation that such stars should be pulsationally stable.

The importance of this star led to a major, multi-site observing effort by Kreidl et al. (1990). They found from hundreds of hours of observations obtained over many observing seasons, that there is *no* evidence for any periodicity in the range of 4.8 min to 2 h, but that the star does have a period of 2.00435 d. The latter is typical of the rotational light variations seen in Ap stars. Thus, this study implies that Ap stars do *not* have $\delta$ Sct pulsation, given that one of the best cases, 21 Com, does not pulsate.

But there are other candidates. Weiss (1983) discussed this problem and concluded: "It now seems well established that few chemically peculiar stars (CP2) of the upper main sequence exist which pulsate in the fundamental and/or low overtone mode[s]." He presented arguments for $\delta$ Sct pulsation in five stars with CP2 classifications. Unfortunately, for each star there is some doubt about the classification. Kreidl (1987) also discussed this problem and remarked on nine possible $\delta$ Sct – Ap stars. No case is certain, but he concluded, similarly to Weiss, that there are a few possible Ap stars which show consistent $\delta$ Sct pulsation.

In the late 1970s, at the time that this discussion of $\delta$ Sct pulsation in magnetic Ap stars was underway, Kurtz (1982) discovered the rapidly oscillating Ap (roAp) stars. These are cool Ap SrCrEu (CP2) stars which pulsate with periods from just under 6 min to about 16 min, and with semi-amplitudes (through a Johnson $B$ filter) up to 8 mmag, although usually much less than this. Some of them seem to be singly periodic; others are multi-periodic and useful for asteroseismology. They are oblique pulsators: they pulsate in non-radial modes (mostly dipole modes) with their pulsation axes aligned with their magnetic axes, both of which are inclined to the rotation axes. There have been many reviews of the roAp stars over the last decade. For much more detail about them and their interpretation, see Weiss (1986), Shibahashi (1987, 1990), Kurtz (1990), Matthews (1991) and Martinez (Martinez et al. 1991; Martinez 1993; Martinez & Kurtz 1994a,b, 1995).

There are 31 known roAp stars as of August 1999. Their existence certainly proves that Ap stars can pulsate, but with their very short periods (implying very high overtone pulsation modes) they do not contribute to our understanding of whether Ap stars can be $\delta$ Sct stars. The pulsation driving mechanism for roAp stars is not certain, but it is thought to be H-driving near the surface which, if true, makes them physically different to the He II-driven $\delta$ Sct stars (Dziembowski & Goode 1996)

Another CP2 star which has generated great interest is the B9p Si star ET And. Claims were made for a 143-min period in this star (Panov 1978, Scholz et al. 1985). Since B9 is much hotter than the otherwise-observed blue border of the $\delta$ Sct instability strip, this was an important and intriguing claim. Then Panov (1984) claimed the discovery of rapid oscillations with periods between 7 and 16 min in

ET And. Since this star is much hotter than the otherwise-hottest roAp star, HD 6532, this is again an important and intriguing claim. Weiss et al. (1998) conducted a major observational re-examination of ET And. They found that the 143-min period definitely belongs to the comparison star used, HD 219891, which lies within the $\delta$ Sct instability strip. They found no evidence for variations on the time-scale of 6 to 16 min. They thus concluded that ET And is stable against pulsation and presents no challenge to the theory of pulsation in magnetic stars. ET And *is* photometrically variable, hence its variable star name, but the only period found by Weiss et al. is the 1.618875-d rotational light variation typical of the CP2 stars.

Much more intriguing now is HD 75425. This star was classified by Houk (1978) as Ap Sr(CrEu) who remarked "weak case; undetected visual double, $P = 288°$, $D = 0.4''$, mags 9.9, 10.0." Martinez (1993) measured the Strömgren indices of this star to be $V = 9.584, b-y = 0.112, m_1 = 0.247, c_1 = 0.805$, and $\beta = 2.864$. The calculated dereddened metallicity and luminosity indices are therefore $\delta[m_1] = -0.064$ and $\delta[c_1] = -0.116$. These are indicative of the strong metallicity and heavy line blocking which are characteristic of the roAp stars, but may also occur in Am and $\rho$ Pup stars. Recently, Martinez & Medupe (1998) discovered HD 75425 to be a $\delta$ Sct star with multi-periodic oscillations; the highest amplitude period is 30 min. This is about twice as long as the longest period roAp star, and is exceptionally short (but not unprecedented) for a $\delta$ Sct star. This is another case where a more detailed examination of the spectrum is called for, but which suggests that magnetic Ap stars may sometimes also be $\delta$ Sct stars.

Stars of this sort continue to be discovered. We have some evidence from high-speed photometry of a 1-h period in the Ap SrEu star, HD 187761. However, we observed this star for four consecutive nights differentially and found only the rotation period of several days with no evidence for the 1-h period. The rotational variation and its time-scale is conclusive evidence that the star is a magnetic Ap star, since the non-magnetic stars are not spotted. If an Ap star *could* be found with both rotational variation and $\delta$ pulsation periods, it would definitively show that magnetic stars can have $\delta$ Sct pulsation. As it stands now, HD 187761 is another case similar to ET And.

### 5.2. Pulsation in Am stars

After Breger (1970) originally noticed and pointed out that Am stars do not pulsate, Kurtz (1976) showed that evolved Am stars (then called $\delta$ Del stars, now known as $\rho$ Pup stars, as discussed in Section 1) near the red border of the $\delta$ Sct instability strip *can* pulsate, although not all do. For those among these stars which do pulsate, it is thought that their abundance anomalies developed by diffusion on the main sequence in a stable Am star. Later, when the star evolved, changes in its internal structure shifted the He II ionization zone more deeply (in mass fraction) into the star, where there was sufficient residual helium left to begin driving low-amplitude pulsation (Cox et al. 1979). This pulsation is then presumed to be sufficiently laminar, i.e. non-turbulent, that the surface manifestation of the earlier diffusion is not completely destroyed by mixing, although it may be weakened.

Following Breger's announcement that none out of a sample of 30 Am stars pulsates, there were several claims of counterexamples. These were discussed and dismissed by Kurtz et al. (1976). The best of these counterexamples at that time was 32 Vir, which Kurtz et al. showed was a double-lined spectroscopic binary. They suggested that the primary is a slowly rotating Am star, and the secondary is a more rapidly rotating $\delta$ Sct star. With this model they preserved their theoretical preconception that Am stars could not pulsate based on the simple view from the diffusion hypothesis that He drains from the He II ionization zone and quenches pulsation in Am stars. However, Mitton & Stickland (1979) re-examined 32 Vir and concluded instead that the primary is the pulsating star, but that it is a $\delta$ Del ($\rho$ Pup) star, rather than a classical Am star. Following the previous discovery of pulsating $\delta$ Del stars, and Cox et al.'s (1979) theoretical interpretation of this within the diffusion hypothesis, Mitton & Stickland's conclusion seemed correct and believable.

Further attempts to find Am stars with $\delta$ Sct pulsation were made. Stickland (1977) searched an additional six Am stars for light variability and found none. González et al. (1980) showed that four Am stars suspected of variability are constant. And Hauck & Lovy (1981) could fine no evidence of variability in the study of the standard deviations of measurements in the Geneva photometric system for *several hundred* Am stars!

All of this gave the appearance that pulsation on the main sequence and metallicism were mutually exclusive, but that Am stars could later begin pulsating when they evolved off of the main sequence to the sub-giant and giant stage. That was confounded by the discovery of $\delta$ Sct pulsation in three marginal Am stars (Am: stars) (Kurtz 1978a, 1984). Then Kurtz (1989) found a classical Am star, HD 1097, which is also a $\delta$ Sct star, albeit one of very low amplitude: only 9 mmag peak-to-peak in Johnson $B$. It might be thought of as being "marginally pulsating."

There would at first look seem to be little doubt about the classical Am nature of HD 1097. Houk (1982) classified it as A3/5mF0-F5, meaning the K-line type is A3 or A5, the H-line type is F0 and the metal-line type is F5. Since classical Am stars are defined to have at least 5 subtypes difference between the K-line and metal-line types, HD 1097 is an extreme case with a full spectral class difference between the K-line and metal-line types. But Slettebak & Brundage (1971) classified HD 1097 as "A7p?" with the comment that the "G-band, Sr II, and Si II lines appear to be visible." Graham & Slettebak (1973) further comment about HD 1097 that it appears to be a "peculiar A star of the $\beta$ CrB type or late Am star." This uncertainty is not difficult to resolve. The K-line is easily visible on the objective prism spectra from which Houk works, and her K-line type of A3/5 with a H-line type of F0 is a clear indication that the star is Am, rather than Ap. Adelman (1973), in his study of the cool magnetic Ap stars (CP2 stars), found that they are *not* deficient in Ca. Hence, the early K-line type in HD 1097 compared to its H-line type is an Am signature. Thus, we conclude that classical Am stars can also be $\delta$ Sct pulsators, although perhaps only with low-amplitudes.

The *Naini Tal – Cape roAp Star Survey* (Martinez et al. 2000), which is currently searching for northern-hemisphere roAp stars, has discovered $\delta$ Sct pulsation in two peculiar stars, HD 13038 (Martinez et al. 1999a) and HD 13079 (Martinez et al. 1999b). HD 13038 has Strömgren indices: $b - y = 0.102$, $m_1 = 0.213$, $c_1 = 0.866$, and $\beta = 2.860$. The H$\beta$ index is consistent with an early type A star. The metallicity index, $\delta m_1 = -0.022$, and the luminosity index, $\delta c_1 = -0.055$, are indicative of the strong line blanketing found in the Am and Ap stars. HD 13038 is definitely either Am or Ap. It has at least two periods of 28 and 34 min which are very short, but not unprecedented, for a $\delta$ Sct star. It is probably a pulsating Am star, although that will not be certain until its spectrum has been studied more thoroughly. It could thus have been included in the last section in the discussion of $\delta$ Sct stars which are possibly magnetic Ap stars.

HD 13079 is a double star. The *Hipparcos* $H_{\rm p}$ magnitudes for the two components are 8.989±0.007 and 11.311±0.057. The separation and position angle are $\rho = 6.173 \pm 0.017''$ and $\theta = 254.4°$, respectively. The Strömgren photometric indices for the combined light of the two stars are: $b - y = 0.203, m_1 = 0.211, c_1 = 0.672, \delta m_1 = -0.023, \delta c_1 = -0.028$, and $\beta = 2.759$. The dereddened metallicity index is $[\delta m_1] = -0.060$ and the dereddened luminosity index is $[\delta c_1] = -0.069$. These indices are typical of the strongly line-blanketed spectra of cool Ap and Am stars. HD 13079 has a pulsation period of 78 min with a peak-to-peak $B$ amplitude of 0.02 mag. The *Hipparcos* parallax, $uvby\beta$ photometry, discovery of Ca-deficiency and pulsations all suggest that it is an Am star near the zero age main sequence, and that it is a fundamental mode pulsator on the red edge of the instability strip. Thus there are now three known classical Am – $\delta$ Sct pulsators: HD 1097, HD 13038 and HD 13079.

The Naini Tal – Cape roAp Star Survey also shows how high-speed photometry, without the use of comparison stars, can detect $\delta$ Sct variables. Figs. 8 and 9 show sample light curves of the Naini Tal data for HD 13038 and HD 13079 where the $\delta$ Sct variations are clear. Obviously, differential photometry is better for detecting and studying $\delta$ Sct pulsations, since the atmosphere can have significant transparency variations on a similar time-scale. With good equipment and stable atmospheric conditions, however, it is possible to produce data such as that in Figs. 8 and 9, thus discovering new variables. As the Naini Tal-Cape Survey is searching for roAp stars, it will continue to turn up new peculiar stars with $\delta$ Sct variability, too.

The story now grows more complex: HD 188136 is classified as a $\delta$ Del star ($\rho$ Pup star) with a very strong metallic line spectrum (Houk & Cowley 1975). Wegner (1981) found the rare earth abundances in HD 188136 to be more characteristic of the cool magnetic Ap stars than the Am stars (the latter of which is suggested by the $\delta$ Del classification). He compared the abundances of the rare earths in HD 188136 with those in the most extreme Ap (CP2) star, HD 101065, also known as Pzybylski's star. Consider, then, that HD 188136 is also a multiperiodic $\delta$ Sct star which pulsates with peak-to-peak light variations of 0.05 mag (Kurtz 1980). This is not the low-amplitude, "marginal" pulsation of HD 1097, but full-blown $\delta$ Sct pulsation in the presence of extreme abundance anomalies. If this star *is* an Ap star, then the presence of global magnetic fields

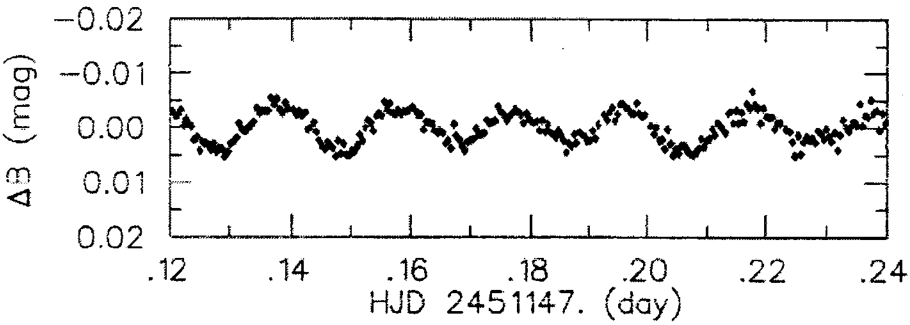


**Fig. 8.** A light curve of HD 13038 obtained at Naini Tal showing the 28.7-min oscillations. This is one of the shortest period $\delta$ Sct stars currently known.

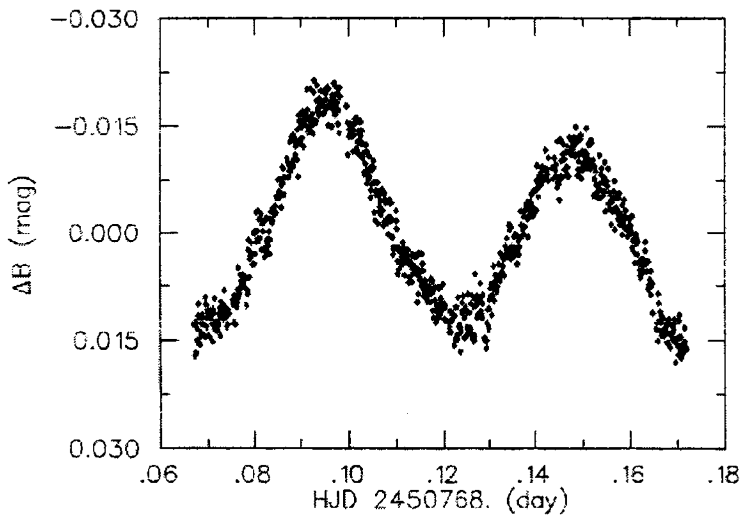


**Fig. 9.** A light curve of HD 13079 obtained at Naini Tal showing the 78-min oscillations. On nights with little sky transparency variation, it is possible to study $\delta$ Sct variability using high-speed (non-differential) photometry.

does *not* prevent $\delta$ Sct pulsation in all cases. Furthermore, whether it is an Ap star or an extreme Am star, then large amplitude pulsation does not cause sufficient turbulence to mix away surface abundance anomalies.

This is further confirmed by the star HD 40765, which is a large amplitude $\delta$ Sct star with peak-to-peak light variations of 0.21 mag in Johnson $B$ and 0.15 mag in Johnson $V$ (Kurtz et al. 1995). Garrison (in Kurtz et al. 1995) found that the spectrum of HD 40765 looks like that of a $\rho$ Pup star with additional peculiarities. The H lines correspond to F5; the G-band corresponds to F2; the K-line type is A7/8. The metals look like about F5 in strength, but the pattern is not perfect; $\lambda 4077$ is stronger than F5 III, $\lambda 4215$ is weaker than F5 III relative to $\lambda 4226$ which is itself slightly weaker than F5 III. The ratio $\lambda 4376/\lambda 4383$ indicates main sequence at any of the above types. The rest of the spectrum looks like an early-F to mid-F dwarf. Hence, Garrison classified the spectrum Fmp (kA7hF5mF5), where the final "p" does not indicate peculiar in the Ap sense, but peculiar in the sense that the spectrum cannot be easily or fully parameterized.

HD 40765 is a $\delta$ Sct star pulsating in the fundamental or first overtone radial mode with a relatively large $B$ amplitude of 0.21 mag peak-to-peak. Martinez (1993) found Strömgren indices of $V = 9.577$, $b - y = 0.271$, $m_1 = 0.241$, $\delta m_1 = -0.059$, $c_1 = 0.666$, $\delta c_1 = 0.009$, and $\beta = 2.739$; the dereddened indices are $[\delta m_1] = -0.108$ and $[\delta c_1] = -0.045$. As we have seen before for HD 13038 and HD 13079, these high $m_1$ and low $c_1$ indices are typical of the strongly line-blanketed spectra of the cool Ap, Am and $\rho$ Pup stars, hence the photometry supports the high-metallicity classification. Because of its higher pulsation amplitude, HD 40765 is a more extreme case of the problem presented by HD 188136.

By comparison with $\rho$ Puppis itself, the surface radial velocity variation in HD 40765 is inferred to be 14 km s$^{-1}$ from the $B$ amplitude. Since the mode is either the fundamental or first overtone, the pulsational radial velocity drops very slowly with decreasing radius. So in the element separation zone in this star the pulsation velocity is about 10 km s$^{-1}$. Thus, either radiative diffusion can occur in the presence of 10 km s$^{-1}$ pulsation without turbulence mixing away the abundance anomalies (this seems truly remarkable given the microturbulent velocities of km s$^{-1}$ measured in the atmospheres of A stars), or diffusion is not the mechanism producing the abundance anomalies – at least in these extreme stars. The competition between diffusion processes and turbulence was discussed theoretically by Vauclair *et al.* (1978), but that was at a time when only pulsation in the evolved $\rho$ Pup stars needed explanation. Vauclair et al.

obtained some important insight, but the more extreme observations of HD 188136 and HD 40765 cry out for further theoretical study.

### *5.3. Conclusions about metallicism and pulsation*

We now know that, in general, stars in the $\delta$ Sct instability strip with Am or Ap abundance anomalies do not pulsate, as originally found by Breger (1970). There are, however, important exceptions:

- Evolved Am stars, the $\rho$ Pup or $\delta$ Del stars, often pulsate. Cox et al. (1979) explained that within the diffusion hypothesis by the evolutionary replenishment of some He in the He II ionization zone – with the presumption that the resulting $\delta$ Sct pulsation, which can be of large amplitude (e.g. $\rho$ Puppis itself), does not mix away the atmospheric peculiarities.
- Marginal Am stars can sometimes be low amplitude $\delta$ Sct stars. It is presumed that the low amplitude does not produce sufficient turbulence to mix away the atmospheric peculiarities, and that sufficient He remains in the He II ionization zone to drive the pulsation.
- Even classical Am stars, such as HD 1097 (and probably HD 13038 and HD 13079), can be low amplitude $\delta$ Sct stars, although this is unusual. It is presumed that the low amplitude does not produce sufficient turbulence to mix away the atmospheric peculiarities.
- Some extremely peculiar stars, such as HD 188136 and HD 40765, can also be relatively large amplitude $\delta$ Sct stars. How such large amplitudes can be laminar and not mix away the atmospheric peculiarities in these stars has not been theoretically modeled.
- It seems probable that some magnetic Ap stars are low amplitude $\delta$ Sct stars, but this is yet to be definitely proved. There is considerable evidence suggesting this, however. There is no theoretical modeling of $\delta$ Sct pulsation in the presence of strong, global magnetic fields.
- Extreme Ap (CP2) peculiarities and strong, global magnetic fields coexist with high-overtone $p$-mode pulsation in the roAp stars.
- No pulsating stars with periods between 16 min (maximum for roAp stars) and 28 min (minimum for $\delta$ Sct stars) have yet been found. But we suspect that they exist, even though they are rare. Their discovery will shed light on all the problems discussed in this section.

## 6. HD 101065 – PRZYBYLSKI'S STAR

The story of HD 101065, Przybylski's star, is one of controversy and prejudice. It is also the story of what led to the discovery of the roAp stars. Science is done by scientists, and our personal feelings and prejudices get in the way, even though we try to overcome them. The story of HD 101065 illustrates this well. Our own prejudices show strongly, and if it should turn out one day that we are wrong about this star, they will be glaring in this record.

HD 101065 was classified as a B5 star in Henry Draper catalogue. Przybylski (1961) discovered HD 101065 *not* to be a B star. He found that the strongest lines in the spectrum are from Ho II, Dy II, Sm II, and Nd II; he said the spectral type from the continuum is K0 and the spectral type from H lines is F8 or G0. Thus was born a forty-year controversy (yet to be fully resolved) over the nature of what is arguably the most peculiar non-degenerate star in the sky. Wegner (1976) published a visible-light spectrum of HD 101065 traced from a photographic spectrum taken by Brian Warner with the Radcliffe 1.9-m telescope in Pretoria. An arresting feature of that spectrum is that the depths of many of the metal lines are greater than that of the Balmer lines! The spectrum is so heavily line-blanketed that it is debatable whether the continuum can be reasonably defined. In fact, this is not just debatable, but has been vigorously debated, as we will see.

Kron & Gordon (1961) measured *UVBGRI* photometry of HD 101065 and concluded that it is "very likely an F8 or G0 dwarf with very strong line blanketing." Their argument was based on the similarity of the magnitudes of HD 101065 in *BGRI* to a late F dwarf standard star, but they noted that the *U* and *B* magnitudes were depressed by line blanketing. Let us state where we stand on this issue at the outset: we believe HD 101065 to have an effective temperature near to that of an F0 star, about 7400 K. So we would argue that line-blanketing has also depressed the *BGRI* magnitudes, and proper correction for this would show a continuum close to that of an F0 star.

From an abundance analysis Przybylski (1966) concluded that HD 101065 has *no* Fe and little Ca. From the *BGRI* observations he found $T_{\rm eff} = 5900\,{\rm K} = T_{\odot}$! The spectral type implied by this is G2Vp, so Przybylski argued that HD 101065 is unique. From another abundance analysis Wegner & Petford (1974) concluded that the rare earths are overabundant by a factor $10^5$, the Fe peak elements are normal, the excitation temperature is $T_{\rm exc} = 6300 \pm 120\,{\rm K}$, and the effective temperature is $T_{\rm eff} = 7000\,{\rm K}$. Therefore the equivalent spectral type is F0Vp. They pointed out that the heavy line blanketing increased the temperature gradient to cause "backwarming" which, if unaccounted for, results in a low estimate of $T_{\rm eff}$.

Typically, $T_{\rm eff}$ can be determined for F and G stars to an accuracy of $\pm 150\,{\rm K}$ from spectral classification or color index. That the dispute over $T_{\rm eff}$ in HD 101065 ranges over 1000 K attests to the extreme peculiarity of this star.

Jones et al. (1974), discovered that the Ap star HD 51418 is rich in Ho II and Dy II, and commented that "the star that most nearly resembles HD 51418 ... is HD 101065." The spectral type of HD 51418 is late B or early A, and it has a measured magnetic field which ranges from $-200\,{\rm G} \leq B_{\rm eff} \leq +750\,{\rm G}$ with a rotational period of $P_{\rm rot} = 5.4379$ d. HD 51418 is obviously an Ap star – one rich in Ho and Dy. Jones et al. suggested that "an attempt should be made to determine whether HD 101065 in any way resembles the Ap stars." They warned that "the temperature of HD 101065 should be carefully reconsidered." By this they meant the low, solar-like $T_{\rm eff}$. And, reasonably, they pointed out that "measurements of magnetic field strength [in HD 101065] would be important." This is because Ap stars have global magnetic fields; solar-type stars have tangled magnetic fields. The incidence of globally organized magnetic fields stops at about F0 because that is where the surface convective envelope starts to grow (with decreasing $T_{\rm eff}$). By mid-F convection drags and entangles whatever global field is present. Hence Jones et al. were suggesting that the discovery of a dipole-like field in HD 101065 would argue strongly for the higher temperature.

Przybylski (1977a) responded to Wegner & Petford by estimating the backwarming correction to be $-460\,{\rm K}$, and concluding that $T_{\rm eff} = 6075 \pm 200\,{\rm K}$ and the spectral type is F8.

Hyland et al. (1975) found from infrared photometry that $T_{\rm eff} \approx 6300 \pm 150\,{\rm K}$ and the spectral type is F5 – F6. HD 101065 is much less line blanketed in the IR, so it was hoped that the controversy could be settled by studying that part of its spectrum. Hyland et al. did not observe their own standards, however. They compared their observations of HD 101065 to standards observed by Ian Glass. We will return to this point below.

