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NONLINEAR BEHAVIOR OF THE PULSATING WHITE DWARF GD 358

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Abstract. Several analyses and qualitative arguments excluded resonant mode coupling and nonlinear driving as the main nonlinear processes responsible for the presence of the numerous cross-frequencies in the temporal spectrum of GD 358. Rather, harmonic distortion is the prime cause. We believe that the nonlinear response of the convection zone to the oscillatory perturbation is the dominant harmonic distortion mechanism.

The amplitude variations recorded in the temporal spectrum of GD 358 might be accounted for by long period beating between eigenmodes and third-order cross-frequencies. This permanent process is referred to as high-order beating.

Key words: stars: interiors, oscillations, individual: GD 358

1. INTRODUCTION

The analysis of the period spectra, also called temporal spectra, of pulsating white dwarfs can reveal a lot about the internal structure of these stars. This is the aim of asteroseismology: using the pulsations in order to probe stellar interiors and extract various stellar parameters.

The asteroseismological models are to date entirely based on the linear adiabatic theory of stellar pulsation, even though all large amplitude pulsating white dwarfs show definite nonlinear behaviour. It is therefore important to study these behaviours, not only to better understand the physics of the stellar pulsations, but also to potentially gain further asteroseismological insight into white dwarf interiors.

GD 358 is the most observed and best understood DB variable white dwarf. Two Whole Earth Telescope (WET) runs have targeted this star in 1990 (Winget et al. 1994) and 1994 (Vuille et al. 1999). The excellent coverage and S/N ratio of these campaigns led to a quite secure mode identification. In this sense, GD 358 is an ideal star on which to perform nonlinear analysis.

2. NONLINEAR BEHAVIOR

Despite the very resemblance between the amplitude spectra obtained from the 1990 and the 1994 WET campaigns (Fig. 1), GD 358 nevertheless displays distinctive nonlinear behaviour. Two specific nonlinear features can be distinguished. First is the presence of numerous harmonics and combination frequencies in the amplitude spectrum (a combination frequency is a linear combination of frequencies of normal modes). Second is the spectral amplitude variations.

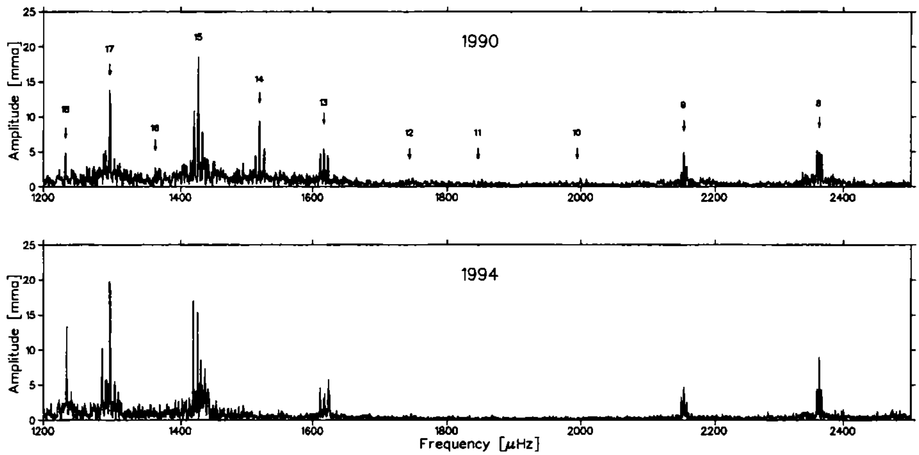

Fig. 1. Comparison of the temporal spectra, in the frequency interval with largest amplitude, from the 1990 *(upper panel)* and 1994 *(lower panel)* WET campaigns. The radial orders k identified in the 1990 spectrum are indicated in the top panel. Reproduced from Vuille et al. (1999).

2.1. Combination frequencies

Several distinctive nonlinear processes may generate combination frequencies in the amplitude spectrum of pulsating white dwarfs. The major such processes are:

- The nonlinear response of the emergent flux to the surface temperature variations ($L = \sigma T^4$).
- The nonlinear response of the convection zone to the oscillatory perturbation travelling through it.
- Resonant mode coupling.
- Nonlinear driving.

