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THEORETICAL AND OBSERVED PERIODS OF THE PULSATING PRE-WHITE DWARFS

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Abstract. I have systematically compared the period spectra of pre-white dwarf evolutionary models to the observed period spectra of the variable PG 1159 stars. I find that no model simultaneously matches both the observed period spacing (considered the primary diagnostic of the stellar mass) and the period offset for *any* pulsating pre-white dwarf star over its spectroscopically constrained temperature range. I discuss implications for previously published asteroseismological mass and distance measurements. Finally, I outline a simple physical interpretation of the problem and suggest possible avenues to pursue the cause of the difficulty.

Key words: stars: white dwarfs – stars: oscillations

1. Introduction

The variable PG 1159 (DOV) type stars are hot, luminous pre-white dwarfs which pulsate in non-radial g-modes. Observations show that their rich power spectra are dominated by $\ell = 1$ modes equally spaced in period (c.f. Winget et al. 1991 and Kawaler et al. 1995). Theory expects this simple pattern in the case of high-radial overtone variations, exemplified by the asymptotic equation

$$P_k = \Delta P \ (k + \epsilon) \qquad k >> 1$$

which is the familiar Wentzel-Kramers-Brillouin (WKB) approximation (c.f. Unno et al. 1989). This formula describes a pattern of periods similar to tick marks on a number line, where ΔP sets the spacing between the ticks and the offset $\epsilon\Delta P$ sets the position of the overall pattern along the line. Kawaler (1987, see also Kawaler and Bradley 1994) showed that the period spacing of stellar models is primarily dependent on the total stellar mass and only slightly dependent on the luminosity and surface layer thickness. Models with radial composition discontinuities showed periodic deviations from exactly equal spacing with increasing k.

Asteroseismology attempts to diagnose the mass, surface layer mass (composed of helium in the PG 1159 stars), and temperature of pulsating stars by matching the period spectra of stellar models to those found in the stars. Specifically, any model which seeks to match one of these stars must display a similar period spacing.

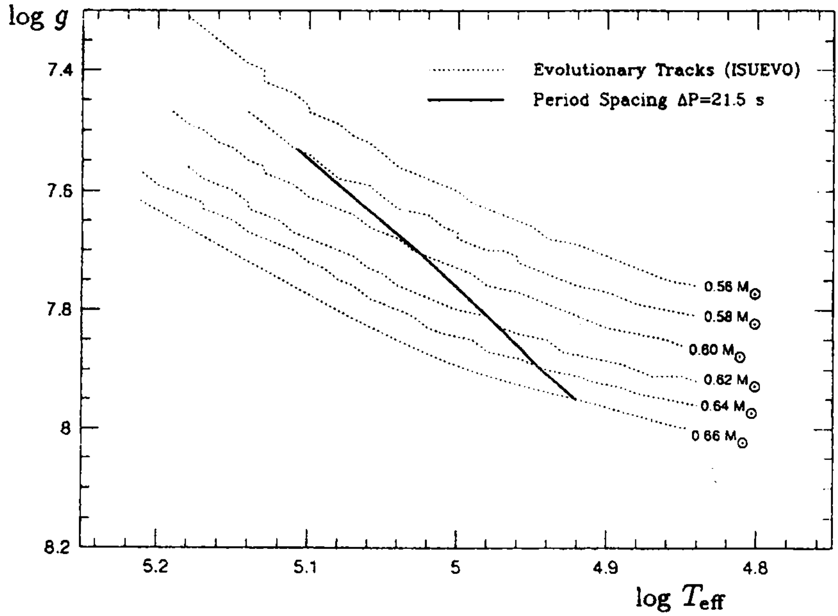

Fig. 1. Logarithmic plot of surface gravity versus effective temperature showing evolutionary tracks based on ISUEVO. Three of four PG 1159 stars lie near the $\Delta P = 21.5$ s line.

2. Comparison of the model periods to reality

As illustrated by the solid line in Fig. 1 (evolutionary tracks computed by ISUEVO, Dehner and Kawaler 1995), any number of models might show similar period spacing. The solid line indicates the location of models with a mean period spacing of 21.5 s, the approximate value found in three of the four variable PG 1159 stars examined in detail so far: PG 1159-035 (Winget et al. 1991), PG 2131+066 (Kawaler et al. 1995), and PG 0122+200 (O'Brien et al. 1995, though Vauclair et al. 1995 suggest a value of ΔP for this star near 16 s). We can eliminate some models because they fall outside the errors of the spectroscopic temperature determination of a given star. In order to do better, we must consider also the period offset. It is not enough for the model to get the period spacing correct; its periods must match those in the real star.

Both ΔP and ϵ can be fit as an average over many modes, and therefore are only affected slightly by changes in the helium layer thickness (trapping cycle). I have thus completed a systematic comparison of these two parameters, as calculated for the models in Fig. 1, to the values measured in each of the three stars PG 1159, PG 2131 and PG 0122. In each case I found that, over the entire allowable temperature range of a given star, our models have a value for ϵ which varies only slightly. For instance, for PG 1159 $\epsilon_{\rm mod}$ varies from 2.20 to 2.35 over the entire range 130 000–140 000 K and (0.56–0.66) $M_\odot$. Furthermore, in each case the value of ϵ in the models does not match the value in the real star (for PG 1159 $\epsilon = 2.0$). Models which are a "good match" to the star's periods achieve the result by fitting ΔP *wrong* to compensate for this mismatch in ϵ.