In 1976 Wolff & Hagen (1976) discovered a $-2200$ G global magnetic field in HD 101065. They pointed out that the presence of the field supported Wegner's higher $T_{\rm eff}$, and they suggested that "...the same mechanism that is responsible for the overabundances of the rare earths in the Ap stars may also be effective in HD 101065..." This is incompatible with the Fe deficiency and the low $T_{\rm eff}$ estimates. The coolest Ap stars are about F0 and no Ap star shows Fe deficiency. In fact, Fe deficiency is theoretically very hard to understand. Fe is very difficult to destroy, and diffusion almost always levitates it in stellar envelopes because it has such a rich absorption spectrum.

In 1975 IAU Colloquium 32, "The Physics of Ap Stars", was held in Vienna. The controversy about HD 101065 boiled over there. Fortunately, the conference organizers taped the discussion and included it in the proceedings, so it is possible to get the real flavor of the discussion. Przybylski (1975) presented a paper in which he claimed for HD 101065 that $T_{\rm eff} = 6075 \pm 200\,{\rm K}$ and that Fe is deficient by 2.5 dex. Let's look at some of the discussion which follows Przybylski's paper (we quote from the proceedings):

Wolff: "Hagen and I ... find that this star has a magnetic field of $-2200$ G. This strengthens the identification of the star as an Ap star."

Przybylski: "The presence of a strong magnetic field is not restricted to hot stars. Therefore, it is not proof that HD 101065 is hotter than I assume. The photometry of HD 101065 leaves little room for any speculation on the effective temperature of the star ($T_{\rm eff} = 6075\,{\rm K}$)."

(You should remember as you read our interjections here that we have taken sides in this controversy: we believe that HD 101065 has $T_{\rm eff} \approx 7400\,{\rm K}$. With that said, we'd like you to note that Przybylski "slipped" in saying above "... hotter than I *assume*." His insistence that there is "little room for any speculation on the effective temperature ..." is absurd in the context of this disagreement.)

During the discussion Przybylski made another presentation on HD 101065. We continue to quote:

Przybylski: "The third slide shows tracings of HD 101065 and Procyon (dashed line) in the interval $\lambda 3820\,$Å to $\lambda 3835\,$Å. Both stars have the same continuum temperature of 6500 K... The second strongest line in the solar spectrum is at $\lambda 3820\,$Å... in HD 101065 this line, if present at all cannot have a depth exceeding 30 % and in this case its equivalent width would be about 100 mÅ..."

Kodaira: "Isn't it possible that the strong Fe lines are masked, because the other lines are so strong? You show that one line coincides with a peak. But even this peak could be the local continuum level."

Przybylski: "The continuum of Procyon can be drawn confidently. On the other hand, some doubts exist about the continuum of HD 101065... Masking of the lines is, of course, a very serious problem ...investigation of the spectrum is an extremely difficult task."

We interrupt this discussion to make our own comments again: Procyon and HD 101065 do not have the same $T_{\rm eff} = 6500\,{\rm K}$. The spectrum of HD 101065 shown during the discussion (see the proceedings) was taken with a Kodak IIaO plate which is rapidly losing sensitivity at $\lambda 3820$ Å, so the spectrum is very noisy. In the last quote above, Przybylski seemed uncharacteristically unsure. This confused Cowley, who was chairing the session, and who was sympathetic to the case for the lower $T_{\rm eff}$.

We will now let the argument run to its [in]conclusion:

Cowley: "Do you agree that this spectrum shows that the Fe is weak or missing? Yes or no? ... O.K. You don't want to commit yourself."

Dworetsky: "It would seem difficult to agree about anything in that spectrum as it is incredibly blended and looks like random noise!"

Cowley: "You have some lines, and if Fe had its normal strength you ought to see it."

Dworetsky: "Yes. Also, there has been some comment on HD 101065 having a higher effective temperature. I am not sure in a star this severely blanketed if our usual idea of what we mean by effective temperature has much meaning..."

Cowley: "...I can tell you that in HR 465 and HD 51418, which have extraordinarily rich spectra and are hot, the Fe lines are strong and stand out and I am not unmoved by this picture of Dr. Przybylski, which fails to show any real evidence of Fe I."

Shore: "When you are working on these stars one of the problems is that if you look in the blue, you can find anything you want to and you can mask almost anything you want to."

Przybylski: "Let us return to the third slide. From the fact that any Fe line at $\lambda 3820$ Å, if present at all, ..." [States his case again]

Cowley: "If I can beg your indulgence. I think we are going over the same ground."

Michaud: "At this meeting I sometimes have the impression of being at a museum of horrors or perhaps errors... it does not seem inconceivable that diffusion would produce HD 101065."

Przybylski: "The results of the six-color photometry and infrared photometry leave very little room for speculation about the effective temperature of the star ..." [States his case again]

Wolff: "I would just like to say that this discussion is essentially fruitless. It is clear that this star is cooler than most Ap stars, whatever its temperature is. It is clear to me that the Fe lines are inconspicuous, at best, and Fe might not be there at all. And since there is only one of us, so far as I know, who owns a red plate, I don't see how we can resolve this issue. This is a case where you have to see it to believe it."

The next year Wegner (1976) found that the backwarming effect is about $-150$ K (but might even be positive). From modeling the hydrogen lines he found from H$\alpha$ that $T_{\rm eff} \approx 7500$ K and from H$\beta$ that $T_{\rm eff} \approx 7100$ K. He reported that Ian Glass had obtained IR photometry of HD 101065 using his own standard stars, and that from Glass's new IR photometry $J$–$K$ implied that $T_{\rm eff} \approx 7000$ K and $J$–$L$ implied $T_{\rm eff} \approx 7500$ K – much hotter than Hyland et al.'s $6300 \pm 150$ K obtained from IR photometry using Glass's standards. Wegner put the spectral type in the range A8–F1.

The controversy continued: Przybylski (1977b) found that Fe is underabundant by 2.4 in the log. Cowley et al. (1977) use a technique called "line coincidence statistics", which is more objective than typical line identification procedure to find that Fe is underabundant by at least 2.4 in the log. They state: "... the strongest Fe lines lie near the plate limit, with equivalent widths of a few tens of milliangstroms. The equivalent widths for these lines range up to 3 Å in the Sun!" And they say, hopefully, "The current study has, we hope, resolved the outstanding question concerning the spectrum of HD 101065: Iron peak elements are present, although ... their spectra are unusually weak."

No such luck!

In 1978 Wegner and Kurtz were privately discussing HD 101065 and Kurtz pointed out that if HD 101065 is as hot as Wegner believes, then it is within the instability strip, and that (at that time) everyone thought that Ap stars and magnetic stars cannot pulsate. This latter claim was because diffusion theory predicted that He sinks from the He II ionization zone which drives $\delta$ Scuti pulsation, hence stabilizes the peculiar stars against pulsation. And it came from the idea that the magnetic field would stabilize the star against pulsation – that it would be difficult for the large scale motions of $\delta$ Scuti pulsation to cross the field lines of a strong global magnetic field, or to drag those field lines along with the pulsation. Typical $\delta$ Scuti pulsational radial velocities are measured in $\mathrm{km\,s^{-1}}$.

At that time Kurtz was observing 12–14 weeks per year; that gave him the freedom to pursue unlikely observational projects, if he felt like it. He decided to test HD 101065 for $\delta$ Scuti pulsation by spending a few hours doing differential photometry on it with the South African Astronomical Observatory 0.5-m telescope at Sutherland. He used two comparison stars and 100-s integration times through a Johnson $V$ filter. The results were that HD 101065 was constant over the hours of observation to a standard deviation of $\sigma$ = 0.003 mag. But he knew the photometer, telescope and observing site well (remember the 12 weeks per year). It was a perfect Sutherland photometric night – the kind of night where the stars blink out when they set against a sharp horizon; the kind of night when $\sigma$ = 0.003 mag is unexpectedly large.

He checked the two comparison stars against each other. They had a standard deviation a bit less than $\sigma$ = 0.002 mag – closer to what he expected. A visual look at the light curve of HD 101065 gave the impression that the points were alternating: up-down, up-down, up-down. He showed this the next afternoon to the other astronomers and everyone agreed that there was no signal, and that HD 101065 was constant to $\sigma$ = 0.003 mag on the time-scale of 8 min which was the cycle time to observe both comparison stars and HD 101065 itself for 100 s each.

That turns out to be true, but not because the star is constant. The next night he decided to observe it through a Johnson $B$ filter with no comparison stars and an integration time of 20 s. The system was old-fashioned: a teletype, punch paper tape, and no on-line plotting. So he plotted the points by hand. In only 30 minutes the 12.15-min pulsation with its 0.01-mag amplitude was obvious (Kurtz 1978b), and he had discovered the first of the rapidly oscillating Ap stars. Fig. 10 shows a light curve for HD 101065 that he obtained with the SAAO 1.9-m telescope in 1984.

Of course, if you find 12-min oscillations in a star, the first test you need to do is to make sure that you haven't discovered 12-min oscillations in your telescope. The following night Wegner and Kurtz observed HD 101065 with both the 1-m and 0.5-m Sutherland telescopes using different filters and photometers. Both telescopes showed the same coherent oscillation. Of course, it was still to be proved that this oscillation was not in the atmosphere above Sutherland, but that was very unlikely, and later observations from other observatories eliminated any such possibility.

Kurtz & Wegner (1979) published their confirmation of the oscillations, and from a new spectroscopic study of the equivalent widths of the Paschen lines found $W_\lambda(\mathrm{P12}) = 4.04\,\mathrm{\AA}$ and concluded that this implied $T_{\mathrm{eff}} = 7400 \pm 300\,\mathrm{K}$. They also pointed out that the Fe I $\lambda 4045$ is expected to have $W_\lambda = 300$ mÅ at this $T_{\mathrm{eff}}$, not the 3 Å Cowley et al. were expecting by comparing to the Sun. They thus doubted the claim that Fe is deficient, frustrating Cowley et al.'s hope that this question was finally resolved. They suggested instead that HD 101065 is an Ap star – the most extreme Ap star – and they further suggested that searches for similar pulsation in other magnetic Ap stars should be made.

In a letter dated 14 April 1980 Przybylski wrote to Kurtz; "With deepest regret, I have to disagree with some statements in your joint paper with Gary Wegner." He claimed that his IR spectra showed much weaker Paschen lines characteristic of a lower temperature. In a letter dated 2 May 1980 Kurtz replied: "I recommend that you publish your [infrared spectra] forthwith. Please be sure to show tracings of your spectra of both HD 101065 and your control stars." To which Przybylski rejoined (22 May 1980): ". ..you doubt whether or not I can have high dispersion spectra which prove that the Paschen series is either absent or weak... You invite me to publish the evidence available to me together with corresponding tracing. I accept this challenge."

Przybylski (1982) found that the equivalent width of P12 was $W_\lambda(\mathrm{P12}) < 300$ mÅ, implying $T_{\mathrm{eff}}(\mathrm{P12}) = 6140$ K. He stated "the attempt by Kurtz and Wegner to revive the controversy about the abundance of iron seems rather unconvincing." Kurtz was not convinced by this paper. One reason was that, even as the non-anonymous referee, he could not persuade Przybylski to publish tracings of his spectra of either HD 101065 or his control star.

In 1983 Wegner et al. (1983) found from IUE UV spectra that the energy distribution of HD 101065 is greatly distorted. They said that the presence of flux below 1900 Å suggests $\mathrm{T} \gg 6000$ K and gave a crude estimate of $7500 \leq T_{\mathrm{eff}} \leq 8000\,\mathrm{K}$, commenting that "this estimate could be too low." They found lines of Fe II, Cr II, Mn II, and Ti II are strongly represented in the 1900–3200 Å range and concluded that "previous reports of the underabundance of iron and the absence of iron-peak elements cannot be supported." They considered all of this to be "... evidence for the classification of HD 101065 as a magnetic Ap star."

We would like to be able to tell you that the story is finished and HD 101065 is the coolest, most peculiar Ap star. We had hopes that asteroseismology, which we will discuss below, would resolve the issue. The asteroseismology of HD 101065 leads to a predicted luminosity. While we waited for the *Hipparcos* parallaxes there was hope that they would confirm or refute the asteroseismic luminosity – that we could pin down the temperature. Our seismic prediction for an assumed $T_{\rm eff} = 7400$ K is $\pi = 7.80 \pm 0.24$ mas; for an assumed $T_{\rm eff} = 5900$ K it is $\pi = 10.4 \pm 0.2$ mas. Alas, the *Hipparcos* parallax is $\pi = 8.0 \pm 1.1$ mas, and nothing is resolved by the asteroseismology (Matthews et al. 1999).

However, continuing work on the abundances in HD 101065 supports the contention that this is the most extreme Ap star. Cowley & Mathys (1998) now find that the iron group abundances are only mildly deficient in HD 101065. They point out again the fact that the H$\alpha$ line wings match $T_{\rm eff} = 7500$ K, but the narrow core does not fit any model. It is clear that the atmosphere of this star is so peculiar that modeling it is a major challenge yet to be accomplished. It is possible that asteroseismic constraints on the atmospheric structure could be a key to the solution of this problem.

The discovery of the 12.15-min oscillations in HD 101065 then led us to the discovery of many more roAp stars. There are now 31 known; HD 101065 is one of them. In our opinion, that, and other observations detailed above, argue that HD 101065 is the coolest, most peculiar of the Ap SrCrEu = CP2 stars.

## 7. RAPIDLY OSCILLATING Ap STARS

The rapidly oscillating Ap (roAp) stars are cool Ap SrCrEu, CP2 stars which pulsate with periods from just under 6 min to about 15 min, and with semi-amplitudes through a Johnson $B$ filter up to 8 mmag, although usually much less than this. Some of them seem to be singly periodic; others are multi-periodic and useful for asteroseismology. They are oblique pulsators: they pulsate in non-radial modes (mostly dipole modes) with their pulsation axes aligned with their magnetic axes.

There have been many reviews of the roAp stars. For much more detail about them and their interpretation, see Weiss (1986), Shibahashi (1987, 1990), Kurtz (1990), Matthews (1991) and Martinez

(Martinez et al. 1991; Martinez 1993; Martinez & Kurtz 1994a,b, 1995). There are 31 known roAp stars as of August 1999. They are listed in Tables 2, 3, and 4 along with relevant observational parameters.

**Table 2.** Brightnesses, spectral types and characteristic pulsation periods and amplitudes for the rapidly oscillating Ap stars.

| HD | $V$ | Spectral type | Period (min) | $\Delta B_{\mathrm{max}}$ (mmag) |
|---|---|---|---|---|
| 6532 | 8.445 | Ap SrCrEu | 7.1 | 5 |
| 9289 | 9.383 | Ap SrEu | 10.5 | 3.5 |
| 12932 | 10.235 | Ap SrEuCr | 11.6 | 4 |
| 19918 | 9.336 | Ap SrEuCr | 14.5 | 2 |
| 24712 | 6.001 | Ap SrEu(Cr) | 6.2 | 10 |
| 42659 | 6.768 | Ap SrCrEu | 9.7 | 0.8 |
| 60435 | 8.891 | Ap Sr(Eu) | 11.4–23.5 | 16 |
| 80316 | 7.782 | Ap Sr(Eu) | 7.4 | 2 |
| 83368 | 6.168 | Ap SrEuCr | 11.6 | 10 |
| 84041 | 9.330 | Ap SrEuCr | 15.0 | 6 |
| 86181 | 9.323 | Ap Sr | 6.2 | 4.6 |
| 99563 | 8.160 | — | 10.7 | 2 |
| 101065 | 7.994 | Controversial | 12.1 | 13 |
| 119027 | 10.022 | Ap SrEu(Cr) | 8.7 | 2 |
| 122970 | 8.310 | — | 11.1 | 2 |
| 128898 | 3.198 | Ap SrEu(Cr) | 6.8 | 5 |
| 134214 | 7.464 | Ap SrEu(Cr) | 5.6 | 7 |
| 137949 | 6.673 | Ap SrEuCr | 8.3 | 3 |
| 150562 | 9.816 | A/F(p Eu) | 10.8 | 0.8 |
| 161459 | 10.326 | Ap EuSrCr | 12.0 | 1.3 |
| 166473 | 7.923 | Ap SrEuCr | 8.8 | 2 |
| 176232 | 5.890 | F0p SrEu | 11.6 | 0.6 |
| 185256 | 9.938 | Ap Sr(EuCr) | 10.2 | 3 |
| 190290 | 9.912 | Ap EuSr | 7.3 | 2 |
| 193756 | 9.195 | Ap SrCrEu | 13.0 | 0.9 |
| 196470 | 9.721 | Ap SrEu(Cr) | 10.8 | 0.7 |
| 201601 | 4.680 | F0p | 12.4 | 3 |
| 203932 | 8.820 | Ap SrEu | 5.9 | 2 |
| 213637 | 9.611 | A(p EuSrCr) | 11.5 | 1.5 |
| 217522 | 7.525 | Ap (Si)Cr | 13.9 | 4 |
| 218495 | 9.356 | Ap EuSr | 7.4 | 1 |

Notes:

[a] The amplitude listed is the typical peak-to-peak variation for a night when the star is “up” in amplitude.

[b] For multi-periodic stars the period of the mode with the highest amplitude is listed.

[c] The spectral types listed are from the Michigan Spectral Catalogue (Houk & Cowley 1975, Houk 1978, 1982, Houk & Smith-Moore 1988).

**Table 3.** Designations, positions, magnetic field strengths and rotation periods for the rapidly oscillating Ap stars.

| HD | HR Name | $\alpha$ (2000) | $\delta$ (2000) | $B_e$ (G) | $P_{\rm rot}$ (day) |
|---|---|---|---|---|---|
| 6532 | | 01 05 56 | −26 43 43 | | 1.944973 |
| 9289 | | 01 31 15 | −11 07 04 | | |
| 12932 | | 02 06 13 | −19 07 16 | | |
| 19918 | | 03 01 24 | −81 54 00 | | |
| 24712 | 1217 | 03 55 16 | −12 05 54 | +400 to +1300 | 12.4572 |
| 42659 | | 06 11 21 | −15 47 32 | | |
| 60435 | | 07 31 00 | −58 00 00 | $<$ 1000 | 7.6793 |
| 80316 | | 09 18 25 | −20 22 13 | | 2.1? |
| 83368 | 3831 | 09 36 25 | −48 45 03 | −700 to +700 | 2.851976 |
| 84041 | | 09 41 34 | −29 22 26 | | 3.69 |
| 86181 | | 09 54 54 | −58 41 00 | | |
| 99563 | | 11 27 43 | −08 55 01 | | |
| 101065 | Przbylski's star | 11 37 37 | -46 43 00 | −2200 | 3.94? |
| 119027 | | 13 41 19 | −28 47 23 | | |
| 122970 | | 14 05 15 | +05 22 21 | | 11.30? |
| 128898 | 5463 $\alpha$ Cir | 14 42 30 | -64 58 00 | −300 | 4.4790 |
| 134214 | | 15 09 03 | −13 59 58 | | |
| 137949 | 33 Lib | 15 29 35 | -17 26 27 | +1400 to +1800 | |
| 150562 | | 16 44 11 | −48 39 16 | | |
| 161459 | | 17 48 29 | −51 55 02 | | |
| 166473 | | 18 12 26 | −37 45 06 | | |
| 176232 | 7167 10 Aql | 18 58 47 | +13 54 00 | | |
| 185256 | | 19 39 24 | −29 44 28 | | |
| 190290 | | 20 14 00 | −78 52 00 | | |
| 193756 | | 20 24 11 | −51 43 25 | | |
| 196470 | | 20 38 10 | −17 30 06 | | |
| 201601 | 8097 $\gamma$ Equ | 21 10 21 | +10 08 00 | −800 to +500 | $>$ 70 yr |
| 203932 | | 21 26 04 | −29 55 45 | | |
| 213637 | | 22 33 14 | −20 02 08 | | |
| 217522 | | 23 01 47 | −44 50 25 | | |
| 218495 | | 23 09 30 | −63 40 00 | | |

Note:

The magnetic field measurements tabulated here are from Table 2 of Kurtz's (1990) review.

**Table 4.** Strömgren photometric indices for the roAp stars.

| HD | $y$ | $b-y$ | $m_1$ | $\delta m_1$ | $c_1$ | $\delta c_1$ | $\beta$ |
|---|---|---|---|---|---|---|---|
| 6532 | 8.445 | 0.088 | 0.214 | −0.014 | 0.879 | −0.051 | 2.880 |
| 9289 | 9.383 | 0.138 | 0.225 | −0.018 | 0.826 | −0.012 | 2.833 |
| 12932 | 10.235 | 0.179 | 0.228 | −0.024 | 0.765 | −0.035 | 2.810 |
| 19918 | 9.336 | 0.169 | 0.216 | −0.010 | 0.822 | −0.058 | 2.855 |
| 24712 | 6.001 | 0.191 | 0.211 | −0.023 | 0.626 | −0.074 | 2.760 |
| 42659 | 6.768 | 0.124 | 0.257 | −0.050 | 0.765 | −0.076 | 2.834 |
| 60435 | 8.891 | 0.136 | 0.240 | −0.034 | 0.833 | −0.047 | 2.855 |
| 80316 | 7.782 | 0.118 | 0.324 | −0.118 | 0.599 | −0.283 | 2.856 |
| 83368 | 6.168 | 0.159 | 0.230 | −0.024 | 0.766 | −0.062 | 2.825 |
| 84041 | 9.330 | 0.177 | 0.233 | −0.026 | 0.797 | −0.061 | 2.844 |
| 86181 | 9.323 | 0.172 | 0.205 | 0.001 | 0.757 | −0.061 | 2.819 |
| 99563 | 8.160 | 0.171 | 0.206 | −0.001 | 0.745 | −0.090 | 2.830 |
| 101065 | 7.994 | 0.431 | 0.387 | −0.204 | 0.002 | −0.370 | 2.641 |
| 119027 | 10.022 | 0.257 | 0.214 | −0.034 | 0.557 | −0.076 | 2.731 |
| 122970 | 8.310 | 0.260 | 0.178 | −0.005 | 0.540 | −0.011 | 2.707 |
| 128898 | 3.198 | 0.152 | 0.195 | 0.012 | 0.760 | −0.077 | 2.831 |
| 134214 | 7.464 | 0.216 | 0.223 | −0.029 | 0.620 | −0.108 | 2.774 |
| 137949 | 6.673 | 0.196 | 0.311 | −0.105 | 0.580 | −0.236 | 2.818 |
| 150562 | 9.816 | 0.301 | 0.212 | −0.015 | 0.659 | −0.087 | 2.783 |
| 161459 | 10.326 | 0.245 | 0.246 | −0.040 | 0.679 | −0.141 | 2.820 |
| 166473 | 7.923 | 0.208 | 0.321 | −0.118 | 0.514 | −0.268 | 2.801 |
| 176232 | 5.890 | 0.150 | 0.208 | −0.004 | 0.829 | 0.031 | 2.809 |
| 185256 | 9.938 | 0.277 | 0.185 | −0.004 | 0.615 | −0.039 | 2.738 |
| 190290 | 9.912 | 0.289 | 0.293 | −0.091 | 0.466 | −0.306 | 2.796 |
| 193756 | 9.195 | 0.181 | 0.213 | −0.008 | 0.760 | −0.040 | 2.810 |
| 196470 | 9.721 | 0.211 | 0.263 | −0.059 | 0.650 | −0.144 | 2.807 |
| 201601 | 4.680 | 0.147 | 0.238 | −0.032 | 0.760 | −0.058 | 2.819 |
| 203932 | 8.820 | 0.175 | 0.196 | 0.004 | 0.742 | −0.020 | 2.791 |
| 213637 | 9.611 | 0.298 | 0.206 | −0.035 | 0.411 | −0.031 | 2.670 |
| 217522 | 7.525 | 0.289 | 0.227 | −0.056 | 0.484 | −0.015 | 2.691 |
| 218495 | 9.356 | 0.114 | 0.252 | −0.049 | 0.812 | −0.098 | 2.870 |

Note:

The photometric data in this Table were acquired as part of the Cape Survey for all but three stars, HD 128898, HD 176232, and HD 201601. For these stars, the photometric data in Table 1 of Kurtz's (1990) review were used.