The first two of these processes, commonly referred to as harmonic distortion effects or "pulse shape effects", originate from the inability of the stellar medium to respond linearly to the pulsations. This naturally redistributes part of the energy contained in the excited eigenmodes into harmonics and cross-frequencies.

In the 1990 and 1994 WET runs, only 8 normal modes were present, while 27 and 62 combination frequencies were respectively identified (Winget et al. 1994, Vuille et al. 1999). Several arguments suggest that these nonlinear peaks owe their presence to the response of the convection zone to the pulsations, and not to the other processes listed above. We devote the rest of this section to this discussion.

Resonant mode coupling can be discarded as the main phenomenon responsible for the presence of the cross-frequencies, simply because too many such peaks have been identified; as the density of the g-mode spectrum decreases with increasing frequency, the forest of sum frequencies observed cannot be all identified with normal g-modes. The analysis of the relative phases of the cross-frequencies provides a sound quantitative support to this conclusion. The relative phase of a (second-order) nonlinear frequency is defined as $\varphi_r = \varphi_c - (\varphi_1 \pm \varphi_2)$, where φ_c, φ_1, and φ_2 are the phases of the combination frequency and of the parent eigenmodes respectively. The "+" and "−" signs refer to sum and difference frequencies. Fig. 2 shows the relative phases of all the second and third-order combination frequencies identified in the 1990 and 1994 data sets.

A clear grouping of the relative phases around the zero line can be seen in Fig. 2, which means that the nonlinear frequencies oscillate in phase with their parent normal modes. No particular phase

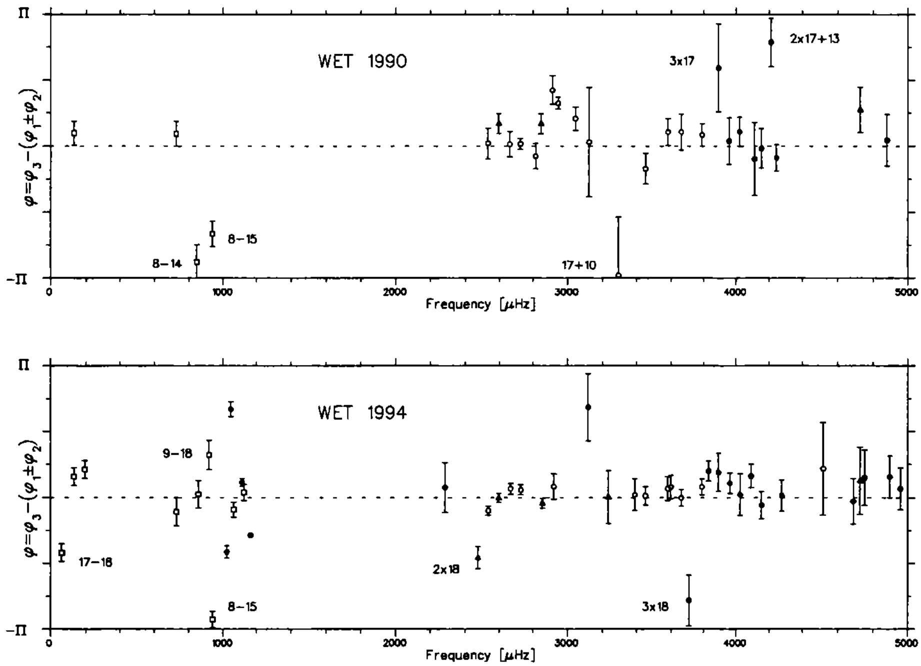

Fig. 2. Relative phases of the combination modes as a function of their frequencies. The difference frequencies are indicated by open squares, the first harmonics by triangles, the sum frequencies by open circles, and the third-order combinations by filled circles. The few cross-frequencies whose relative phases show a definite non-zero value are labelled, for the sake of discussion, by the k-order of the normal modes that combine to form them. Reproduced from Vuille & Brassard (1999).

relationship is expected between eigenmodes involved in a resonant process, because mode coupling is only dependent on the relative amplitudes of the modes coupled to each other. Some phase relationship may exist in certain cases (Kovács & Buchler 1989), but it would not necessarily be an "in phase" one.