In addition, the amount by which $\epsilon_{\rm mod}$ misses the value of $\epsilon_{\rm star}$ is greater than the error in $\epsilon_{\rm star}$ and similar in magnitude to the spread in ϵ over all model masses of the appropriate temperature. Again, the total spread in $\epsilon_{\rm mod}$ for PG 1159 is 2.20 to 2.35 while $\epsilon_{\rm star} = 2.0 \pm 0.1$. I therefore maintain that we cannot realistically choose between models with the correct period spacing; all of them are demonstrably wrong. The mass (or range along the 21.5 s line) is only constrained by the *spectroscopically* determined temperature range, and the models themselves tell us nothing about the temperature. (In fact, without the spectroscopic temperature constraint, the models would not constrain the mass *at all.*) This is the case for PG 2131, PG 0122, and PG 1159.

We must then ask why models have previously been found which satisfactorily match the observed periods of the three stars mentioned above? The answer is that, given their failure to reproduce the period offset, these "best fit models" succeed in fitting the observed periods with an *incorrect* period spacing. In other words, a systematic difference exists within the residuals of each fit. The fractional change in ΔP needed to minimize these residuals is small, so in every case the difference between the period spacing in the star and the period spacing of the published "best-fit model" is slight, but we should really use the models which fit the observed period spacing exactly – and try to learn why none of the models correctly fit the period offset.

3. Implications

First of all it must be stated that these results do not significantly affect the actual values of the best estimates for the mass, luminosity and distance of the three stars mentioned above. It is clear that a model can always be found within the allowed temperature region of a star, with a period spacing only slightly wrong, which reproduces the observed periods in a least-squares sense. Such are the previously published "best fit models," and they will evidently not differ significantly in either mass or luminosity from models within the same temperature region which fit ΔP exactly. However, *the uncertainties placed on the mass and luminosity estimates should be reconsidered.* The implications vary from slight to significant, depending primarily on the temperature of the star. Fig. 1 shows that, at high enough temperature, lines of constant period spacing become almost parallel to evolutionary tracks. This means that in the case of PG 1159, for instance, models which correspond to the correct period spacing have only a very small range of mass – so small, in fact, that the previously published mass uncertainty (Winget et al. 1991) probably stands unchanged by these results. Both PG 2131 and PG 0122, however, lie at such low temperatures ($\sim$ 80 000 K) that the 21.5 s period spacing line crosses evolutionary tracks with a wide range of mass between the spectroscopic temperature limits of each star. Thus the error in the asteroseismological mass of both stars becomes a sensitive function of the spectroscopic temperature uncertainty. The errors in the published mass of PG 2131, for instance, are enlarged from $\pm 3\,\%$ (Kawaler et al. 1995) to $\pm 8\,\%$.

The asteroseismological distances should not suffer so severely, since the combination of the period spacing with spectroscopic temperature measurements still constrains the luminosity of each star to a much narrower range than does the spectroscopic surface gravity. However, the dependence of these distances (as well as the masses) on spectroscopic temperatures suggests the need for more accurate temperature determinations, especially for the cooler PG 1159 stars.

It might also be prudent to continue to compare asteroseismological distance measurements to parallax determinations.

4. Discussion and conclusions

The apparent failure of PG 1159 models to match the observed periods suggests either that some input quantity, such as composition, is wrong in the models, or that a problem exists in the assumed physics. Interestingly, a similar problem has been pointed out by Gough (1990) with regard to helioseismology and the p-mode spectrum of solar models. In this case, no reasonable models reproduce both the spacing and offset (in frequency – not period – for p-modes) of the observed solar p-mode spectrum. Gough concludes that the problem must derive from errors in the physics of the standard solar model, and suggests using the offset parameter (which plays the same role for p-mode oscillations which ϵ plays for g-mode pulsations) to diagnose the problem.

I propose to use ϵ to calibrate the pre-white dwarf models. Though the problem could involve an error in the physics assumed in their construction, a simple initial approach will be to test the effect differing model composition has on ϵ. In the WKB approximation of quantum mechanics, ϵ is a measure of the stiffness of the walls of a resonant cavity, or the extent to which trapped waveforms penetrate the walls of a square well. In our "resonant cavity" these boundary conditions are the degenerate core and the stellar surface. If ϵ turns out to depend primarily on the core carbon-to-oxygen ratio, for instance, then the possibility arises of constraining this ratio in the stars themselves. Just as ΔP is sensitive primarily to mass and temperature, ϵ must also be sensitive to some stellar parameters; if so, we should find and, if possible, measure them. Note that this presumed set of parameters does not apparently include the temperature, since ϵ in our models seems to show little or no dependence on this quantity.

If the problem does not concern the calibration of any simple input parameter, then we are faced with the opportunity to learn something new about the physics of stellar interiors. Problems of this nature, once solved, invariably find application in unexpected areas. Other problems which remain unsolved concerning the pulsating pre-white dwarf stars include: the trend toward shorter period with decreasing temperature, the exact pulsation mechanism, and the curious coincidence that three of the four stars for which a period spacing has been identified have the *same* spacing (this last point raises the question: could $\Delta P = 21.5$ s be a condition for pulsation in the PG 1159 stars?). All of these questions are undoubtedly related, and show that, in spite of previous theoretical and observational victories, our understanding of the PG 1159 stars is far from complete. The one certainty is that more data are needed. A Whole Earth Telescope observing run scheduled for 1996 will clarify the period spacing of PG 0122, and the author is currently looking at data from other pre-white dwarfs (e.g. PG 1707) to see whether the models encounter the same difficulty matching their observed periods.

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