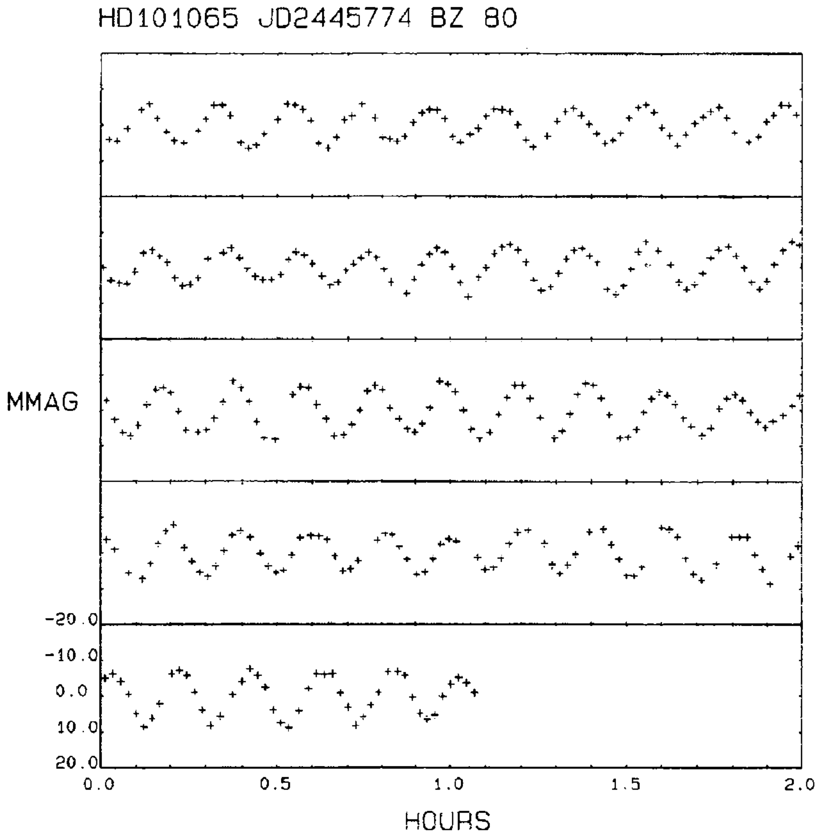


**Fig. 10.** A light curve of HD 101065 obtained using the SAAO 1.9-m telescope.

## 8. ASTEROSEISMOLOGY OF roAp STARS

The roAp stars pulsate in high-overtone p modes. Many of them are multi-periodic. For modes such as these there is an asymptotic relation which describes the frequency separations as a function of the radial overtone, $n$, and the spherical harmonic degree, $\ell$:

$$\nu_{n\ell} = \Delta\nu_0 \left(n + \tfrac{\ell}{2} + \varepsilon\right) + \delta\nu \tag{3}$$

where

$$\Delta\nu_0 = \left(2\int_0^R \frac{dr}{c\,(r)}\right)^{-1} \tag{4}$$

(Tassoul 1980, 1990).

The "large spacing", $\Delta\nu_0$, is the inverse of the sound travel time across the star; for the roAp stars it can be used to determine an asteroseismic luminosity. From Eq. (3) we see that $\Delta\nu_0$ is the spacing of consecutive overtones $(n, n+1, n+2, ...)$ for a given $\ell$. Modes of alternating even and odd $\ell$ have spacing $\frac{1}{2}\Delta\nu_0$. The "small spacing", $\delta\nu$, is sensitive to the sound speed in the core of the star which in turn depends on the $\mu$, the mean molecular weight which changes with age.

### *8.1. HR 1217 - a test of roAp star seismology*

HR 1217 is the roAp star which most closely resembles the Sun in the character of its amplitude spectrum. Kurtz et al. (1989) observed HR 1217 for 365 h at eight observatories over a time-span of three months in 1986. Fig. 11 is an amplitude spectrum of a high-duty-cycle subset of their data. It is easy to see the characteristic alternating spacing of 33.5 $\mu$Hz, 34.5 $\mu$Hz, 33.5 $\mu$Hz, 34.4 $\mu$Hz expected for alternating even and odd $\ell$-modes, as well as the rotational side-lobes generated by the oblique pulsation (discussed in detail in Section 9 below for HR 3831).

From standard A star models Shibahashi & Saio (1985) produced an echelle diagram which is shown in Fig. 12. They calculated that:

$$\begin{aligned}(\nu_{n,1} - \nu_{n-1,2}) - (\nu_{n-1,2} - \nu_{n-1,1}) &\approx 6\ \mu\text{Hz} \\ \neq (\nu_4 - \nu_3) - (\nu_3 - \nu_2) &= -1.41\ \mu\text{Hz}\end{aligned} \tag{5}$$

and

$$\begin{aligned}(\nu_{n,0} - \nu_{n-1,1}) - (\nu_{n-1,1} - \nu_{n-1,0}) &\approx 2\ \mu\text{Hz} \\ = (\nu_5 - \nu_4) - (\nu_4 - \nu_3) &= 1.59\ \mu\text{Hz}\end{aligned} \tag{6}$$

Equations 5 and 6 show the actual spacings from Fig. 11 in comparison with the calculated spacings in Fig. 12. The approximate equality in Eq. (6) argues that the even $\ell$-modes, $\nu_3$ and $\nu_5$, are $\ell = 0$ radial modes, rather than quadrupole modes which give the wrong spacing in Eq. (5). Two problems arise. The first is that this interpretation of the frequencies in Fig. 11 as even and odd $\ell$ modes leaves the sixth frequency at $\nu_6 - \nu_5 = 3/4\Delta\nu_0$. That makes no sense in terms of Eq. (3) which admits no possibility of $3/4\Delta\nu_0$ frequency separations. The other problem is that $\nu_3$ and $\nu_5$ both have

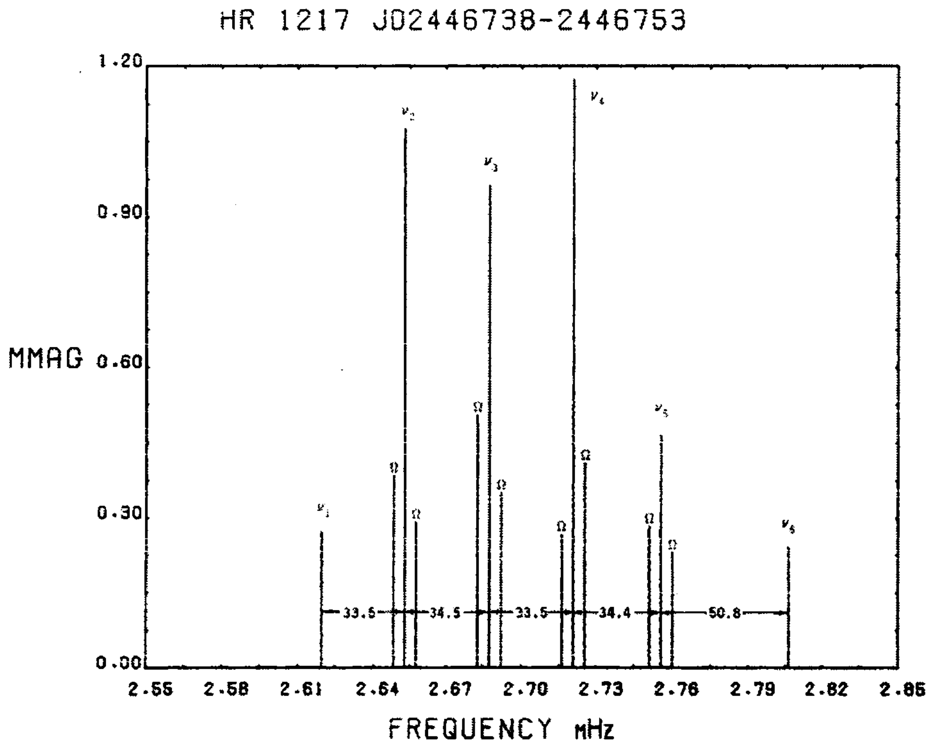


**Fig. 11.** A schematic amplitude spectrum for HR 1217 showing the frequency spacing characteristic of alternating even and odd $\ell$ modes, and similar to the pattern seen in the Sun. The frequency spacings are given in $\mu$Hz. The peaks labeled $\Omega$ are sidelobes generated by oblique pulsation. From Kurtz et al. (1989).

rotational sidelobes which indicate amplitude modulation. Radial, $\ell = 0$, modes should not show amplitude modulation in an oblique pulsator.

Asteroseismology provides an additional test. Fig. 13 shows a theoretical HR diagram on which lines of constant $\Delta\nu_0$ and evolutionary tracks have been placed. Many of the roAp stars which are multiperiodic, and for which $\Delta\nu_0$ can be measured, are placed on the diagram. Because their position in this diagram measures their luminosity, their distances can be calculated and an asteroseismic parallax predicted. HR 1217 is pointed out in Fig. 13 for the case where its modes are alternating even and odd modes. That gives $\Delta\nu_0 = 68$ $\mu$Hz, which then implies $\pi_{\rm seismic} = 19.1 \pm 0.5$ mas. If the

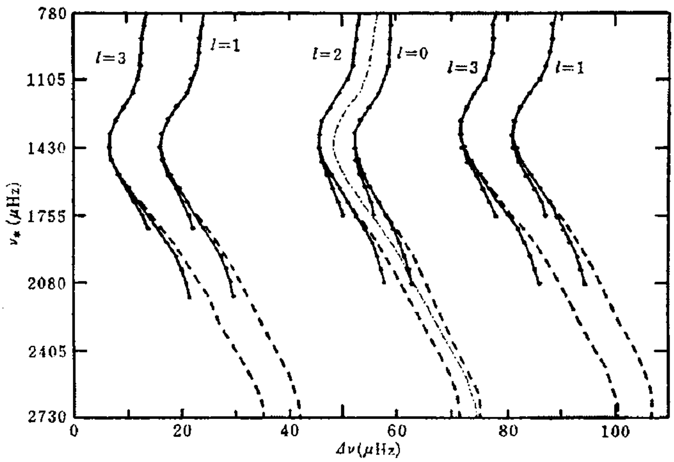


**Fig. 12.** This is an echelle diagram for HR 1217 from a standard 2-$M_\odot$ A star model. This shows how the frequency spacing from Eq. (3) changes with increasing frequency. From Shibahashi & Saio (1985).

modes in HR 1217 were all of the same $\ell$ for consecutive overtones, then $\Delta\nu_0 = 34$ $\mu$Hz and $\pi_{\rm seismic} = 11.0 \pm 0.5$ mas. *Hipparcos* clearly resolves the ambiguity with its measured parallax of $\pi = 20.4 \pm 0.8$ mas (Matthews et al. 1999).

What could the explanation for $\nu_6$ be then? One possibility is that the frequency of $\nu_6$ is only coincidentally near to $3/4\Delta\nu_0$. It is known that the pulsation frequencies in HR 1217 are near to, or even greater than, the critical frequency, beyond which standing wave pulsation modes can only be maintained by strong driving. There is some indication in the work of Gautschy et al. (1998) that the frequency spacings in HR 1217 are greatly distorted by non-adiabatic effects.

The rotational sidelobes for $\nu_3$ and $\nu_5$ must mean that they are distorted radial modes, i.e. they cannot be described by a single normal mode. An examination of Table 6 of Kurtz et al. (1989) supports this. In that table it is shown that the ratio of the amplitudes of the rotational sidelobes compared to the central frequency, $(A_{+1} + A_{-1})/A_0$, for $\nu_2$ and $\nu_4$ are as predicted from the oblique pulsator model and the independent magnetic field observations. For $\nu_3$ and $\nu_5$, $(A_{+1} + A_{-1})/A_0$ does not conform to $\ell = 0$, 1 or 2 modes,

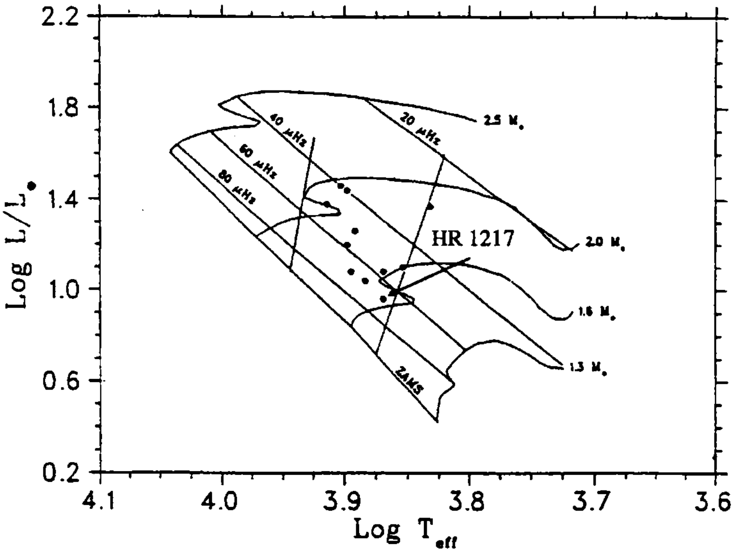


**Fig. 13.** A theoretical HR diagram showing the position of the $\delta$ Scuti instability strip (vertical lines), lines of constant $\Delta\nu_0$, evolutionary tracks, and the positions of roAp stars for which $\Delta\nu_0$ has been measured. From Martinez (1993) and Heller & Kawaler (1988).

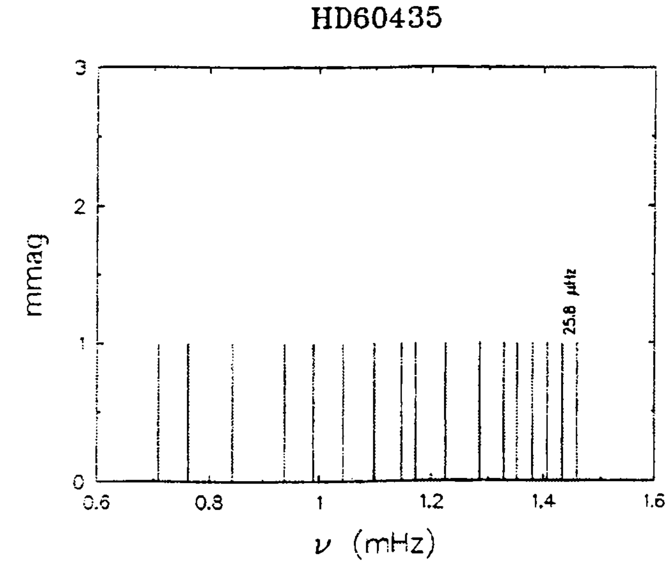


**Fig. 14.** A schematic amplitude spectrum for HD 60435 showing its many pulsation frequencies and basic frequency spacings of $\Delta\nu_0/2 = 25.8$ $\mu$Hz. From Matthews et al. (1986, 1987).

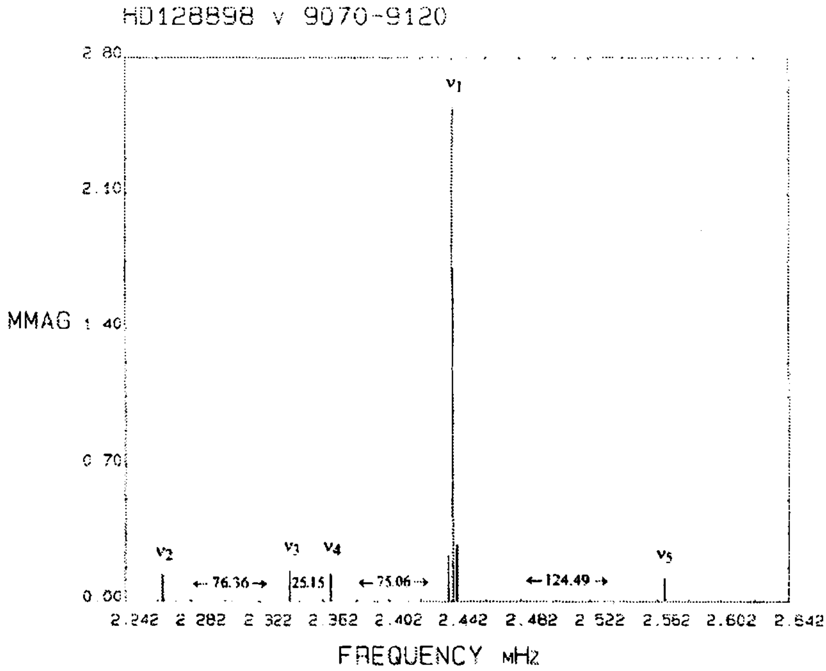


**Fig. 15.** A schematic amplitude spectrum for $\alpha$ Cir showing its main pulsation frequency and rotational sidelobes, plus four low amplitude frequencies which suggest $\Delta\nu_0 = 50\ \mu$Hz. From Kurtz et al. (1994b).

supporting the contention that those frequencies belong to distorted radial modes.

### *8.2. $\Delta\nu_0$ for other roAp stars*

To give a flavor of the variety of frequency patterns in the roAp stars, and to convince you that we do measure $\Delta\nu_0$, we will show here amplitude spectra for several other stars. The first, in Fig. 14, is HD 60435 which has the richest amplitude spectrum of any roAp star. The frequency spacing for this star is 25.8 $\mu$Hz, so we take $\Delta\nu_0 = 52\ \mu$Hz (Matthews et al. 1986, 1987). This star rotates with a period of $P_{\rm rot} = 7.6793$ d, and its pulsation modes have lifetimes less than a rotation period. Further multi-site observing campaigns on this star would be very fruitful.

The frequency pattern for $\alpha$ Cir is intriguing, and also a serious problem. Fig. 15 shows several low amplitude frequencies with separations that suggest $\Delta\nu_0/2 = 25\ \mu$Hz (Kurtz et al. 1994b). This leads to a discrepancy with the *Hipparcos* parallax (Matthews et al. 1999). We have some confidence in the identification of those low amplitude frequencies, but they are not certain. This star is in great need of another multi-site observing campaign to clarify this problem.

A first analysis of HD 119027 led to a clear identification of $\Delta\nu_0/2 = 26\ \mu$Hz (Martinez et al. 1993). Fig. 16 shows this. It also shows a pair of closely spaced frequencies with a separation of only 2 $\mu$Hz. That seemed to have the potential to be identified with $\delta\nu$, the small separation in Eq. (3), or with magnetic effects on the frequencies, so further observations of this star were carried out. The new study (Martinez et al. 1998b) found the same pair of close frequencies with the 2 $\mu$Hz separation, but discovered an ambiguity for $\Delta\nu_0/2$, which may be either 26 $\mu$Hz, as previously found, or possibly half of that, 13 $\mu$Hz. This is a strong message that the roAp stars need more observations, even for seemingly well-determined cases of $\Delta\nu_0$ as seen in Fig. 16.

And finally, Fig. 17 shows the amplitude spectrum for HD 203932 with a clear case for $\Delta\nu_0/2 = 32\ \mu$Hz. These examples show that we are finding the large spacing in roAp stars. That the luminosities derived from these spacings agree with the *Hipparcos* luminosities argues that we are correctly identifying $\Delta\nu_0$, and that the magnetic perturbation to the pulsation frequencies is probably not distorting $\Delta\nu_0$ appreciably. Dziembowski & Goode (1996) have examined the magnetic perturbation theoretically and find that it can be of order 10 $\mu$Hz – much larger than the small separation and enough to be worrisome in the determination of the large separation.

## 9. HR 3831 AND THE OBLIQUE PULSATOR MODEL

### *9.1. Properties of HR 3831*

HR 3831 (HD 83368) is a bright, $V = 6.168$, southern peculiar A star classified Ap SrCrEu by Houk (1978), who remarked on its strong Sr lines. It is a visual binary with a separation of 3.29 arcsec and a difference in brightness between the components of $\Delta V = 2.84$ according to Hurley & Warner (1983) who obtained *UBV* photometry for the resolved components using an area scanner. They found for HR 3831A: $V = 6.25$, $B - V = 0.25$, and $U - B = 0.12$; and for

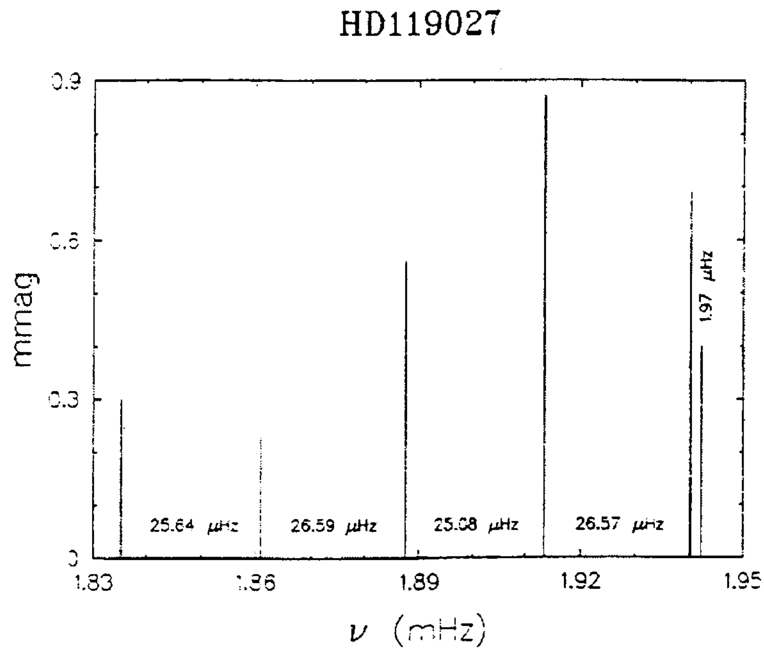


**Fig. 16.** A schematic amplitude spectrum for HD 119027 showing a clear $\Delta\nu_0/2 = 26$ $\mu$Hz and a possible small separation of 2 $\mu$Hz. Further work has shown $\Delta\nu_0$ to be ambiguous between 26 $\mu$Hz and 13 $\mu$Hz. From Martinez et al. (1993).

HR 3831B: $V = 9.09$, $B - V = 0.64$, and $U - B = 0.15$. (Throughout this paper HR 3831 used by itself means HR 3831A.)

From its colors HR 3831B is close to being a solar analogue, presumably less than $10^9$ yr old, since HR 3831A is a main sequence A star. The *Hipparcos* parallax is $\pi = 13.80 \pm 0.76$ mas, which gives a distance of $72 \pm 4$ pc. This then gives for the absolute magnitude of HR 3831B $M_V = 4.75$ including a bolometric correction of 0.05 mag and assuming no interstellar extinction. This is slightly fainter than the Sun, as is expected given the relative youth of HR 3831B compared to the Sun.

Strömgren and H$\beta$ indices have been measured for the unresolved binary system by Martinez (1993) who found $V = 6.168$, $b - y = 0.159$, $m_1 = 0.230$, $c_1 = 0.766$, and $\beta = 2.825$, with standard deviations of about 0.006 mag for all indices. If we sum the intensities derived from Hurley & Warner's $V$ measure-

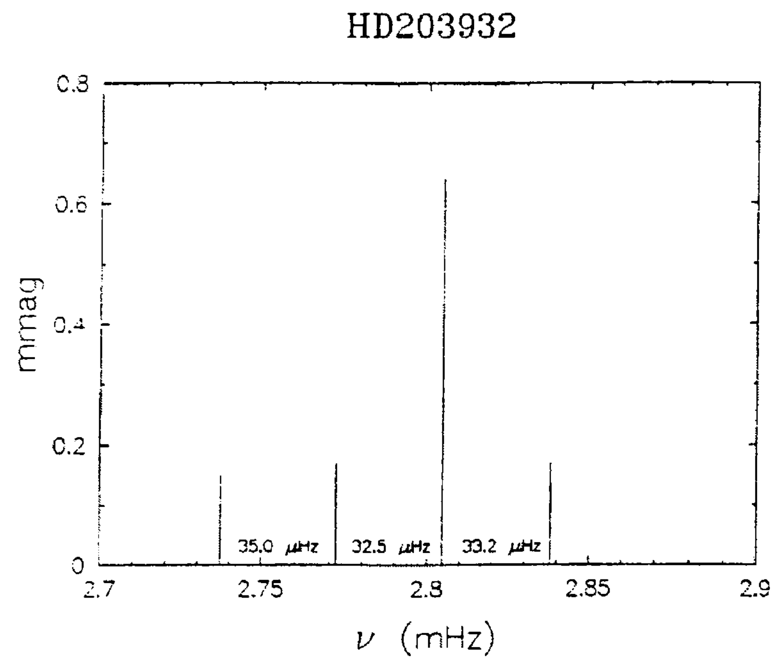


**Fig. 17.** A schematic amplitude spectrum for HD 203932 showing a clear $\Delta\nu_0/2 = 32$ $\mu$Hz. From Martinez et al. (1990).

ments of the components, $V_{\rm A} = 6.25$ and $V_{\rm B} = 9.09$, we find $V_{\rm A+B} = -2.5 \log \left(10^{\frac{6.25}{-2.5}} + 10^{\frac{9.09}{-2.5}}\right) = 6.17$, in perfect agreement with Martinez's $V = 6.168$. We can calculate that $B - V = 0.22$ for HR 3831A after removing the slight reddening caused by the companion. The $b - y$ and $\beta$ indices must also be slightly reddened by the contribution of the companion to the composite indices.

