Baltic Astronomy, vol. 8, 411–424, 1999.

NONLINEAR STABILITY OF EQUILIBRIA OF THE MANEV-TYPE TWO-BODY PROBLEM

Vasile Mioc and Magda Stavinschi

Astronomical Institute of the Romanian Academy,
Str. Cuţitul de Argint 5, RO-75212 Bucharest, Romania

Received May 5, 1999.

Abstract. The nonlinear stability of the equilibrium orbits corresponding to a potential function $A/r + B/r^2$ (r = distance between bodies, A and B = real parameters) is being investigated. The method uses a combination of the block-diagonalization technique with the reduction procedure. The test points out certain nonlinearly stable equilibria, and is irrelevant for the remaining equilibria. The latter ones are treated via linearization. With a single exception (which shows linear stability), all such cases prove linear instability. The nonlinearly stable equilibria remain stable under any perturbation which preserves the conserved momentum.

Key words: celestial mechanics

1. INTRODUCTION

The orbital stability of heavenly bodies always constituted one of the main topics of celestial mechanics (and of astronomy, in general). The objects on which the investigations are focused are especially natural and artificial satellites, planetary rings, asteroids,comets, binary stars, and stellar satellites (from planets to particles in surrounding disks).

In this paper we shall deal with the nonlinear stability of the equilibria of the Manev-type two-body problem. Such problems are associated to a potential of the form $A/r + B/r^2$, where r is the distance between the bodies, while A and B are real parameters.

The Manev-type problems model many concrete physical and astronomical situations; for a survey of such cases, see Mioc and Stoica (1995c, 1997), Delgado et al. (1996), Diacu et al. (1999), and the references therein. This is the reason for which the recent interest in Manev's field is "monotonically increasing". The contributions of Lacomba et al. (1991), Casasayas et al. (1993), Diacu (1993, 1996), Diacu et al. (1995, 1999), Mioc and Stoica (1995a,b,c, 1996, 1997), Stoica and Mioc (1996, 1997) used modern mathematical results (like KAM theory, Melnikov method, etc.) as well as classical techniques, or went into the physical and astronomical significance of the model. From the mathematical standpoint, the Manev-type law opens a new research field. Up to now it has provided surprising results concerning the dynamics of particles, which disagree with the classical ones (when the motion neighbors singularities), or enrich them by new types of motion. Last, but not least, the genuine Manev model seems to build a bridge between classical mechanics and general relativity (see Diacu 1996; Diacu et al. 1999).

In the study of the nonlinear stability of the equilibria, a very efficient tool is provided by a combination of the block-diagonalization method proposed by Marsden et al. (1989) - and developed by Maddocks (1991) and Simó et al. (1991) - with the classical reduction procedure. This technique, particularized by Zombro and Holmes (1993) to systems with a finite number of degrees of freedom and with a single rotational symmetry, was applied, for instance, to the study of the relative equilibrium configurations (steadily rotating states) in the $(n+1)$-body problem; see, e.g. Elmabsout (1988, 1990, 1994, 1996).

We use this technique to reduce the Hamiltonian of the problem. The (relative and relative rest) equilibria of the corresponding amended potential lie in a fixed plane.

Considering the equilibria previously pointed out by Stoica and Mioc (1996), Mioc and Stoica (1997), or Diacu et al. (1999), for the whole interplay between field parameters, level of energy, and angular momentum, we tackle the Liapunov nonlinear stability of these equilibria. We find nonlinearly stable cases, which remain stable for the whole class of perturbations that do not affect the conserved angular momentum. We also find situations in which the nonlinear stability test (Zombro and Holmes 1993) is inconclusive. Applying the classical linearization method to these cases, we conclude that all corresponding equilibrium orbits (with one exception) are linearly unstable.

Of course, every kind of relative equilibrium or relative rest equilibrium is interpreted in terms of real motion.