The phase relationship seen in Fig. 2 is precisely what is expected from harmonic distortion (Vuille 1999), which strongly suggests that such a process is responsible for the presence of the combination frequencies in the temporal spectrum. This contention is further supported by the fact that nonlinear driving, as opposed to a sinusoidal driving of the pulsations, can be excluded as the process generating these nonlinear frequencies, on the following grounds:

• The clear phase relationship observed in Fig. 2 indicates that the normal modes and the cross-frequencies are physically related to each other. Such a correlation would not be expected if the driving was itself nonlinear, because all the different frequencies would then be individually excited.

• Nonlinear features appear only in the light curve of large amplitude variable white dwarfs (e.g. Robinson 1979), which is exactly what is expected from harmonic distortion, but certainly not from an intrinsically nonlinear oscillator. An illuminating analogy is given by heart beats (Fig. 3) to which stellar pulsations are often compared. Heart beats are clearly nonlinearly driven, in the sense that the general shape of the cardiac cycle is the same, no matter what the amplitude of the pulsations are, i.e. no matter what the size of the animal considered is. Pulsating white dwarfs should thus be compared to loud-speakers, rather than to heart beats; loud-speakers behave linearly if the amplitude is small, but saturate if the amplitude is too high, thus producing a distorted output signal.

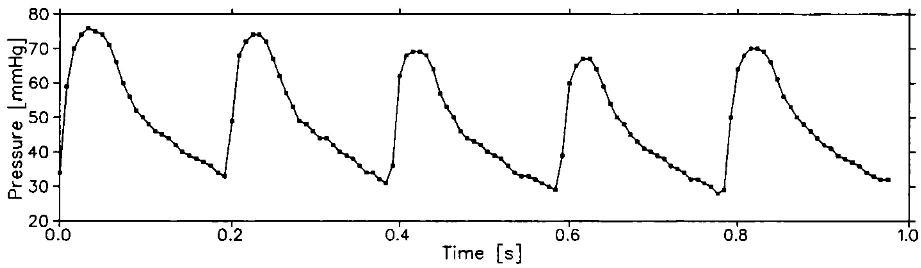

Fig. 3. Pressure in the aorta of a rat during five cardiac cycles.

• The excellent signal-to-noise ratio obtained during the 1994 WET campaign enabled the identification of virtually all the possible second and third-order combination frequencies (Vuille et al. 1999), which is also consistent with harmonic distortion effects. There would be, a priori, no reason for a nonlinear driving process to systematically excite all the combination frequencies at all orders of perturbation.

• Clemens, Van Kerkwijk, & Wu (1999) showed that, in the DAV G 29-38, the combination frequencies have no associated horizontal velocities, whereas the normal modes do. This indicates that these two categories of frequencies bear a different physical origin, which thus excludes nonlinear driving as being the cause of the cross-frequencies. No similar analysis has been conducted yet on GD 358, or on any other DB variable, but the above conclusion could possibly hold.

The above discussions securely established that harmonic distortion is the major process responsible for the presence of cross-frequencies in the period spectrum of GD 358. The same conclusion was reached in the case of the DAV G 29-38 (Vuille 1999), and this is likely to hold true for all large amplitude pulsating white dwarfs.

To try and determine which, if any, of the different harmonic distortion processes is dominating in GD 358 is not a trivial problem. As none of these processes bear a distinctive signature, they are difficult to disentangle. We nevertheless assert that the response of the convection zone to the oscillatory perturbation is the dominant nonlinear process in action, and that the radiative transfer has a negligible impact in comparison. This contention is based on the following arguments:

• The model of Wu (1998), which treats the nonlinear response of the convection zone to the pulsations, reproduces quite well the nonlinearities observed in the amplitude spectrum of GD 358.