From $\beta = 2.825$ we find that $T_{\rm eff} = 8000$ K for HR 3831A from the calibration of the Strömgren and H$\beta$ indices by Moon & Dworetsky (1985); this temperature is essentially independent of the luminosity of HR 3831. The bolometric correction for a late A star is 0.2 mag, so from the *Hipparcos* parallax we find $M_{\rm bol}$(HR 3831A) $= 1.76$, assuming no interstellar reddening. Hence, if we take HR 3831B to be a G2 V star of $M_{\rm bol} = 4.75$, $T_{\rm eff} = 5800$ K and $R = 1\,{\rm R}_\odot$, then we calculate $L_{\rm A}/L_{\rm B} = 10^{\frac{1.76-4.75}{-2.5}} = 15.70$ and $R_{\rm A}/R_{\rm B} = (L_{\rm A}/L_{\rm B})^{\frac{1}{2}} \left(T_{\rm B}/T_{\rm A}\right)^2 = 2.1$, so the radius of HR 3831A is

$R = 2.1 \pm 0.1\,R_\odot$, where we have estimated the error by assuming that the temperature error is $\pm 200\,\mathrm{K}$ and the magnitude error is $\pm 0.01$ mag.

From Crawford's (1979) calibration of Strömgren photometry (for chemically normal stars) we find for HR 3831 that $\Delta V(\mathrm{ZAMS}) = 0.7$ mag. Thus we find that HR 3831 is an A7 V star with $T_{\mathrm{eff}} = 8000\,\mathrm{K}$ lying 0.7 mag above the zero age main sequence.

The Strömgren $\delta c_1$ index is generally not a reliable luminosity index for the Ap stars; line blanketing reduces its numerical value compared to that of normal stars of the same luminosity. HR 3831 is a good demonstration of this. The $\delta c_1$ value calculated from Crawford's calibration is $\delta c_1 = -0.062$ which, taken at face value, indicates that HR 3831 lies about 0.5 mag *below* the zero age main sequence. Thus, the $\delta c_1$ index underestimates the luminosity of HR 3831 by about 1.2 mag. Other luminosity indicators also fail for the roAp stars because of the spectral peculiarities of these stars. This is the reason why the asteroseismic luminosities discussed in Section 8 are so important.

HR 3831 is a known magnetic variable. The effective longitudinal magnetic field was measured 9 times by Thompson (1983), 12 times by Mathys (1991) and 12 times by Mathys & Hubrig (1997) who find a sinusoidal variation with a mean of $H_0 = 17 \pm 74$ G and a semi-amplitude of $H_1 = 576 \pm 100$ G. Thus HR 3831 has a magnetic field that reverses polarity about a mean that is indistinguishable from zero. Within the oblique rotator model this means that the rotational inclination, $i$, and/or the magnetic obliquity, $\beta$, is near $90°$.

The sinusoidal variation indicates a dipolar magnetic field. From Section 3.2 we have for a centred dipolar magnetic field:

$$H_{\mathrm{eff}} = \frac{1}{20}\frac{15+\mu}{3-\mu} H_{\mathrm{p}} \left(\cos i \cos\beta + \sin i \sin\beta \cos\Omega t\right) \tag{7}$$

where $H_{\mathrm{p}}$ is the polar magnetic field strength, $i$ and $\beta$ are the rotational inclination and magnetic obliquity, $\mu$ is the limb-darkening coefficient and $\Omega$ is the rotation frequency. It is easy to see from Eq. (7) that the maximum magnetic field strength is

$$H_{\mathrm{eff}}(\max) = \frac{1}{20}\frac{15+\mu}{3-\mu} H_{\mathrm{p}} \left(\cos i \cos\beta + \sin i \sin\beta\right), \tag{8}$$

which occurs when $\cos \Omega t = 1$, and the minimum magnetic field strength is

$$H_{\mathrm{eff}}(\min) = \frac{1}{20}\frac{15+\mu}{3-\mu} H_{\mathrm{p}} \left(\cos i \cos \beta - \sin i \sin \beta\right), \tag{9}$$

which occurs when $\cos \Omega t = -1$. It is traditional to define a term

$$r \equiv \frac{H_{\mathrm{eff}}(\min)}{H_{\mathrm{eff}}(\max)} = \frac{H_0 - H_1}{H_0 + H_1} \tag{10}$$

with $r$ restricted to the range $-1 \leq r \leq +1$. Dividing Eq. (9) by Eq. (8) then yields with a bit of manipulation

$$\tan i \tan \beta = \frac{1-r}{1+r}. \tag{11}$$

From Mathys & Hubrig's (1997) values of $H_0$ and $H_1$ given above we find $r = -0.94 \pm 0.16$. This does not put much of a constraint on the magnetic geometry.

HR 3831 also shows mean light variations. Kurtz et al. (1992) studied these; their results, coupled with a later study by Kurtz et al. (1997), gives a rotation period for HR 3831 of $P_{\mathrm{rot}} = 2.851976 \pm 0.000003$ d. Kurtz et al. (1992) found that the spots that give rise to the mean light variations are not concentric about the magnetic poles, but lag behind them in rotational longitude by $0.055 \pm 0.011$ rotation periods. This will be an important point when we discuss the pulsation radial velocities later in this paper in comparison to the photometric variations.

The rotational velocity of HR 3831 is $v_{\mathrm{eq}} \sin i = 32.6 \pm 2.6$ km s$^{-1}$. With our calculated radius of $R = 2.1 \pm 0.1\ R_{\odot}$ and $P_{\mathrm{rot}} = 2.851976$ d, this gives $v_{\mathrm{eq}} = 37 \pm 2$ km s$^{-1}$. That then yields $\sin i = 0.88 \pm 0.08$, or $i \geq 40°$ at the $3\sigma$ confidence level. Again, this is not a strong constraint.

### *9.2. The rapid oscillations*

Following the discovery of the first rapidly oscillating Ap star, HD 101065, Kurtz was intensively observing the second such star he had discovered, HR 1217. As the observing season for that star wore on, and it began to set before the end of the observing night, he paged through the Bright Star Catalogue for a late Ap star in the southern sky that was rising as HR 1217 was setting. HR 3831 was

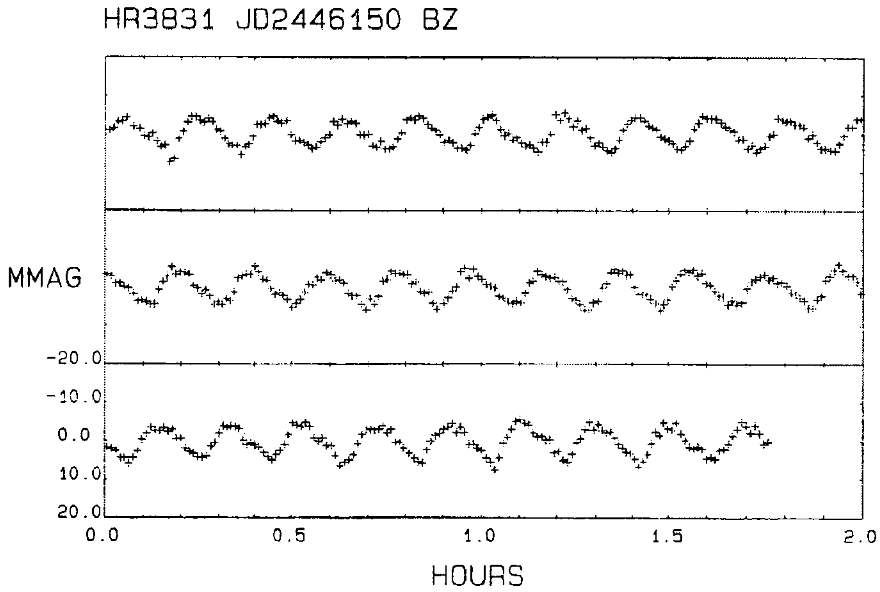


**Fig. 18.** The light curve of HR 3831 near the time of pulsation maximum. Each panel is 0.04 mag high; the panels read like lines of print, so this is a continuous light curve nearly 6 h long.

that star and its 11.67-min oscillations were immediately obvious at the telescope, making it the third member of the then new class to be discovered.

Fig. 18 shows the light curve obtained at amplitude maximum for HR 3831. The data plotted in this light curve were obtained with the South African Astronomical Observatory (SAAO) 0.75-m telescope. Each point represents a 40-s integration. The data have been corrected for mean extinction, dead-time, sky background and low frequency transparency variations. The latter, however, are very small. This same light curve can be seen in its original form without the low frequency corrections in Kurtz & Shibahashi (1986). The quality of these data attest to the exceptionally high photometric transparency of the Sutherland site and to the careful maintenance of the SAAO equipment by the dedicated technical staff.

Fig. 19 is an amplitude spectrum of that light curve in which the clear non-linearity of the light curve is shown by the presence of

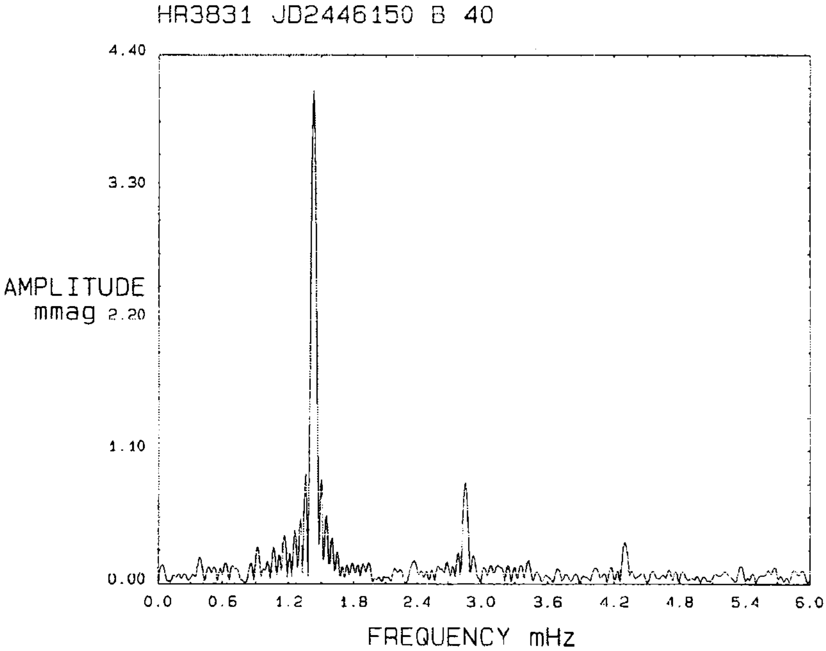


**Fig. 19.** The amplitude spectrum of the light curve shown in Fig. 18. The amplitudes in this and all other amplitude spectra shown in this paper are in millimagnitudes (mmag). The frequencies are in milliHertz (mHz). From Kurtz & Shibahashi (1986).

the first and second harmonics. All amplitudes in this paper are in millimagnitudes (mmag) and are semi-amplitudes as in the relation $\Delta m = A\cos(2\pi f t + \varphi)$. So the highest peak in Fig. 19 at 1.4 mHz has a semi-amplitude of just over 4 mmag; this can then be related directly to the nearly 0.01 mag peak-to-peak variation seen in Fig. 18.

The data discussed in this paper have all been obtained through a Johnson $B$ filter. This broad-band filter gives high count rates, suffers less extinction than the $U$ filter, and is at the wavelength where the roAp stars have their highest amplitudes (Medupe & Kurtz 1998). The observations are all obtained without reference to comparison stars. This is because the variations are too rapid to allow time to cycle through the program star and comparison star, and still get enough data on the program star. At the high count rates needed to get the precision seen in Figs. 18 and 19, CCD photometers cannot yet match photoelectric photometers, so it is not possible to observe the variable simultaneously with a comparison star. Seldom is a bright enough comparison available in a small CCD field in any case. This latter problem means that two-channel photometers also seldom provide any additional benefit.

The technique used is called high-speed photometry. It does not particularly refer to the shortness of the integration times - 10-s integrations are generally used, and for many other purposes (e.g. pulsar observations) this is not "fast". The "high-speed" refers to continuous observations which are not interrupted for observing comparison stars. Because the noise at the frequencies of interest is dominated by scintillation, not sky transparency variations, this technique works well at observatories with high photometric transparency. *It does not work well from hazy or dusty observing sites.*

HR 3831 is the best-studied roAp star. As of the end of 1997 we had 846 h of observations obtained on 388 nights over 18 yr. These data are being used to improve our knowledge of the pulsation frequencies, amplitudes and phases for HR 3831, test and extend the oblique pulsator model and study cyclic frequency variability. HR 3831 has a low frequency septuplet at $\nu = 1428$ $\mu$Hz ($P = 11.67$ min), a first harmonic quintuplet, a second harmonic triplet, and a third harmonic singlet. All of these will be discussed in considerable detail in the next subsections.

### *9.3. The frequency analysis of the HR 3831 data.*

The frequency analysis of HR 3831 is complex for a variety of reasons. We will first look at the frequency analysis of all of the 1980 – 1995 data together. The times of these data have been modified to remove the cyclic frequency variability (see section 10.1 below). Much of the analysis presented here follows that of Kurtz et al. (1993, hereafter KKM) and Kurtz et al. (1997). HR 3831 is part of a University of Cape Town – South African Astronomical Observatory long-term monitoring project. The star is observed for 1 h on every possible night when we have the telescope throughout the observing season. The last few years of data are discussed in Section 10.1 in the context of the frequency variability, but they have not yet been included in the frequency analysis.

We will now look at the amplitude spectra for the 777 h of data obtained from 1980 to 1995. The frequency variability has already been removed by adjusting the times of the observations. Peaks in the amplitude spectrum are identified one at a time. Frequency, amplitude and phase for each new frequency, and all previously identified frequencies, are then optimized by a combination of linear least-squares and non-linear least-squares fitting. The identified frequencies are then removed in the time domain in a process known as prewhitening, and the amplitude spectrum of the residuals is examined for the next highest peak. This is continued until no new significant frequency peak can be identified unambiguously.

The data we are analyzing have many gaps in them; they are unequally spaced in time. We have written the program to produce the amplitude spectra by modifying slightly Deeming's (1975) Discrete Fourier Transform (= DFT) to increase the computing speed (Kurtz 1985). In the amplitude spectrum produced by a DFT for data with gaps, there are "aliases", false peaks which are generated by the gaps in the data. They represent cycle count ambiguities in the gaps where there are no data. For data such as we are about to examine, the alias pattern, known as the spectral window, can be extremely complex, and potentially confusing.

There are automatic frequency analysis programs which remove the confusing pattern of the spectral window and leave, ideally, only the actual frequencies present in the data. One widely used program is known as CLEAN (see Foster (1995) for an improved version of this known as CLEANEST). Jan Hogboom originally wrote CLEAN for spatial reconstruction of radio interferometry – to eliminate spatial sidelobes. It can be used for temporal elimination of sidelobes, and is mandatory for data-sets such as those found in helioseismology where millions of frequencies are present. But it should be used with the utmost caution on small data-sets with only a few frequencies to be extracted.

This is because CLEAN hides information. The temporal sidelobes in an amplitude spectrum tell you how confident you can be that the highest peak is a real peak. If the general noise level has highest peaks which are greater than the difference in the amplitudes of the central peak of the spectral window and its first sidelobes, then you cannot be sure you have the correct frequency when you pick the highest peak. This depends on how the noise spectral window and real frequency spectral window add in complex space.

For any data-set where prewhitening is possible, we recommend visual inspection and prewhitening over the use of CLEAN. The process of prewhitening and examining each piece of the amplitude spectrum gives you a thorough knowledge of the frequency content of the amplitude spectrum you are trying to understand. We also strongly recommend the use of the amplitude spectrum, rather than the power spectrum (power is amplitude squared). The power spectrum makes the signal-to-noise *appear* better than the amplitude spectrum, but that can fool you. Amplitude also relates directly to the light curve.

It is important to know and remember that when the peaks in an amplitude spectrum are ambiguous – when you cannot be sure which peak is the real highest peak because the aliases differ in amplitude by less than the highest noise peaks – that there is no sure-fire technique to remove the problem. The problem is caused by insufficient data, and no period-searching technique can overcome that problem. The only solution is to obtain more data in the gaps to reduce the amplitude of the aliases. The order in which peaks are removed in prewhitening does not change this fundamental limitation.

KKM obtained a 41% duty-cycle (duty cycle is defined to be observed time/total time) for HR 3831 over 17 d in 1991 from CTIO and SAAO. This reduced the 1 $d^{-1}$ aliases substantially compared to single-site data. Fig. 20 shows their data-set. The rotation period of HR 3831 is $P_{\rm rot} = 2.851976$ d. The amplitude of the pulsation is modulated twice per rotation cycle.

Fig. 21 shows the light curve from the KKM data for the night of JD 2448313 at much higher resolution than in Fig. 20. In particular, notice in the fourth panel that the SAAO and CTIO data overlap, and they agree in amplitude and phase. This is unequivocal proof that the variations are in the star, and are not instrumental or atmospheric, since data from two separate observatories separated by 6 h of longitude agree.

Fig. 22 shows a low-resolution amplitude spectrum of the 1991 41%-duty-cycle data-set. Notice the fundamental frequencies (unresolved), and the first, second and third harmonics. Note well the 8.33-mHz sidereal tracking rate of the SAAO 0.75-m telescope. This shows up because the photometer was not perfectly adjusted, so that a small, periodic wobble in the position of the defocused (through a Fabry lens) stellar image on the photocathode caused by a periodic tracking error in the drive gear (a common problem) translated into a periodic intensity variation. This problem occurs in many telescope/photometer combinations, usually at periods of 2 or 4 sidereal minutes. It can be eliminated by correct adjustments of the photometer so that there is no sensitivity variation as a function of position on the photocathode. If that proves to be impossible (sometimes the problem cannot be found), then an autoguider, which holds the

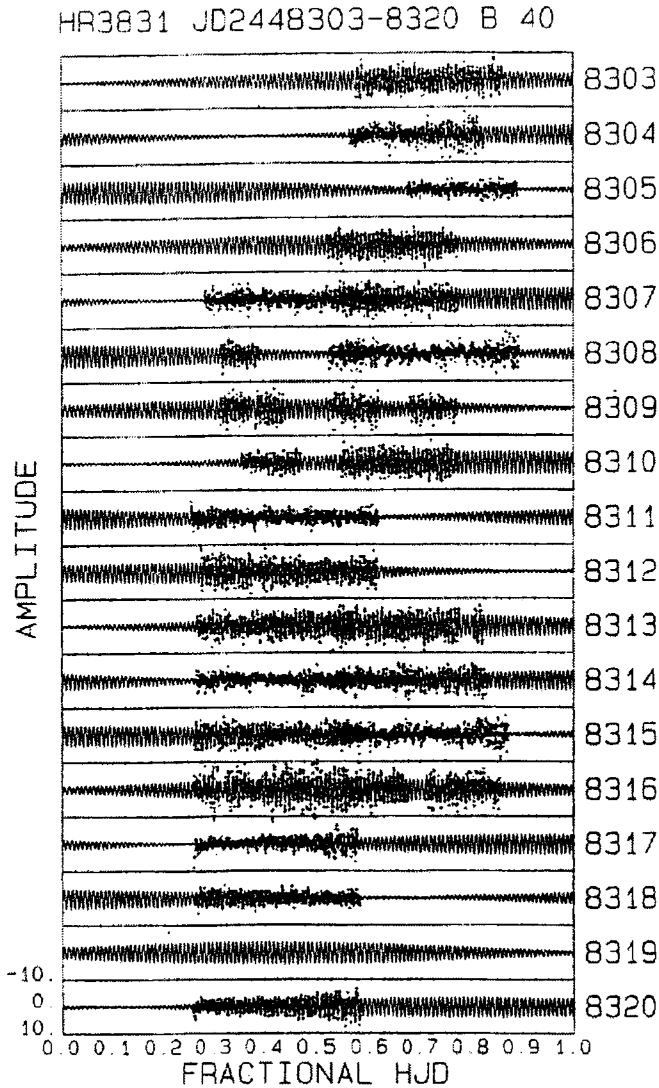


**Fig. 20.** The light curves of HR 3831 obtained by KKM. Each panel is 24 h long. The JDs are along the right. Data obtained from about 0.3 to 0.6 in fractional JD is from SAAO; that from 0.6 to 0.9 is from CTIO. The solid line is the fit of the frequency solution. It clearly shows the $P_{\rm rot}/2 = 1.43$-d amplitude modulation. The data are compressed in this view so they look noisy. Fig. 21 shows the JD 2448313 panel at higher resolution. From Kurtz et al. (1993 - KKM).

stellar image fixed on the photocathode can reduce, or eliminate the problem. Care must be taken not to identify such frequencies as real pulsation frequencies in the star.

The bottom panel of Fig. 22 shows the noise remaining after the frequency solution given in Table 5 has been fitted to the data. The highest peaks have amplitudes of only about 0.08 mmag. The inset shows the spectral window, but it is not resolved at this scale.

For the fundamental frequency septuplet in HR 3831 the frequencies are all clustered near to 1.428 mHz. So we will now expand the frequency scale compared to Fig. 22 to look in detail at these fundamental frequencies. Fig. 23 shows the spectral window for the 15-yr data-set. It has been produced by sampling a pure, noise-free sinusoidal variation, with the frequency and amplitude of the highest peak, at the actual times of the observations.

There are standard methods of calculating the spectral window (see Deeming 1975), but this method is better. It allows you to compare the spectral window directly with the actual amplitude spectrum. At low frequencies this method includes the effects of the spill-over of the negative amplitude spectrum into positive frequency space. The standard method fails to show that. For HR 3831 the frequencies are so far from zero frequency that no significant part of the negative amplitude spectrum is present, so the two methods are virtually indistinguishable in this case.

Fig. 23 is what a spectral window looks like for data with many gaps. There is only one frequency here at the highest central peak. All the other peaks are aliases. The principal aliases occur at $\pm 1\,\mathrm{d}^{-1}$, $\pm 2\,\mathrm{d}^{-1}$, $\pm 3\,\mathrm{d}^{-1}$, etc., since the dominant gaps are caused by the diurnal day-night cycle. In the case of these data (see bottom panel of Fig. 22), the highest noise peaks have amplitudes less than 0.1 mmag. This is much less than the difference in amplitude between the highest peak and its first aliases, making the identification of the central peak of the alias pattern unambiguous for a single, resolved frequency. All frequency analysis techniques, including CLEAN, work well in this circumstance.

Fig. 24 shows the amplitude spectrum for the fundamental frequencies. The highest peak and the second highest peak, plus their window patterns are clear. Both peaks and their $\pm 1\,\mathrm{d}^{-1}$ aliases are marked. The word "fix" in the header to Fig. 24 refers to the 15-yr data-set adjusted for the frequency variability (see section 10.1). The third highest peak, $\nu_3$, is also visible in Fig. 24, but difficult to iden-

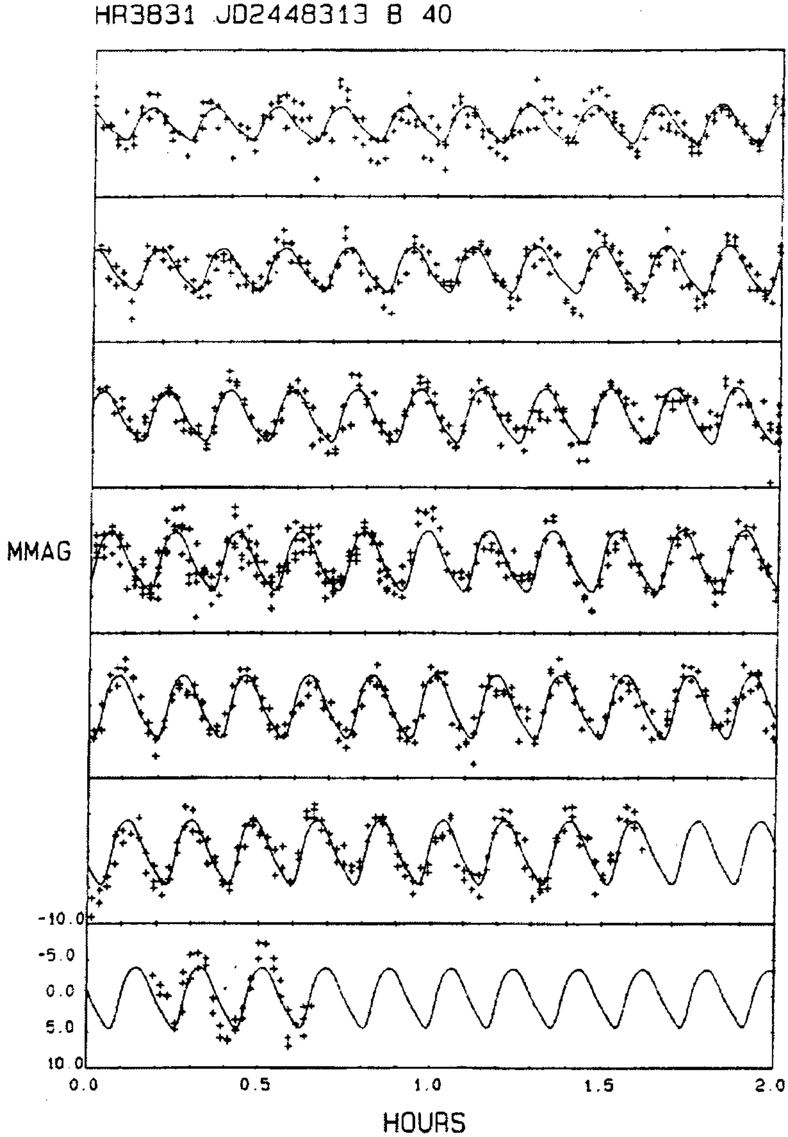


**Fig. 21.** The light curve of HR 3831 obtained by KKM on JD2448313. Each panel is 2-h long. This is typical of the data. (Compare with Fig. 18 which shows some of the best data.) Note in the fourth panel that the SAAO and CTIO data overlap, and they agree in amplitude and phase. From Kurtz et al. (1993 – KKM).