2. AMENDED POTENTIAL

We fix the origin of the coordinates in the field-generating body and study the relative motion of the other body with respect to it. The corresponding equations of motion are

$$\begin{aligned} \dot{\mathbf{q}} &= \partial H(\mathbf{q},\mathbf{p})/\partial \mathbf{p}, \\ \dot{\mathbf{p}} &= -\partial H(\mathbf{q},\mathbf{p})/\partial \mathbf{q}, \end{aligned} \tag{1}$$

where $\mathbf{q} = (q_1, q_2, q_3) \in \mathbf{R}^3 \backslash \{(0,0,0)\}$ is the configuration vector, while $\mathbf{p} = (p_1, p_2, p_3) \in \mathbf{R}^3$ is the momentum vector. The class of Hamiltonians we deal with has the form (in suitably chosen units):

$$H(\mathbf{q},\mathbf{p}) = \frac{|\mathbf{p}|^2}{2} - \frac{A}{|\mathbf{q}|} - \varepsilon V\left(\sqrt{q_1^2+q_2^2}, q_3\right), \tag{2}$$

in which $A \in \mathbf{R}$, and ε is a small parameter. It is needless to say that the first two terms in the right-hand side of (2) describe the unperturbed problem, whereas the third term represents the perturbation.

Let us pass to cylindrical coordinates (r, θ, z) and corresponding momenta (p_r, p_θ, p_z) via the transformations

$$\begin{aligned} (q_1, q_2, q_3) &= (r\cos\theta, r\sin\theta, z), \\ (p_1, p_2, p_3) &= \left(p_r\cos\theta - \frac{p_\theta}{r}\sin\theta,\ p_r\sin\theta + \frac{p_\theta}{r}\cos\theta,\ p_z\right). \end{aligned} \tag{3}$$

Observe that, under this change of variables, the motion equations preserve their Hamiltonian character (however, in the general case, the Hamiltonian character is sacrificed). Applying (3) to (2), the latter one becomes

$$H = \frac{1}{2}\left(p_r^2 + p_z^2 + \frac{p_\theta^2}{r^2}\right) - \frac{A}{(r^2+z^2)^{1/2}} - \frac{B}{r^2+z^2}, \tag{4}$$

where we kept the notation H for the new Hamiltonian depending on the variables r, z, θ, p_r, p_z, p_θ.

This Hamiltonian has the property (see, e.g., Delgado et al. 1996)

$$H = h/2 = \text{constant}, \tag{5}$$

which provides the first integral of energy, h being the energy constant.

Also notice that H does not depend on θ. This means that the momentum conjugate to θ is conserved. Actually, this is nothing but the conservation of the angular momentum (with the corresponding first integral). Therefore we may apply the reduction, confining the Hamiltonian to the level set $C := p_\theta = \text{constant}$.

According to Zombro & Holmes (1993), the amended potential of (2) will have the expression

$$W = \frac{C^2}{2r^2} - \frac{A}{(r^2+z^2)^{1/2}} - \varepsilon V(r,z), \tag{6}$$

where we kept the same notation V for the new function of r and z. Taking into account the estimates for B in different cases (e.g. Diacu et al. 1995; Mioc & Stoica 1995c), we may identify $\varepsilon V(r,z) = B/(r^2+z^2)$, so that (6) becomes

$$W = \frac{C^2}{2r^2} - \frac{A}{(r^2+z^2)^{1/2}} - \frac{B}{r^2+z^2}. \tag{7}$$

By (4), (5), and (7), the integral of energy acquires the expression

$$p_r^2 + p_z^2 + \frac{C^2}{r^2} - \frac{2A}{(r^2+z^2)^{1/2}} - \frac{2B}{r^2+z^2} = h. \tag{8}$$

Lastly, we provide the relative motion equations in the more explicit form

$$\begin{aligned}
\dot r &= p_r, \\
\dot z &= p_z, \\
\dot\theta &= \frac{p_\theta}{r^2}; \\
\dot p_r &= -\frac{Ar}{(r^2+z^2)^{3/2}} - \frac{2Br}{(r^2+z^2)^2} + \frac{C^2}{r^3}, \\
\dot p_z &= -\frac{Az}{(r^2+z^2)^{3/2}} - \frac{2Bz}{(r^2+z^2)^2}, \\
\dot p_\theta &= 0.
\end{aligned} \tag{9}$$

3. EQUILIBRIA OF THE PROBLEM

The equilibria of the Manev-type two-body problem correspond to

$$\dot{\theta} = \text{constant}, \qquad (10)$$
$$p_\theta = C$$

(which is obvious, taking into account the angular momentum conservation) and

$$\frac{Ar}{(r^2+z^2)^{3/2}} + \frac{2Br}{(r^2+z^2)^2} - \frac{C^2}{r^3} = 0, \qquad (11)$$

$$\frac{Az}{(r^2+z^2)^{3/2}} + \frac{2Bz}{(r^2+z^2)^2} = 0, \qquad (12)$$

$$p_r = 0 = p_z. \qquad (13)$$

The relations (11) and (12) derive from (9) and (10), observing the condition for equilibrium. As to (13), it is trivial.