• The models of Wu (1998) and of Brickhill (1992), which both treat the response of the convective layer, predict the amplitudes of the cross-frequencies to the correct order of magnitude for the DAV G 29-38, whereas the model of Brassard et al. (1995), which treats the radiative transfer problem, gives predictions two orders of magnitude too low (Vuille & Brassard 1999). Because GD 358 is a DB variable, the visible part of its emission spectrum falls in the Rayleigh-Jeans tail of its energy distribution, where the emergent flux is more or less proportional to the temperature ($L \propto T$). The nonlinearities generated by the radiative transfer are therefore expected to be even smaller in DB variables than in DA variables where an $L \propto T^4$ relation holds.

• The models of Wu (1998) and of Brickhill (1992) both predict that the combination frequencies should have phases slightly in advance compared to that of their parent normal modes ($0 < \varphi_r \ll \pi$), whereas the model of Brassard et al. (1995) predicts the nonlinear frequencies to be perfectly in phase with their parent modes ($\varphi_r = 0$). A closer look at Fig. 2 shows that the vast majority of the relative phases measured are slightly above zero, indicating a phase advance. In order to determine whether this advance is statistically significant, we measured the average relative phase over all the data points in Fig. 2 (after having removed the few obviously off points). This gave $<\varphi_r>= 0.17 \pm 0.06$, indicating a significant phase advance, which therefore supports the models of Wu and of Brickhill.

2.2. Amplitude variations

Even though the 1990 and 1994 temporal spectra look qualitatively very similar, each mode has experienced significant amplitude variations (Fig. 1). Such variations were even recorded during the course of these campaigns (Vuille & Brassard 1999). These two WET runs are respectively 170 and 270 hours long, while the beating between the different components of the identified eigentriplets have periods not exceeding 80 hours. Beating between the normal modes cannot therefore be held responsible for these observed spectral changes.

Although mode coupling processes, such as 1:1 resonances or simultaneous coupling of the identified eigenfrequencies with invisible modes could possibly account for these amplitude variations, the process of *high-order beating* appears as a more natural candidate. The physics of this phenomenon, discussed briefly in Vuille et al. (1999) and in full detail in Vuille & Brassard (1999), is chiefly the following.

If a star, such as GD 358, shows asymmetric eigenmultiplets and has large enough pulsational amplitude to generate harmonic distortion effects, numerous third-order combination frequencies (i.e. combinations of three normal modes, $f_c = f_1 \pm f_2 \pm f_3$) are then expected to fall within the frequency range of each eigen-multiplet. These third-order frequencies are much smaller than the normal modes, and cannot individually affect the latter. However, they are so numerous (in the case of GD 358, 153 of them are expected to blend each eigentriplet) that their combined amplitudes may generate significant beating processes with the normal modes. Indeed, these third-order peaks are so closely flanking the eigenmodes, that several months of continuous data would be necessary to resolve their structure, i.e. to resolve the beating processes that they inevitably generate.

High-order beating might account for the mild amplitude variations observed in between, and during the WET campaigns on

GD 358. It cannot however account for drastic spectral variations where the qualitative aspect of the amplitude spectrum of a star changes drastically from an observing run to the next. The DAV G 29-38 (Kleinman et al. 1998) is an example of such a star.

3. CONCLUSION

We have shown that most, if not all, the combination frequencies identified in the period spectrum of GD 358 owe their presence to harmonic distortion, rather than to mode coupling or to nonlinear driving. The actual physical cause of this pulse shape distortion is believed to lie in the inability of the convection zone to respond linearly to the oscillatory perturbation travelling through it.

Although mode coupling cannot be strictly excluded as the processes responsible for the amplitude variations observed in the period spectrum of GD 358, the process of *high-order beating* could account for these changes in a more natural way; natural in the sense that it is a permanent phenomenon and it does not require any resonant condition to operate.

Even though intrinsic nonlinear behaviour has been reported for years in variable white dwarfs, harmonic distortion might possibly account for both the presence of the combination frequencies and the amplitude variations observed in most of these star. In particular, this might explain why small amplitude pulsators have sinusoidal light curve and are fairly stable, while the large amplitude pulsators always show many nonlinear peaks and are never stable.

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