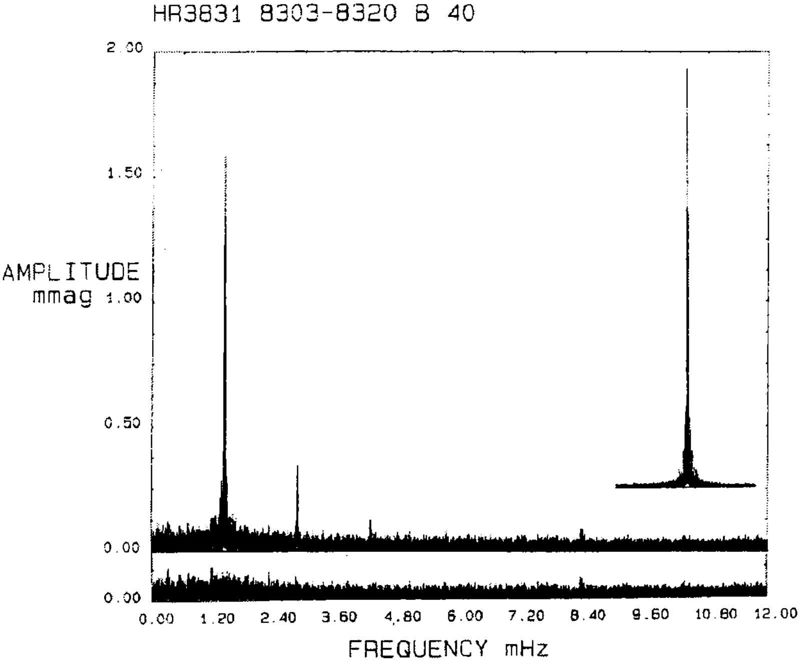


**Fig. 22.** This is a low resolution amplitude spectrum of the 41%-duty-cycle KKM data. The inset shows the spectral window; the lower panel shows the residual noise after the data have been prewhitened by the frequency solution given in Table 5. From Kurtz et al. (1993 – KKM).

tify until $\nu_1$ and $\nu_2$ have been prewhitened. If you examine Fig. 25, then return to this plot, you will be able to identify $\nu_3$.

We will now look at the amplitude spectrum as each new peak is identified and then prewhitened. Each time the entire window pattern associated with the new peak is removed in the prewhitening process. Remember that all the frequencies have their amplitudes and phases optimized simultaneously by least squares; we will return to this point later. Fig. 25 shows the amplitude spectrum of the residuals after prewhitening by $\nu_1$. The highest peaks and their alias patterns belong to $\nu_2$ and $\nu_3$ and are marked. Fig. 26 shows the amplitude spectrum of the residuals after prewhitening by $\nu_1$ to $\nu_2$ (top left), by $\nu_1$ to $\nu_3$ (bottom left) (note the change of ordinate scale from Figs. 24 and 25), by $\nu_1$ to $\nu_4$ (top right) and by $\nu_1$ to $\nu_5$ (bottom right) (note again the change of ordinate scale).

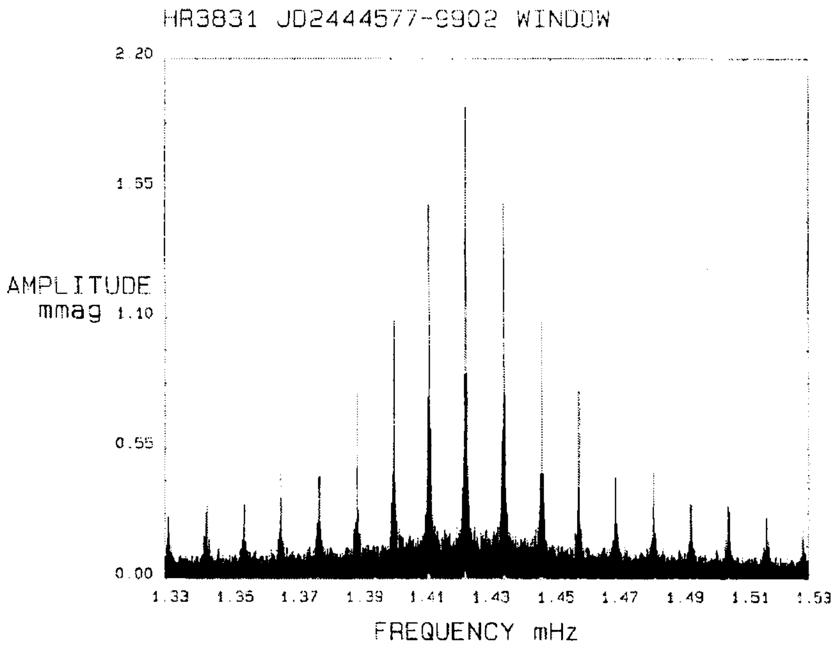


**Fig. 23.** This is a high resolution spectral window of the 15-yr data-set in the frequency range of the fundamental frequency septuplet. From Kurtz et al. (1997).

Finally, after prewhitening by $\nu_1$ to $\nu_6$, there is no further peak in this frequency range of the amplitude spectrum. This is shown in Fig. 27 where the highest peaks in the noise have remarkably low amplitudes of only 0.06 mmag.

As each new frequency is identified, it and all previously determined frequencies are fitted simultaneously to the data by least squares. The amplitudes and phases of each of the frequencies are then compared to the amplitudes and phases that were determined before the new frequency was added to the solution. In this test, there should be no shifts in the amplitudes and phases by more than $3\sigma$, if the spectral windows of the components are not interfering with each other. If the new amplitudes and phases do shift by more than $3\sigma$ compared to the old values, then there is cross-talk in the window patterns of the frequencies, and the error estimates in amplitudes and phases must be increased because of this external error

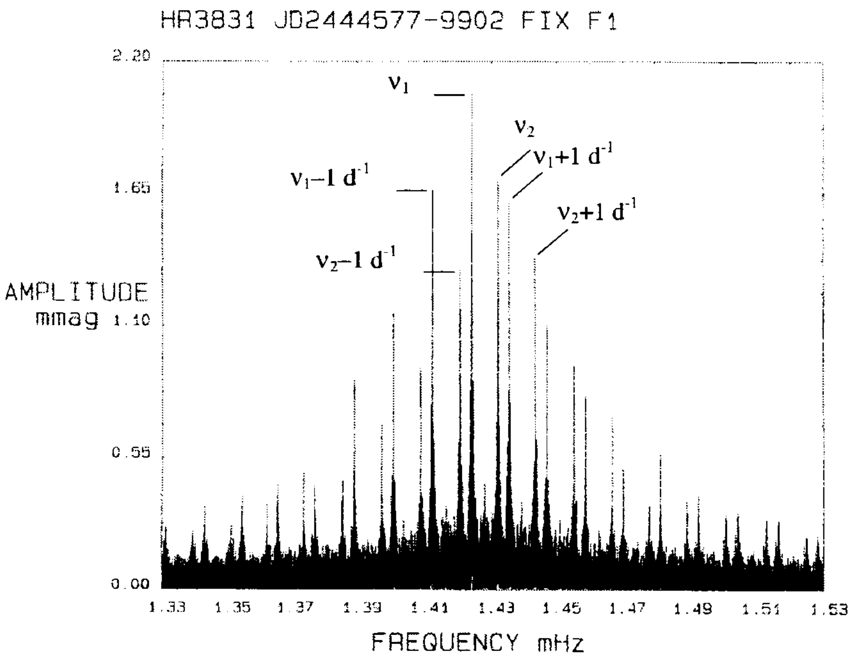


**Fig. 24.** This is a high resolution amplitude spectrum of the 15-yr data-set in the frequency range of the fundamental frequency septuplet. The two highest amplitude peaks and their $\pm 1\,\mathrm{d}^{-1}$ aliases are marked. From Kurtz et al. (1997).

source. See Kurtz et al. (1993 – KKM) to follow this process step-by-step for the 41%-duty-cycle data-set of HR 3831.

This same process of prewhitening has been performed on the harmonic frequencies. Table 5 shows the full frequency solution from Kurtz et al. (1997); see that paper for a complete discussion of the solution. The frequency separations are correct, but the actual values of the frequencies have been shifted by the process of removing the frequency variability. Therefore, the relative values of the frequencies are precise to the last two digits given; their absolute values are not as precise.

Fig. 28 shows the frequency solution in schematic amplitude spectra.

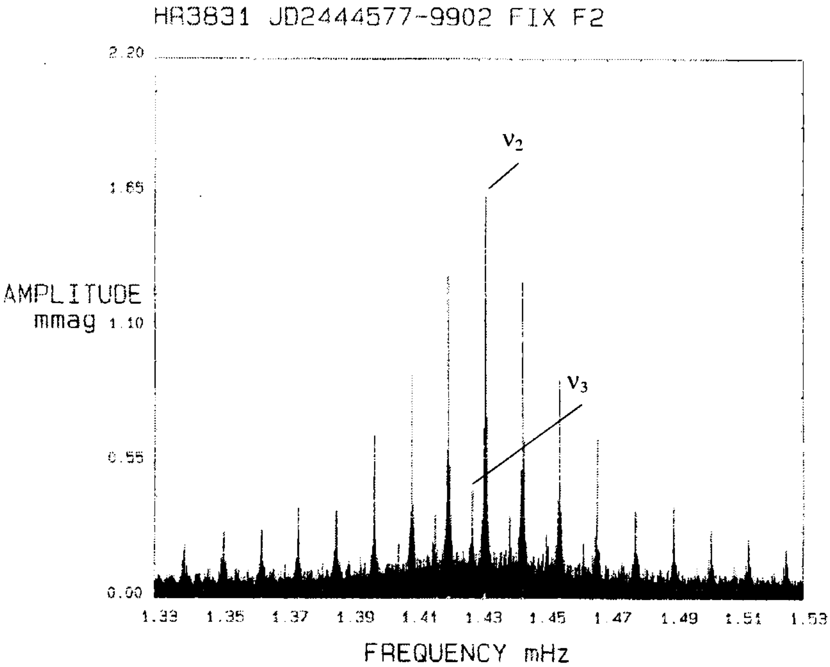


**Fig. 25.** The same as Fig. 24 after prewhitening by $\nu_1$; $\nu_2$ and $\nu_3$ are marked. From Kurtz et al. (1997).

### *9.4. The oblique pulsator model*

The seven frequencies of the low frequency septuplet are split by a frequency very close to the rotation frequency. Why not interpret them as rotationally perturbed $m$-modes with $\ell = 3$ and $m = -3$, $-2$, $-1$, 0, $+1$, $+2$, $+3$? The first-order splitting of such modes is given by

$$\nu_m = \nu_0 + m\,(1 - C_{n\ell})\,\Omega \tag{12}$$

where $C_{n,\ell}$ is a constant which depends on stellar structure, but for roAp stars is close to zero, and $\Omega$ is the rotation frequency (Ledoux 1951). With $C_{n,\ell}$ so close to zero, we expect that the frequency multiplets in HR 3831 should be split by a value very close to the rotation frequency, $\Omega$.

This was, in fact, what Kurtz thought he had discovered when he first found the central three frequencies of the septuplet in HR 3831.

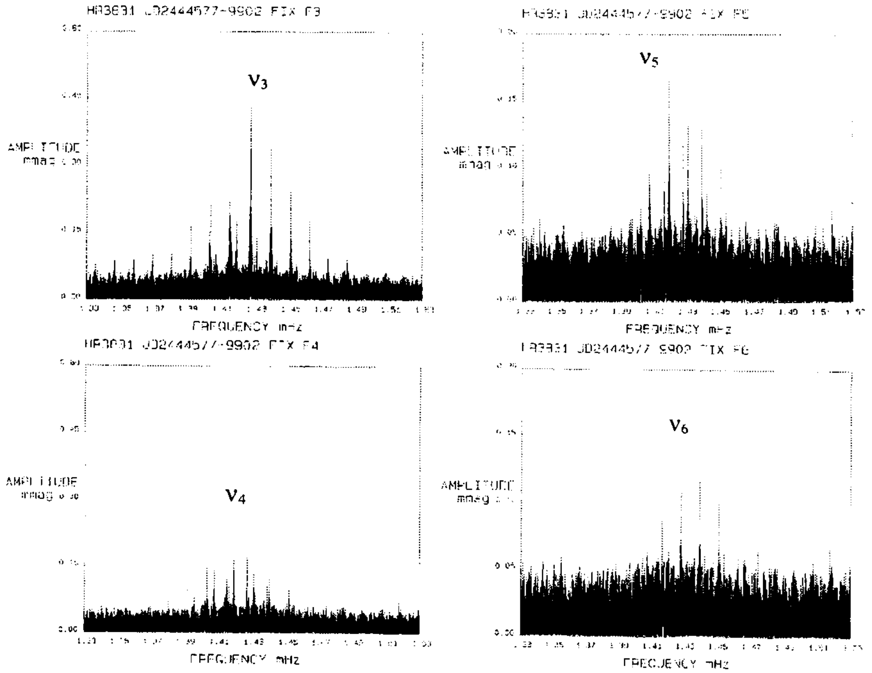


**Fig. 26.** Steps in the prewhitening.

Since the rotation frequency in Ap stars can be determined independently from the mean light variations, this promised to allow the observational determination of $C_{n,\ell}$ for the first time, since all other quantities are measured in Eq. (12). However, this did not happen.

The rotational period of HR 3831 is known from the mean light variations to be $\nu_{\rm rot} = 4058.265 \pm 0.004$ nHz (Kurtz et al. 1997). From the splitting of the observed frequencies and the use of Eq. (12), $C_{n,\ell} \leq 6 \times 10^{-6}$ at the $3\sigma$ confidence level. This is so much smaller than the theoretically expected value, which is $C_{n,\ell} \approx 0.001$ (Shibahashi & Takata 1993), that it rules out the interpretation of the frequency multiplets as rotationally perturbed $m$-modes.

Furthermore, the times of pulsation maximum in the roAp stars coincide with the times of magnetic maximum. The magnetic period is the rotation period. If the pulsation amplitude modulation period were *not* exactly the rotation period (a condition which requires that $C_{n,\ell}$ to be exactly zero, if Eq. (12) is applied), then the pulsation and magnetic maxima would precess with respect to each other and would

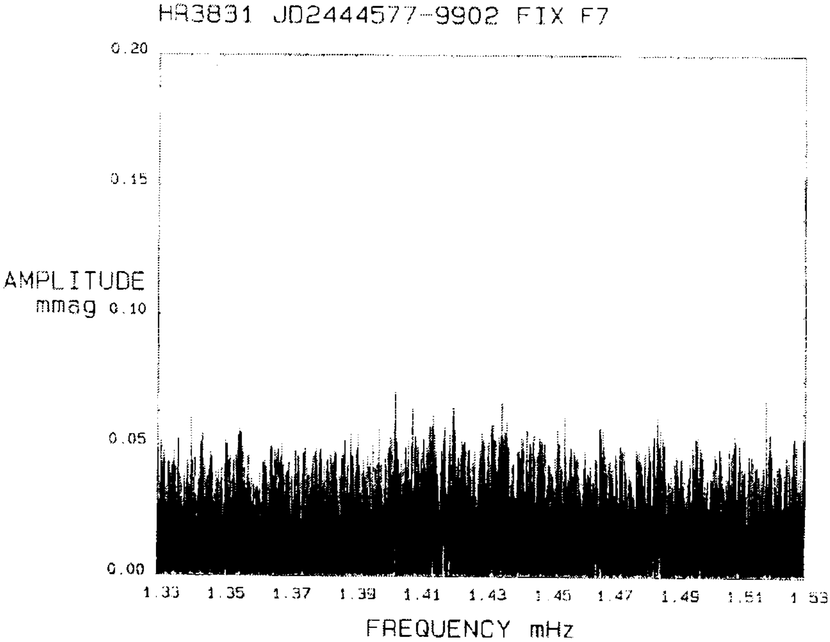


**Fig. 27.** Amplitude spectrum of the residuals after prewhitening by $\nu_1$ to $\nu_6$. Only noise is left with the highest noise peaks having an amplitude of only 0.06 mmag.

only coincidentally come into phase once per precession cycle. This also rules out the $m$-mode interpretation.

This argument led Kurtz (1982) to propose the *oblique pulsator model* in which the pulsation modes in the roAp stars are non-radial modes with their pulsation axes aligned with the magnetic axes of the stars, so that the mode is seen from varying aspect with the rotation of the star, thus showing amplitude and phase modulation. Fig. 1 shows the geometry of the oblique pulsator model as well as the oblique rotator model – they are the same since the magnetic and pulsation axes are aligned. The inclination of the rotation pole is $i$, the obliquity of the magnetic axis is $\beta$, and the variable angle between the magnetic pole and the line-of-sight is $\alpha$.

In a simple beginning to the oblique pulsator model, Kurtz (1982) showed that for an oblique dipole mode, a frequency triplet is expected which is split by exactly the rotation frequency. The angle between the pulsation pole and the line-of-sight varies as

**Table 5.** The linear least squares solution to the 15-yr data-set, JD2444577–2450240, for HR 3831 after removal of the frequency variability.

| Identification | Frequency $\mu$Hz | Amplitude mmag | Phase radians |
|---|---|---|---|
| | The fundamental septuplet | | |
| $\nu - 3\ \nu_{rot}$ | 1415.834274 | 0.194±0.009 | -0.009±0.044 |
| $\nu - 2\ \nu_{rot}$ | 1419.892539 | 0.180±0.009 | -1.436±0.048 |
| $\nu - 1\ \nu_{rot}$ | 1423.950803 | 1.985±0.009 | -0.370±0.004 |
| $\nu$ | 1428.009068 | 0.420±0.009 | -2.317±0.021 |
| $\nu + 1\ \nu_{rot}$ | 1432.067332 | 1.635±0.009 | -0.370±0.005 |
| $\nu + 2\ \nu_{rot}$ | 1436.125597 | 0.075±0.009 | +2.443±0.115 |
| $\nu + 3\ \nu_{rot}$ | 1440.183861 | 0.121±0.009 | +0.387±0.071 |
| | The first-harmonic quintuplet | | |
| $2\nu - 2\ \nu_{rot}$ | 2847.901607 | 0.121±0.009 | -2.117±0.058 |
| $2\nu - 1\ \nu_{rot}$ | 2851.959872 | 0.067±0.009 | -3.133±0.128 |
| $2\nu$ | 2856.018136 | 0.390±0.009 | -2.047±0.022 |
| $2\nu + 1\ \nu_{rot}$ | 2860.076401 | 0.018±0.009 | +2.072±0.486 |
| $2\nu + 2\ \nu_{rot}$ | 2864.134665 | 0.132±0.009 | -1.876±0.065 |
| | The second-harmonic triplet | | |
| $3\nu - 1\ \nu_{rot}$ | 4279.968940 | 0.072±0.009 | -2.929±0.119 |
| $3\nu$ | 4284.027204 | 0.041±0.009 | +1.567±0.211 |
| $3\nu + 1\ \nu_{rot}$ | 4288.085469 | 0.102±0.009 | -2.747±0.084 |
| | The third-harmonic singlet | | |
| $4\nu$ | 5712.036272 | 0.052±0.009 | +2.691±0.165 |
| | $\sigma$ = 1.577 mmag $t_0$=HJD 2448312.23606 | | |

$$\cos\alpha = \cos i \cos\beta + \sin i \sin\beta \cos\Omega t\,. \tag{13}$$

The brightness variation is then expected to go as

$$\Delta L/L \propto Y_\ell^m(\theta,\varphi)\cos(\omega t+\varphi) \tag{14}$$

which for an oblique dipole mode is

$$\Delta L/L \propto \cos\alpha\cos(\omega t+\varphi) \tag{15}$$

where the rotational phase is define to be zero at the time of pulsation maximum, i.e. when $\alpha$ is minimum.

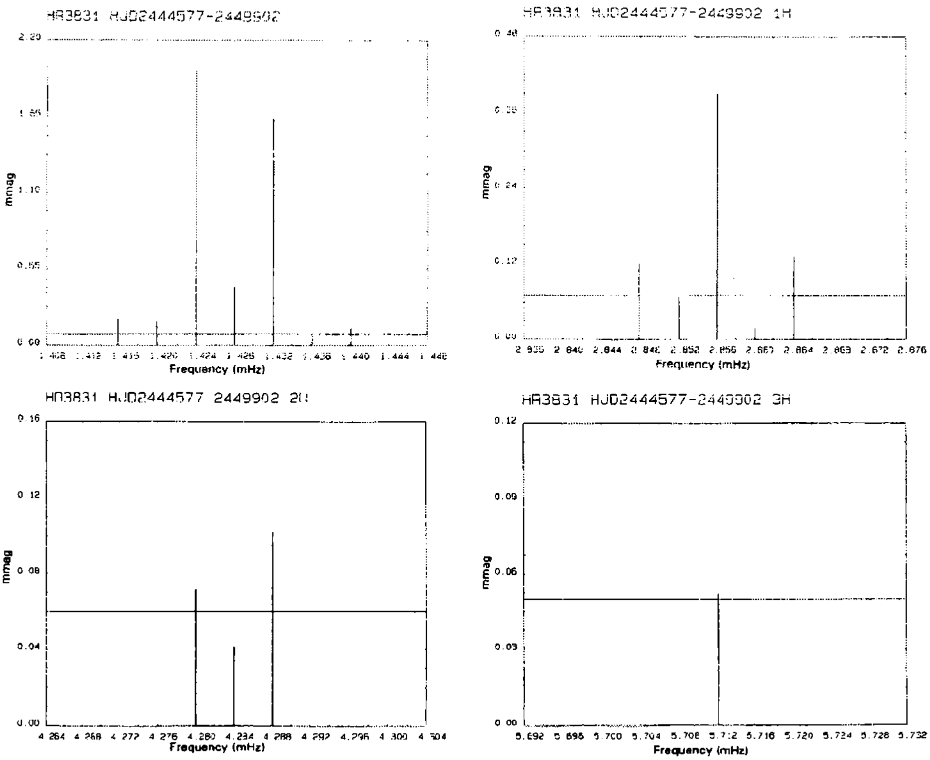


**Fig. 28.** The schematic amplitude spectra showing the low frequency septuplet, the first harmonic quintuplet, the second harmonic triplet, and the third harmonic singlet. Note that not all of the frequencies in the multiplets are actually identified; some of them are surmised to be present, but have amplitudes below the height of the highest noise peaks (shown by the horizontal lines). Note the changes of ordinate scales. From Kurtz et al. (1997).

For this dipole mode it is easy to expand Eq. 15 to give a frequency triplet,

$$\begin{aligned}\Delta L/L \propto A_{-1}\cos\left[(\omega-\Omega)\ t+\varphi\right]+A_0\cos\left[\omega t+\varphi\right]\\ +\ A_{+1}\cos\left[(\omega+\Omega)\ t+\varphi\right]\end{aligned} \tag{16}$$

where the central component has amplitude

$$A_0 = \cos i \cos \beta \tag{17}$$

and the two outer components have equal amplitudes

$$A_{+1} = A_{-1} = \tfrac{1}{2}\sin i \sin \beta. \tag{18}$$

The rotational inclination, $i$, and magnetic obliquity, $\beta$, are constrained by

$$\frac{A_{+1} + A_{-1}}{A_0} = \tan i \tan \beta. \tag{19}$$

Note that this is the same geometric constraint provided by Eq. (11) for a dipole magnetic field.

Shibahashi & Saio (1985) examined the oblique pulsator model in this form and put it on much firmer physical and mathematical foundation. The simplistic model correctly predicted frequency multiplets split by exactly the rotation frequency. But it also predicted *incorrectly* that $A_{+1} = A_{-1}$. Dolez & Gough (1982) questioned the oblique pulsator model. They pointed out that an axisymmetric dipole mode ($\ell = 1$, $m = 0$) aligned with the pulsation axis can be written as a linear sum of ($\ell = 1$, $m = -1, 0, +1$) modes aligned with the rotation axis, and this should lead us to expect Eq. (12) for rotationally perturbed $m$-modes to apply.