Definition 3.1. We shall call *relative equilibria* the solutions of (9) which fulfil (10)-(13) with $C \neq 0$.

Definition 3.2. We shall call *relative rest equilibria* the solutions of (9) which fulfil (10)-(13) with $C = 0$.

Remark 3.3. The relative equilibria constitute steadily rotating states of the "satellite" body with respect to the central body.

Remark 3.4. The relative rest equilibria represent rest of the "satellite" body with respect to the central body.

Denote

$$R^2 := r^2 + z^2. \qquad (14)$$

We shall distinguish two situations: $z = 0$ $(R = r)$ and $z \neq 0$ $(R > r)$, which will separately be analyzed in Sections 5 and 6, respectively.

Remark 3.5. The above defined quantity R is nothing but the Euclidean norm of the radius vector of the "satellite" body with respect to the central body. In other words, it is the distance between the two bodies.

Proposition 3.6. *At equilibria, for $z \neq 0$ the "satellite" body does not rotate around the central body ($C = 0$).*

Proof. Multiplying the fourth and the fifth equations (10) by $-z$ and r, respectively, then adding the results together, it follows immediately that $C = 0$.

Corollary 3.7. *At equilibria, for $z = 0$, the energy level is*

$$h = A^2/(2B). \tag{15}$$

Proof. Taking into account (8), (13), (14), and Proposition 3.6, the relation (15) follows immediately.

Remark 3.8. It is clear from Proposition 3.6 that for $z \neq 0$ the critical points are relative rest equilibria. In physical terms, the "satellite" body is at rest with respect to the central body.

Theorem 3.9. *Regardless to the angular momentum value, the equilibria of the Manev-type two-body problem are located at*

$$R = -A/h. \tag{16}$$

Proof. By (8), (13), and (14), we have

$$\frac{C^2}{r^2} - \frac{2A}{R} - \frac{2B}{R^2} = h. \tag{17}$$

On the other hand, the fourth equation (9), written for equilibria, provides (taking into account (14)):

$$\frac{C^2}{r^3} - \frac{Ar}{R^3} - \frac{2Br}{R^4} = 0. \tag{18}$$

Eliminating C between (17) and (18), we obtain

$$hR^4 + AR^3 + (AR + 2B)z^2 = 0. \tag{19}$$

For $z = 0$ (which implies $R = r$), formula (16) follows immediately. For $z \neq 0$, using the fifth equation (9) written for equilibrium, we get

$$R = -2B/A. \tag{20}$$

Resorting to (15), one easily obtains (16). The theorem is proved.

4. THE NONLINEAR STABILITY TEST

Taking into account the results presented in Section 3, we are in the position to apply the algorithm described by Zombro and Holmes (1993). This technique allows one to formulate conclusions about the nonlinear stability of the equilibria via the examination of the positive definiteness of the matrix

$$D^2W\big|_e := \begin{bmatrix} \partial^2 W/\partial r^2 & \partial^2 W/\partial r\partial z \\ \partial^2 W/\partial r\partial z & \partial^2 W/\partial z^2 \end{bmatrix}_e, \tag{21}$$

where the subscript "e" fixes the respective values at equilibrium ($R = R_e$, provided by (16), $r = r_e$, $z = z_e$).

The nonlinear stability is entailed by the conditions

$$D_1 > 0, \quad D_2 > 0, \tag{22}$$

where

$$D_1 := \left.\frac{\partial^2 W}{\partial r^2}\right|_e, \qquad D_2 := \det\left(D^2W\big|_e\right). \tag{23}$$

If both conditions (22) are fulfilled, the respective equilibrium is nonlinearly stable. Else, one can say nothing about the nonlinear stability. In such cases, when the test is inconclusive, one must resort to linearization.

5. STABILITY OF EQUILIBRIA FOR $z = 0$

For $z = 0$ we have $R = r$. Checking whether the conditions (22) are fulfilled or not, we can state

Proposition 5.1. *If $z = 0$, the following implications hold*

$$\begin{aligned} h < 0 &\Longrightarrow D_1 > 0,\ D_2 > 0, \\ h \geq 0 &\Longrightarrow D_1 \leq 0,\ D_2 \leq 0, \end{aligned} \tag{24}$$

or, equivalently,

$$\begin{aligned} C^2 > 2B &\Longrightarrow D_1 > 0,\ D_2 > 0, \\ C^2 \le 2B &\Longrightarrow D_1 \le 0,\ D_2 \le 0. \end{aligned} \tag{25}$$