Following Dziembowski & Goode (1984) and Gough & Taylor (1984), Dziembowski & Goode (1985) developed a generalization of the oblique pulsator model in which the effects of both the magnetic field and the rotation were taken into account. They assumed that the perturbations to the eigenfrequencies by the magnetic field dominates that of rotation and showed that this leads naturally to the expectation that the pulsation axis should be rigidly locked to the magnetic axis. Including the effects of the Coriolis force and the Lorentz force, which were neglected in the simple oblique pulsator model, explains the inequality $A_{+1} \neq A_{-1}$. The difference in those amplitudes leads to a measure of the magnetic field strength integrated over the volume of the star.

It is assumed that the eigenfrequencies in the presence of an axisymmetric magnetic field can be approximately described by the spherical harmonics $Y_\ell^m(\theta_B, \varphi_B)$, where $\theta_B = 0, \pi$ are the magnetic poles. Since the magnetic field is symmetric with respect to the prograde and retrograde $m$-modes, the perturbation to the eigenfrequencies depends on $|m|$ and is $\omega^{(1)\mathrm{mag}}_{n\ell|m|}$. The perturbed frequencies are, therefore, $\omega = \omega^{(0)}_{n\ell} + \omega^{(1)}_{nl|m|}$, where $\omega = 2\pi\nu$. It is assumed that the magnetic field dominates over the Coriolis force, i.e. that $\omega^{(1)\mathrm{mag}}_{n\ell|m|} >> C_{n\ell}\Omega$. This assumption locks the pulsation axis to the magnetic axis and will be justified below. The observed variations for a single mode labeled by ($n$, $\ell$, $m$) is then (Shibahashi 1986; Kurtz & Shibahashi 1986; Kurtz et al. 1990):

$$\frac{\Delta L}{L} \propto \sum_{m'=-\ell}^{\ell} (-1)^{m'} \left\{ d^{(\ell)}_{mm'}(\beta) + C_{n\ell}\Omega \sum_{k=-\ell}^{\ell} \frac{d^{(\ell)}_{km'}(\beta)}{\omega^{(1)\mathrm{mag}}_{|m|} - \omega^{(1)\mathrm{mag}}_{|k|}} \sum_{p=-\ell}^{\ell} p d^{(\ell)}_{kp}(\beta) d^{(\ell)}_{mp}(\beta) \right\}$$
$$\times d^{(\ell)}_{m0}(i) \cos\left( \left[ \omega^{(0)}_{n\ell} + \omega^{(1)\mathrm{mag}}_{nl|m|} + mC_{n\ell}\Omega\cos\beta - m'\Omega \right] t + \varphi \right) \tag{20}$$

where the $d^{(\ell)}_{ij}(\beta)$ matrices are standard spherical harmonic rotation matrices (e.g. see Edmonds 1957). The explicit form for $\ell = 1$ is given by Kurtz & Shibahashi (1986) and for $\ell = 2$ by Kurtz et al. (1989). Forms for $\ell = 1$, 2 and 3 are given by Shibahashi & Takata (1993).

For an axisymmetric dipole ($\ell = 1$, $m = 0$) mode Eq. (14) can be expanded as

$$\frac{\Delta L}{L} \propto \frac{1}{2}\sin i \sin\beta \left( 1 - \frac{C_{n\ell}\Omega}{\omega^{(1)\mathrm{mag}}_{1} - \omega^{(1)\mathrm{mag}}_{0}} \right) \cos\left[ \left( \omega_0 + \omega^{(1)\mathrm{mag}}_{0} - \Omega \right) t + \varphi \right]$$
$$+ A_0 \cos\left[ \left( \omega_0 + \omega^{(1)\mathrm{mag}}_{0} \right) t + \varphi \right]$$
$$+ \frac{1}{2}\sin i \sin\beta \left( 1 + \frac{C_{n\ell}\Omega}{\omega^{(1)\mathrm{mag}}_{1} - \omega^{(1)\mathrm{mag}}_{0}} \right) \cos\left[ \left( \omega_0 + \omega^{(1)\mathrm{mag}}_{0} + \Omega \right) t + \varphi \right] \tag{21}$$

where

$$\frac{A_{+1} + A_{-1}}{A_0} = \tan i \tan\beta \tag{22}$$

exactly as in Eq. (19), but now

$$\frac{A_{+1} - A_{-1}}{A_{+1} + A_{-1}} = \frac{C_{n\ell}\Omega}{\omega^{(1)\mathrm{mag}}_{1} - \omega^{(1)\mathrm{mag}}_{0}}. \tag{23}$$

From the data in Table 5 for the central frequency triplet in HR 3831 we find $\frac{A_{+1}-A_{-1}}{A_{+1}+A_{-1}} = -0.097 \pm 0.004$, and for the roAp star HD 6532 we find $\frac{A_{+1}-A_{-1}}{A_{+1}+A_{-1}} = -0.247 \pm 0.027$ (Kurtz et al. 1996). These values justify the original assumption that $\omega^{(1)\mathrm{mag}}_{n\ell|m|} \gg C_{n\ell}\Omega$.

Dziembowski & Goode (1985) specify in the case of a dipole magnetic field

$$\omega_{|m|}^{(1)\mathrm{mag}} \propto \frac{\ell\,(\ell+1) - 3m^2}{4\ell\,(\ell+1) - 3} K^{\mathrm{mag}} \tag{24}$$

where the coefficient $K^{\mathrm{mag}}$ follows from a $|Y_\ell^m|^2$-weighted integration over the distortion by the field.

Equations 23 and 24 can then be used to obtain a measure of $K^{\mathrm{mag}}$ for HR 3831 and HD 6532. Values of $C_{n,\ell}$ have been calculated for A-star models by Takata & Shibahashi (1995), and the rotation frequencies of the two stars are known (see Kurtz et al. 1996). All this gives $K^{\mathrm{mag}}(\mathrm{HR}\,3831) = 3.5 \pm 0.2$ $\mu$Hz, and $K^{\mathrm{mag}}(\mathrm{HD}\,6532) = 1.0 \pm 0.1$ $\mu$Hz. Presuming that the surface magnetic field is proportional to $K^{\mathrm{mag}}$ leads to the expectation that the measured magnetic field of HD 6532 should be about one third of that in HR 3831.

Available measurements of the longitudinal magnetic fields are consistent with this; measurements of the quadratic fields are probably inconsistent with it. The mean longitudinal field $\langle H_z \rangle$ of HR 3831 varies from about $-600$ G to about $+600$ G, as we noted in section 9.1. A single observation of HD 6532 yielded $\langle H_z \rangle = -517 \pm 273$ G (Mathys & Hubrig 1997), which is effectively a null result. Measurements of the quadratic field strengths, $\sqrt{H^2 + H_z^2}$, give $21.9 \pm 4.4$ kG (Mathys & Hubrig 1997) for HD 6532 and $11.4 \pm 0.4$ kG for HR 3831 (Mathys 1995). Thus, the few available magnetic observations do not confirm the interpretation of $K^{\mathrm{mag}}$ as a measure of the integrated internal magnetic field strength. More work is needed on this problem, both theoretically and observationally.

The oblique pulsator model as presented so far leads to the expectation of a frequency triplet for dipole mode pulsation. HR 3831 has a septuplet (Table 5) and HD 6532 a quintuplet, so the model is insufficient. Kurtz (1992) showed how the formulation of the oblique pulsator model by Kurtz, Shibahashi & Goode (1990) can be used to decompose the frequency septuplet in HR 3831 into contributions from a spherical harmonic series of axisymmetric modes with $\ell = 0$, 1, 2, 3. This is shown graphically is Fig. 29.

The same technique can be used to decompose the frequency quintuplet in HD 6532 into contributions from a spherical harmonic series of axisymmetric modes with $\ell = 0$, 1, 2. This is shown graphically is Fig. 30.

Figs. 29 and 30 beautifully demonstrate that HR 3831 and HD 6532 are oblique pulsators with modes that are close to dipole

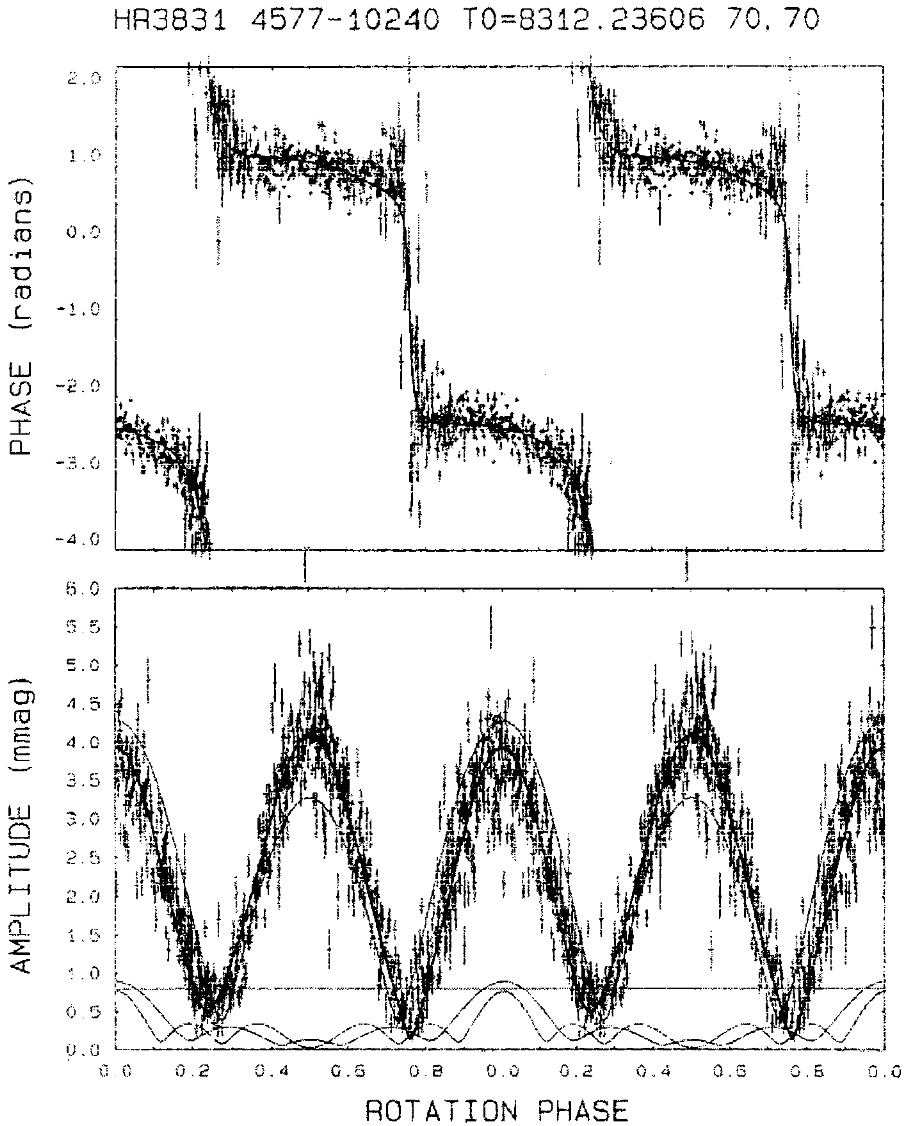


**Fig. 29.** Rotational phase and amplitude modulation in HR 3831. Each point in this diagram shows the pulsation amplitude (bottom) and pulsation phase (top) determined from a least squares fit of the central frequency of the septuplet to 1 h of data. The heavy solid line fits are for a spherical harmonic decomposition of the pulsation mode into $\ell$ = 0, 1, 2, 3 components. The thin lines in the amplitude plot show those component contributions to the overall fit.

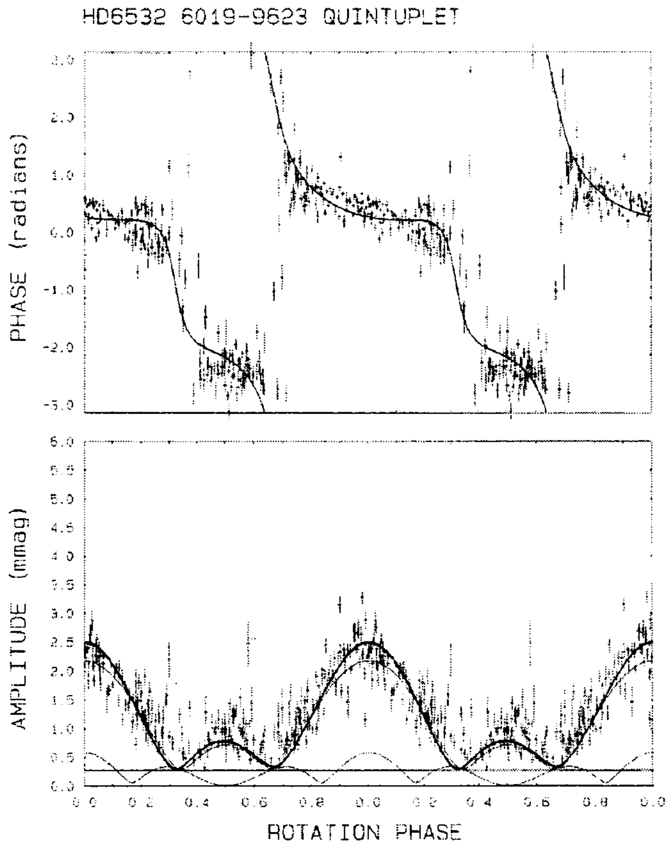


**Fig. 30.** Rotational phase and amplitude modulation in HD 6532. Each point in this diagram shows the pulsation amplitude (bottom) and pulsation phase (top) determined from a least-squares fit of the central frequency of the quintuplet to 1 h of data. The heavy solid line fits are for a spherical harmonic decomposition of the pulsation mode into $\ell = 0, 1, 2$ components. The thin lines in the amplitude plot show those component contributions to the overall fit.

modes. Note in particular in the top parts of the diagrams that the pulsation phase flips by $\pi$ radians as the star goes through quadrature. This is exactly as expected for oblique pulsation in a dipole mode. Since the two pulsation poles are in anti-phase, the apparent pulsation phase reverses by $\pi$ radians as one pole goes out of view and the other comes into view. In the case of HR 3831, where either $i$ or $\beta$ is near 90°, both poles are seen for 1/2 a rotation cycle. In the case of HD 6532, where neither $i$ nor $\beta$ is near 90°, one pole is in sight for about 2/3 of a cycle and the other pole for 1/3 of a cycle.

In the bottom halves of the plots which show how the pulsation amplitude modulates with the star's rotation, it can be seen that there are significant contributions to the overall fitted model from the $\ell = 0$, 2, and 3 components for HR 3831 and from the $\ell = 0$ and 2 components for HD 6532. Note carefully that we call them "contributions", not "modes". There is no suggestion here that there are multiple modes present. In these stars it is supposed that only one pulsation mode is observed, but that the magnetic field (presumably) distorts the mode so that it cannot be described by a single normal mode (i.e. by a single spherical harmonic).

The above descriptions are empirical. Shibahashi & Takata (1993) gave them a more physical basis. In the previous form of the oblique pulsator model, only the perturbation to the eigenfrequency was considered. Shibahashi & Takata examined the effect of the first-order perturbations of the magnetic field on the eigenfunctions. They found that this induced an octupole component, as well as the dipole component. Their new theory thus predicted a septuplet of frequencies for an axisymmetric dipole mode:

$$
\begin{aligned}
&\tfrac{\Delta L_{n,1,0}}{L} \propto \\
&N_3\left(\tfrac{5}{16}\tilde{\beta}_{3,0}\sin^3\beta + \tfrac{5\sqrt{3}}{16}\tilde{\beta}_{3,1}\sin^2\beta\right)\sin^3 i\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} + 3\Omega\right]t \\
&+N_3\left(\tfrac{15}{8}\tilde{\beta}_{3,0}\sin^2\beta\cos\beta + \tfrac{5\sqrt{3}}{8}\tilde{\beta}_{3,1}\sin 2\beta\right)\sin^2 i\cos i\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} + 2\Omega\right]t \\
&+\left[\begin{array}{l} N_1\left\{\tfrac{1}{2}\left(1+\tilde{\beta}_{1,0} - \tfrac{C_{n,1}\Omega}{\omega_0^{(1)\mathrm{mag}} - \omega_1^{(1)\mathrm{mag}}}\right)\sin\beta + \tfrac{\sqrt{2}}{2}\tilde{\beta}_{1,1}\right\}\sin i \\ +N_3\left\{\tfrac{3}{256}\tilde{\beta}_{3,0}\left(\sin\beta + 5\sin 3\beta\right) + \tfrac{\sqrt{3}}{128}\tilde{\beta}_{3,1}\left(5\cos 2\beta + 3\right)\right\}\left(\sin i + 5\sin 3i\right)\end{array}\right] \\
&\times\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} + \Omega\right]t \\
&+\left[N_1\left(1+\tilde{\beta}_{1,0}\right)\cos\beta\cos i + N_3\tfrac{1}{64}\tilde{\beta}_{3,0}\left(5\cos 3\beta + 3\cos\beta\right)\left(5\cos 3i + 3\cos i\right)\right] \\
&\times\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}}\right]t \\
&+\left[\begin{array}{l} N_1\left\{\tfrac{1}{2}\left(1+\tilde{\beta}_{1,0} + \tfrac{C_{n,1}\Omega}{\omega_0^{(1)\mathrm{mag}} - \omega_1^{(1)\mathrm{mag}}}\right)\sin\beta - \tfrac{\sqrt{2}}{2}\tilde{\beta}_{1,1}\right\}\sin i \\ +N_3\left\{\tfrac{3}{256}\tilde{\beta}_{3,0}\left(\sin\beta + 5\sin 3\beta\right) - \tfrac{\sqrt{3}}{128}\tilde{\beta}_{3,1}\left(5\cos 2\beta + 3\right)\right\}\left(\sin i + 5\sin 3i\right)\end{array}\right] \\
&\times\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} - \Omega\right]t \\
&+N_3\left(\tfrac{15}{8}\tilde{\beta}_{3,0}\sin^2\beta\cos\beta - \tfrac{5\sqrt{3}}{8}\tilde{\beta}_{3,1}\sin 2\beta\right)\sin^2 i\cos i\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} - 2\Omega\right]t \\
&N_3\left(\tfrac{5}{16}\tilde{\beta}_{3,0}\sin^3\beta - \tfrac{5\sqrt{3}}{16}\tilde{\beta}_{3,1}\sin^2\beta\right)\sin^3 i\cos\left[\omega_{n1}^{(0)} + \omega_0^{(1)\mathrm{mag}} - 3\Omega\right]t
\end{aligned}
\tag{25}
$$

where

$$N_1 = \frac{1}{4}\sqrt{\frac{3}{\pi}\frac{4-\mu}{3-\mu}} \qquad \text{and} \qquad N_3 = \frac{1}{8}\sqrt{\frac{7}{\pi}\frac{\mu}{3-\mu}}$$

describe the limb-darkening, and the $\tilde{\beta}_{ij}$ are expansion coefficients which are derived, but not tabulated, by Shibahashi & Takata (1993).

While this theory does produce a frequency septuplet, that septuplet is composed of dipole ($\ell = 1$) and octupole ($\ell = 3$) components only. However, HR 3831 shows radial ($\ell = 0$) and quadrupole ($\ell = 2$) components, as well. To account for this Takata & Shibahashi (1995) next looked at the effects of a quadrupole component of the magnetic field on the oscillations. They found, in this case, that $\ell = 0$, 1, 2, 3, 4 and 5 components are induced, but because of the averaging over the observed hemisphere only the $\ell = 0$, 1, 2 and 3 components are expected to contribute significantly to the observed variations. The equations are more complex than Eq. (25), so it is easier to show the components now in graphical form in Fig. 31.

Reasonable values of $i$ and $\beta$ can match the observed *amplitudes*, but cannot completely account for the phases. Non-axisymmetric field components may provide the answer. As was discussed in Section 3.2, many Ap stars do need completely decentred dipole fields to describe their magnetic variations; such fields can be described by spherical harmonic series including non-axisymmetric components.

Work continues on the oblique pulsator model. Dziembowski & Goode (1996) have questioned the perturbation approach. They find that the effect of the field is very significant. At kG photospheric strength, the magnetic field perturbation to the frequencies is 10 – 20 $\mu$Hz. This is comparable to, or greater than, the asymptotic small separations, $\delta\nu$. The observed modes depart significantly from normal modes, thus they expect frequency multiplets with many components.

### *9.5. Spots and the spotted pulsator model: proof of the oblique pulsator model*

HR 3831 has spots which are *not* concentric about its magnetic poles. This is not uncommon in Ap stars. In HR 3831 mean light observations from 1975 to 1987 and pulsation observations from 1981 to 1995 give the same rotation period independently; they agree to $1\sigma$. Together they give $P_{\rm rot} = 2.851976 \pm 0.000003$ d. But they are *not in phase*! Fig. 32 shows this using an earlier value of $P_{\rm rot}$.

Least squares fits give the following times of maxima:

$t_0$(pulsation) = HJD 244 4576.150 $\pm$ 0.004,
$t_0$(mean light) = HJD 244 4576.327 $\pm$ 0.006,
$t_0$(magnetic) = HJD244 4576.210 $\pm$ 0.054.

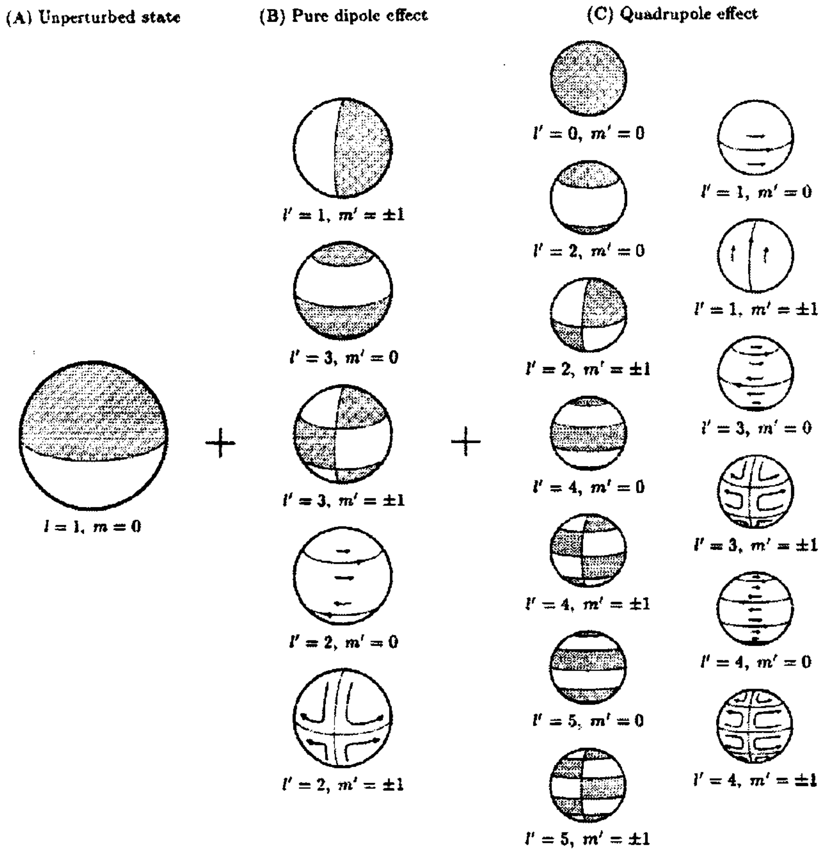


**Fig. 31.** This shows the components of a dipole mode: On the left, column (A) is the unperturbed dipole as described in Eq. (15); column (B) shows the components induced by a pure dipole magnetic field as described by Eq. (25); column (C) shows the components induced by the quadrupole component of the magnetic field. Each of these effects adds to the others. The modes with horizontal arrows are *toroidal* modes which are not expected to be observable. Likewise, the $\ell = 4$ and 5 components in column (C) are not expected to be observable. From Takata & Shibahashi (1995).

These differ by:

$$t_0(\text{pulsation}) - t_0(\text{mean light}) = -0.177 \pm 0.007 \text{ d},$$
$$t_0(\text{pulsation}) - t_0(\text{magnetic}) = -0.060 \pm 0.054 \text{ d},$$
$$t_0(\text{mean light}) - t_0(\text{magnetic}) = +0.117 \pm 0.054 \text{ d},$$

which shows that the spots which produce the mean light variations lag behind the pulsation pole by $0.062 \pm 0.002$ rotation periods. The

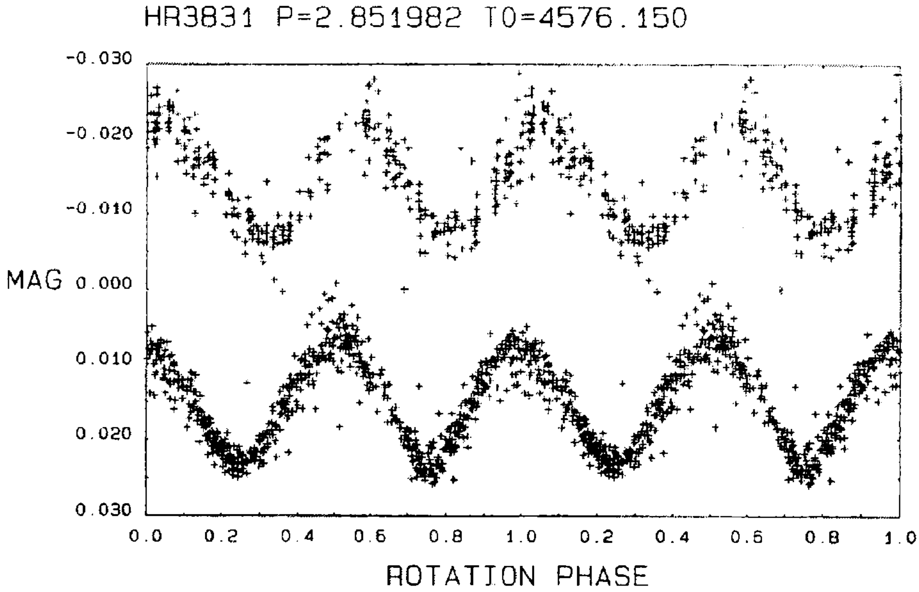


**Fig. 32.** This shows the relative rotational phase of the mean light variations (top) and the pulsation amplitude variations (bottom), both of which have been normalized in amplitude for the purposes of this diagram. The time of mean light maximum brightness clearly lags behind the time of pulsation amplitude maximum by $0.062 \pm 0.002$ rotation periods. From Kurtz et al. (1992).

data are consistent with the pulsation pole and magnetic pole coinciding. Because of the spots, and because an axisymmetric dipole mode ($\ell = 1$, $m = 0$) aligned with the pulsation axis can be written as a linear sum of ($\ell = 1$, $m = -1, 0, +1$) modes aligned with the rotation axis, Mathys (1985) developed the *spotted pulsator model.* He showed that an oscillation of a roAp star can be mathematically modeled with a normal mode pulsating with its axis aligned with the rotation axis of the star, if $f$, the flux-to-radius variation amplitude, and $\psi$, the flux-to-radius variation phase-lag are allowed to vary over the surface of the star within reasonable ranges. This theory predicts that the radial velocity will not be modulated with rotation.

We now have the observations to rule out the spotted pulsator model in favor of the oblique pulsator model. Fig. 33 shows the pulsational radial velocity variations in HR 3831 as a function of rotation phase. The fitted curves are derived from the light variations. Several details are clear from this diagram: (1) the amplitude is modulated with rotation in the same manner as that for the light variations. This is consistent with the oblique pulsator model and in clear contradiction with the spotted pulsator model. (2) The pulsation radial velocity reverses phase by $\pi$ radians just as the pulsation light variation phase does. (3) The pulsation radial velocity amplitude reaches maximum following the time of light maximum by $0.065 \pm 0.014$ rotation periods.

This means that radial velocity maximum seems to be centered on the spot, rather than the magnetic pole. That is a surprise, and we give here only a first guess at the cause (suggested by Jaymie Matthews): The radial velocities were measured from the H$\alpha$ line of hydrogen. It is thought that He sinks preferentially in the spots in the observable atmosphere (see the discussion of the diffusion hypothesis in Section 4). If that is correct, then the deficiency of He translates into an overabundance of H so that the radial velocity measurements in H$\alpha$ are naturally skewed towards the spot.

## 10. FREQUENCY VARIABILITY IN roAp STARS

A new problem arose in the study of roAp stars when intensive multi-site observations of HR 1217 led to a frequency spectrum that could not be fully solved (Kurtz et al. 1989). Fig. 11 in Section 8.1 shows the schematic amplitude spectrum for HR 1217 for 15 d of multi-site data. That project, however, obtained 46 d of data, and when they were all combined, new frequencies, which were called "secondary frequencies", appeared. These were not fully resolved from the principal peaks, and were not explained. Fig. 34 shows the schematic amplitude spectrum for the full 46 d of data. It can be compared with Fig. 11 to see where the new frequencies appear.

Kurtz et al. (1989) speculated that the secondary frequencies indicated amplitude modulation on a time-scale longer than the time-span of their data. Another possibility, however, is that they indicate frequency modulation, rather than amplitude modulation. This question is yet to be resolved for HR 1217, but observations of several other roAp stars indicate that frequency variability is the norm for these stars, so we suspect that it is the case for HR 1217. Because of the six pulsation modes in HR 1217, this will take some effort to prove.

In the roAp stars HR 3831, $\alpha$ Cir and HD 134214 the case is clear: they show long-term frequency modulation. We will show in this section how we came to discover that, and how we have been

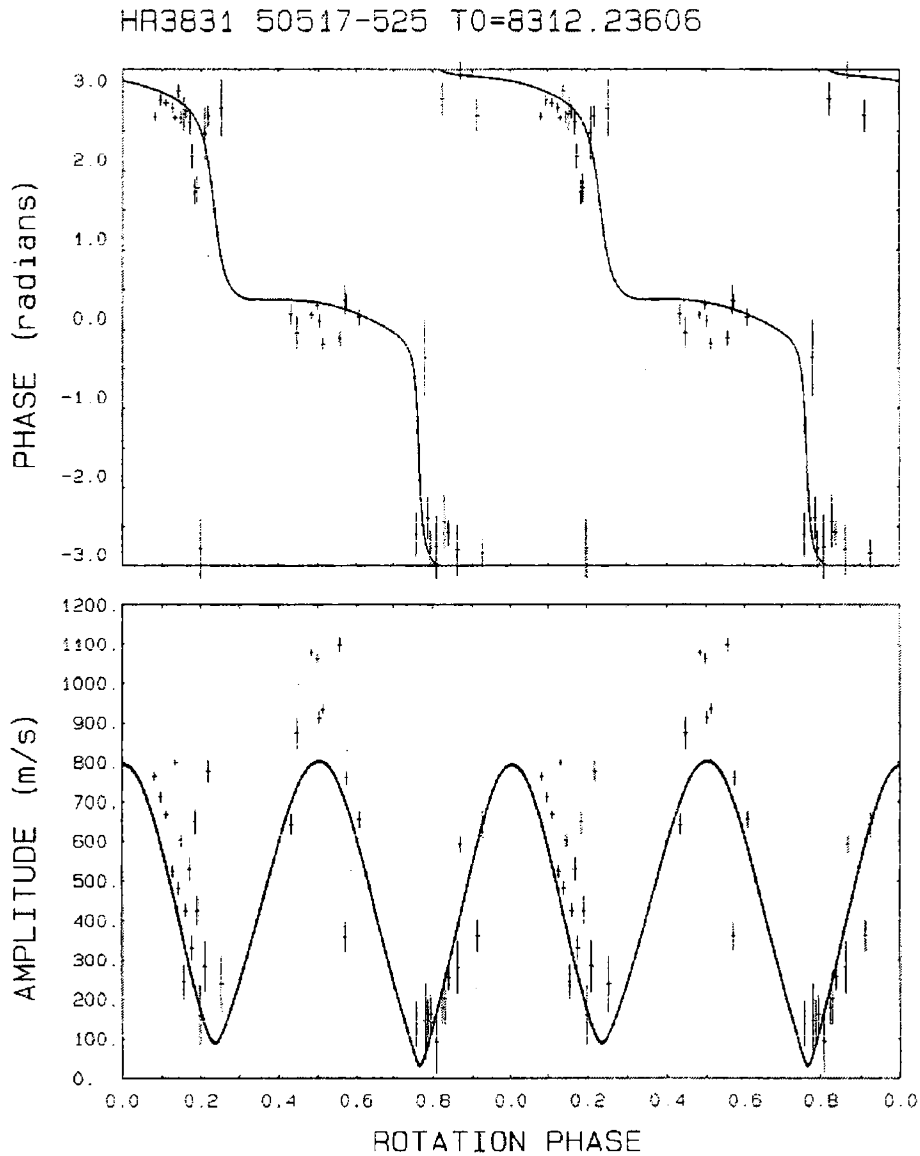


**Fig. 33.** The radial velocity pulsation phase and amplitude as a function of rotation phase for HR 3831. The amplitude variation rules out the spotted pulsator model. Note that the radial velocity pulsation maximum lags behind the pulsation light maximum (solid curve) by $0.065 \pm 0014$ $P_{\rm rot}$, i.e. radial velocity maximum measured in the H$\alpha$ line seems to coincide with the time of mean light maximum. See the text for a discussion. From Baldry et al. (1998).

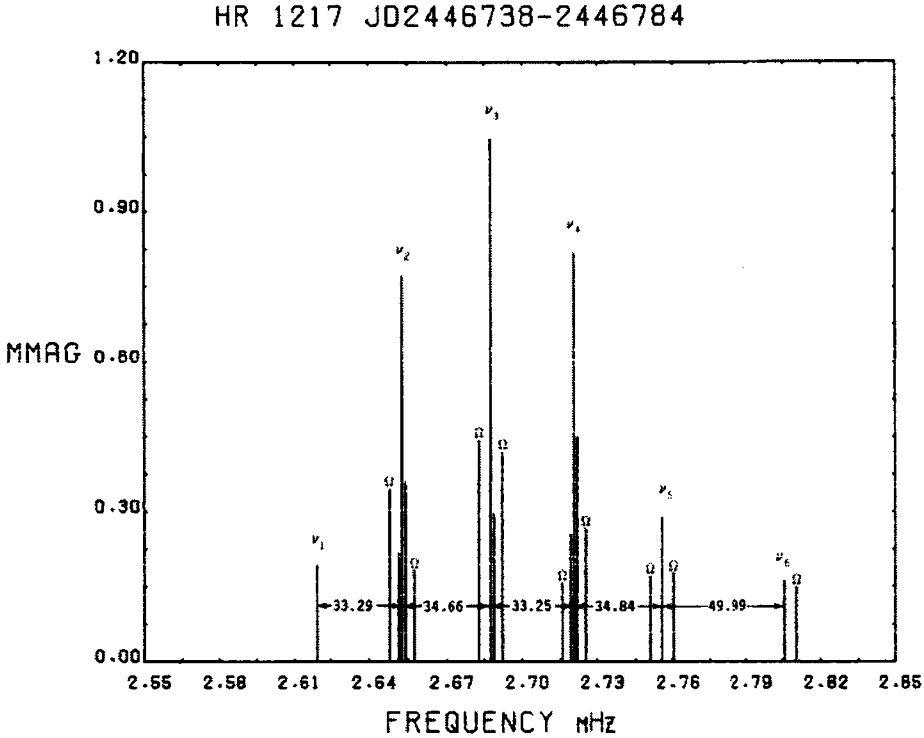


**Fig. 34.** A schematic amplitude spectrum for HR 1217 showing the frequency spacing characteristic of alternating even and odd $\ell$ modes, and similar to the pattern seen in the Sun. The frequency spacings are given in $\mu$Hz. The peaks labeled $\Omega$ are sidelobes generated by oblique pulsation. Notice the closely spaced "secondary frequencies" near to, and unresolved from, $\nu_2$, $\nu_3$, and $\nu_4$. They probably are the result of frequency modulation, although this is yet to be proved for this star. From Kurtz et al. (1989).

monitoring this variability continuously for many years now. The cause of this frequency variability is still not known.

### *10.1. Frequency Variability in HR 3831*

The roAp star HR 3831 is discussed in great detail in Section 9. The frequency analysis presented there was performed after the frequency variability was "removed" by adjusting the times of the observations, and it was promised that this procedure would be explained in this section.

The problem of frequency variability in HR 3831 was first noticed by Kurtz et. al (1994a) who could not "phase" data spanning 12 yr for this star. An amplitude spectrum of all the data produced unresolved "secondary" frequencies, as in HR 1217. HR 3831 has simpler pulsational behavior than HR 1217, though. It has only one pulsation mode, although rotational modulation of the amplitude and phase of that mode generates a frequency septuplet, as is discussed in Section 9.3 in considerable detail.

Kurtz et al. (1994a) fitted that frequency septuplet to yearly data-sets and found that there was little, or no modulation of the amplitude, as is shown in Fig. 35. With amplitude modulation ruled out, the only other explanation for the secondary frequencies is that the pulsation phase is variable. This is equivalent to frequency variability.

Traditionally, such frequency variability in pulsating stars is studied using O–C (observed minus calculated) diagrams. Essentially, times of observed maxima for a pulsating star are compared to calculated times of maxima based on a perfect clock. The difference is the O–C, and it can be used to extract useful information about the frequency variability.

For HR 3831 we chose to examine the frequency variability using a variation of the O–C diagram; the difference being that we plotted the pulsation phase versus time, instead of the observed time of maximum. This has the advantage that the entire light curve is used to determine the pulsation phase, rather than only the data around the time of maximum, thus increasing the signal-to-noise ratio.

For HR 3831 there is a complication, however. As Fig. 36 illustrates, the pulsation phase is a function of rotational phase because of the changing aspect from which the distorted dipole mode is seen in an oblique pulsator. Before it is possible to study the long-term behavior of the frequency of HR 3831 in an O-C diagram, this rotational modulation of the pulsation phase must be modeled and removed. Fig. 29 showed how the rotational amplitude and phase modulation in HR 3831 could be modeled in terms of $\ell = 0$, 1, 2 and 3 components. Fig. 36 shows a comparison between that modeling procedure and a direct fit of the frequency solution. There are small differences which do not significantly affect the calculation of the O–C values.

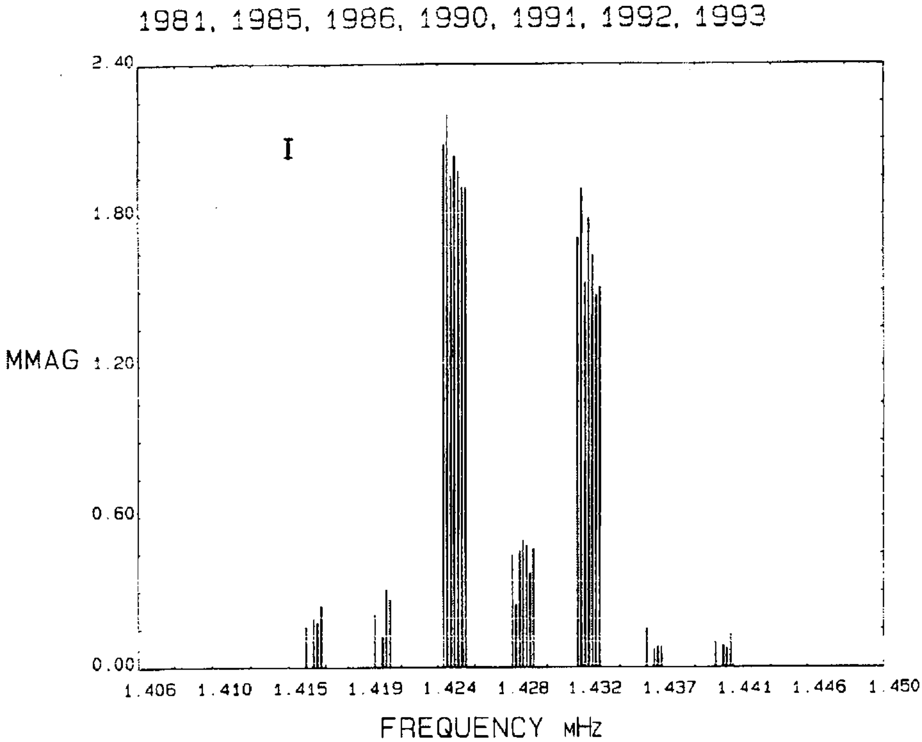


**Fig. 35.** A schematic amplitude spectrum for the fundamental frequency septuplet of HR 3831 showing the amplitudes for seven independent data sets. The amplitude spectrum for each data-set is shifted artificially by a small amount in frequency to display all seven amplitude spectra on the same diagram. For each frequency the left-most peak is for the 1981 data, the second peak for 1985, etc. For three of the data-sets, 1985, 1992 and 1993, the window pattern is too complicated to allow the derivations of any but the three central frequencies. The internal error bars are $\pm 0.1$ mmag, showing that the amplitude of HR 3831 has little, or no amplitude modulation over the time-span of the data-sets. From Kurtz et al. (1994a).

To study the long-term phase behavior of HR 3831 we must remove the rotational phase variability. This is an iterative procedure for which the final results are shown in Fig. 37. We begin by fitting an analytic relation to the rotational phase variability for the short-time span, high duty cycle data-set obtained from JD2448303–8320 (Kurtz et. al 1993 – KKM). There are two ways to determine the an-

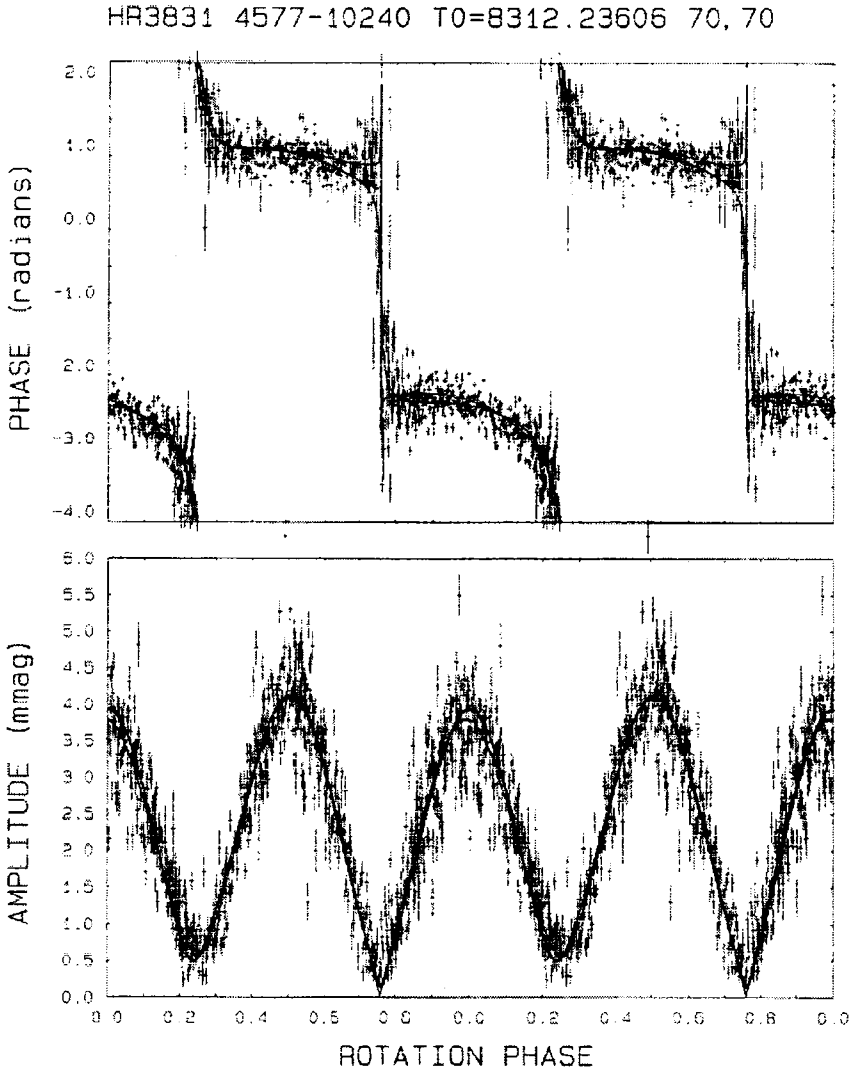


**Fig. 36.** Rotational phase and amplitude modulation in HR 3831. Each point in this diagram shows the pulsation amplitude (bottom) and pulsation phase (top) determined from a least squares fit of the central frequency of the septuplet to 1 h of data. The heavy solid line fits are for a spherical harmonic decomposition of the pulsation mode into $\ell = 0, 1, 2, 3$ components, and the thin solid line is a direct 7-frequency solution of the amplitude spectrum. A similar curve to the top one for 41%-duty-cycle data of KKM defines the "calculated" phase from which the O–C values are determined for HR 3831.

alytical relation used to fit the data. One is calculated directly from the frequencies, amplitudes and phases of the frequency septuplet. The other is from a decomposition of the frequency septuplet into a spherical harmonic series using the technique of Kurtz (1992). (Both are shown in Fig. 36.) We then find the pulsation phase for each night by fitting the central frequency of the septuplet by least squares to the data. The difference between that phase and the analytical representation of the rotational phase from the JD2448303–8320 data is what we call the phase O–C for that night. If there were no frequency variability in HR 3831, this procedure would result in a set of non-variable O–Cs scattered about a mean of zero.

The frequency variability which is present for HR 3831 means that a linear ephemeris will not fit all of the data. We remove the frequency variability in the time domain by shifting the observed times by an amount which forces the phase O–Cs to zero. In practice, we do this by fitting polynomials to short sections of the O–C diagram shown in Fig. 37. With the frequency variability removed from the data, we then solve for the best fitting frequency septuplet to give the analytical form of the rotational phase variation shown in Fig. 36. The procedure is then repeated from the start to remove the rotational phase variations and show the long-term phase O–C, which illustrates the frequency variability.

The results of these procedures are shown in Fig. 37 which is the phase O–C diagram for the last seven years during which we have been monitoring HR 3831 intensively in a long-term observing program at the South African Astronomical Observatory (SAAO). During these seven years we have been observing this star throughout its observing season (late October through early July) for one hour (five pulsation cycles) on several nights each week whenever one of us has the 0.5-m telescope. For the collaborators involved in this work – primarily Fred Marang, Francois van Wyk and Greg Roberts of the SAAO – this is for the greater part of the year. Other observers contribute light curves as the opportunity presents itself. Kurtz et al. (1997) showed the latest published O–C diagram; Fig. 37 shows an even more up-to-date picture.

Clearly, the frequency is variable. A constant frequency would produce a straight line in an O–C diagram. The largest range of the frequency variations is between the first two years of data shown in Fig. 37 where the slopes are largest; that represents a change in the pulsation frequency of 0.12 $\mu$Hz.

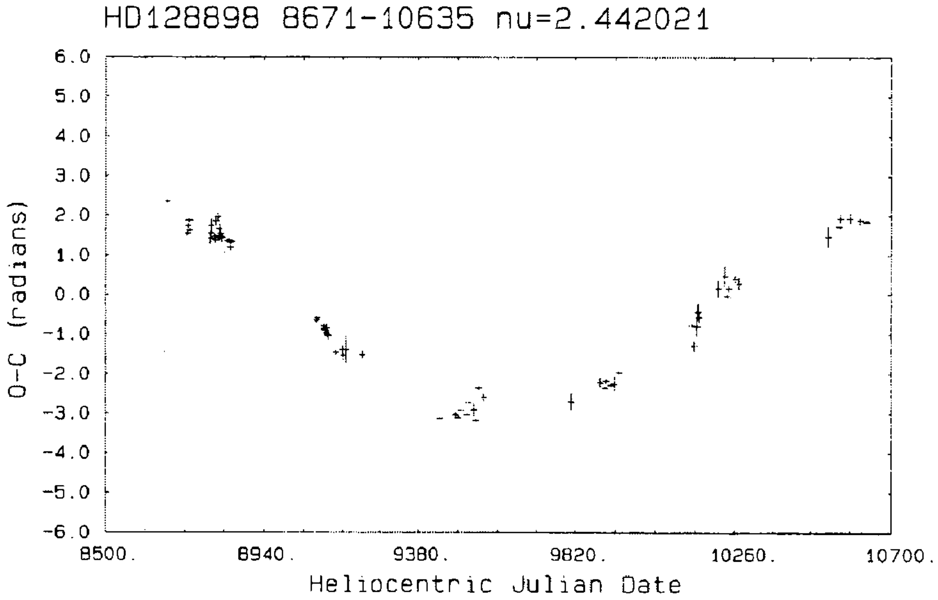


**Fig. 37.** The O–C diagram for HR 3831 for data obtained from 1992–1998. Each point represents the pulsation phase for one night of observation (usually one hour, or 5 pulsation cycles) minus the calculated pulsation phase from a fitted curve for the 41%-duty-cycle data of KKM. See the text for a discussion. The observations are primarily those of Fred Marang, Francois van Wyk and Greg Roberts of the South African Astronomical Observatory.

To us the frequency variability looks to be cyclic on a time-scale of about 1.6 yr. It is clearly not periodic, but there does appear to be a characteristic time-scale. This is, of course, speculative. Only more data will show whether it is correct, or not. We are patiently and methodically gathering those data.