Proof. By (7), (21), and (23), we have at equilibrium

$$D_1 = \frac{1}{r_e^3}\left[-2A + \frac{3(C^2 - 2B)}{r_e}\right], \tag{26}$$

$$D_2 = \frac{1}{r_e^6}\left[-2A^2 + \frac{A(3C^2 - 10B)}{r_e} + \frac{6B(C^2 - 2B)}{r_e^2}\right]. \tag{27}$$

The fourth equation (9), written for equilibrium ($r = r_e$, $z = 0$), provides

$$r_e = (C^2 - 2B)/A. \tag{28}$$

Combining (16) (where$R_e = r_e$) with (28), and using (26) and (27), we obtain directly

$$h = -A^2/(C^2 - 2B); \tag{29}$$

$$D_1 = -h^3/A^2 = A^4/(C^2 - 2B)^3; \tag{30}$$

$$D_2 = -C^2h^7/A^6 = C^2A^8/(C^2 - 2B)^7. \tag{31}$$

Implications (24) and (25) follow immediately. The proposition is proved.

This proposition implies immediately

Corollary 5.2. *The equilibria for $z = 0$ are nonlinearly stable only if $h < 0$ or $C^2 > 2B$.*

Now, let us survey the equilibria of the Manev-type two-body problem for the whole interplay among the field parameters (A and B), energy level (h) and angular momentum (C). Recall that we are in the case $z = 0$ ($R = r$).

Proposition 5.3. *If $z = 0$, the equilibria of the Manev-type two-body problem occur for*

(i) $B > 0$, $A > 0$, $h < 0$, $C = \pm\sqrt{2B - A^2/h}$;

(ii) $B > 0$, $A = 0$, $h = 0$, $C = \pm\sqrt{2B}$;

(iii) $B > 0,\ A < 0,\ h = A^2/(2B),\ C = 0$;
(iv) $B > 0,\ A < 0,\ h > A^2/(2B),\ C = \pm\sqrt{2B - A^2/h}$;
(v) $B = 0,\ A > 0,\ h < 0,\ C = \pm A/\sqrt{-h}$;
(vi) $B = 0,\ A = 0,\ h = 0,\ C = 0$;
(vii) $B < 0,\ A > 0,\ h > A^2/(2B),\ C = 0$;
(viii) $B < 0,\ A > 0,\ A^2/(2B) < h < 0,\ C = \pm\sqrt{2B - A^2/h}$.

Proof. The analysis performed by Stoica and Mioc (1996), Mioc and Stoica (1997), or Diacu et al. (1999), points out the fact that the only equilibria of the problem take place for the combinations (i)-(viii).

Taking into account Proposition 5.3 and Corollary 5.2, we can state

Theorem 5.4 *For $z = 0$, the equilibria corresponding to the cases (i), (v), (vii), and (viii) are nonlinearly stable.*

As regards the remaining cases, the nonlinear stability test is inconclusive. In terms of the conditions (22), this can be expressed as

Proposition 5.5. *The cases in which the conditions (22) are not fulfilled, namely (ii), (iii), (iv), and (vi), imply respectively:*

$$D_1 = 0, D_2 = 0;\ \ D_1 < 0, D_2 < 0;\ \ D_1 < 0, D_2 > 0;\ \ D_1 = 0, D_2 = 0.$$

Proof. These results are immediate by either direct calculation or using Proposition 5.1.

In such cases we can say nothing about the nonlinear stability, therefore we have to resort to linearization. Starting from (9) with the equations for θ and p_θ discarded, the linearized system near equilibria reads

$$\begin{aligned}
\dot{\varrho} &= p_\varrho, \\
\dot{z} &= p_z, \\
\dot{p}_\varrho &= \left[\frac{2A}{r_e^3} - \frac{3(C^2 - 2B)}{r_e^4}\right]\varrho,
\end{aligned} \qquad (32)$$

$$\dot{p}_z = \left[-\frac{A}{r_e^3} - \frac{2B}{r_e^4}\right] z,$$

where $\varrho = r - r_e$. Recalling that $z = 0$, and using (26), the system (32) reduces to

$$\begin{aligned} \dot{\varrho} &= p_\varrho, \\ \dot{p}_\varrho &= -D_1 \varrho, \end{aligned} \tag{33}$$

with the characteristic equation

$$\lambda^2 + D_1 = 0. \tag{34}$$

On this basis we can state

Theorem 5.6. *For $z = 0$, the equilibria corresponding to the cases (ii), (iii), (iv), and (vi) are linearly unstable.*

Proof. By virtue of Proposition 5.5, equation (34) yields $\lambda^2 = 0$ for the cases (ii) and (vi), that means degenerate equilibria, which are linearly unstable. Within the same framework, $\lambda^2 > 0$ for the cases (iii) and (iv), which means that at least one root has positive real part, hence these equilibria are linearly unstable, too.