### *10.2. Frequency variability in $\alpha$ Cir (HD 128898; HR 5463) and HD 134214*

Once we knew about the frequency variability in HR 3831, we decided to start long-term monitoring of two other roAp stars, $\alpha$ Cir and HD 134214. The latter is singly periodic, with the shortest known period in an roAp star, 5.65 min (Kreidl et al. 1994), and the former, $\alpha$ Cir, has one, large-amplitude frequency with several very-much-smaller-amplitude frequencies which do not perturb the

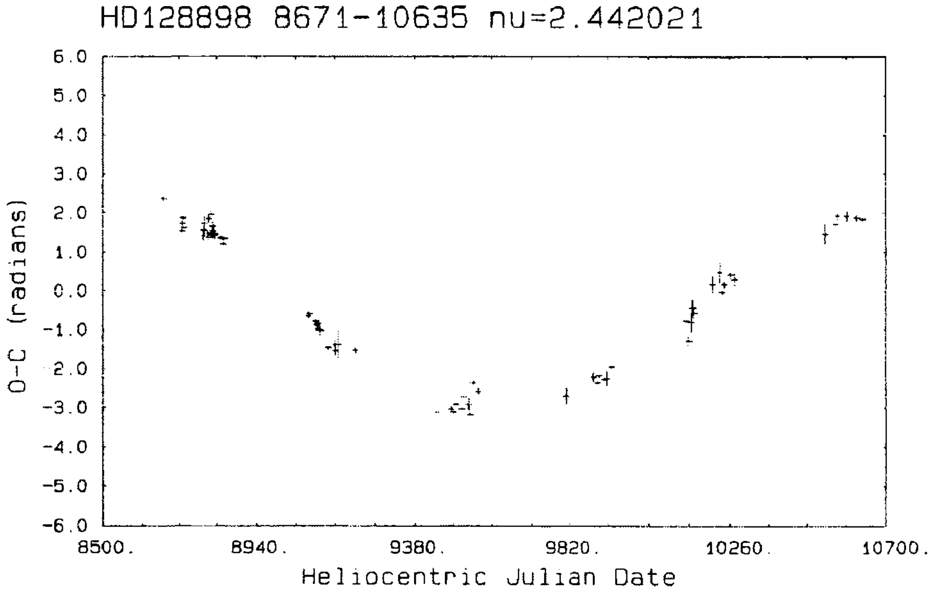


**Fig. 38.** The O–C diagram for $\alpha$ Cir for data obtained from 1993–1998. Each point represents the pulsation phase for one night of observation (usually one hour, or about 9 pulsation cycles) minus the calculated pulsation phase for a constant frequency. The observations are primarily those of Fred Marang, Francois van Wyk and Greg Roberts of the South African Astronomical Observatory.

principal frequency significantly (Kurtz et al. 1994b). This means that phase O–C values for these stars can be calculated directly without the need of the rotational modeling that is necessary for the more complicated case of HR 3831. Figs. 38 and 39 show the O–C diagrams for the last six years for $\alpha$ Cir and HD 134214.

### *10.3. Discussion of the frequency variability*

An obvious first thought when confronted with frequency variability in a pulsating star is that it might be caused by a Doppler shift as a result of orbital motion in a binary, or multiple star system, or in a planetary system (for small enough frequency changes). Kurtz et al. (1994a) showed the first two seasons of intensive monitoring for HR 3831, which had the look of a sinusoidal variation in the phase O–C diagram (see the first two seasons of data in Fig. 37). They argued that it was not plausible that this frequency variation could

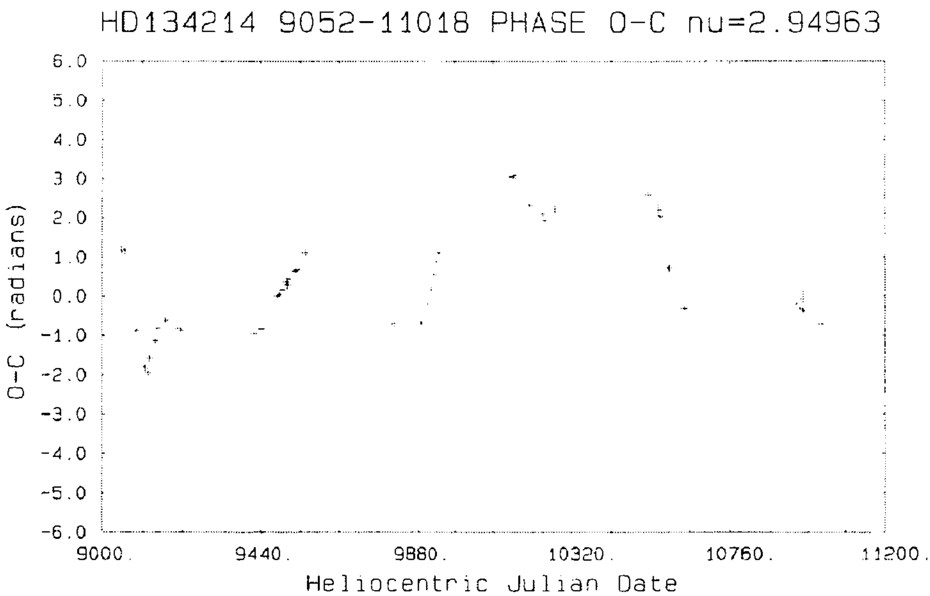


**Fig. 39.** The O–C diagram for HD 134214 for data obtained from 1993–1998. Each point represents the pulsation phase for one night of observation (usually one hour, or about 11 pulsation cycles) minus the calculated pulsation phase for a constant frequency. The observations are primarily those of Fred Marang, Francois van Wyk and Greg Roberts of the South African Astronomical Observatory.

be caused by a Doppler shift from binary motion, because of the upper limit to the radial velocity variations of 3 km s$^{-1}$, and because the magnetic Ap stars have a very low binary frequency (see Wolff 1983). Fig. 37 lends extremely strong support to those arguments. Any interpretation of the frequency variability in terms of Doppler shifts would now need to postulate many orbiting companions, and still could not surmount the radial velocity limitation. Similar arguments apply to the complex frequency variability of HD 134214 shown in Fig. 39. Thus, it is reasonably certain that the frequency variability is intrinsic to these stars, and is not caused by Doppler shifts.

Some physical property of the pulsation cavities in these stars is changing on a time-scale of many months to years. There may also be some shorter time-scale variations in Figs. 37, 38 and 39, but we are uncertain about their reality. Frequency variability is also observed in other roAp stars. Similar, though less extensive, results are reported for HD 12932 (Martinez et al. 1994b), and, in fact, there is no roAp star which has been monitored over several seasons which shows a constant frequency. Frequency variability seems to be the norm in these stars.

The question obviously arises about whether there is any amplitude variability in conjunction with the frequency variability in these stars. Fig. 35 shows that there is little, or no, amplitude variability in HR 3831 by comparing the yearly amplitude spectra. Kurtz et al. (1997) used the more sensitive measure of amplitude variability, the amplitude O–C diagram. In the determination of the nightly phases by least-squares fitting of the central frequency, they also determined the nightly pulsation amplitudes. This amplitude also varies with the rotation cycle of the star, so, completely analogously with the procedure used to produce the phase O–C diagrams, they computed the *amplitude* O–C by taking the difference of the nightly amplitude to the analytical fit of the frequency solution shown in Fig. 36. The result is that there is marginal evidence for small amplitude variations.

This amplitude variability has not been explained. Because it seems to be correlated with the frequency variability, its cause is probably the same as the (unexplained) cause of the frequency variability. The amplitude variability gives further proof that Doppler shifts cannot explain the frequency variability. Doppler shifts do not affect amplitude.

An interesting speculation is that the roAp stars may have magnetic cycles. The magnetic field in these stars is thought to be a fossil field "frozen in" during the formation of the star. There is no large convection zone in the stellar envelope to support a dynamo of the kind that is thought to generate the magnetic fields of cooler stars. But the cores of the A stars are convective, and it is not impossible that there may be some small dynamo contribution to the magnetic field which could be modulated cyclically.

Following the work of O. C. Wilson (1978), Baliunas & Vaughan (1985) have continued monitoring solar-type stars with a photometric measure of the emission cores in the Ca H and K lines, which they call $S$. In the Sun these emission cores are known to be correlated with the solar cycle, so when cyclic variations in the emission measure $S$ are seen in other stars, that is taken to be strong evidence of solar-type magnetic cycles in those stars. Fig. 40 shows the results of long-term monitoring of $S$ in a sample of four solar type stars from over 100 stars monitored by Wilson, then Baliunas & Vaughan.

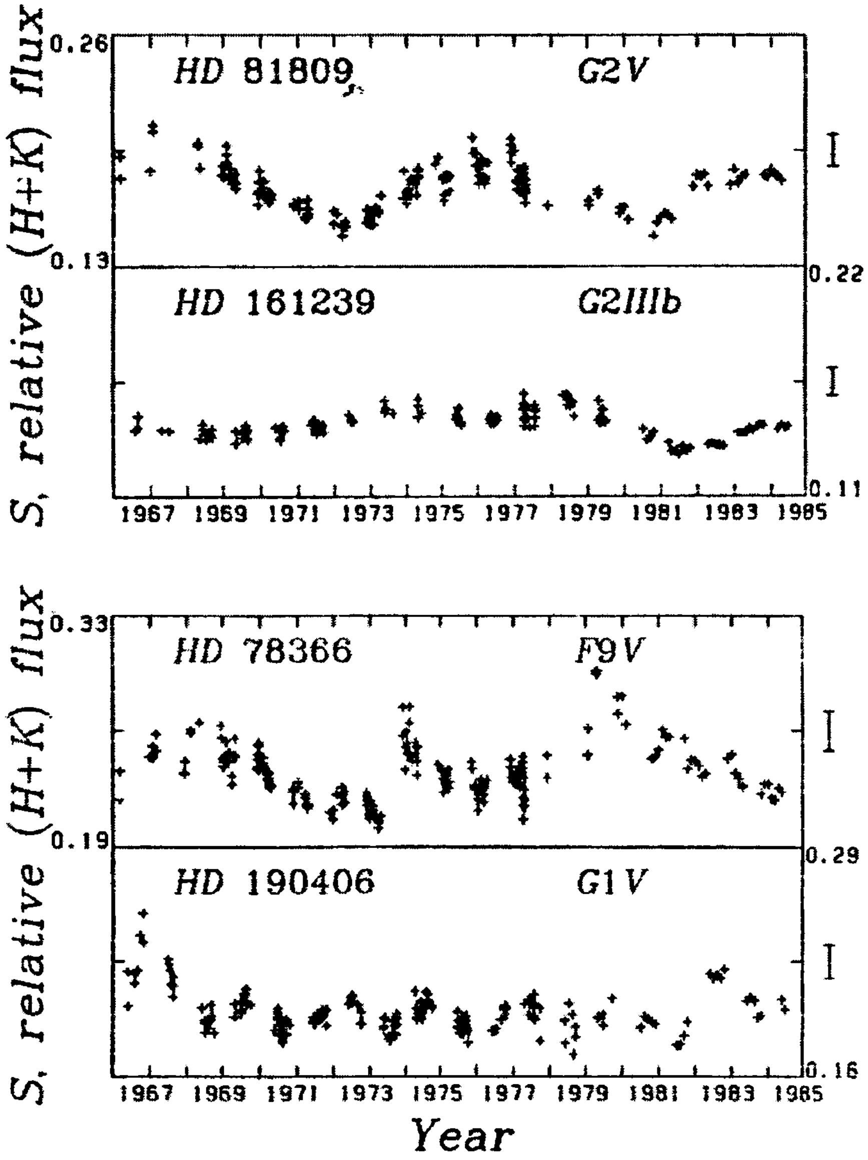


**Fig. 40.** Long-term variability in the magnetic cycle proxy index, $S$, a measure of the emission line reversals in the centers of the Ca H and K lines. Note the cyclic, but not perfectly periodic, behavior of $S$ which is expected for magnetic cycles. The cyclic time-scales range from about 2.5 yr for the HD 190406 to roughly 13 yr for HD 161239. From Baliunas & Vaughan (1985)

The pulsation frequencies in the Sun also vary with the solar cycle by about a tenth of a $\mu$Hz (for the low solar frequencies which are similar to those in the roAp stars). If we take these frequency variations to be a proxy measure of the solar cycle, we are led to the speculation that 0.1 $\mu$Hz variations in the roAp stars on time scales similar to the magnetic cycle time-scales of Fig. 40 may indicate that there are magnetic cycles in the roAp stars. Unfortunately, this speculation is very difficult to test. There is no known chromospheric activity which is measurable in the Ap stars, and which might provide an independent measure of cyclic variability.

These observations of frequency variability do show that there are intrinsic changes in the pulsation cavities of the roAp stars on time scales of months to years. The frequency changes are orders of magnitude larger than those expected from stellar evolution in the roAp stars (Heller & Kawaler 1988), so there is little hope of measuring evolutionary time-scales directly from period changes in these stars. It has become clear that the same is true for period changes in the $\delta$ Scuti stars (Breger & Pamyatnikh 1998).

Whatever the mechanism of the frequency variability, it is an intriguing problem which is very likely to pay off in increased understanding of these stars ... once we eventually figure out what causes it.

## 11. OBSERVING roAp STARS WITH WET: COMMON PITFALLS AND INSTRUMENTAL SOURCES OF ERROR

As part of the Whole Earth Telescope (WET) fifth extended coverage run in 1990 May, the rapidly oscillating Ap (roAp) star HD 166473 was observed as a tertiary target. This was eventually termed a "learning experience": no science resulted from it. The problems were (i) a poor duty cycle because HD 166473 was a third-priority target, (ii) the very low amplitude of HD 166473 (less than a mmag for the highest amplitude modes), and (iii) photometry from many WET sites in which the noise level was too high to detect any signal. In the decade since then WET has not attempted observations of roAp stars again. We believe that exciting science *can* successfully be done by WET on roAp stars, provided that due cognizance is taken of the care and precision required to produce useful roAp star photometry.

Because of the extremely low amplitudes of the roAp stars it is imperative that roAp star photometrists be aware of, and overcome, a wide variety of faults which are often tolerated in less demanding observations, such as:

1. The use of small apertures causes light losses from tracking errors and seeing fluctuations. Generally, apertures $\geq 30$ arcsec should be used, unless a *tested* autoguider is being used. Even then, apertures should generally be $\geq 20$ arcsec. *If the star is bright, you don't need a small aperture.*
2. Careless manual guiding causes spurious dips in the light curve which have to be excised. Such gaps reduce the duty cycle and introduce unwanted sidelobes in the Fourier spectrum of the time series. Similar problems occur when manual dome rotation is not monitored carefully.
3. In some telescopes moonlight reflects off internal parts of the telescope causing problems, especially when the moon is close to the program star. This often causes step discontinuities in the light curve when the dome is rotated; blackening the inner edges of the dome shutters often solves this problem. Since roAp star are bright, they are often observed during bright time. HR 1217 is a prime WET candidate; at $\delta = -12°$ the full moon can pass close to the star at the time of year when it transits at local midnight – the best time for a WET campaign.
4. For photometers without dedicated sky channels bad monitoring of sky background causes the sky subtraction process to introduce spurious trends which manifest themselves as low-frequency noise. This is especially true for observations acquired near astronomical twilight, during a lunar eclipse or at moon rise/set.
5. Sensitivity variations across the aperture cause guiding corrections to introduce discontinuities in the light curve. Such variations also exacerbate the deleterious effects of the periodic drive error of the telescope. It is easy enough to test the flatness of the aperture by positioning the star in the four quadrants of a large aperture and noting the count rates at each position, or performing drift scans. Such sensitivity variations are *common* in many photometers but may not have been noticed with high photon statistical noise with fainter stars.
6. In cooled photometers, damp photomultiplier tube bases are often caused by power failures unnoticed by the observer during the day. Often power is restored with a minimal delay, but if the power is down for long enough, the cold-box starts to warm

up and moisture condenses on the photomultiplier tube base thus providing an electrical conduction path for inter-electrode leakage. This causes greatly increased scatter in the count rates, higher dark counts and erratic excursions in the light curve. The only solution is to bake out the moisture from the PMT and base. The presence of dirt or grease on the base of PMT can lead to similar problems. Another consequence of daytime power cuts is that the PMT sensitivity may not stabilize by nightfall.

7. Some photomultiplier tubes may have sensitivity drifts for 10 to 30 min when first exposed to bright starlight. This can happen even when the counts are well below the maximum safe rate for the tube. Test your tube. If this is a known characteristic of your tube, expose the tube to bright starlight during twilight so it is settled by the time observations commence.
8. In refrigerated systems moisture can also condense on the Fabry lens or glass window of the photocathode thus fogging these surfaces. To prevent this, the Fabry lens should be maintained at a slightly elevated temperature with respect to its surroundings in the coldbox. The power connections for Fabry heating are often tiny, delicate wires which can easily be dislodged. Check them.
9. Dirt on filter surfaces exacerbates problems with the drive error of the telescope at best and produces useless data at worst. Another good reason for checking the filters at the start of an observing run is that often the filters in the instrument may not be the ones the observer thinks! Alternatively, the filters might be scratched or otherwise damaged or have deteriorated. Interference filters (e.g. Strömgren) deteriorate with time. This deterioration is accelerated in a humid environment and may be retarded by storing filters in a desiccator.
10. Dirt sometimes accumulates on the glass window of the photocathode. Many observers keep their filter sets scrupulously clean only to place them in a photometer whose photocathode is dirty. Check for dirt introduced by the dark slide or resulting from abrasion of the dark slide itself.
11. Vignetting can be a frustrating source of error to find. Several years ago it was discovered that the baffle on the secondary mirror of one of the SAAO telescopes was vignetting the beam. The problem was so subtle that it took two years to find its cause.
12. Vignetting problems can also arise in the photometer through misalignments of various moving prisms, mirrors, filters and

apertures, usually because these components are not well seated in their detentes.

13. The long time series acquired in high-speed photometry often reveal flexure problems in photometers. Flexure in the photometer is indicated if the program star gradually drifts off-center in the aperture while the guide star is kept centred. This problem is eliminated by recentering the program star every hour or so.
14. Spurious periodicities are often introduced into the data by the drive oscillations which are present in most telescopes. The amplitude of the drive error goes as the cosine of the declination. These oscillations should present no problem if the detector response is flat across the aperture and there is no dirt or vignetting in the photometer. Drive errors are indicated if the data show an oscillation with a period equal to an integral number of sidereal minutes. The most common telescope drive errors are those with periods of 2 or 4 sidereal minutes (8.356 or 4.178 mHz, respectively). In the case of HD 60435, a 4-mHz oscillation was "studied" for several years before it was realized that this was an oscillation in the telescope and not in the star!
15. A telescope that is too finely balanced "floats", causing erratic excursions of the star in the aperture. It is best to have a slight imbalance so that the worm pushes against a load on the wormwheel; not the other way around!
16. Electronic malfunctions such as drifting or fluctuating HT supply introduce spurious trends in the data. The very large variation in gain produced by a small variation in voltage makes it essential to use a very well regulated voltage supply. This may be a particular problem for some WET sites in countries with undependable national power grids.
17. Inadequately shielded photomultiplier tubes are affected by variations in the local magnetic field as when, for example, an iron observing ladder is moved close to the instrument. Test for this at the start of a run with an unfamiliar instrument.
18. It is not uncommon to have electrical switching noise caused by coldbox coolers, lights, dome rotation motors, wind-blind motors, printers and radios causing spikes in the data stream. Test for this at the start of a run with an unfamiliar instrument.
19. Arcing sometimes arises in PMT cable connectors in old cables or pulled cables. This gives rise to spikes or erratic fluctuations in PMT count rates. Make sure your electrical cables are in good condition. Check for possible damage where cable wrap

can occur. Someone else may have caused such damage since you last used the telescope.

20. Poor thermal regulation in cold boxes can lead to high dark current as the PMT warms up. This often appears as a periodic oscillation in the count rates as the cooler unit switches on and off with some characteristic period. One should also remember that temperature variations cause changes in the spectral characteristics of the photocathode, with a general increase in red sensitivity with increasing temperature. For $U$-band photometry this means that the red leak will increase as the PMT temperature increases.
21. Inaccurate time is also an insidious source of error. It is vital that the observer should have reliable means of checking the time accurate to $\simeq 0.1$ sec independent of the data acquisition system. Computing the Heliocentric times to sufficient precision is vital to producing useful roAp star photometry, especially when subtle period changes or frequency shifts are being sought.
22. Errors of 1 day in HJD are easy to make. Record in your log the date in two forms: HJD and DD/DD+1/MM/YY local date. At SAAO, a prime WET site, the data acquisition program used for WET runs sets the civil date in the header wrong by one day when the program is started after midnight local time. DRED then gets the HJD wrong by one day. This problem has persisted for a decade. Other observatories may have similar problems. Always check!

The above sources of error arise in the instrumentation and can all be eliminated or minimized by careful maintenance. We end this section with the comment that a graphical on-line data display is absolutely essential to detecting many of these problems at the telescope in time for remedial action to be taken.

## 12. WHICH roAp STAR SHOULD WET OBSERVE?

### *12.1. HR 1217*

A three-week WET run on HR 1217 could do the following:

- Redetermine the frequency spectrum to higher precision. The frequencies are separated by $\approx 3$ d$^{-1}$ giving mediocre results even from the previous study with its 29% duty cycle; WET is needed. All frequencies are at, or above, the calculated critical frequency; models are poor and probably inadequate; next to nothing is known about the precise frequency spacing expected, so precision determination is a first step to motivate the modeling. Theoreticians in the WET consortium will have an unsolved problem to work on.

- Determine the amplitudes and phases of the rotational sidelobes to measure the distortion of the modes from pure spherical harmonics. This is particularly interesting for the modes thought to be $\ell = 0$. The problem of how magnetic fields interact and modify pulsation modes in the atmospheres of stars is essentially unstudied. The only other star for which there are data is HR 3831, and it has a single distorted dipole mode. The presence of both distorted dipole modes and radial modes should constrain the interaction more than has been previously possible. Little has been done previously.
- Remeasure the separation of $\nu_5$ and $\nu_6$ to see if it really is 50 $\mu$Hz $= 3/4\Delta\nu_0$. If it is, an explanation is needed.
- Find out if $\nu_6$ is a rotational doublet (see Fig. 34); if so, an explanation is needed.
- Study possible frequency variability on a time-scale as short as the WET run.
- Use the frequencies to model the star using the best Ap models available. The atmospheres of Ap stars are abnormal. HR 1217 has been studied spectroscopically for abundances, magnetic field and abundance patches; frequency fitting asteroseismically may give better knowledge of the atmospheric structure which would then feed back to better understanding of the abundances. Ryabchikova et al. (1997) found "a systematic difference between surface gravities obtained from spectroscopy and from both asteroseismology and evolutionary tracks [...] for the roAp stars [HR 1217], $\alpha$ Cir, and $\gamma$ Equ."
- Search for additional frequencies from a better amplitude spectrum, with lower noise levels, than in the previous study.

### *12.2. Some problems*

**HR 1217 is bright.**

A major problem for WET is that HR 1217 is too bright for some of the telescopes in the network, if a Johnson $B$ filter is used. A possible solution to this is to buy and distribute Strömgren $v$ filters to all participants. The count rate for HR 1217 at the

SAAO 1-m telescope with a $B$ filter is about $10^6$ s$^{-1}$. Strömgren $v$ will reduce this by about a factor of 5 to a manageable $2 \times 10^5$ s$^{-1}$. Even so, with a 2-m telescope the problem is still there with the Strömgren $v$ filter (although one could use a neutral density filter to overcome this problem).

For roAp stars the main noise source is scintillation, so larger aperture is an advantage. The solution is therefore not simply to avoid the larger telescopes – they give better s/n. Also, at the high frequency of HR 1217 (2.6 mHz) the sky is well-behaved except at the worst sites, or on very poor nights.

**Moonlight.**

Because of its brightness and the need for a three-week run, HR 1217 may need to be observed near full moon. In November and December 2000 the nearly full moon is at declination +17 when it passes through the same right ascension as HR 1217. That means they will be less than 30° apart on 13 November and 10 December 2000 – this could result in problems of internally scattered moonlight in some telescopes in the network.

**Uncertainty in the rotational ephemeris.**

The rotational ephemeris of HR 1217 is in dispute by a small amount. This means we can only calculate the time of maximum pulsation to within a few days. We suggest the Mathys ephemeris. That gives magnetic (hence pulsation) maxima at HJD 2451854.4 (6 November 2000), 2451866.9 (18 November 2000), 2451879.4 (1 December 2000) and 2451891.8 (13 December 2000). The Kurtz & Marang ephemeris give maxima which precede those given by about 3 d. If a three-week run should be chosen, two maxima are better than two minima in terms of potential signal-to-noise.

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