To end this section, let us interpret the equilibria for $z = 0$ in terms of real motion. First, the following result is to be pointed out:

Proposition 5.7. *The cases (i), (ii), (iv), (v), and (viii) are relative equilibria. The remaining cases are relative rest equilibria.*

Proof. Taking into account Definitions 3.1 and 3.2, as well as Proposition 5.3, the proof is obvious.

Finally, we state

Theorem 5.8. *In terms of real motion, the equilibria of the Manev-type two-body problem for $z = 0$ correspond to:*
(i) *stable circular motion at distance $(C^2 - 2B)/A$;*
(ii) *unstable circular motion at any distance;*
(iii) *unstable rest at distance $-2B/A$;*
(iv) *unstable circular motion at distance $(C^2 - 2B)/A$;*
(v) *stable circular motion at distance C^2/A;*

(vi) *unstable rest at any distance;*
(vii) *stable rest at distance* $-2B/A$;
(viii) *stable circular motion at distance* $(C^2 - 2B)/A$,
where (i)-(viii) *denote the cases pointed out by Proposition 5.3.*

Proof. As regards the stability of the motion, it is easy to see by the linearized system that the cases (i), (v), (vii), and (viii) are centers, (iii) and (iv) are saddles, whereas (ii) and (vi) are degenerate equilibria. The nature of the motion (steady rotation or rest) is specified by Proposition 5.7.

As to the distance of the equilibria, it can be determined from Theorem 3.9, as well as from the relations (28) and (29) (see also Stoica and Mioc 1996; Mioc and Stoica 1997; Diacu et al. 1999). This completes the proof.

6. STABILITY OF EQUILIBRIA FOR $z \neq 0$

First we identify the combinations (A, B, h) for which there are equilibria. The angular momentum is not to be considered, because $z \neq 0$ implies $C = 0$ (see Proposition 3.6).

Proposition 6.1. *If* $z \neq 0$, *the equilibria of the Manev-type two-body problem occur for:*
(iii) $B > 0$, $A < 0$, $h = A^2/(2B)$;
(vi) $B = 0$, $A = 0$, $h = 0$;
(vii) $B < 0$, $A > 0$, $h = A^2/(2B)$.

Proof. Taking into account Theorem 3.9 and the formula (20), the proof is immediate.

Remark 6.2. The numbering in Proposition 6.1 was chosen to agree with Proposition 5.3.

All these critical points are relative rest equilibria (see Remark 3.8). Applying the nonlinear stability test, we get

Theorem 6.3. *If* $z \neq 0$, *the conditions for nonlinear stability are not fulfilled.*

Proof. Taking into account (7), (21), and (23), we obtain by straightforward calculation

$$D_1 = -2Br_e^2/R_e^6, \qquad D_2 = 0. \tag{35}$$

By virtue of (22), the theorem is proved.

As previously, we must resort to linearization. The system (9) linearized in the neighborhood of equilibria, with the equation for θ and p_θ dropped, has the form

$$\begin{aligned}
\dot{\varrho} &= p_r, \\
\dot{\zeta} &= p_z, \\
\dot{p}_r &= \left[-\frac{A}{R_e^3} - \frac{2B}{R_e^4} + \left(\frac{3A}{R_e^5} + \frac{8B}{R_e^6}\right) r_e^2\right] \varrho + \left[\left(\frac{3A}{R_e^5} + \frac{8B}{R_e^6}\right) r_e z_e\right] \zeta, \\
\dot{p}_z &= \left[\left(\frac{3A}{R_e^5} + \frac{8B}{R_e^6}\right) r_e z_e\right] \varrho + \left[-\frac{A}{R_e^3} - \frac{2B}{R_e^4} + \left(\frac{3A}{R_e^5} + \frac{8B}{R_e^6}\right) z_e^2\right] \zeta,
\end{aligned} \tag{36}$$

where $\varrho = r - r_e$ and $\zeta = z - z_e$. Taking into account (20), the characteristic equation reads

$$\lambda^2 \left(\lambda^2 - \frac{A^4}{8B^3}\right) = 0. \tag{37}$$

All these results lead to

Theorem 6.4. *For $z \neq 0$, the equilibria corresponding to the cases (iii) and (vi) are linearly unstable, while the case (vii) is linearly stable.*

To end, we state

Theorem 6.5. *In terms of real motion, the equilibria of the Manev-type two-body problem for $z \neq 0$ correspond to:*
(iii) *unstable rest at distance $-2B/A$;*
(vi) *unstable rest at any distance;*
(vii) *linearly stable rest at distance $-2B/A$.*

Proof. Taking into account the results presented in this section, as well as formula (20), the proof is immediate.

7. CONCLUDING REMARKS

Summarizing, the Manev-type two-body problem admits eight classes of equilibria for $z = 0$, and three classes of equilibria for $z \neq 0$.The classes (i), (ii), (iv), (v), and (viii) represent relative equilibria (steadily rotating states). They occur only for $z = 0$. The classes (iii), (vi) and (vii) are relative rest equilibria, regardless to the value of z.

For $z = 0$, the classes of equilibria (i), (v), (vii), and (viii) are nonlinearly stable, while (ii), (iii), (iv), and (vi) are linearly unstable. For $z \neq 0$, the classes (iii) and (vi) are linearly unstable, whereas the class (vii) is linearly stable.

An important result concerning the classes of equilibria (i), (v), (vii), and (viii) for $z = 0$ can be stated as

Theorem 7.1. *The nonlinearly stable equilibria for $z = 0$* remain stable under the influence of every perturbation which does not affect the conserved angular momentum.

Proof. Zombro and Holmes (1993) showed that the nonlinearly stable equilibrium orbits of the problem remain stable under perturbations of this kind.

To end, we have to emphasize that the perturbations that observe the conditions of Theorem 7.1, namely those which let the equilibrium orbits be nonlinearly stable, may have any nature. This fact makes the class of concrete astronomical situations corresponding to this model considerably more rich.

REFERENCES

Casasayas J., Fontich E., Nunes A. 1993, in *Hamiltonian Systems and Celestial Mechanics*, Advanced Series in Nonlinear Dynamics, Vol. 4, World Scientific, Singapore, p. 35

Delgado J., Diacu F. N., Lacomba E. A., Mingarelli A., Mioc V., Perez E., Stoica C. 1996, J. Math. Phys., 37, 2748

Diacu F. N. 1993, J. Math. Phys., 34, 5671

Diacu F. N. 1996, J. Diff. Eq., 128, 58

Diacu F. N., Mioc V., Stoica C. 1999, *Nonlinear Analysis* (in press)

Diacu F. N., Mingarelli A., Mioc V., Stoica A. 1995, in *Dynamical Systems and Applications*, World Scientific Series in Applicable Analysis, Vol. 4, ed. R. P. Agarwal, World Scientific, Singapore, p. 213

Elmabsout B. 1988, Celest. Mech., 41, 131

Elmabsout B. 1990, Celest. Mech., 49, 219

Elmabsout B. 1994, Dyn. Stabil. Syst., 9, 305

Elmabsout B. 1996, Rom. Astron. J., 6, 61

Lacomba E. A., Llibre J., Nunes A. 1991, in *The Geometry of Hamiltonian Systems*, ed. T. Ratiu, Proc. Workshop held on June 5–16, 1989, Springer-Verlag, New York, p. 373

Maddocks J. 1991, IMA J. Appl. Math., 46, 71

Marsden J., Simó J., Lewis D., Posbergh T. 1989, Contemporary Math., 8, 297

Mioc V., Stoica C. 1995a, C. R. Acad. Sci. Paris, 320, sér. I, 645

Mioc V., Stoica C. 1995b, C. R. Acad. Sci. Paris, 321, sér. I, 961

Mioc V., Stoica C. 1995c, Bull. Astron. Belgrade Obs., No.152, 43

Mioc V., Stoica C. 1996, Rendiconti Sem. Fac. Sci. Univ. Cagliari, 66, 101

Mioc V., Stoica C. 1997, Baltic Astron., 6, 637

Simó J., Lewis D., Marsden J. 1991, Arch. Rat. Mech. Anal., 115, 15

Stoica C., Mioc V. 1996, Bull. Astron. Belgrade Obs., No.154, 1

Stoica C., Mioc V. 1997, Bull. Astron. Soc. India, 25, 217

Zombro B., Holmes P. 1993, Dyn. Stabil. Syst., 8, 41